Appendices to:
Revisiting the Welfare Effects of Eliminating Business Cycles

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A The integration principle applied to the baseline model

The 4×4 Markov transition matrix for (ϵ, z), in its calibrated form, reads

$$\begin{pmatrix} 0.8507 & 0.1159 & 0.0243 & 0.0091 \\ 0.1229 & 0.8361 & 0.0021 & 0.0389 \\ 0.5833 & 0.0313 & 0.2917 & 0.0938 \\ 0.0938 & 0.3500 & 0.0313 & 0.5250 \end{pmatrix},$$

with ordering (e,g), (e,b), (u,g), (u,b). This implies probabilities of finding a job, conditional on last period's employment status and on the aggregate shocks in the current and in the last period. These job-finding probabilities can be ranked from least to most lucky as follows: the probability of becoming employed next period ($\epsilon' = 1$) is,

1. conditional on $\epsilon = 0$, $z = g$, and $z' = b$, $0.0313/0.1250 = 0.2504 \equiv \bar{i}_1$;
2. conditional on $\epsilon = 0$, $z = b$, and $z' = b$, $0.3500/0.8750 = 0.4000 \equiv \bar{i}_2$;
3. conditional on $\epsilon = 0$, $z = g$, and $z' = g$, $0.5833/0.8750 = 0.6666 \equiv \bar{i}_3$;
4. conditional on $\epsilon = 0$, $z = b$, and $z' = g$, $0.0938/0.1250 = 0.7504 \equiv \bar{i}_4$;
5. conditional on $\epsilon = 1$, $z = g$, and $z' = b$, $0.1159/0.1250 = 0.9272 \equiv \bar{i}_5$;
6. conditional on $\epsilon = 1$, $z = b$, and $z' = b$, $0.8361/0.8750 = 0.9555 \equiv \bar{i}_6$;
7. conditional on $\epsilon = 1$, $z = g$, and $z' = g$, $0.8507/0.8750 = 0.9722 \equiv \bar{i}_7$; and
8. conditional on $\epsilon = 1$, $z = b$, and $z' = g$, $0.1229/0.1250 = 0.9832 \equiv \bar{i}_8$.

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Thus, our idiosyncratic shock i_t can end up in 9 relevant subintervals, defined by the cutoff values $\bar{i}_1$–$\bar{i}_9$, in each period. Let us take an example: if $i \in [\bar{i}_6, \bar{i}_7)$, the agent would have been employed currently only if the aggregate and idiosyncratic shocks leading up to the last period made the agent employed in that period and the current aggregate state would have been bad (independently of what the state was last period).

For a realized sequence of idiosyncratic shocks $\{i_s\}_{s=1}^t$ one can then compute an average employment outcome in period t by brute-force averaging across all $\{z_s\}_{s=1}^t$ sequences (appropriately weighted by probabilities): given each such sequence of aggregate shocks, together with an employment status in period 0, the employment outcomes in all periods up to and including t are known: they follow applying the cutoff values above in every time period.

The resulting employment process will have long memory in terms of the idiosyncratic shocks: one generally needs to know *all* prior values of i_s, $s < t$, in order to know what the average employment outcome is at t. However, it is possible to represent the new employment process recursively. To this end, let P_{gt} denote the probability (or fraction of the time) that, among all possible outcomes of the aggregate process, (i) the individual would have been employed in time t, given his initial (time-0) employment status and an initial (time-0) value for the aggregate state AND (ii) the aggregate state at time t would have been good. Similarly define P_{bt} as the probability that the agent would have been employed in t jointly with a bad aggregate state in that period. Letting π_t denote the probability of a good aggregate state in period t given z_0, these definitions imply that $\pi_t - P_{gt}$ is the probability that the agent would have been unemployed in t jointly with a good aggregate state in t and similarly that $1 - \pi_t - P_{bt}$ is the probability that the agent would have been unemployed in t jointly with a bad aggregate state in t. The key insight now is that the variables $P_t \equiv (P_{gt}, P_{bt})$ summarize all there is to know from history in order to know the expected (average) value for employment in period $t+1$ given a value for i_{t+1}. I.e., P_t summarizes all the relevant knowledge about $\{i_1, i_2, \ldots, i_t\}$. This representation is possible because the joint underlying process for employment and the aggregate state is first-order Markov.

The recursive structure needs to update P_t into P_{t+1} given a value for i_{t+1}, and it needs to assign a value for the average employment outcome in period $t+1$ conditional on the state variable P_t summarizing the individual's idiosyncratic history and i_{t+1}, the new idiosyncratic shock. The latter is easy: the average value of employment across the aggregate shock outcomes will be $\epsilon_{t+1}^{w/o} = P_{g,t+1} + P_{b,t+1}$, because g and b are disjoint outcomes.

To understand how to update P_t given i_{t+1} (in order to obtain P_{t+1}), note that in any given period in the economy with cycles the consumer is in one of four possible states: he is either employed or unemployed and the aggregate state is either good or bad. Denote these states by $(1, g)$, $(1, b)$, $(0, g)$, and $(0, b)$. (As noted above, the probabilities of these four states can be deduced from knowledge of P_{gt}, P_{bt}, and the probability that the aggregate state is good in period t.) Suppose that the consumer is in state $(1, g)$ in period t and that the aggregate state in period $t+1$ is also good. If the consumer's luck in period $t+1$, i_{t+1}, is sufficiently good, he will also be in state $(1, g)$ in period $t+1$. In this case, the consumer's luck is "sufficiently good" if $i_{t+1} < \pi_{1|1gg} \equiv P(\epsilon_{t+1} = 1 | \epsilon_t = 1, z_t = g, z_{t+1} = g)$. The conditional probability $\pi_{1|1gg}$, therefore, is a "cutoff" that determines whether the consumer's luck is good or bad, given that he is employed in t and that the aggregate state is good in both t and $t+1$. In total, there are eight such cutoffs, one for each of the eight possible permutations of $(\epsilon_t, z_t, z_{t+1})$, and these eight cutoffs define the 9 regions described above.

In period t, the consumer could also be in state $(1, b)$, $(0, g)$, or $(0, b)$. In each case, if the aggregate state in period $t+1$ is good and the consumer's luck in $t+1$ is below the relevant cutoff, he will be in state $(1, g)$ in period $t+1$. $P_{g,t+1}$ is then a weighted average of four indicator functions (each of which indicates whether i_{t+1} is above or below the appropriate cutoff). The weights corresponding to the period-t probabilities of the four states $(1, g)$, $(1, b)$, $(0, g)$, and $(0, b)$, multiplied in each case by the conditional probability of transiting to a good aggregate state in $t+1$ given the aggregate state in t. Similarly, $P_{b,t+1}$ is also the weighted average of four indicator functions, appropriately weighted.

As described above, in the baseline model the variable i_{t+1} can fall into any one of the 9 regions defined by the cutoffs $\bar{i}_1$–$\bar{i}_9$ above. The exact updating formulas for the baseline model are:

1. $i_{t+1} \in [0, \bar{i}_1)$: $P_{g,t+1} = \pi_{t+1}$ and $P_{b,t+1} = 1 - \pi_{t+1}$;
2. $i_{t+1} \in [\bar{i}_1, \bar{i}_2)$: $P_{g,t+1} = \pi_{t+1}$ and $P_{b,t+1} = P_{gt}\pi_{b|g} + (1 - \pi_t)\pi_{b|b}$
3. $i_{t+1} \in [\bar{i}_2, \bar{i}_3)$: $P_{g,t+1} = \pi_t$ and $P_{b,t+1} = P_{gt}\pi_{b|g} + P_{bt}\pi_{b|b}$;
4. $i_{t+1} \in [\bar{i}_3, \bar{i}_4)$: $P_{g,t+1} = P_{gt}\pi_{g|g} + (1 - \pi_t)\pi_{g|b}$ and $P_{b,t+1} = P_{gt}\pi_{b|g} + P_{bt}\pi_{b|b}$;
5. $i_{t+1} \in [\bar{i}_4, \bar{i}_5)$: $P_{g,t+1} = P_{gt}\pi_{g|g} + P_{bt}\pi_{g|b}$ and $P_{b,t+1} = P_{gt}\pi_{b|g} + P_{bt}\pi_{b|b}$;
6. $i_{t+1} \in [\bar{i}_5, \bar{i}_6)$: $P_{g,t+1} = P_{gt}\pi_{g|g} + P_{bt}\pi_{g|b}$ and $P_{b,t+1} = P_{bt}\pi_{b|b}$;
7. $i_{t+1} \in [\bar{i}_6, \bar{i}_7)$: $P_{g,t+1} = P_{gt}\pi_{g|g} + P_{bt}\pi_{g|b}$ and $P_{b,t+1} = 0$;
8. $i_{t+1} \in [\bar{i}_7, \bar{i}_8)$: $P_{g,t+1} = P_{bt}\pi_{g|b}$ and $P_{b,t+1} = 0$; and
9. $i_{t+1} \in [\bar{i}_8, 1]$: $P_{g,t+1} = P_{b,t+1} = 0$.

One needs to spell through these carefully to see that they are correct. We have, and for verification we have also (i) simulated this process for various draws of the idiosyncratic process $\{i_t\}_{t=1}^T$ (where T is large) and, based on the resulting $\{P_t\}_{t=1}^T$ sequence, computed the associated employment outcomes and (ii) made sure that the resulting values are replicated for the same $\{i_t\}_{t=1}^T$ draws by a brute-force averaging across aggregate shock processes. They do.

Appendix B displays the general updating formula (expressed as a weighted average of indicator functions), for the baseline case (expressed as a weighted average of indicator functions), and Appendix D displays it for the model with short- and long-term unemployment.

B Computational algorithm for the benchmark model

B.1 General algorithm

This section outlines the computational algorithm applying the integration principle to our model and computing the transition path.[1] Note that the business cycle is eliminated in the beginning of period 1, after all the period-1 shocks are realized. Thus z_1 and $\bar{k}_1$ are given. At the individual level, the distribution of k_1, ϵ_1, and $\tilde{\beta}_1$ are given from one point in the simulation that corresponds to z_1 and $\bar{k}_1$. ϵ_1 provides the initial conditions for each individuals: if $z_1 = g$ and $\epsilon_1 = 1$, $P_{g1} = 1$ and $P_{b1} = 0$; if $z_1 = b$ and $\epsilon_1 = 1$, $P_{g1} = 0$ and $P_{b1} = 1$; and if $\epsilon_1 = 0$, $P_{g1} = 0$ and $P_{b1} = 0$. (Recall that P_{gt} is the joint probability of $z_t = g$ and $\epsilon_t = 1$ and that P_{bt} is the joint probability of $z_t = b$ and $\epsilon_t = 1$.)

The general computational strategy is to first postulate the time path of aggregate capital, solve for the agents' decisions given this path, and then verify that that the time path for aggregate capital implied by agents' aggregated decisions matches the postulated time path.

We postulate the time path for 600 periods. Then we divide the 600 periods into the first 125 periods and the final 475 periods. We solve the consumer's problem backwards—first solve for the final 475 periods and then for the first 125 periods. After the optimization problem is solved, we simulate the economy with many consumers (we use 90,000 consumers) and generate the path for aggregate capital by summing up individual savings. Finally, we check whether this simulated path of capital stock is the same as the initially postulated time path. The following explains these steps more in detail.

1. First, postulate the path of the aggregate capital stock for 600 periods. We use the average of the law of motions of the capital stock in the fluctuating economy to generate the initial guess.

2. We solve the consumer's problem backwards. In the final 475 periods, the exogenous variables, such as z, u, and π_z are set to their limit values ($z = 1$, $u = (u_g + u_b)/2$, and $\pi_z = 0.5$). Thus, we treat this economy as a stationary on except for the movement in $\bar{k}$ ($\bar{k}$ settles much more slowly than do the exogenous variables). We summarize the movement of the capital stock by the law of motion $\bar{k}' = H(\bar{k})$. In practice, we use a (log-)linear function for $H(\cdot)$, with the initial value of the law of motion obtained by applying ordinary least squares to the data on aggregate capital for the final 475 periods. The Bellman equation is:

$$V(k, P_g, P_b, \tilde{\beta}; \bar{k}) = \max_{c,k'} \{U(c) + \tilde{\beta} E[V(k', P'_g, P'_b, \tilde{\beta}'; \bar{k}') | P_g, P_b, \tilde{\beta}]\}$$

subject to

$$c + k' = r(\bar{k}, 1 - \bar{u})k + w(\bar{k}, 1 - \bar{u})(P_g + P_b) + g(1 - P_g - P_b) + (1 - \delta)k,$$

[1] Also see Mukoyama and Şahin (2005, Appendix C and D) for a detailed exposition of the implementation of the integration principle. Note that Krusell and Smith (2002) and Mukoyama and Şahin (2005, 2006) use Markov approximations to the P processes while here we use the Ps directly in the computation.

and

$$\bar{k}' = H(\bar{k}).$$

As is explained in Appendix A, P'_g and P'_b are functions of a random variable $i' \sim U[0,1]$.

$$\begin{aligned} P'_g(i') &= \Pr[z' = g, \epsilon' = 1|i'] \\ &= \sum_{z=g,b}[\Pr[z' = g, \epsilon' = 1|i', z, \epsilon = 1]P_z + \Pr[z' = g, \epsilon' = 1|i', z, \epsilon = 0](1/2 - P_z)] \\ &= \sum_{z=g,b}[\Pr[\epsilon' = 1|i', z' = g, z, \epsilon = 1]\pi_{g|z}P_z + \Pr[\epsilon' = 1|i', z' = g, z, \epsilon = 0]\pi_{g|z}(1/2 - P_z)] \\ &= \sum_{z=g,b}[I(i' \leq \pi_{11|zg})\pi_{g|z}P_z + I(i' \leq \pi_{01|zg})\pi_{g|z}(1/2 - P_z)] \end{aligned}$$

and

$$\begin{aligned} P'_b(i') &= \Pr[z' = b, \epsilon' = 1|i'] \\ &= \sum_{z=g,b}[\Pr[z' = b, \epsilon' = 1|i', z, \epsilon = 1]P_z + \Pr[z' = b, \epsilon' = 1|i', z, \epsilon = 0](1/2 - P_z)] \\ &= \sum_{z=g,b}[\Pr[\epsilon' = 1|i', z' = b, z, \epsilon = 1]\pi_{b|z}P_z + \Pr[\epsilon' = 1|i', z' = b, z, \epsilon = 0]\pi_{b|z}(1/2 - P_z)] \\ &= \sum_{z=g,b}[I(i' \leq \pi_{11|zb})\pi_{b|z}P_z + I(i' \leq \pi_{01|zb})\pi_{b|z}(1/2 - P_z)], \end{aligned}$$

where $I(\cdot)$ is the indicator function equaling 1 if the statement is true and 0 if it is false. Note that this is just a compact way of writing the transition rules in Appendix A. The prices are:

$$r(\bar{k}, 1 - \bar{u}) = \alpha \bar{k}^{\alpha-1}(1 - \bar{u})^{1-\alpha}$$

and

$$w(\bar{k}, 1 - \bar{u}) = (1 - \alpha)\bar{k}^{\alpha}(1 - \bar{u})^{-\alpha}.$$

The expectation operator in the Bellman equation is taken over i' and $\tilde{\beta}'$ values. In practice, we divide the $[0, 1]$ into $4 \times 2 + 1 = 9$ subintervals when we take an expectation with respect to the i's, as is explained in Appendix A. The dynamic-programming problem is solved in a similar way to that used in Krusell and Smith (1998). We use 6 grids in each P direction (or, more precisely, on the conditional probability $\hat{P}$, as is detailed below) and apply linear interpolation to the value function when evaluating the values under P' (which are usually not on the grid).

3. For $t = 1, ..., 125$, we solve backwards for the path. Now the exogenous parameters move over time and we treat each period differently (for example, the value function has the index t). First, we provide the terminal value function: $V_{126}(k, P_g, P_b, \tilde{\beta}) = V(k, P_g, P_b, \tilde{\beta}; \bar{k}_{126})$, and we then calculate the probability that the aggregate state is z at time t, π_t^z, for $t = 1, ..., 125$: $\pi_1^g = 1$ if $z_1 = g$ and $\pi_1^g = 0$ if $z_1 = b$. It is always the case that $\pi_t^b = 1 - \pi_t^g$, and $\pi_{t+1}^g = \pi_t^g \pi_{g|g} + (1 - \pi_t^g)\pi_{g|b}$. z_t can be calculated using $z_t = \pi_t^g g + (1 - \pi_t^g)b$. u_t can also be calculated as $z_t = \pi_t^g u_g + (1 - \pi_t^g)u_b$. Thus the prices are

$$r_t = \alpha z_t \bar{k}_t^{\alpha-1}(1 - u_t)^{1-\alpha}$$

and

$$w_t = (1 - \alpha)z_t \bar{k}_t^{\alpha}(1 - u_t)^{-\alpha}.$$

Given these, we can solve the consumer's problem, working backwards. The Bellman equation is:

$$V_t(k, P_g, P_b, \tilde{\beta}) = \max_{c,k'}\{U(c) + \tilde{\beta}E_t[V_{t+1}(k', P'_g, P'_b, \tilde{\beta}')|P_g, P_b, \tilde{\beta}]\}$$

subject to

$$c + k' = r_t k + w_t(P_g + P_b) + g(1 - P_g - P_b) + (1 - \delta)k,$$

Again, P'_g and P'_b are functions of a random variable $i' \sim U[0, 1]$.

$$\begin{aligned} P'_g(i') &= \Pr[z' = g, \epsilon' = 1|i'] \\ &= \textstyle\sum_{z=g,b}[\Pr[z' = g, \epsilon' = 1|i', z, \epsilon = 1]P_z + \Pr[z' = g, \epsilon' = 1|i', z, \epsilon = 0](\pi_t^z - P_z)] \\ &= \textstyle\sum_{z=g,b}[\Pr[\epsilon' = 1|i', z' = g, z, \epsilon = 1]\pi_{g|z}P_z + \Pr[\epsilon' = 1|i', z' = g, z, \epsilon = 0]\pi_{g|z}(\pi_t^z - P_z)] \\ &= \textstyle\sum_{z=g,b}[I(i' \leq \pi_{11|zg})\pi_{g|z}P_z + I(i' \leq \pi_{01|zg})\pi_{g|z}(\pi_t^z - P_z)] \end{aligned}$$

and

$$\begin{aligned} P'_b(i') &= \Pr[z' = b, \epsilon' = 1|i'] \\ &= \textstyle\sum_{z=g,b}[\Pr[z' = b, \epsilon' = 1|i', z, \epsilon = 1]P_z + \Pr[z' = b, \epsilon' = 1|i', z, \epsilon = 0](\pi_t^z - P_z)] \\ &= \textstyle\sum_{z=g,b}[\Pr[\epsilon' = 1|i', z' = b, z, \epsilon = 1]\pi_{b|z}P_z + \Pr[\epsilon' = 1|i', z' = b, z, \epsilon = 0]\pi_{b|z}(\pi_t^z - P_z)] \\ &= \textstyle\sum_{z=g,b}[I(i' \leq \pi_{11|zb})\pi_{b|z}P_z + I(i' \leq \pi_{01|zb})\pi_{b|z}(\pi_t^z - P_z)]. \end{aligned}$$

4. Having solved the consumer's decision problem, we can simulate the economy. We assign an initial distribution for the individual state variables and then simulate consumers' decisions (we use 90,000 consumers). Adding up the implied saving choices, we obtain the time series for $\bar{k}_t$. Using this path, we check whether it reproduces the initially postulated path. If not, we update $\bar{k}_2, ..., \bar{k}_{126}$ and the law of motion $H(\bar{k})$ by ordinary least squares. We repeat until convergence.

B.2 Issues in actual implementation

In practice, we work on the conditional probabilities for Ps rather than with the joint probabilities. We define $\hat{P}_z$ as the probability of being employed *conditional on* the aggregate state z. Here, in the first step (since the probability of each aggregate state is 1/2),

$$\hat{P}_g = \frac{P_g}{1/2} = 2P_g \tag{1}$$

and

$$\hat{P}_b = \frac{P_b}{1/2} = 2P_b. \tag{2}$$

The new problem becomes (with the new value function $\hat{V}$)

$$\hat{V}(k, \hat{P}_g, \hat{P}_b, \tilde{\beta}; \bar{k}) = \max_{c,k'}\{U(c) + \tilde{\beta}E[\hat{V}(k', \hat{P}'_g, \hat{P}'_b, \tilde{\beta}'; \bar{k}')|\hat{P}_g, \hat{P}_b, \tilde{\beta}]\}$$

subject to

$$c + k' = r(\bar{k}, 1 - \bar{u})k + w(\bar{k}, 1 - \bar{u})\left(\frac{1}{2}\hat{P}_g + \frac{1}{2}\hat{P}_b\right) + g\left(1 - \frac{1}{2}\hat{P}_g - \frac{1}{2}\hat{P}_b\right) + (1 - \delta)k,$$

and

$$\bar{k}' = H(\bar{k}).$$

From (1) and (2), $\hat{P}_g'(i')$, $\hat{P}_g'(i')$ can be calculated by

$$\begin{aligned}\hat{P}_g'(i') &= 2P_g'(i') \\ &= 2\sum_{z=g,b}[I(i' \leq \pi_{11|zg})\pi_{g|z}P_z + I(i' \leq \pi_{01|zg})\pi_{g|z}(1/2 - P_z)] \\ &= \sum_{z=g,b}[I(i' \leq \pi_{11|zg})\pi_{g|z}\hat{P}_z + I(i' \leq \pi_{01|zg})\pi_{g|z}(1 - \hat{P}_z)]\end{aligned}$$

and

$$\begin{aligned}\hat{P}_b'(i') &= 2P_b'(i') \\ &= 2\sum_{z=g,b}[I(i' \leq \pi_{11|zb})\pi_{b|z}P_z + I(i' \leq \pi_{01|zb})\pi_{b|z}(1/2 - P_z)] \\ &= \sum_{z=g,b}[I(i' \leq \pi_{11|zb})\pi_{b|z}\hat{P}_z + I(i' \leq \pi_{01|zb})\pi_{b|z}(1 - \hat{P}_z)].\end{aligned}$$

The advantage of using conditional probabilities is that we can ensure that the labor-income terms in the budget constraint, $w(\bar{k}, 1-\bar{u})(\hat{P}_g/2 + \hat{P}_b/2)$ and $g(1 - \hat{P}_g/2 - \hat{P}_b/2)$, are positive as long as $\hat{P}_g, \hat{P}_b \in [0,1]$, and we can utilize the entire $[0,1]$ domain for $\hat{P}_g$ and $\hat{P}_b$. (Here, it is not a big advantage since we can instead just restrict $P_g, P_b \in [0, 0.5]$. However, the advantage is much larger in the second step, since the corresponding domain becomes time-variant.)

Similarly, in the second step of the optimization, define

$$\hat{P}_g = \frac{P_g}{\pi_t^g} \tag{3}$$

and

$$\hat{P}_b = \frac{P_b}{\pi_t^b}. \tag{4}$$

Note that we have to be careful about the initial point—we cannot divide when $\pi_t^z = 0$. To avoid this, we can start from $\pi_t^g = 1 - \varepsilon$ and $\pi_t^b = \varepsilon$ for a very small ε, for example.

The problem becomes

$$\hat{V}_t(k, \hat{P}_g, \hat{P}_b, \tilde{\beta}) = \max_{c,k'}\{U(c) + \tilde{\beta}E_t[\hat{V}_{t+1}(k', \hat{P}_g', \hat{P}_b', \tilde{\beta}')|\hat{P}_g, \hat{P}_b, \tilde{\beta}]\}$$

subject to

$$c + k' = r_t k + w_t(\pi_t^g\hat{P}_g + \pi_t^b\hat{P}_b) + g(1 - \pi_t^g\hat{P}_g - \pi_t^b\hat{P}_b) + (1-\delta)k.$$

Again, P_g' and P_b' are functions of a random variable $i' \sim U[0,1]$:

$$\begin{aligned}\hat{P}_g'(i') &= \tfrac{1}{\pi_{t+1}^g}P_g'(i') \\ &= \sum_{z=g,b}\left[I(i' \leq \pi_{11|zg})\tfrac{\pi_{g|z}}{\pi_{t+1}^g}P_z + I(i' \leq \pi_{01|zg})\tfrac{\pi_{g|z}}{\pi_{t+1}^g}(\pi_t^z - P_z)\right] \\ &= \sum_{z=g,b}\left[I(i' \leq \pi_{11|zg})\tfrac{\pi_t^z}{\pi_{t+1}^g}\pi_{g|z}\hat{P}_z + I(i' \leq \pi_{01|zg})\tfrac{\pi_t^z}{\pi_{t+1}^g}\pi_{g|z}(1 - \hat{P}_z)\right]\end{aligned}$$

and

$$\begin{aligned}\hat{P}_b'(i') &= \tfrac{1}{\pi_{t+1}^b}P_b'(i') \\ &= \sum_{z=g,b}\left[I(i' \leq \pi_{11|zb})\tfrac{\pi_{b|z}}{\pi_{t+1}^b}P_z + I(i' \leq \pi_{01|zb})\tfrac{\pi_{b|z}}{\pi_{t+1}^b}(\pi_t^z - P_z)\right] \\ &= \sum_{z=g,b}\left[I(i' \leq \pi_{11|zb})\tfrac{\pi_t^z}{\pi_{t+1}^b}\pi_{b|z}\hat{P}_z + I(i' \leq \pi_{01|zb})\tfrac{\pi_t^z}{\pi_{t+1}^b}\pi_{b|z}(1 - \hat{P}_z)\right].\end{aligned}$$

C Calculating the welfare gain

As in Lucas (1987), our measure of the welfare gain from eliminating the business cycle, λ, satisfies (we now change the notation of the discount factor so that we can be explicit about its stochastic nature)

$$E_0\left[\sum_{t=0}^{\infty}\left(\prod_{t=0}^{t}\beta(j)\right)\log((1+\lambda)c_t)\right] = E_0\left[\sum_{t=0}^{\infty}\left(\prod_{t=0}^{t}\beta(j)\right)\log(\tilde{c}_t)\right],$$

where $\beta(j)$ is the discount factor from time $j-1$ to j (known at time j) and $\beta(0)=1$. $\beta(1)$ is known at time 0—it is an initial condition. c_t is consumption in the economy with business cycles and $\tilde{c}_t$ is consumption in the economy with business cycles.

λ can be calculated as follows.[2]

$$\lambda = \exp\left(\frac{\mathcal{V}-\tilde{\mathcal{V}}}{d}\right) - 1,$$

where

$$\mathcal{V} = E_0\left[\sum_{t=0}^{\infty}\left(\prod_{t=0}^{t}\beta(j)\right)\log(c_t)\right]$$

and

$$\tilde{\mathcal{V}} = E_0\left[\sum_{t=0}^{\infty}\left(\prod_{t=0}^{t}\beta(j)\right)\log(\tilde{c}_t)\right]$$

can easily be calculated from the value functions. d is defined as

$$d \equiv E_0\left[\sum_{t=0}^{\infty}\left(\prod_{t=0}^{t}\beta(j)\right)\right].$$

Let $\bar{d}_i$ be the value of d when $\beta(1)=\beta_i$ ($i \in \{h,m,l\}$). Let the vector D be defined as

$$D \equiv \begin{pmatrix} \bar{d}_h \\ \bar{d}_m \\ \bar{d}_l \end{pmatrix}.$$

D satisfies the following equation:

$$D = I + \mathbf{B}\Omega D,$$

where

$$I \equiv \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix},$$

$$\Omega \equiv \begin{pmatrix} \omega_{hh} & \omega_{hm} & \omega_{hl} \\ \omega_{mh} & \omega_{mm} & \omega_{ml} \\ \omega_{lh} & \omega_{lm} & \omega_{ll} \end{pmatrix},$$

[2] The welfare measure in the main text is in percentage points, i.e. $\lambda \times 100$.

and

$$\mathbf{B} \equiv \begin{pmatrix} \beta_h & 0 & 0 \\ 0 & \beta_m & 0 \\ 0 & 0 & \beta_l \end{pmatrix}.$$

Therefore,

$$D = (\mathbf{I} - \mathbf{B}\Omega)^{-1} I,$$

where $\mathbf{I}$ is the 3×3 identity matrix.

D Algorithm for the model with short- and long-run unemployment

D.1 Notation

Let $\epsilon \in \{l, f, s, e\}$ denote long-term unemployment after the second unemployment period, long-term unemployment in the first unemployment period ("fired"), short-term unemployment, and employment, respectively. $z = g$ is the good state and $z = b$ is the bad state (from the second consecutive period). The transition matrix for z is now

The individual employment states evolve according to the following matrices.

- For $(z, z') = (g, g)$,

$$\begin{pmatrix} \pi_{ll|gg} & \pi_{lf|gg} & \pi_{ls|gg} & \pi_{le|gg} \\ \pi_{fl|gg} & \pi_{ff|gg} & \pi_{fs|gg} & \pi_{fe|gg} \\ \pi_{sl|gg} & \pi_{sf|gg} & \pi_{ss|gg} & \pi_{se|gg} \\ \pi_{el|gg} & \pi_{ef|gg} & \pi_{es|gg} & \pi_{ee|gg} \end{pmatrix} = \begin{pmatrix} 0.50 & 0 & 0 & 0.50 \\ 0.50 & 0 & 0 & 0.50 \\ 0.25 & 0 & 0 & 0.75 \\ 0 & 0 & 0.03 & 0.97 \end{pmatrix}.$$

- For (g, b),

$$\begin{pmatrix} \pi_{ll|gb} & \pi_{lf|gb} & \pi_{ls|gb} & \pi_{le|gb} \\ \pi_{fl|gb} & \pi_{ff|gb} & \pi_{fs|gb} & \pi_{fe|gb} \\ \pi_{sl|gb} & \pi_{sf|gb} & \pi_{ss|gb} & \pi_{se|gb} \\ \pi_{el|gb} & \pi_{ef|gb} & \pi_{es|gb} & \pi_{ee|gb} \end{pmatrix} = \begin{pmatrix} 0.94 & 0 & 0 & 0.06 \\ 0.94 & 0 & 0 & 0.06 \\ 0.75 & 0 & 0 & 0.25 \\ 0 & 0.04 & 0.03 & 0.93 \end{pmatrix}.$$

- For (b, g),

$$\begin{pmatrix} \pi_{ll|bg} & \pi_{lf|bg} & \pi_{ls|bg} & \pi_{le|bg} \\ \pi_{fl|bg} & \pi_{ff|bg} & \pi_{fs|bg} & \pi_{fe|bg} \\ \pi_{sl|bg} & \pi_{sf|bg} & \pi_{ss|bg} & \pi_{se|bg} \\ \pi_{el|bg} & \pi_{ef|bg} & \pi_{es|bg} & \pi_{ee|bg} \end{pmatrix} = \begin{pmatrix} 0.17 & 0 & 0 & 0.83 \\ 0.17 & 0 & 0 & 0.83 \\ 0.03 & 0 & 0 & 0.97 \\ 0 & 0 & 0.03 & 0.97 \end{pmatrix}.$$

- For (b, b),

$$\begin{pmatrix} \pi_{ll|bb} & \pi_{lf|bb} & \pi_{ls|bb} & \pi_{le|bb} \\ \pi_{fl|bb} & \pi_{ff|bb} & \pi_{fs|bb} & \pi_{fe|bb} \\ \pi_{sl|bb} & \pi_{sf|bb} & \pi_{ss|bb} & \pi_{se|bb} \\ \pi_{el|bb} & \pi_{ef|bb} & \pi_{es|bb} & \pi_{ee|bb} \end{pmatrix} = \begin{pmatrix} 0.99 & 0 & 0 & 0.01 \\ 0.99 & 0 & 0 & 0.01 \\ 0.03 & 0 & 0 & 0.97 \\ 0 & 0 & 0.03 & 0.97 \end{pmatrix}.$$

Computation of equilibrium for the economy with aggregate shocks is more involved than in the homogeneous unemployment case, since now we have 3 aggregate states and 4 individual states, but does not significantly depart in complexity or difficulty from Krusell and Smith (1998).

D.2 Transition dynamics

The general computational strategy is the same as for the case of homogeneous unemployment in Appendix B, though it is more involved because of the many state variables.

1. First we postulate the path for the aggregate capital stock.

2. As in the previous section, we start from $t \geq 126$. The Bellman equation is:

$$V(k, \mathbf{P}, \tilde{\beta}; \bar{k}) = \max_{c,k'} \{U(c) + \tilde{\beta} E[V(k', \mathbf{P}', \tilde{\beta}'; \bar{k}')|\mathbf{P}, \tilde{\beta}]\}$$

subject to

$$c + k' = r(\bar{k}, 1-\bar{u})k + w(\bar{k}, 1-\bar{u}) \sum_{z=g,b} P_z^e + \psi_s \sum_{z=g,b} (P_z^s + P_z^f) + \psi_l \sum_{z=g,b} P_z^l + (1-\delta)k,$$

and

$$\bar{k}' = H(\bar{k}).$$

Here, $\mathbf{P}$ is the vector of P_z^ϵs (the joint probability that the aggregate state is z and the individual state is ϵ, if there were aggregate fluctuations). We do not need to keep track of all the Ps: it is sufficient to have P_g^e, P_b^e, P_g^s, P_b^s, and P_b^f as state variables.[3] This is because $P_z^l = \pi^z - P_z^e - P_z^s - P_z^f$ and $P_g^f = 0$. P_z^ϵ evolves according to

$$\begin{aligned} P_{z'}^{\epsilon'}(i') &= \Pr[z', \epsilon'|i'] \\ &= \textstyle\sum_{z=g,b} \sum_{\epsilon=e,s,f,l} \Pr[z', \epsilon'|i', z, \epsilon] P_z^\epsilon \\ &= \textstyle\sum_{z=g,b} \sum_{\epsilon=e,s,f,l} \Pr[\epsilon'|i', z', z, \epsilon] \pi_{z'|z} P_z^\epsilon. \end{aligned} \tag{5}$$

Calculating $\Pr[\epsilon'|i', z', z, \epsilon]$ is harder in this case. Before, we set $\Pr[\epsilon' = 1|i', z', z, \epsilon] = 1$ if $i' \leq \pi_{\epsilon\epsilon'|zz'}$ and zero otherwise; and $\Pr[\epsilon' = 0|i', z', z, \epsilon] = 1$ if $i' > \pi_{\epsilon\epsilon'|zz'}$ and zero otherwise.

Now we have four idiosyncratic states instead of two, so we adopt the following cutoff rule:

$$\Pr[\epsilon' = e|i', z', z, \epsilon] = \begin{cases} 1 \text{ if } i' \in [0, \pi_{\epsilon e|zz'}] \\ 0 \text{ otherwise} \end{cases}$$

$$\Pr[\epsilon' = s|i', z', z, \epsilon] = \begin{cases} 1 \text{ if } i' \in (\pi_{\epsilon e|zz'}, \pi_{\epsilon e|zz'} + \pi_{\epsilon s|zz'}] \\ 0 \text{ otherwise} \end{cases}$$

$$\Pr[\epsilon' = f|i', z', z, \epsilon] = \begin{cases} 1 \text{ if } i' \in (\pi_{\epsilon e|zz'} + \pi_{\epsilon s|zz'}, \pi_{\epsilon e|zz'} + \pi_{\epsilon s|zz'} + \pi_{\epsilon f|zz'}] \\ 0 \text{ otherwise} \end{cases}$$

$$\Pr[\epsilon' = l|i', z', z, \epsilon] = \begin{cases} 1 \text{ if } i' \in (\pi_{\epsilon e|zz'} + \pi_{\epsilon s|zz'} + \pi_{\epsilon f|zz'}, 1] \\ 0 \text{ otherwise} \end{cases}.$$

We keep the structure that a low i is "lucky" and a high i is "unlucky." In the computation, we divide the interval of possible is in $[0, 1]$ into subintervals by these cutoff thresholds when we take expectations in the Bellman equation.

[3]Note that here we do not need to make distinctions between the z_b state and the z_d state.

Again, in the actual computation, we work with conditional probabilities (but not conditional only on the aggregate states); we define these conditional probabilities, $\hat{P}_z^\epsilon$, as follows.

$$\hat{P}_g^e \equiv \frac{P_g^e}{\pi_t^g}, \tag{6}$$

$$\hat{P}_b^e \equiv \frac{P_b^e}{\pi_t^b}, \tag{7}$$

$$\hat{P}_g^s \equiv \frac{P_g^s}{\pi_t^g(1-\hat{P}_g^e)}, \tag{8}$$

$$\hat{P}_b^s \equiv \frac{P_b^s}{\pi_t^b(1-\hat{P}_b^e)}, \tag{9}$$

$$\hat{P}_b^f \equiv \frac{P_b^f}{\pi_t^b(1-\hat{P}_b^e)(1-\hat{P}_b^s)}. \tag{10}$$

Then, the problem becomes

$$\hat{V}(k,\hat{\mathbf{P}},\tilde{\beta};\bar{k}) = \max_{c,k'}\{U(c) + \tilde{\beta}E[V(k',\hat{\mathbf{P}}',\tilde{\beta}';\bar{k}')|\hat{\mathbf{P}},\tilde{\beta}]\}$$

subject to

$$c + k' = r(\bar{k}, 1-\bar{u})k + \mathcal{I}_w + \mathcal{I}_{u1} + \mathcal{I}_{u2} + (1-\delta)k$$

and

$$\bar{k}' = H(\bar{k}).$$

Here,

$$\mathcal{I}_w \equiv w(\bar{k}, 1-\bar{u}) \sum_{z=g,b} P_z^e = w(\bar{k}, 1-\bar{u}) \sum_{z=g,b} \pi_t^z \hat{P}_z^e,$$

$$\mathcal{I}_{u1} \equiv \psi_s \sum_{z=g,b} (P_z^s + P_z^f) = \psi_s \left[\sum_{z=g,b} \pi_t^z (1-\hat{P}_z^e)\hat{P}_z^s + \pi_t^b(1-\hat{P}_b^e)(1-\hat{P}_b^s)\hat{P}_b^f \right],$$

and

$$\begin{aligned} \mathcal{I}_{u2} &\equiv \psi_l \textstyle\sum_{z=g,b} P_z^l \\ &= \psi_l \textstyle\sum_{z=g,b}(\pi_t^z - P_z^e - P_z^s - P_z^f) \\ &= \psi_l \left[1 - \pi_t^g(\hat{P}_g^e + (1-\hat{P}_g^e)\hat{P}_g^s) - \pi_t^b(\hat{P}_b^e + (1-\hat{P}_b^e)\hat{P}_b^s + (1-\hat{P}_b^e)(1-\hat{P}_b^s)\hat{P}_b^f)\right]. \end{aligned}$$

The evolution of $\hat{P}_z^\epsilon$ can be obtained from the above relationships. In particular, we can convert $\hat{P}$s into Ps using the equations (6)-(10), calculate the transition of Ps by (5), and transform the Ps back to $\hat{P}$s using (6)-(10) once again. Here, since all the exogenous variables are already settled, we can use $\pi_t^z = 1/2$.

Again, the advantage of working on the conditional distributions is that we can ensure that $\mathcal{I}_w$, $\mathcal{I}_{u1}$, and $\mathcal{I}_{u2}$ are all positive for $\hat{P}_g^e, \hat{P}_b^e, \hat{P}_g^s, \hat{P}_b^s, \hat{P}_b^f \in [0,1]$. We put 5 grid points in each $\hat{P}$ direction and linearly interpolate the value functions when evaluating the value at $\hat{P}'$.

3. The other steps are similar to those in Appendix B. For the first 125 periods, π_t^z changes over time: it evolves according to $\pi_{t+1}^{z'} = \sum_{z=g,b} \pi_{z'|z}\pi_t^z$.

E More tables

E.1 One type of unemployment

	Utility gain in percentage consumption							
	< 1	1–5	5–25	25–50	50–75	75–95	95–99	> 99
All	0.861	0.261	0.070	−0.050	−0.093	0.228	1.088	1.689
$\epsilon = 1$	0.263	0.156	0.045	−0.057	−0.096	0.228	1.090	1.691
$\epsilon = 0$	1.691	0.671	0.254	0.033	−0.057	0.233	1.067	1.643

Average utility gains by wealth group ($\bar{k} = 11.2$, $z = z_b$)

	Utility gain in percentage consumption							
	< 1	1–5	5–25	25–50	50–75	75–95	95–99	> 99
All	0.278	0.175	0.043	−0.059	−0.078	0.219	0.993	1.552
$\epsilon = 1$	0.247	0.166	0.040	−0.060	−0.078	0.219	0.994	1.551
$\epsilon = 0$	0.579	0.345	0.113	−0.013	−0.054	0.201	0.979	1.531

Average utility gains by wealth group ($\bar{k} = 12.3$, $z = z_g$)

	Utility gain in percentage consumption							
	< 1	1–5	5–25	25–50	50–75	75–95	95–99	> 99
All	0.393	0.207	0.050	−0.056	−0.079	0.224	1.020	1.577
$\epsilon = 1$	0.250	0.171	0.035	−0.064	−0.083	0.226	1.022	1.566
$\epsilon = 0$	0.821	0.416	0.169	0.012	−0.044	0.206	1.000	1.667

Average utility gains by wealth group ($\bar{k} = 12.3$, $z = z_b$)

	Wealth percentile						
Type of agent	constr.	0.005	0.05	0.5	0.95	0.995	0.999
$\epsilon = 1$, $\beta =$ low	0.636	0.470	0.303	0.080	0.432	1.202	1.510
$\epsilon = 1$, $\beta =$ middle	0.316	0.181	0.046	−0.095	0.668	1.479	1.787
$\epsilon = 1$, $\beta =$ high	0.120	0.033	−0.022	0.014	1.041	1.864	2.176
$\epsilon = 0$, $\beta =$ low	3.808	1.867	0.732	0.182	0.419	1.199	1.509
$\epsilon = 0$, $\beta =$ middle	2.890	1.314	0.390	−0.033	0.653	1.476	1.787
$\epsilon = 0$, $\beta =$ high	2.107	0.884	0.183	0.006	1.025	1.860	2.175

Utility gains for different types of agents ($\bar{k} = 11.2$, $z = z_b$)

	Wealth percentile						
Type of agent	constr.	0.005	0.05	0.5	0.95	0.995	0.999
$\epsilon = 1$, $\beta =$ low	0.563	0.356	0.241	0.099	0.369	1.069	1.338
$\epsilon = 1$, $\beta =$ middle	0.264	0.090	0.010	-0.076	0.599	1.343	1.613
$\epsilon = 1$, $\beta =$ high	0.089	0.001	-0.007	0.040	0.970	1.723	1.995
$\epsilon = 0$, $\beta =$ low	1.508	0.644	0.420	0.164	0.360	1.067	1.337
$\epsilon = 0$, $\beta =$ middle	1.034	0.321	0.138	-0.039	0.588	1.340	1.612
$\epsilon = 0$, $\beta =$ high	0.681	0.131	0.033	0.022	0.959	1.720	1.995

Utility gains for different types of agents ($\bar{k} = 12.3$, $z = z_g$)

	Wealth percentile						
Type of agent	constr.	0.005	0.05	0.5	0.95	0.995	0.999
$\epsilon = 1$, $\beta =$ low	0.648	0.364	0.228	0.094	0.386	1.107	1.382
$\epsilon = 1$, $\beta =$ middle	0.328	0.095	0.008	-0.080	0.619	1.382	1.659
$\epsilon = 1$, $\beta =$ high	0.134	0.004	-0.009	0.035	0.992	1.765	2.044
$\epsilon = 0$, $\beta =$ low	3.022	0.850	0.507	0.188	0.375	1.104	1.381
$\epsilon = 0$, $\beta =$ middle	2.259	0.487	0.208	-0.024	0.605	1.380	1.658
$\epsilon = 0$, $\beta =$ high	1.625	0.256	0.074	0.015	0.977	1.762	2.043

Utility gains for different types of agents ($\bar{k} = 12.3$, $z = z_b$)

E.2 Short- and long-term unemployment

	Utility gain in percentage consumption							
	< 1	1–5	5–25	25–50	50–75	75–95	95–99	> 99
All	7.415	2.640	1.225	0.525	0.397	1.136	4.463	6.960
$\epsilon = e$	1.504	0.917	0.825	0.336	0.267	1.092	4.464	6.951
$\epsilon = s$	2.057	1.221	1.096	0.453	0.348	1.167	4.509	6.749
$\epsilon = f$	14.512	9.071	6.059	3.020	2.355	1.746	4.566	6.976
$\epsilon = l$	18.973	10.971	6.335	3.205	2.555	1.719	4.253	7.098

Average utility gains by wealth group ($\bar{k} = 11.3$, $z = z_d$)

	Utility gain in percentage consumption							
	< 1	1–5	5–25	25–50	50–75	75–95	95–99	> 99
All	20.632	9.994	1.325	0.421	0.332	1.137	4.233	6.595
$\epsilon = e$	1.579	1.417	0.917	0.393	0.323	1.116	4.237	6.580
$\epsilon = s$		1.958	1.139	0.510	0.411	1.080	4.074	6.614
$\epsilon = l$	20.804	11.068	6.805	3.290	2.437	1.676	4.249	6.673

Average utility gains by wealth group[4] ($\bar{k} = 11.3$, $z = z_b$)

	Utility gain in percentage consumption							
	< 1	1–5	5–25	25–50	50–75	75–95	95–99	> 99
All	2.092	1.684	0.838	0.456	0.392	1.135	4.264	6.672
$\epsilon = e$	1.727	1.599	0.795	0.432	0.377	1.131	4.270	6.670
$\epsilon = s$	3.207	3.057	1.613	0.925	0.791	1.210	4.062	6.811
$\epsilon = l$	4.830	3.810	1.861	1.278	1.091	1.371	4.213	5.937

Average utility gains by wealth group ($\bar{k} = 12.1$, $z = z_g$)

	Utility gain in percentage consumption							
	< 1	1–5	5–25	25–50	50–75	75–95	95–99	> 99
All	3.385	2.010	1.026	0.576	0.447	1.148	4.212	6.627
$\epsilon = e$	1.493	1.414	0.680	0.373	0.323	1.107	4.194	6.629
$\epsilon = s$	1.925	1.722	0.855	0.472	0.410	1.106	4.301	6.827
$\epsilon = f$	9.100	8.185	4.679	2.936	2.435	1.615	4.347	6.246
$\epsilon = l$	11.167	8.670	4.504	3.141	2.645	1.690	4.502	6.460

Average utility gains by wealth group ($\bar{k} = 12.1$, $z = z_d$)

[4] In the simulated data, there are no agents with $\epsilon = s$ and asset holdings below the first percentile.

	Utility gain in percentage consumption							
	< 1	1–5	5–25	25–50	50–75	75–95	95–99	> 99
All	5.339	2.153	1.188	0.594	0.361	1.137	4.215	6.620
$\epsilon = e$	1.517	1.424	0.704	0.372	0.323	1.104	4.212	6.596
$\epsilon = s$	1.777	1.713	0.894	0.481	0.413	1.211	4.052	6.989
$\epsilon = l$	11.122	8.517	4.411	3.143	2.663	1.637	4.304	6.655

Average utility gains by wealth group ($\bar{k} = 12.1$, $z = z_b$)

	Wealth percentile						
Type of agent	constr.	0.005	0.05	0.5	0.95	0.995	0.999
$\epsilon = e$, $\beta =$ low	1.960	1.844	1.524	0.885	1.191	4.017	5.101
$\epsilon = e$, $\beta =$ middle	1.062	0.957	0.695	0.277	2.089	5.540	6.669
$\epsilon = e$, $\beta =$ high	0.636	0.615	0.612	0.884	4.353	8.033	9.212
$\epsilon = s$, $\beta =$ low	2.749	2.508	1.904	1.027	1.181	4.014	5.104
$\epsilon = s$, $\beta =$ middle	1.685	1.477	0.988	0.370	2.071	5.537	6.668
$\epsilon = s$, $\beta =$ high	1.054	0.922	0.744	0.879	4.331	8.030	9.212
$\epsilon = f$, $\beta =$ low	22.119	16.464	10.074	4.093	1.191	4.008	5.102
$\epsilon = f$, $\beta =$ middle	16.336	12.143	7.258	2.551	1.996	5.529	6.670
$\epsilon = f$, $\beta =$ high	10.755	7.913	4.497	1.606	4.218	8.020	9.213
$\epsilon = l$, $\beta =$ low	30.194	21.159	11.223	4.333	1.182	4.006	5.101
$\epsilon = l$, $\beta =$ middle	22.226	15.627	8.150	2.737	1.982	5.527	6.669
$\epsilon = l$, $\beta =$ high	14.676	10.277	5.125	1.683	4.200	8.018	9.212

Utility gains for different types of agents ($\bar{k} = 11.3$, $z = z_d$)

	Wealth percentile						
Type of agent	constr.	0.005	0.05	0.5	0.95	0.995	0.999
$\epsilon = e$, $\beta =$ low	2.066	1.981	1.643	0.966	1.155	3.606	4.617
$\epsilon = e$, $\beta =$ middle	1.153	1.076	0.793	0.338	1.940	5.052	6.112
$\epsilon = e$, $\beta =$ high	0.663	0.644	0.625	0.861	4.072	7.429	8.527
$\epsilon = s$, $\beta =$ low	2.865	2.685	2.039	1.113	1.153	3.603	4.616
$\epsilon = s$, $\beta =$ middle	1.778	1.628	1.099	0.436	2.021	5.048	6.111
$\epsilon = s$, $\beta =$ high	1.085	0.984	0.768	0.860	4.191	7.425	8.526
$\epsilon = l$, $\beta =$ low	30.340	23.461	11.716	4.471	1.161	3.593	4.616
$\epsilon = l$, $\beta =$ middle	22.347	17.335	8.539	2.850	2.038	5.035	6.110
$\epsilon = l$, $\beta =$ high	14.728	11.397	5.371	1.711	4.211	7.409	8.526

Utility gains for different types of agents ($\bar{k} = 11.3$, $z = z_b$)

	Wealth percentile						
Type of agent	constr.	0.005	0.05	0.5	0.95	0.995	0.999
$\epsilon = e$, $\beta =$ low	2.469	1.833	1.632	1.051	1.126	3.703	4.812
$\epsilon = e$, $\beta =$ middle	1.451	0.927	0.775	0.390	1.995	5.179	6.338
$\epsilon = e$, $\beta =$ high	0.861	0.727	0.730	0.904	4.209	7.613	8.821
$\epsilon = s$, $\beta =$ low	5.854	3.526	2.994	1.700	1.121	3.699	4.813
$\epsilon = s$, $\beta =$ middle	4.056	2.215	1.797	0.834	1.967	5.174	6.339
$\epsilon = s$, $\beta =$ high	2.575	1.413	1.218	1.011	4.171	7.607	8.822
$\epsilon = l$, $\beta =$ low	8.334	4.741	3.966	2.141	1.111	3.695	4.812
$\epsilon = l$, $\beta =$ middle	5.969	3.162	2.545	1.143	1.941	5.169	6.338
$\epsilon = l$, $\beta =$ high	4.056	1.964	1.604	1.080	4.138	7.602	8.821

Utility gains for different types of agents ($\bar{k} = 12.1$, $z = z_g$)

	Wealth percentile						
Type of agent	constr.	0.005	0.05	0.5	0.95	0.995	0.999
$\epsilon = e$, $\beta =$ low	2.052	1.588	1.433	0.964	1.115	3.621	4.598
$\epsilon = e$, $\beta =$ middle	1.144	0.750	0.635	0.333	1.967	5.069	6.093
$\epsilon = e$, $\beta =$ high	0.661	0.627	0.651	0.860	4.138	7.455	8.515
$\epsilon = s$, $\beta =$ low	2.781	1.949	1.728	1.108	1.105	3.619	4.597
$\epsilon = s$, $\beta =$ middle	1.722	1.030	0.858	0.430	1.949	5.066	6.092
$\epsilon = s$, $\beta =$ high	1.052	0.751	0.732	0.858	4.117	7.451	8.514
$\epsilon = f$, $\beta =$ low	21.258	9.674	7.974	4.172	1.120	3.611	4.598
$\epsilon = f$, $\beta =$ middle	15.720	6.961	5.627	2.619	1.882	5.056	6.092
$\epsilon = f$, $\beta =$ high	10.330	4.269	3.347	1.609	4.012	7.451	8.512
$\epsilon = l$, $\beta =$ low	27.883	10.670	8.656	4.405	1.112	3.609	4.597
$\epsilon = l$, $\beta =$ middle	20.573	7.737	6.164	2.800	1.868	5.053	6.091
$\epsilon = l$, $\beta =$ high	13.572	4.817	3.715	1.685	3.997	7.435	8.511

Utility gains for different types of agents ($\bar{k} = 12.1$, $z = z_d$)

Type of agent	Wealth percentile constr.	0.005	0.05	0.5	0.95	0.995	0.999
$\epsilon = e$, $\beta =$ low	2.060	1.609	1.439	0.964	1.107	3.621	4.647
$\epsilon = e$, $\beta =$ middle	1.149	0.766	0.639	0.334	1.963	5.074	6.147
$\epsilon = e$, $\beta =$ high	0.671	0.635	0.660	0.872	4.144	7.469	8.583
$\epsilon = s$, $\beta =$ low	2.790	1.977	1.734	1.109	1.098	3.618	4.645
$\epsilon = s$, $\beta =$ middle	1.729	1.051	0.862	0.430	1.946	5.070	6.146
$\epsilon = s$, $\beta =$ high	1.063	0.763	0.742	0.869	4.123	7.466	8.582
$\epsilon = l$, $\beta =$ low	27.910	10.884	8.661	4.389	1.105	3.608	4.646
$\epsilon = l$, $\beta =$ middle	20.592	7.903	6.167	2.787	1.865	5.058	6.145
$\epsilon = l$, $\beta =$ high	13.591	4.940	3.723	1.688	4.003	7.450	8.580

Utility gains for different types of agents ($\bar{k} = 12.1$, $z = z_b$)

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