Near-Rational Alternatives and the Empirical Evaluation of Real Business Cycle Models

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Abstract

This paper shows that if the consumers in a "standard" real business cycle (RBC) model are permitted to use near-rational decision rules—that is, decision rules from which they suffer trivial utility losses—then it is difficult to distinguish the standard model from several competing extensions on the basis of the models' implications for a small set of key time series statistics. Moreover, there exist near-rational alternatives to the standard model which reproduce *exactly* the values of these statistics in observed time series. These findings suggest that RBC researchers should examine much larger sets of second moments when evaluating the empirical performance of RBC models.

1 Introduction

This paper examines the implications of near-rationality for the empirical evaluation of real business cycle (RBC) models. In particular, this paper argues that the following methodological principles are mutually incompatible:

1. The purpose of RBC studies is to "sort out" models: that is, the goal of RBC studies is to identify the structure of technology and preferences underlying observed aggregate time series.

2. RBC studies should focus on a limited set of moments of aggregate time series.

3. Economic models should not dogmatically rule out "sloppiness" of economic agents, from which agents suffer trivial utility losses.

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Hansen and Wright (1992)'s survey of the labor market market in real business cycle theory provides an excellent illustration of how methodological principles #1 and #2 guide research efforts in real business cycles. Hansen and Wright (1992) trace, in particular, the development of a variety of RBC models designed to improve the labor market performance of a "standard", or baseline, RBC model. The various models are compared to each other and to the observed data by focusing on the models' implications for a small set of time series statistics, with special emphasis on two key labor market statistics: the variability of hours relative to wages and the contemporaneous correlation of hours and wages. As Hansen and Wright (1992) document, the baseline model does a relatively poor job of matching the observed values of the two labor market statistics. Hansen and Wright (1992) exposit four extensions to the baseline model, each of which improves the ability of the baseline model to match the observed values of the two labor market statistics.

The line of research summarized by Hansen and Wright (1992) shows the practical import of the first two methodological principles. First, RBC researchers seek to find specifications of RBC models which are consistent with the properties of aggregate times series (methodological principle #1). Second, RBC researchers typically focus on a small set of aggregate time series statistics when assessing the distance between model and data (methodological principle #2).

The purpose of this paper is to argue that the first two methodological principles are incompatible with the third: if the consumers in real business cycle models are permitted to use near-rational decision rules, then RBC researchers will not be able to sort out different specifications of RBC models on the basis of their implications for a limited set of time series moments. Specifically, this paper shows that the key time series moments of a baseline RBC model—the divisible labor model of Hansen (1985)—are very sensitive to near-rational deviations from optimal behavior (see also Cochrane (1989), Smith (1992), and Krusell and Smith (1993a)).

To give a concrete example, consider five key aggregate time series statistics: the variabilities of consumption, investment, and hours relative to the variability of output, the variability of hours relative to that of average productivity (wages), and the contemporaneous correlation of hours and wages. In their survey article, Hansen and Wright (1992) report values of these statistics for the baseline model was well as for four variations on the baseline model—models with, respectively, nonseparable leisure, indivisible labor, government spending, and home production. They also report the values of these five statistics for two different sets of U.S. observed time series (depending on whether hours is measured using establishment or household survey data). The values of these statistics vary widely in some cases, and none of the models comes close to matching simultaneously the three labor market statistics in the observed data.

This paper shows that, for each of these six sets of statistics, one can find decision rules (i.e., functions mapping the current state into choices for capital and labor) which reproduce *exactly* the given values of the five statistics, and yet carry trivial utility losses. In particular, for each of these near-rational decision rules, a small consumer gains less than 3/100 of one percent of per period consumption by switching to the optimal rule (given the near-rational behavior of all other consumer in the economy). In a well-defined sense, therefore, the range of time series statistics generated by the set of RBC models within a small perturbation of the baseline model is very large: under well-defined near-rational alternatives, the baseline model can reproduce not only the time series statistics generated by other models, but also both sets of observed time series statistics.

These results suggest that, once one allows for the possibility of near-rational behavior, it is difficult, on the basis of a limited set of time series moments, to distinguish the baseline model from models with richer specifications of technology and preferences. This inability to distinguish models renders problematic the task of deciding which specification of technology and preferences underlies observed time series.

These results have important implications for the empirical evaluation of RBC models. Few researchers will argue with the importance of methodological principle #1. Some may reject principle #3, but it is difficult to justify ruling out behavior with trivial welfare consequences. Instead, RBC researchers should modify principle #2 by examining much larger sets of moments—such as impulse response functions or the entire autocovariance function of the vector of variables of interest—when evaluating RBC models. For example, when the number of statistics increases from five to eight, the welfare losses associated with decision rules chosen to match these statistics also increase, in some cases by as much as a factor of six. This example suggests that as the set of moments expands, the welfare costs of decision rules designed to reproduce these moments can increase dramatically. The results in this paper therefore provide support for "full-system" methods of evaluating the goodness-of-fit of equilibrium business cycle models, such as those pioneered by Hansen and Sargent (1980) and applied to RBC models in Hansen and Sargent (1988) and Smith (1993).

The paper is organized as follows. Section 2 presents the baseline RBC model and the four extensions considered in this paper. Section 3 describes and implements the numerical procedure for constructing decision rules which reproduce a given set of time series statistics. Section 4 describes and implements the numerical procedure for computing the welfare gain that a small consumer realizes by switching to an optimal decision rule, given the suboptimal behavior of all other consumers in the economy. Section 5 concludes briefly.

2 The Baseline RBC Model and Four Extensions

The five RBC models considered in this paper—the baseline model and four extensions—are drawn directly from the survey article by Hansen and Wright (1992). This section briefly discusses each of the five models. The reader is referred to Hansen and Wright (1992) for complete details.

The baseline real business cycle model is similar to the divisible labor model of Hansen (1985). It takes the form of the following social planning problem, where C_t is period t aggregate consumption, X_t is period t aggregate investment, H_t is period t aggregate hours, K_t is the aggregate capital stock at the beginning of period t, and z_t is the period t technology shock:

$$\max_{\{K_t\}_{t=1}^{\infty}, \{H_t\}_{t=0}^{\infty}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \, (\log(C_t) + A \, \log(1 - H_t)) \tag{1}$$

subject to:

$$\begin{aligned} &K_0 \text{ and } z_0 \text{ given} \\ &C_t + X_t = K_t^{\alpha} \, H_t^{1-\alpha} \, \exp(z_t) \\ &K_{t+1} = (1 - \delta) \, K_t + X_t \\ &z_{t+1} = \rho \, z_t + \epsilon_{t+1} \\ &\epsilon_{t+1} \sim iidN(0, \sigma_{\epsilon}^2) \end{aligned}$$

The depreciation rate δ, the discount parameter β, the autoregressive parameter ρ, capital's share of income α, and the utility parameter A satisfy: $0 \leq \delta \leq 1$, $0 < \beta < 1$, $|\rho| < 1$, $0 < \alpha < 1$, and $A > 0$.[1] The period t state variables in the baseline economy defined by Problem (1) are K_t and z_t; the period t choice variables are K_{t+1} and H_t. The solution to Problem (1) can be expressed as a pair of time-invariant decision rules f and g mapping the state variables into the choice variables: $K_{t+1} = f(K_t, z_t)$ and $H_t = g(K_t, z_t)$.

The four extensions to the baseline model, surveyed by Hansen and Wright (1992), are designed to improve the labor market performance of the baseline model. Specifically, the variability of hours relative to output is lower in the baseline model than in the observed data; the variability of hours relative to the variability of wages is lower in the model than in the data; and the contemporaneous correlation of hours and wages is higher in the model than in the data. The four extensions to the baseline model improve the performance of the baseline model along one or more of these three dimensions. Briefly, the four extensions can be described as follows:

1. *Nonseparable leisure* (see Kydland and Prescott (1982)): Instantaneous utility depends on a weighted average of current and past leisure. This extension increases the willingness of individuals to engage in intertemporal substitution of leisure.

2. *Indivisible labor* (see Hansen (1985)): Individuals are constrained, in each period, to work either zero hours or some fixed number of hours greater than zero. Under this extension, the instantaneous utility function of the representative agent is linear in aggregate hours, leading to a large intertemporal substitution effect.

3. *Government spending* (see Christiano and Eichenbaum (1992)): Stochastic government spending, modelled as a pure drain on output, is introduced into the baseline model. The introduction of a shock which shifts the labor supply curve lowers the correlation between hours and wages.

[1] The specific parameter values for the baseline model are: $\delta = 0.025$, $\beta = 0.99$, $\rho = 0.95$, $\alpha = 0.36$, $\sigma_{\epsilon} = 0.007$, and $A = 1.721365$ (A is chosen so that the deterministic steady state value of H_t is one-third).

4. *Home production* (see Benhabib, Rogerson, and Wright (1991)): A household (nonmarket) production sector is added to the baseline model. This modification increases the volatility of hours worked in the market sector, since individuals can now substitute between the market and nonmarket sectors of the economy in response to technology shocks in the two sectors.

The reader is referred to the excellent exposition in Hansen and Wright (1992) for further details concerning the functional forms, parameter values, etc. underlying each of the above extensions.

Table 1 reports time series statistics for each of the five models, as well as for two different sets of observed time series, depending on whether hours worked is measured using establishment survey data or household survey data. This paper focuses on eight key statistics: σ_c/σ_y, σ_x/σ_y, σ_h/σ_y, σ_h/σ_w, σ_{hw}, σ_{wy}, σ_{cy}, and σ_{hy}, where c denotes consumption, h denotes hours, x denotes investment, y denotes output, w denotes wages (i.e., the marginal product of labor), σ_i is the unconditional standard deviation of series i, and σ_{ij} is the contemporaneous correlation between series i and series j.[2]

The observed sample consists of postwar U.S. aggregate time series for the period 1947:1 to 1991:3, a total of 179 quarters. The time series statistics implied by a given model are calculated by generating 100 statistically independent simulations of the model's time series, with 179 observations in each time series, and then computing sample means of the relevant statistics across the 100 simulations. For both observed and simulated data, the Hodrick-Prescott (HP) filter is applied separately to the natural logarithm of each of the time series prior to the computation of time series statistics. The models are solved using standard techniques, in particular, the linear-quadratic solution method pioneered by Kydland and Prescott (1982) and used by, among others, Hansen (1985), Christiano (1990), and McGrattan (1990).

TABLE 1: EIGHT STATISTICS OF INTEREST

	σ_c/σ_y	σ_x/σ_y	σ_h/σ_y	σ_h/σ_w	σ_{hw}	σ_{wy}	σ_{cy}	σ_{hy}
Household data	0.45	2.78	0.78	1.37	0.07	0.63	0.71	0.82
Establishment data	0.45	2.78	0.96	2.15	−0.14	0.31	0.71	0.90
Baseline model	0.31	3.15	0.49	0.94	0.93	0.99	0.89	0.98
Nonseparable leisure	0.29	3.23	0.65	1.63	0.80	0.92	0.87	0.97
Indivisible labor	0.29	3.25	0.76	2.63	0.76	0.87	0.87	0.98
Government spending	0.54	3.08	0.55	0.90	0.49	0.88	0.55	0.85
Home production	0.51	2.73	0.75	1.92	0.49	0.75	0.71	0.94

[2]The numbers in Table 1 are drawn directly from Hansen and Wright (1992), with the exception of the last three statistics for each of the five models, which Hansen and Wright (1992) do not report. To verify the accuracy of the model statistics, I solved each of the models independently, replicating the numbers reported in Hansen and Wright (1992), while at the same time generating values for the remaining three statistics.

3 Finding Decision Rules to Match the Moments

This section describes and implements the numerical procedure for finding decision rules which reproduce some, or all, of the statistics in a given row of Table 1.

As a first step, replace the optimal decision rules f and g with arbitrary log-linear functions which depend not only on the current state variables but also on lagged capital and lagged hours.[3] These decision rules, together with the constraints to Problem (1), lead to the following dynamic equations of motion (given initial conditions K_0, z_0, K_{-1}, and H_{-1}):

$$\log K_{t+1} = a_0 + a_1 \log K_t + a_2 z_t + a_3 \log K_{t-1} + a_4 \log H_{t-1} \tag{2}$$

$$\log H_t = b_0 + b_1 \log K_t + b_2 z_t + b_3 \log K_{t-1} + b_4 \log H_{t-1} \tag{3}$$

$$X_t = K_{t+1} - (1-\delta) K_t \tag{4}$$

$$Y_t = K_t^{\alpha} H_t^{1-\alpha} \exp(z_t) \tag{5}$$

$$C_t = Y_t - X_t \tag{6}$$

$$w_t = (1-\alpha) Y_t / H_t \tag{7}$$

$$z_{t+1} = \rho z_t + \epsilon_{t+1}, \ \epsilon_{t+1} \sim iidN(0, \sigma_\epsilon^2) \tag{8}$$

In equation (7), w_t is the period t wage rate (i.e., the marginal product of labor in period t).

The decision rule coefficients (a_i, b_i), $i = 0, \ldots, 4$, in equations (2) and (3) are free parameters to be chosen so that the equations of motion (2)–(8) reproduce a given set of time series statistics.[4] Since the statistics in Table 1 depend only on second moments, and since the decision rule coefficients a_0 and b_0 have no effect on the second moment properties of the equations of motion (2)–(8), restrict these coefficients as follows:

$$a_0 = (1 - a_1 - a_3) \log \bar{K} - a_4 \log \bar{H} \tag{9}$$

and

$$b_0 = (1 - b_4) \log \bar{H} - (b_1 + b_3) \log \bar{K}, \tag{10}$$

where $\bar{K}$ and $\bar{H}$ are the deterministic steady state values of capital and hours in Problem (1). These restrictions ensure that, no matter what the values of (a_i, b_i), $i = 1, \ldots, 4$, the unconditional means of K_t and H_t are, respectively, $\bar{K}$ and $\bar{H}$, i.e., precisely what they would be if the optimal decision rules were used in place of the arbitrary decision rules (2) and (3).

Given a vector of decision rule parameters $\Psi \equiv [\, a_0 \ a_1 \ a_2 \ a_3 \ a_4 \ b_0 \ b_1 \ b_2 \ b_3 \ b_4 \,]'$, simulation methods can be used to calculate consistent estimates of the second moments implied by the equations of motion (2)–(8). For example, to calculate a consistent estimate of the contemporaneous

[3] I am assuming that all consumers use the *same* decision rules, so that there is no need, as there is in Section 4, to make a distinction between individual and aggregate variables.

[4] Note that if the optimal decision rules are approximated by log-linear functions of the current state variables, then $a_3 = a_4 = b_3 = b_4 = 0$, while a_0, a_1, a_2, b_0, b_1, and b_2 can be expressed as nonlinear functions of the underlying model (structural) parameters.

correlation between the HP-filtered log of output and the HP-filtered log of consumption in samples of size T, generate N statistically independent simulations of the equations of motion (2)–(8) (with T time periods in each simulation), compute the desired statistic in each of the N simulations, and then compute the sample mean of these statistics across the N simulations.[5]

Suppose that one is interested in choosing the decision rule parameters Ψ to match an $s \times 1$ vector of statistics $\bar{S}$. For example, $\bar{S}$ might consist of the first five statistics in a given row of Table 1.[6] Let $S_N(\Psi)$ be the corresponding vector of statistics implied by the equations of motion (2)–(8) when the decision rule parameters are equal to Ψ. Each element of the vector $S_N(\Psi)$ is computed using the simulation procedure described in the previous paragraph; the subscript 'N' indicates that N independent simulations are used to calculate $S_N(\cdot)$.

To find a set of decision rule parameters which generates the vector of statistics $\bar{S}$, solve the following minimization problem:

$$\bar{\Psi}_N \equiv \arg\min_{\Psi} \; (\bar{S} - S_N(\Psi))' \, (\bar{S} - S_N(\Psi)) \tag{11}$$

subject to (9) and (10). The simulation error in $\bar{\Psi}_N$ can be made as small as desired by increasing N, the number of independent simulations of equations (2)–(8) used to calculate $S_N(\cdot)$.

If s, the dimension of $\bar{S}$, is equal to eight, then $\bar{\Psi}_N$ is exactly identified: there are eight free decision rule parameters which can be varied to match eight statistics of interest, so that the minimum value of the objective function on the right hand side of (11) is zero.[7] If $s < 8$, then $\bar{\Psi}_N$ is underidentified, in which case one must impose additional identifying restrictions. For the case $s = 5$, which I investigate below, I set $a_3 = a_4 = b_3 = 0$.

For each row of Table 1 (except the row corresponding to the baseline model), Problem (11) is solved for two different sets of statistics $\bar{S}$. For the first set of statistics, $s = 5$ and $\bar{S}$ consists of the statistics in the first five columns of Table 1: in particular, σ_c/σ_y, σ_x/σ_y, σ_h/σ_y, σ_h/σ_w, and σ_{hw}, with all time series logged and HP-filtered prior to the computation of statistics. For this case, as noted above, there are five free decision rule parameters. For the second set of statistics, $s = 8$ and $\bar{S}$ consists of the eight statistics in a given row of Table 1: in particular, the five statistics listed above, together with σ_{wy}, σ_{cy}, and σ_{hy}. For this case, there are eight free decision rule parameters.

Appendix A reports the results. The number of independent simulations N is set to 100 (the same number of simulations used to construct the last five rows of Table 1) and the length T of each simulation is set to 179 (the number of observations in the observed time series). In every case, it is possible to find a set of decision rule parameters to match *exactly* the required set of statistics. Appendix A tabulates the decision rule coefficients for each case. For purposes of comparison,

[5]Note that this is precisely the same procedure that is used to compute the statistics in the last five rows of Table 1.

[6]Note that sampling error in $\bar{S}$ is ignored here: the vector $\bar{S}$ is treated simply as a fixed set of numbers.

[7]I am assuming, of course, that there exists a solution to Problem (11), i.e., that there exists a $\bar{\Psi}_N$ which satisfies all of the identifying restrictions. In the numerical results I report below, this assumption is always satisfied.

Appendix A also gives the decision rule coefficients for a log-linear approximation to the optimal decision rules f and g associated with Problem (1).

4 Computing Welfare Gains

This section computes the welfare gain that a small consumer realizes by switching to the optimal decision rule, given that all other consumers in the economy are using one of the suboptimal decision rules computed in Section 3 and tabulated in Appendix A. Put differently, this section provides a quantitative answer to the following question: To what extent can a rational consumer take advantage of the irrationality of all other consumers in the economy?

To answer this question, consider the decision problem faced by a small (measure zero) rational consumer. The consumer faces the following budget constraint in any given period:

$$c_t + x_t = r_t\, k_t + w_t\, h_t, \tag{12}$$

where c_t is the consumer's period t consumption, x_t is the consumer's period t investment in capital, k_t is the consumer's holdings of capital at the beginning of period t, h_t is the consumer's period t hours of work, r_t is the period t rental price of capital, and w_t is the period t wage rate. The consumer's holdings of capital accumulate according to:

$$k_{t+1} = (1 - \delta)\, k_t + x_t. \tag{13}$$

The prices r_t and w_t are determined by the first-order conditions to the static optimization problem faced by the economy's constant-returns-to-scale firm (whose production function is $Y_t = K_t^{\alpha}\, H_t^{1-\alpha}\, \exp(z_t)$):

$$r_t = \alpha\, K_t^{\alpha-1}\, H_t^{1-\alpha}\, \exp(z_t) \tag{14}$$

and

$$w_t = (1 - \alpha)\, K_t^{\alpha}\, H_t^{-\alpha}\, \exp(z_t), \tag{15}$$

where K_t is the period t aggregate capital stock and H_t is period t aggregate hours of work. Since the small consumer's actions do not affect the aggregates K_t and H_t, the consumer takes the prices r_t and w_t as given when solving his optimization problem.

Finally, the consumer knows the (common) decision rules used by measure one of the economy's consumers:

$$K_{t+1} = \tilde{f}(K_t, K_{t-1}, H_{t-1}, z_t) \tag{16}$$

and

$$H_t = \tilde{g}(K_t, K_{t-1}, H_{t-1}, z_t). \tag{17}$$

These decision rules determine the dynamic behavior of the economy's aggregates K_t and H_t.[8]

The optimization problem faced by a measure zero rational consumer is:

$$\max_{\{k_t\}_{t=1}^{\infty},\, \{h_t\}_{t=0}^{\infty}} \quad E_0 \ \sum_{t=0}^{\infty} \beta^t \ (\log(c_t) + A \ \log(1 - h_t)) \,, \tag{18}$$

given k_0, K_0, K_{-1}, H_{-1}, and z_0, subject to (12)–(17) and the stochastic law of motion (8) for the technology shock z_t. The state variables for this problem are k_t, K_t, K_{t-1}, H_{t-1}, and z_t; the choice variables are k_{t+1} and h_t. The solution to this problem consists of a pair of time-invariant decision rules expressing the choice variables as functions of the state variables. Note that the consumer's decision rules depend on lagged capital K_{t-1} and lagged hours H_{t-1} if and only if these variables enter the decision rules $\tilde{f}$ and $\tilde{g}$ in equations (16) and (17).

Log-linear approximations to the consumer's optimal decision rules are computed using a solution method based on linear-quadratic approximations (see, for example, Kydland and Prescott (1982), Hansen (1985), Christiano (1990), and McGrattan (1990)). This solution method, in brief, works as follows: substitute the constraints (12)–(17) into the period t utility function, approximate the period t utility function by a second-order Taylor series about the deterministic steady state, and then solve the resulting linear-quadratic programming problem by iterating on the matrix Ricatti equation (see Chapter 1 of Sargent (1987)).

The consumer's optimal decision rules vary with the decision rules $\tilde{f}$ and $\tilde{g}$ used by all other consumers in the economy. Appendix B reports optimal decision rules for twelve different choices for $\tilde{f}$ and $\tilde{g}$: in particular, the twelve decision rules listed in Appendix A. For example, the first pair of decision rules in Appendix B characterizes the optimal behavior of a small rational consumer in an economy whose aggregate dynamics replicate the the first five statistics listed in Table 1 for the establishment survey data.

How much does a small rational consumer gain by using an optimal decision rule rather than the decision rule used by all other consumers in the economy? To answer this question in an economically meaningful way, express the increase in expected utility realized by a small rational consumer in terms of a uniform percentage increase in consumption across all periods of the planning horizon. Specifically, solve for λ in the following equation:

$$E_0 \sum_{t=0}^{\infty} \beta^t \, (\log((1+\lambda)\, \tilde{c}_t) + A \ \log(\tilde{h}_t)) = E_0 \sum_{t=0}^{\infty} \beta^t \, (\log(c_t^\star) + A \ \log(h_t^\star)), \tag{19}$$

where $\{c_t^\star\}$ and $\{h_t^\star\}$ are the optimal consumption and hours sequences chosen by the small rational consumer and $\{\tilde{c}_t\}$ and $\{\tilde{h}_t\}$ are the consumption and hours sequences generated by the decision

[8]Note that all consumers in the economy face the same budget constraint (12). This implies that, no matter what the decision rules $\tilde{f}$ and $\tilde{g}$ used by the nonoptimizing consumers, markets clear and the firm maximizies profits so long as prices are determined according to equations (14) and (15). Thus consumers using $\tilde{f}$ and $\tilde{g}$ to make their investment and hours decisions can be viewed as operating in the same decentralized environment as the small rational consumer whose decision problem is considered in this section.

rules $\tilde{f}$ and $\tilde{g}$ used by all other consumers in the economy. The solution is:

$$\lambda = 1 - \exp[(1-\beta)\,(U^\star - \tilde{U})], \tag{20}$$

where $U^\star \equiv E_0 \sum_{t=0}^{\infty} \beta^t \,(\log(c_t^\star) + A\,\log(h_t^\star))$ and $\tilde{U} \equiv E_0 \sum_{t=0}^{\infty} \beta^t\,(\log(\tilde{c}_t) + A\,\log(\tilde{h}_t))$. The increase in expected utility $U^\star - \tilde{U}$ realized by a small rational consumer is equivalent to a $100 \times \lambda\%$ increase in consumption, uniformly across all periods and states. Simulation methods are used to compute consistent estimates of $U^\star$ and $\tilde{U}$. These estimates are then inserted into equation (20) to yield a consistent estimate of λ. The standard error of the estimated value of λ is computed using standard results from asymptotic theory.

Table 2 tabulates consistent estimates of the welfare gains associated with the optimal decision rules reported in Appendix B.

TABLE 2: ESTIMATED WELFARE GAINS

	Number of statistics matched (s)	
	$s = 5$	$s = 8$
Establishment survey data	0.054% (0.002)	0.332% (0.020)
Household survey data	0.063% (0.002)	0.372% (0.029)
Nonseparable leisure model	0.0049% (0.0005)	0.0049% (0.0005)
Indivisible labor model	0.0104% (0.0007)	same as s = 5
Home production model	0.043% (0.002)	0.088% (0.006)
Government spending model	0.048% (0.002)	0.109% (0.009)

Notes: The first number in each cell is a consistent estimate of λ (expressed as a percentage); the second number in each cell (in parentheses) is the estimated standard error of the point estimate. For example, if measure one of the economy's consumers use decision rules which reproduce the first five statistics listed in Table 1 for the establishment survey data, then a small rational consumer gains an equivalent of 0.054% of per period consumption by using an optimal decision rule (given the decision rules used by all other consumers in the economy).

The welfare gains tabulated in Table 2 show that if the number of statistics matched is equal to five (i.e., if the decision rules used by measure one of consumers reproduce the first five statistics in a given row of Table 1), then a small rational consumer gains very little, in terms of increased consumption, by switching to the optimal decision rule: at most 0.063% of per period consumption for the statistics associated with the household survey data. Assuming annual consumption expenditures of $30,000, this increase amounts to less than $5 per quarter. If the number of statistics matched increases to eight (i.e., if the decision rules used by measure one of consumers reproduce all eight statistics in a given row of Table 1), then the gains realized by the small rational consumer increase by as much as a factor of six. Nonetheless, the welfare increases, measured in terms of increased consumption, remain small: less than (in most cases considerably less than) $30 per quarter (once again assuming annual consumption expenditures of $30,000).

These results show that, although the twelve pairs of decision rules tabulated in Appendix A are suboptimal, an individual consumer nonetheless gains very little by attempting to exploit the suboptimal behavior of all other consumers in the economy. Thus the economies in which aggregate dynamics are driven by consumers using one of the twelve pairs of decision rules in Appendix A can be viewed as "near-rational" equilibria in the sense of Akerlof and Yellen (1985): "An equilibrium . . . is termed *near rational* if no nonmaximizer stands to gain a significant amount by becoming a maximizer" (p. 708). To quote Akerlof and Yellen (1985) quoting Lucas: "there are no $500 bills on the sidewalk[s]" (p. 708) of these economies.

Table 2 also reveals that, in most cases, the extent to which a nonmaximizer can gain by becoming a maximizer increases as the number of statistics matched increases from five to eight. This finding suggests a more general result: as the set of statistics reproduced by nonmaximizers' decision rules becomes larger, the welfare gain realized by maximizers increases. Further research is needed to explore this conjecture more deeply.

In any case, the findings in this paper do show the following: if one allows for the possibility of near-rational behavior, then a set of five, or even eight, key time series statistics is not large enough to allow an investigator to discriminate between a baseline RBC model and several competing extensions. Moreover, one can find near-rational alternatives to a baseline RBC model which reproduce the values of these statistics in observed aggregate time series.

5 Conclusion

This paper shows, by means of specific quantitative examples, that a baseline RBC model makes few predictions concerning a set of key time series statistics that are robust to small departures from rationality. In particular, near-rational alternatives to the baseline model make the same predictions for these statistics as do several competing extensions to the baseline model. This lack of robustness suggests that it is difficult, on the basis of the models' implications for a limited set of aggregate time series moments, to identify the "true" model underlying observed aggregate time series. Finally, this paper contains numerical results which suggest that one way to address this problem of identification is to examine larger sets of second moments when evaluating the empirical performance of RBC models.

A key item for future research is to investigate the effect of heterogeneity and incomplete markets on these results. The baseline RBC model considered in this paper assumes that consumers face only aggregate risk: in effect, there exist complete markets which allow individual consumers to insure against idiosyncratic risk. In the presence of large, uninsurable risk, the welfare costs of suboptimal decision rules such as those in Appendix A could rise dramatically. Krusell and Smith (1993b) begin to address some of these issues.

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Appendix A

This appendix tabulates the decision rule coefficients which reproduce a given $s \times 1$ vector of statistics $\bar{S}$ associated with a given row of Table 1. For each row, two sets of statistics are considered: the first set consists of the first five statistics (this case is labelled $s = 5$ below) and the second set consists of all eight statistics (this case is labelled $s = 8$ below). For purposes of comparison, this appendix also gives the decision rule coefficients for a log-linear approximation to the optimal decision rules f and g associated with Problem (1).

• Log-linear decision rules for the baseline model:
$\log K_{t+1} = 0.118 + 0.954 \log K_t + 0.113\, z_t$
$\log H_t = -0.481 - 0.243 \log K_t + 0.707\, z_t$

• Match statistics for establishment survey data
$\underline{s = 5}$: $\log K_{t+1} = 0.517 + 0.796 \log K_t + 0.118\, z_t$
$\log H_t = 8.535 - 3.937 \log K_t + 1.366\, z_t - 0.330 \log H_{t-1}$
$\underline{s = 8}$: $\log K_{t+1} = 0.252 + 0.832 \log K_t + 0.142\, z_t + 0.069 \log K_{t-1} + 0.001 \log H_{t-1}$
$\log H_t = 6.234 - 3.513 \log K_t + 1.263\, z_t + 0.721 \log K_{t-1} + 0.222 \log H_{t-1}$

• Match statistics for household survey data
$\underline{s = 5}$: $\log K_{t+1} = 0.750 + 0.705 \log K_t + 0.157\, z_t$
$\log H_t = 11.441 - 4.939 \log K_t + 2.024\, z_t + 0.002 \log H_{t-1}$
$\underline{s = 8}$: $\log K_{t+1} = 0.163 + 0.747 \log K_t + 0.120\, z_t + 0.189 \log K_{t-1} + 0.001 \log H_{t-1}$
$\log H_t = 6.221 - 3.594 \log K_t + 0.908\, z_t + 0.675 \log K_{t-1} - 0.083 \log H_{t-1}$

• Match statistics for nonseparable leisure model
$\underline{s = 5}$: $\log K_{t+1} = 0.163 + 0.936 \log K_t + 0.136\, z_t$
$\log H_t = 0.401 - 0.587 \log K_t + 1.086\, z_t + 0.009 \log H_{t-1}$
$\underline{s = 8}$: $\log K_{t+1} = 0.162 + 0.937 \log K_t + 0.136\, z_t - 0.0003 \log K_{t-1} - 0.00002 \log H_{t-1}$
$\log H_t = 0.401 - 0.587 \log K_t + 1.086\, z_t - 0.0001 \log K_{t-1} + 0.008 \log H_{t-1}$

• Match statistics for indivisible labor model
$\underline{s = 5}$: $\log K_{t+1} = 0.148 + 0.942 \log K_t + 0.155\, z_t$
$\log H_t = 0.111 - 0.477 \log K_t + 1.471\, z_t$
$\underline{s = 8}$: Same as for the case $s = 5$
(Note: These decision rules are log-linear approximations to the optimal decision rules for the indivisible labor model.)

• Match statistics for home production model

$\underline{s=5}$: $\log K_{t+1} = 0.451 + 0.822\,\log K_t + 0.131\,z_t$

$\log H_t = 3.928 - 1.909\,\log K_t + 1.261\,z_t + 0.165\,\log H_{t-1}$

$\underline{s=8}$: $\log K_{t+1} = 0.206 + 0.704\,\log K_t + 0.136\,z_t + 0.215\,\log K_{t-1} + 0.001\,\log H_{t-1}$

$\log H_t = 2.971 - 1.665\,\log K_t + 1.050\,z_t + 0.155\,\log K_{t-1} + 0.216\,\log H_{t-1}$

• Match statistics for government spending model

$\underline{s=5}$: $\log K_{t+1} = 0.585 + 0.769\,\log K_t + 0.120\,z_t$

$\log H_t = 3.494 - 1.695\,\log K_t + 0.667\,z_t + 0.264\,\log H_{t-1}$

$\underline{s=8}$: $\log K_{t+1} = 0.192 + 0.514\,\log K_t + 0.131\,z_t + 0.411\,\log K_{t-1} + 0.001\,\log H_{t-1}$

$\log H_t = 2.555 - 1.551\,\log K_t + 0.536\,z_t + 0.206\,\log K_{t-1} + 0.217\,\log H_{t-1}$

Appendix B

This section tabulates the decision rules chosen by a small rational consumer when all other consumers in the economy use one of the pairs of decision rules tabulated in Appendix A. For example, the first pair of decision rules tabulated below characterizes the optimal behavior of a small rational consumer when the rest of the economy's consumers use the pair of decision rules tabulated in Appendix A for the case: establishment survey data, $s = 5$. (Note: $\tilde{K}_t \equiv \log K_t$, where K_t is the aggregate capital stock, and $\tilde{H}_t \equiv \log H_t$, where H_t is aggregate hours worked.)

• Match statistics for establishment survey data

$\underline{s=5}$: $\log k_{t+1} = -0.371 + 0.999\,\log k_t + 0.131\,z_t + 0.153\,\tilde{K}_t + 0.015\,\tilde{H}_{t-1}$

$\log h_t = -7.820 - 0.103\,\log k_t + 0.712\,z_t + 2.854\,\tilde{K}_t + 0.240\,\tilde{H}_{t-1}$

$\underline{s=8}$: $\log k_{t+1} = 0.037 + 0.999\,\log k_t - 0.109\,z_t + 0.026\,\tilde{K}_t - 0.045\,\tilde{K}_{t-1} - 0.010\,\tilde{H}_{t-1}$

$\log h_t = 6.234 - 0.103\,\log k_t - 1.690\,z_t + 1.458\,\tilde{K}_t - 0.645\,\tilde{K}_{t-1} - 0.159\,\tilde{H}_{t-1}$

• Match statistics for household survey data

$\underline{s=5}$: $\log k_{t+1} = -0.502 + 0.999\,\log k_t + 0.109\,z_t + 0.198\,\tilde{K}_t - 0.0001\,\tilde{H}_{t-1}$

$\log h_t = -9.915 - 0.103\,\log k_t + 0.314\,z_t + 3.575\,\tilde{K}_t - 0.001\,\tilde{H}_{t-1}$

$\underline{s=8}$: $\log k_{t+1} = 0.201 + 0.999\,\log k_t - 0.145\,z_t - 0.004\,\tilde{K}_t - 0.074\,\tilde{K}_{t-1} + 0.004\,\tilde{H}_{t-1}$

$\log h_t = -1.407 - 0.103\,\log k_t - 1.965\,z_t + 1.173\,\tilde{K}_t - 0.924\,\tilde{K}_{t-1} + 0.058\,\tilde{H}_{t-1}$

• Match statistics for nonseparable leisure model

$\underline{s=5}$: $\log k_{t+1} = 0.067 + 0.999\,\log k_t + 0.092\,z_t - 0.026\,\tilde{K}_t - 0.0004\,\tilde{H}_{t-1}$

$\log h_t = -1.232 - 0.103\,\log k_t + 0.391\,z_t + 0.153\,\tilde{K}_t - 0.006\,\tilde{H}_{t-1}$

$\underline{s=8}$: $\log k_{t+1} = 0.068 + 0.999\,\log k_t + 0.090\,z_t - 0.027\,\tilde{K}_t + 0.00003\,\tilde{K}_{t-1} - 0.0004\,\tilde{H}_{t-1}$

$\log h_t = -1.214 - 0.103\,\log k_t + 0.376\,z_t + 0.146\,\tilde{K}_t + 0.0004\,\tilde{K}_{t-1} - 0.006\,\tilde{H}_{t-1}$

• Match statistics for indivisible labor model

$\underline{s=5}$: $\log k_{t+1} = 0.083 + 0.999 \log k_t + 0.068\, z_t - 0.033\, \tilde{K}_t$

$\log h_t = -0.985 - 0.103 \log k_t + 0.042\, z_t + 0.058\, \tilde{K}_t$

$\underline{s=8}$: Same as for the case $s = 5$

• Match statistics for home production model

$\underline{s=5}$: $\log k_{t+1} = -0.155 + 0.999 \log k_t + 0.167\, z_t + 0.058\, \tilde{K}_t - 0.007\, \tilde{H}_{t-1}$

$\log h_t = -4.412 - 0.103 \log k_t + 1.109\, z_t + 1.358\, \tilde{K}_t - 0.116\, \tilde{H}_{t-1}$

$\underline{s=8}$: $\log k_{t+1} = 0.134 + 0.999 \log k_t + 0.007\, z_t - 0.018\, \tilde{K}_t - 0.039\, \tilde{K}_{t-1} - 0.010\, \tilde{H}_{t-1}$

$\log h_t = -1.229 - 0.103 \log k_t - 0.456\, z_t + 0.520\, \tilde{K}_t - 0.432\, \tilde{K}_{t-1} - 0.154\, \tilde{H}_{t-1}$

• Match statistics for government spending model

$\underline{s=5}$: $\log k_{t+1} = -0.180 + 0.999 \log k_t + 0.200\, z_t + 0.066\, \tilde{K}_t - 0.011\, \tilde{H}_{t-1}$

$\log h_t = -4.547 - 0.103 \log k_t + 1.596\, z_t + 1.382\, \tilde{K}_t - 0.185\, \tilde{H}_{t-1}$

$\underline{s=8}$: $\log k_{t+1} = 0.213 + 0.999 \log k_t + 0.004\, z_t - 0.020\, \tilde{K}_t - 0.068\, \tilde{K}_{t-1} - 0.010\, \tilde{H}_{t-1}$

$\log h_t = -0.320 - 0.103 \log k_t - 0.357\, z_t + 0.475\, \tilde{K}_t - 0.746\, \tilde{K}_{t-1} - 0.154\, \tilde{H}_{t-1}$