

Dynamic credit relationships in general equilibrium[☆]

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Received 4 February 2005; accepted 25 February 2005

Available online 15 May 2006

Abstract

We construct a general equilibrium model with private information in which borrowers and lenders enter into long-term dynamic credit relationships. Each new generation of ex ante identical individuals is divided in equilibrium into workers and entrepreneurs. Workers save through financial intermediaries in the form of interest-bearing deposits and supply labor to entrepreneurs in a competitive labor market. Entrepreneurs borrow from financial intermediaries to finance projects which produce privately observed sequences of random returns. Each financial intermediary holds deposits from a large number of workers and operates a portfolio of dynamic contracts with different credit positions. We calibrate the model to the U.S. economy and find that dynamic contracting is very effective at mitigating the effects of private information. Moreover, restricting borrowers and lenders to use static (one-period) contracts with a costly monitoring technology has adverse effects both on the level of aggregate economic activity and on individual welfare unless monitoring costs are very small. Finally, the optimal provision of intertemporal incentives leads to increasing consumption inequality over time within generational cohorts as in U.S. data.

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Keywords: Dynamic contracts; Asymmetric information; General equilibrium; Inequality

[☆]For helpful comments, we would like to thank Wouter den Haan, Martin Gervais, Jeremy Greenwood, Burton Hollifield, Per Krusell, John Laitner, Nancy Stokey, Robert Lucas, Ted Temzelides, Neil Wallace, Steve Williamson, Eric Young, and seminar participants at Carnegie Mellon University, Institute for International Economic Studies, Pennsylvania State University, Rutgers University, University of Illinois at Urbana-Champaign, University of Pennsylvania, University of Pittsburgh, University of Rochester, the 1999 Annual Meetings of the Society for Economic Dynamics, the Northwestern Summer Workshop on Macroeconomics, the Stanford Institute for Theoretical Economics, and the 1999 Annual Meetings of the Canadian Macroeconomics Study Group. The first author gratefully acknowledges financial support from the National Science Foundation.

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1. Introduction

The last 20 years have seen many efforts at modelling explicitly private information and the financial contracting process in the benchmark neoclassical growth model. In particular, a substantial part of the literature addresses the question of how the presence of private information contributes to the propagation of aggregate economic uncertainties. [Bernanke and Gertler \(1989\)](#), for example, argue that with private information, swings in the firm's balance sheet are a potential source of persistent output dynamics. In [Williamson \(1987b\)](#), monitoring costs of financial intermediaries are important for the propagation of aggregate disturbances. [Kiyotaki and Moore \(1997\)](#) also study business cycle dynamics propagated through the financial contracting process between entrepreneurs and investors, although their story is based on limited commitment instead of private information. [Carlstrom and Fuerst \(1997\)](#) develop a computable general equilibrium model based on [Bernanke and Gertler \(1989\)](#) to address quantitatively the importance of agency costs for the propagation of aggregate shocks. [Cooley and Nam \(1998\)](#) incorporate a problem of debt contracting with asymmetric information into a quantitative monetary business cycle model to generate a persistent liquidity effect induced by monetary disturbances.

An obvious yet serious limitation of most of the existing literature on financial contracting and business cycles is that the financial borrowing and lending process is modelled as a one-shot game.¹ In practice, financial intermediaries often engage in long-term relationships, rather than interact only once with their borrowers. Modelling the financial lending process as a one-period contract may severely restrict the contracting parties' ability to achieve risk-sharing, and hence may have important implications for how successfully the contract can be used in a dynamic macroeconomic setting as an explicit description of the financial lending process. In addition, from a technical perspective, in a standard dynamic general equilibrium macroeconomic model, borrowers and lenders are all infinitely lived agents, and very special assumptions have to be made in order to fit the static contracting relationship into the rest of the economy (which is fully dynamic).²

The above limitation of the literature was first pointed out by [Gertler \(1992\)](#), who developed a model in which lenders and borrowers can enter into long-term but finite contractual relationships and used his model to show that shifts in aggregate economic fundamentals can be amplified through the process of long-term contracting. Yet, as Gertler himself pointed out, "a major limitation of this model is that it lies well short of a fully dynamic framework that can be matched to data. While allowing for multi-period contracts ... is a helpful step in this important direction, there is still a long way to go" (p. 470).

What this paper attempts to undertake can be viewed as just another step in the long way that [Gertler \(1992\)](#) pointed out. Instead of being ambitious in providing a theory that explains business cycles dynamics, this paper constructs a quantitative dynamic general equilibrium model with no aggregate uncertainty but in which long-lived economic agents can enter into fully dynamic financial lending contracts. We find that, relative to static

¹One notable exception is [Cooley et al. \(2004\)](#).

²See [Carlstrom and Fuerst \(1997\)](#) and [Cooley and Nam \(1998\)](#). Note that the static contracting relationship can be more comfortably embedded in an OLG framework ([Boyd and Smith, 1997](#)). This perhaps is the main reason why most of the theoretical contributions in the literature have used an OLG structure with two-period-lived agents.

contracts, dynamic contracts generate equilibrium allocations with higher output and higher welfare.

In our model, the economy is populated by a sequence of overlapping generations of agents who potentially can live forever. Each new generation of *ex ante* identical individuals is divided, in equilibrium, into workers and entrepreneurs. Workers save through financial intermediaries in the form of interest-bearing deposits and supply labor to entrepreneurs in a competitive labor market. Entrepreneurs borrow from financial intermediaries to finance projects which produce idiosyncratic sequences of random returns. The entrepreneur's returns are not observed by any other parties. Financial intermediaries arise as institutions to facilitate financial borrowing and lending by providing risk-sharing for entrepreneurs and workers. Financial intermediaries are competitive, each holding deposits from a large number of workers and operating a portfolio of dynamic contracts with different credit positions.

A notable feature of our model is that we model occupational choice as an equilibrium phenomenon. In the model, *ex ante* identical economic agents *choose* to become entrepreneurs (borrowers) and workers (depositors). Occupational choice plays a central role in our model as the link between the labor and asset markets and the determination of aggregate output.

Our model provides a vehicle for evaluating both qualitatively and quantitatively the implications of dynamic contracting and financial intermediation for the operation of the macroeconomy.³ How does dynamic contracting operate in general equilibrium? What are the implications of dynamic contracting for the equilibrium dynamics of the balance sheets of financial intermediaries? What role does private information play in the determination of the general equilibrium levels of aggregate quantities such as output and the capital stock? What are the implications of dynamic contracting for individual welfare (relative to static contracting under private information and full risk-sharing under complete information)? How does dynamic contracting affect inequality through the provision of intertemporal incentives? How important is costly monitoring for the performance of the macroeconomy?

A distinctive feature of our model is that, in equilibrium, financial intermediaries hold positive amounts of assets. These assets arise in our model even though we assume a competitive financial intermediation industry, so that financial intermediaries earn zero profits. The existence of the assets held by financial intermediaries in our model is an equilibrium outcome associated with the dynamic lending process.⁴ In the model here, the equilibrium provision of intertemporal incentives in the dynamic contract implies that the entrepreneurs make more repayments in earlier stages rather than later stages during the credit contracting process. The assets held by the financial intermediaries are essentially savings of the entrepreneurs. As we will show in the paper, among other factors, the equilibrium size of the assets held by financial intermediaries depends critically on the rate of interest.

A key quantitative finding of the paper is that dynamic contracting is very effective at mitigating the effects of private information: even when informational asymmetries are

³There is now a large literature on dynamic private information, including Townsend (1982), Green (1987), Spear and Srivastava (1987), Phelan and Townsend (1991), Atkeson and Lucas (1992), Wang (1995), Aiyagari and Williamson (1999), and Cole and Kocherlakota (2001), and Khan and Ravikumar (2001).

⁴Williamson (1987b) also models financial intermediation in a competitive market but the financial intermediaries in his model do not hold assets in equilibrium.

large, the economy can achieve close to a first-best outcome. One implication of this finding is that costly state verification (CSV), which is a key feature of many static models of the borrowing and lending process, is not important quantitatively in the presence of fully dynamic credit relationships. The model of CSV, originally developed by Townsend (1979), has been used widely as a vehicle for studying the relation between financial intermediation and macroeconomic aggregates (Bernanke and Gertler, 1989; Williamson, 1987b; Boyd and Smith, 1997).

We also demonstrate the importance of dynamic contracting for the determination of macroeconomic aggregates in our model by forcing borrowers and lenders to use static (one-period) contracts with a costly monitoring technology rather than dynamic contracts. We show that moving from dynamic to static contracting (i.e., restricting the ability of contracts to use intertemporal incentives) has adverse effects both on the level of aggregate economic activity and on the welfare of individuals unless monitoring costs are very small. In other words, this paper offers strong quantitative evidence supporting the argument (Gertler, 1992) that modelling the financial lending relationship as a one-shot game can be very misleading in explaining macroeconomic activities.

We also show that the quantitative framework we construct can partially account for the increasing consumption inequality over time within generational cohorts that Deaton and Paxson (1994) document in U.S. data. That private information and incomplete insurance provide a potential explanation for consumption inequality has been the theme of several recent studies. These studies, including Green (1987), Atkeson and Lucas (1992, 1995), and Banerjee and Newman (1991), are all theoretical investigations and have not addressed the relationship between incentives and consumption inequality using a quantitative growth model that features dynamic contracting such as the one that we develop in this paper.⁵ Of course, our finding that private information and incentives can account only partially for the observed pattern of consumption inequality is well anticipated. The literature on inequality and its determinants shows that other factors, including ex ante heterogeneity in ability, uninsurable idiosyncratic shocks, and borrowing constraints, also play important roles in determining consumption inequality (see, e.g., Becker and Tomes, 1979; Deaton and Paxson, 1994; Storesletten et al., 2004).

Finally, we illustrate how our model economy can provide a vehicle for conducting various types of policy experiments. In particular, we use our model to analyze the effects of requiring financial intermediaries to hold a fraction of their deposits as reserves. We find that the cost of holding 10% of total deposits as required reserves is roughly 1% of aggregate output. In equilibrium the economy holds more capital in total in response to an increase in required reserves.

Section 2 presents the model, Section 3 discusses computation of the model, Section 4 explains how we calibrate the model, Section 5 presents the results, and Section 6 concludes.

2. The model economy

Time is discrete and lasts forever. The economy consists of a sequence of overlapping generations, each of which contains a continuum of individuals. Each individual faces a

⁵Our model bears some resemblance to the model of Banerjee and Newman (1991) in that all individuals are ex ante identical, risk-bearing is a consequence of the individual's occupational choice, and only entrepreneurs bear risk.

time-invariant probability Δ of surviving into the next period. The total measure of individuals in the economy is equal to one. We assume that each new generation has measure $1 - \Delta$, so that the number of births and the number of deaths are equal at any point in time. There is one good per period in the economy; this good can be used both for consumption and investment. Each individual, when alive, is endowed with one unit of time in each time period. An individual's preferences over streams of consumption and leisure are given by

$$E_0 \sum_{t=0}^{\infty} (\beta \Delta)^t U(c_t, \ell_t), \quad (1)$$

where c_t denotes period t consumption, ℓ_t denotes period t leisure, and $\beta \in [0, 1)$ is the discount rate. Let $\mathbf{C}$ denote the space of consumption from which c_t takes on values.

Each newly born individual decides to become either a worker or an entrepreneur. Each individual's "career" choice is irreversible. Workers save through financial intermediaries in the form of interest-bearing deposits and supply labor to entrepreneurs in a competitive labor market. Entrepreneurs use fixed amounts of capital and labor to operate risky projects. In order to finance their projects, entrepreneurs sign dynamic lending contracts with financial intermediaries. Financial intermediaries compete for new contracts, so that new contracts earn zero profits in equilibrium.

2.1. Workers

Let r be the interest rate paid by the financial intermediary on deposits and let $R = 1 + r$. Let ω be the wage rate per unit of time. In our stationary equilibrium (to be described in Section 3), the prices r and ω do not vary. Workers take these prices as given when making their decisions. Workers begin life with zero assets and seek to maximize expected lifetime utility subject to a sequence of period-by-period budget constraints.

Following Blanchard (1985) and Ríos-Rull (1996), workers take part in an annuities market to insure themselves against mortality risk. Specifically, competitive insurance firms sell securities which pay one unit of the good in the next period if the agent is alive and pay zero otherwise. Since agents do not value bequests, agents will use all of their savings to purchase these securities. Equilibrium in the insurance industry requires that the price of these securities be equal to the survival rate Δ .

The worker's dynamic problem takes the recursive form:

$$V_w(d; r, \omega) = \max_{c, d', h} [U(c, 1 - h) + \beta \Delta V_w(d'; r, \omega)] \quad (2)$$

subject to:

$$c + \Delta d' = Rd + \omega h,$$

and $d' \geq 0$, where d is deposits at the beginning of the period, d' is deposits at the beginning of next period, h is the amount of time spent working, and $V_w(d; r, \omega)$ is the worker's lifetime expected utility given that he currently has deposits (or savings) equal to d and that he faces prices r and ω .⁶

⁶We do not allow workers to borrow since we restrict d' to be nonnegative. In the calibrated version of our economy, it turns out that this restriction is not binding.

Let $d' = f(d; r, \omega)$ and $h = g(d; r, \omega)$ be the optimal decision rules associated with this problem. In the steady state, these decision rules and the birth/death rate $1 - \Delta$ give rise to a stationary distribution of workers across deposit holdings. In each period, fraction $1 - \Delta$ of workers die and are replaced with an equal number of new workers, each of whom begins life with zero assets. Consequently, fraction $1 - \Delta$ of workers have zero deposits, fraction $(1 - \Delta)\Delta$ have deposits equal to $d_1(r, \omega) = f(0; r, \omega)$, fraction $(1 - \Delta)\Delta^2$ have deposits equal to $d_2(r, \omega) = f(d_1(r, \omega); r, \omega)$, and, in general, fraction $(1 - \Delta)\Delta^t$ have deposits equal to $d_t(r, \omega)$. Suppose for the moment that the population (or measure) of workers is 1.⁷ The total amount of deposits $D(r, \omega)$ held by workers is then:

$$D(r, \omega) = (1 - \Delta) \sum_{t=0}^{\infty} \Delta^t d_t(r, \omega), \quad (3)$$

where $d_0(r, \omega) = 0$. Similarly, the total amount of labor supplied by workers is given by

$$H(r, \omega) = (1 - \Delta) \sum_{t=0}^{\infty} \Delta^t g(d_t; r, \omega). \quad (4)$$

Note that $D(r, \omega)$ and $H(r, \omega)$ can also be viewed as per capita quantities; i.e., $D(r, \omega)$ is deposits per worker and $H(r, \omega)$ is hours per worker in the steady-state equilibrium of our economy.

2.2. Entrepreneurs

Each entrepreneur operates a long-lived project that produces output in each period of the life of the project. Each entrepreneurial project requires in each period K units of capital and L units of labor (in addition to the entrepreneur's own labor). Project returns are random and idiosyncratic. Specifically, let θ be the amount of output produced by a project in any given period. We assume that θ takes on values from the finite set $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$ where $\theta_1 < \theta_2 < \dots < \theta_n$. Let $\pi_i = \Pr\{\theta = \theta_i\} > 0$, $\sum \pi_i = 1$. Project returns are independent and identically distributed across time. We assume that the entrepreneur alone directly observes the level of his output in any given period. In other words, outside parties cannot observe the entrepreneur's output.⁸

A new entrepreneur, like a new worker, has no wealth. In order to finance his project, therefore, the entrepreneur must borrow in the credit market. Although each entrepreneur has the same preferences as a typical worker, entrepreneurs, unlike workers, do not make a labor-leisure decision; instead, each entrepreneur spends $\bar{h}$ units of his time endowment working and $\bar{\ell} = 1 - \bar{h}$ units of his time endowment in leisure. Entrepreneurs (and their projects) survive into the next period with (time-invariant) probability Δ .

2.3. The credit market and financial intermediation

Financial intermediaries arise in our model as institutions to provide risk-sharing for the risk-averse workers and entrepreneurs. In each period, financial intermediaries raise funds from a short-term credit market (by accepting deposits from the workers) in which all

⁷In Section 3, where we describe the equilibrium conditions of our stationary economy, the measure of workers is determined endogenously.

⁸See Green (1987), Thomas and Worrall (1990), or Atkeson and Lucas (1992) for an analysis of the problem of dynamic contracting under idiosyncratic risk with private information.

agents can participate. Financial intermediaries also participate in a long-term credit market in which they provide funds for entrepreneurs in exchange for payments. Since financial intermediaries can hold a portfolio of contracts with a large number of depositors and entrepreneurs, in the steady-state equilibrium on which we will focus our attention in this paper, standard deposit contracts are optimal and there is no default risk for the depositors. That is, in each period, for each unit of savings that the worker deposits in the financial intermediary, the financial intermediary pays $1 + r$ units at the end of the period, where r is the endogenously determined interest rate on deposits.

Since a typical entrepreneur's project is long-lived and since the returns on this project are private information, it is optimal for financial intermediaries and entrepreneurs to enter into long-term dynamic credit relationships, instead of entering into a sequence of short-term contracts, or having the entrepreneur borrow directly from the short-term credit market each period. The structure of the optimal dynamic contract will be discussed in the following section, but each dynamic contract specifies a history-dependent repayment scheme to which the entrepreneur is committed, and in exchange for which the financial intermediary promises to provide the resources that the entrepreneur needs to operate his project throughout his productive life.

There is free entry into the competitive financial intermediation industry. As a result, in equilibrium all financial intermediaries make zero profits. In principle, any coalition of agents can establish a financial intermediary. Since financial intermediaries make zero profits, however, who owns financial intermediaries is immaterial. Moreover, it is without loss of generality to view the financial intermediation industry as consisting of a single representative financial intermediary as we will do in the remainder of the paper.

2.4. *The dynamic lending contract*

As discussed in the previous section, financial intermediaries lend resources to entrepreneurs so that investment and production can take place. The informational asymmetry between financial intermediaries and entrepreneurs and the long-lived nature of entrepreneurs' projects imply that the optimal credit relationship between financial intermediaries and entrepreneurs is itself long-lived.

We assume that both the entrepreneurs and the financial intermediaries are fully committed to the contract. That is, neither the entrepreneurs nor the intermediaries are allowed to leave the contract in any ex post state of the world.⁹ We also assume that entrepreneurs cannot engage in side trades with outside parties. We assume that the intermediaries are able to control the entrepreneurs' consumptions perfectly, and the entrepreneurs cannot hold deposits or trade on the annuity market.

With the above qualifications, we describe the dynamic lending contract governing the credit relationship between a typical financial intermediary and a typical entrepreneur. The financial intermediary seeks to maximize the net present value of the stream of payments from the entrepreneur to the financial intermediary subject to incentive compatibility (truth-telling) and promise-keeping constraints. The financial intermediary discounts the future at the interest rate r , which it takes as given. In addition, when discounting future payments the financial intermediary recognizes that, in any given period, a project will die with time-invariant probability $1 - \Delta$.

⁹To examine the wealth effects of limited commitment would be a useful extension of the paper.

Following Spear and Srivastava (1987) and others, the state variable in the financial intermediary's dynamic problem is w , the amount of future utility that the financial intermediary promises to deliver to the entrepreneur. The solution to the financial intermediary's problem consists of a pair of functions $m(\theta_i, w)$ and $W(\theta_i, w)$, where $m(\theta_i, w)$ specifies the entrepreneur's payment if his current promised utility is w and he reports that his output is θ_i and $W(\theta_i, w)$ specifies the entrepreneur's promised utility at the beginning of the next period if his current promised utility is w and he reports that his output is θ_i .

Let $v(w)$ be the value function associated with the financial intermediary's dynamic contracting problem; $v(w)$ is the expected net present value of the stream of payments from the entrepreneur to the financial intermediary given that the entrepreneur's current level of promised utility is w . Promised utilities take on values in the state space $W \subseteq [(1 - \beta\delta)^{-1} U(\underline{c}, \bar{\ell}), (1 - \beta\delta)^{-1} U(\bar{c}, \bar{\ell})]$, where $\underline{c}$ and $\bar{c}$ are the minimum and maximum levels of consumption in the entrepreneur's consumption space C . The following recursive dynamic programming problem determines the value function v and the optimal contract (i.e., the optimal $m(\theta_i, w)$ and $W(\theta_i, w)$) for all $w \in W$,

$$v(w) = \max_{(m_i, W_i)_{i=1}^n} \sum_{i=1}^n \pi_i [m_i + R^{-1} \Delta v(W_i)] \quad (5)$$

subject to:

$$U(\theta_i - m_i, \bar{\ell}) + \beta \Delta W_i \geq U(\theta_i - m_j, \bar{\ell}) + \beta \Delta W_j, \quad \forall \theta_i, \theta_j, \quad (6)$$

$$\sum_{i=1}^n \pi_i [U(\theta_i - m_i, \bar{\ell}) + \beta \Delta W_i] = w, \quad (7)$$

$$\theta_i - m_i \in C, \quad \forall \theta_i, \quad (8)$$

$$W_i \in W, \quad \forall \theta_i. \quad (9)$$

Eq. (6) is the truth-telling constraint: given that the agent receives θ_i , he has no incentive to report θ_j . Eq. (7) is the promise-keeping constraint; it ensures that the amount of utility delivered to the entrepreneur (the left-hand side) is equal to the amount that has been promised (the right-hand side). Finally, Eq. (8) states that the entrepreneur's consumption lies in the consumption space C and Eq. (9) states that tomorrow's promised utility must lie in the state space W .

Since an entrepreneur's utility function is strictly concave, the optimal dynamic contract provides (partial) risk-sharing to the entrepreneur. In other words, for all w , the payment schedule m is increasing in θ . Moreover, for least one pair of output levels θ_i and θ_j satisfying $\theta_i < \theta_j$, it must be the case that $m(\theta_i, w)$ is strictly smaller than $m(\theta_j, w)$. These facts, together with Eq. (6), imply that $W(\theta_i, w) \leq W(\theta_j, w)$ whenever $\theta_i < \theta_j$ and that $W(\theta_i, w) < W(\theta_j, w)$ for at least one pair of output levels satisfying $\theta_i < \theta_j$. As a result, the promised utilities of a cohort of entrepreneurs each of whom begins with the same promised utility will spread out over time. Thus the optimal provision of intertemporal incentives necessarily requires that inequality within a cohort of entrepreneurs increase over time.

3. Equilibrium

Let λ be the fraction of newly born individuals who choose to become entrepreneurs; λ is an endogenous variable in our stationary equilibrium. Since the total measure of individuals in the economy is equal to one, λ is, in addition, the measure of entrepreneurs in the economy and $1 - \lambda$ is the measure of workers in the economy.¹⁰

In equilibrium, each new individual must be indifferent between becoming a worker and becoming an entrepreneur. Let w_0 be the promised expected lifetime utility of a new entrepreneur. Indifference between becoming a worker and becoming an entrepreneur requires that the following condition hold:

$$V_w(0; r, \omega) = w_0. \quad (10)$$

In other words, a new worker's lifetime utility is equal to the amount of utility that a financial intermediary promises to deliver to a new entrepreneur.

Market-clearing in the labor market requires that the following condition hold:

$$\lambda L = (1 - \lambda)H(r, \omega), \quad (11)$$

where the left-hand side is the aggregate demand for labor by entrepreneurs (recall that each entrepreneurial project requires K units of capital and L units of labor) and the right-hand side is the aggregate supply of labor by workers.

As noted in Section 2.3, we allow free entry into the financial intermediation sector. In equilibrium, therefore, the value of a new contract to a financial intermediary is equal to zero. The cost associated with a new contract is the net present value of the resources that the financial intermediary must commit to the entrepreneur. We assume that capital used in production depreciates at the rate δ , so the cost of providing capital to an entrepreneur in any given period is $(r + \delta)K$. In addition, an entrepreneur's labor costs in any given period are equal to ωL . The benefit associated with a new contract is the net present value of the stream of payments from the entrepreneur to the financial intermediary. The financial intermediary discounts future cash flows at the rate $R^{-1}\Delta$. The zero-profit condition, therefore, reads

$$v(w_0) = \frac{(r + \delta)K + \omega L}{1 - R^{-1}\Delta}. \quad (12)$$

In our stationary equilibrium, the representative financial intermediary holds a stationary (time-invariant) portfolio of contracts. Each contract in the portfolio is indexed by the current level of promised utility associated with the contract. Let the distribution of promised utilities of the t -year-old entrepreneurs in the financial intermediary's portfolio be denoted Γ_t . The distribution Γ_t is determined by the law of motion (or decision rule) $W(\theta, w)$, the initial promised utility w_0 , and the stochastic process governing the evolution of θ . For example, Γ_0 puts unit mass on w_0 , Γ_1 puts mass π_i on $W(\theta_i, w_0)$, $i = 1, \dots, n$, Γ_2 puts mass $\pi_i\pi_j$ on $W(\theta_j, W(\theta_i, w_0))$, $i = 1, \dots, n$ and $j = 1, \dots, n$, and so on. The economy's representative financial intermediary holds contracts for a portfolio of entrepreneurs of

¹⁰In our model, it is critical that λ be an endogenous variable. Given the fixed amounts of capital and labor that each entrepreneur can operate, specifying λ exogenously would, in effect, imply that the level of aggregate output is exogenous. An alternative modelling strategy is to fix λ exogenously but allow entrepreneurs to choose the size of their projects, as in Bernanke and Gertler (1989). Allowing such a choice in our model would add another dimension to the state space of the dynamic contracting problem.

different ages; let Γ denote the (time-invariant) distribution of promised utilities for the contracts in the financial intermediary's portfolio. Given our demographic assumptions,

$$\Gamma = \sum_{t=0}^{\infty} (1 - \Delta) \Delta^t \Gamma_t. \quad (13)$$

That is, the distribution Γ can be viewed as the weighted average of the distributions associated with different cohorts of entrepreneurs, where the weights correspond to the sizes of the cohorts.

In each period, the financial intermediary makes total payments equal to E to the entrepreneurs in its portfolio. These payments are defined by

$$E = (r + \delta)K + \omega L. \quad (14)$$

Since each entrepreneurial project is identical, E can also be viewed as payments per entrepreneur in each period. In each period, the financial intermediary also receives total payments equal to M from the entrepreneurs in its portfolio. Formally,

$$\begin{aligned} M &= \int \bar{m}(w) d\Gamma(w) \\ &= \sum_{t=0}^{\infty} (1 - \Delta) \Delta^t \int \bar{m}(w) d\Gamma_t(w), \end{aligned} \quad (15)$$

where

$$\bar{m}(w) \equiv \sum \pi_i m(\theta_i, w)$$

is the average payment that the financial intermediary receives from entrepreneurs whose expected utility is w at the beginning of the period.

Let

$$X_t = \int (\bar{m}(w) - E) d\Gamma_t(w). \quad (16)$$

X_t represents the per capita net payments made to the financial intermediary by the cohort of entrepreneurs who are t periods old. Next, let

$$B_{t+1} = RB_t + (1 - \Delta) \Delta^t X_t, \quad t = 0, 1, 2, \dots \quad (17)$$

B_t is the present value of the assets that are deposited at the financial intermediary by the entrepreneurs in its portfolio who are t periods old. Newly born entrepreneurs have no wealth, so $B_0 = 0$. The total amount of deposits B_1 that a cohort of new entrepreneurs carries into the next period is then $(1 - \Delta)X_0$, i.e., the size of the cohort multiplied by the per capita net payments of new entrepreneurs. Similarly, the total amount of deposits B_2 that a cohort of one-year-old entrepreneurs carries into the next period is given by $RB_1 + (1 - \Delta)\Delta X_1$, i.e., the principal and interest on the capital with which the one-year-old cohort begins the period plus the total payments made to the financial intermediary by the cohort of one-year-old entrepreneurs. Finally, let

$$B = \sum_{t=0}^{\infty} B_t.$$

B is the total amount of assets deposited by the entrepreneurs of all ages in the financial intermediary's portfolio.

One way to think about B_t and B is the following. Suppose that the optimal dynamic contract leads to front-loaded (net) payments from a cohort of entrepreneurs, i.e., suppose that X_t decreases monotonically with t . Since the intermediary makes zero expected profit on the contract, X_t must be positive for t sufficiently low and negative for t sufficiently high. Thus, there is a sense in which the young entrepreneurs (with $X_t > 0$) are saving by depositing at the financial intermediary while the old entrepreneurs (with $X_t < 0$) are dissaving by withdrawing deposits from the financial intermediary. In this case, both B_t (the total savings of a t -year-old cohort of entrepreneurs) and B (the total savings of all entrepreneurs) are positive. As discussed briefly below and demonstrate in Appendix A, in equilibrium B_t converges to 0 as t goes to infinity. That is, as a cohort of entrepreneurs ages, its total deposits at the financial intermediary vanish in the limit.

Suppose instead that payments are back-loaded, i.e., suppose that X_t increases monotonically with t . Then X_t is negative for t sufficiently low and positive for t sufficiently high. That is, entrepreneurs are borrowing from the intermediaries when young and paying back their debt when old, so that both B_t and B are negative. In this case, in order for the intermediaries to be able to lend to the entrepreneurs, the intermediaries must borrow from the workers who receive in each period interest payments equal to $-rB$. For this case, too, we show in Appendix A that B_t converges to 0 as t goes to infinity.

Appendix A contains a thorough discussion of the determinants of B , the assets that entrepreneurs deposit with financial intermediaries. There, we discuss three mechanisms—the “interest rate effect,” the “incentive effect,” and the “concavity effect”—that determine how X_t varies with time. We also show that B is finite (implying that B_t converges to 0) provided that new contracts earn zero profits and an entrepreneur's consumption space C is bounded.

Given the definition of B , it follows that:

$$rB + M - E = 0. \quad (18)$$

This equation states the financial intermediary's budget is balanced at any point in time.¹¹ Specifically, in each time period, the interest earned on the financial intermediary's capital exactly balances the difference between payments to and from the entrepreneurs in the financial intermediary's portfolio.

Market-clearing in the asset market requires that the following condition hold:

$$\lambda K = (1 - \lambda)D(r, \omega) + \lambda B, \quad (19)$$

where B is given by Eq. (18). The left-hand side of this equation is the aggregate demand for capital (assets) by entrepreneurs, while the right-hand side is the aggregate supply of capital by workers and financial intermediaries.

¹¹To see that $rB + M - E = 0$, simply add up both sides of Eq. (17):

$$\sum_{t=0}^{\infty} B_{t+1} = R \sum_{t=0}^{\infty} B_t + \sum_{t=0}^{\infty} (1 - \Delta) \Delta^t X_t.$$

Since $B_0 = 0$, $\sum B_{t+1} = \sum B_t = B$. Also, using Eqs. (15) and (16) $\sum (1 - \Delta) \Delta^t X_t = M - E$. So $B = RB + M - E$, from which it follows that $rB + M - E = 0$.

We can now give a formal definition of a steady-state equilibrium:

Definition 1. A steady-state equilibrium consists of value functions V_w and v , decision rules f , g , W , and m , prices r and ω , an initial promised utility w_0 , a distribution of promised utilities Γ , deposits D , labor supply H , the amount of assets held by financial intermediaries B , and a fraction of entrepreneurs λ satisfying:

1. Given r and ω , the value function V_w and the decision rules f and g solve the worker's problem (2), and D and H are consistent with f , g , and the birth/death rate $1 - \Delta$ (i.e., D and H are determined by Eqs. (3) and (4)).
2. Given r , the value function v and the decision rules W and m solve the dynamic contracting problem (5), and the distribution of promised utilities Γ is the distribution determined by the initial promised utility w_0 , the decision rule W , the stochastic process for θ , and the birth/death rate $1 - \Delta$, (i.e., Γ is given by Eq. (13)).
3. New individuals are indifferent between becoming a worker and becoming an entrepreneur (i.e., Eq. (10) holds).
4. The financial intermediary's budget is balanced in each period (i.e., Eq. (18) holds).
5. The value of a new contract is equal to zero (i.e., Eq. (12) holds).
6. The labor market and the credit market clear (i.e., Eqs. (11) and (19) hold).

Finally, we note that when the equilibrium conditions are satisfied, the goods market also clears in each period, i.e., aggregate output is divided between aggregate consumption of workers, aggregate consumption of entrepreneurs, and aggregate investment. To see this, let Y denote output per entrepreneur, let C_W denote consumption per worker, and let C_E denote consumption per entrepreneur. By definition, $Y = C_E + M$, i.e., entrepreneurial output is divided between entrepreneurial consumption and payments from entrepreneurs to financial intermediaries. Inserting the definition of E (see Eq. (14)) into Eq. (18), we can, therefore, obtain the following expression:

$$rB = (r + \delta)K + \omega L - Y + C_E. \quad (20)$$

Now multiply both sides of the market-clearing condition (19) for the asset market by r and replace the term rB with the right-hand side of (20) to obtain

$$\lambda rK = (1 - \lambda)rD + \lambda[(r + \delta)K + \omega L - Y + C_E].$$

Rearranging this expression and using the market-clearing condition for the labor market (11) to substitute for λL yields

$$\lambda Y = (1 - \lambda)(rD + \omega H) + \lambda C_E + \lambda \delta K.$$

Finally, note that the workers' budget constraints imply that $C_W = rD + \omega H$ in a steady-state equilibrium. Thus, aggregate output λY is divided between aggregate consumption of workers (i.e., $(1 - \lambda)C_W$), aggregate consumption of entrepreneurs (i.e., λC_E), and aggregate investment (i.e., $\lambda \delta K$).

3.1. Equilibrium under complete information

It is useful to consider as a benchmark the behavior of our economy under complete information, that is, under the assumption that entrepreneurial output is publicly observable. Under complete information, Definition 1 continues to apply, except that the

incentive constraint (6) in the firm's optimal contracting problem need not be satisfied any more. In this case, the optimal dynamic contract, which now achieves full risk-sharing, solves the following deterministic dynamic program:

$$v(w) = \max_{c, w'} (\bar{\theta} - c + R^{-1} \Delta v(w'))$$

subject to $c \in \mathbf{C}$ and

$$U(c, \bar{l}) + \beta \Delta w' = w,$$

where $\bar{\theta} = \sum \pi_i \theta_i$.

Under complete information, however, dynamic contracting and financial intermediation are not essential for achieving the equilibrium allocation. Specifically, in an earlier version of the paper (Smith and Wang, 2000), we show that under complete information, the steady-state allocation defined by Definition 1 can be achieved with a more standard market structure in which entrepreneurs are allowed to trade state-contingent securities, save and earn interest in a short-term credit market, and take part in an annuities market to insure themselves against mortality risk. We show that, under complete information, the total amount of assets held by financial intermediaries in the economy with dynamic contracting and financial intermediation is exactly equal to the total amount of entrepreneurial savings in the economy with state-contingent securities. This further confirms our interpretation that in the economy with private information, the assets held by financial intermediaries can be viewed simply as entrepreneurial savings. In the case of private information, however, this saving is delegated to the financial intermediaries, and the motives for saving are more complicated. As we explain in Appendix A, with private information assets held by financial intermediaries are essential for the efficient provision of intertemporal incentives and hence essential for the working of the dynamic lending relationship.

Finally, as we show in Appendix A, the equilibrium assets B of financial intermediaries are positive under complete information in our economy. This result obtains because the equilibrium interest rate must be greater than the rate of time preference of the entrepreneurs and workers in order for the equilibrium amount of capital in the economy to be positive. Assets held by financial intermediaries, or, equivalently, entrepreneurial savings, reflect the desire of entrepreneurs to shift consumption from earlier to later in their lives.

4. Numerical analysis of the model

In this section, we use numerical methods to study a realistically calibrated version of our economy.

4.1. Calibration and computation

Assume the output produced by a project in any given period can be either low (θ_L) or high (θ_H). We assume that θ is equal to $\theta_L = (1 - \sigma)\bar{\theta}$ with probability π and is equal to $\theta_H = (1 + \sigma)\bar{\theta}$ with probability $1 - \pi$, where $\sigma > 0$ and $\pi \in (0, 1)$. In any given period, therefore, the expected value of an entrepreneur's output is $\bar{\theta}$.

We let the instantaneous utility function $U(c, \ell) = \log(c) + \eta \log(\ell)$. Given this parameterization, we must choose values for the following model parameters: β (discount rate), Δ (survival rate), η (relative weight on leisure in the utility function), π (the probability that the entrepreneur has high output), $\bar{\theta}$ (expected value of an entrepreneur's output), σ (parameter governing the variation of an entrepreneur's output), K (units of capital per entrepreneurial project), L (units of labor per entrepreneurial project), δ (rate of depreciation of capital), and $\bar{h}$ (hours of work supplied by an entrepreneur in each period).

We let a period in the model correspond to one year. We let $\Delta = 0.98$, so that the expected value of the length of an individual's (working) life is 50 years. We set $\beta = 0.9633$; this choice implies that the equilibrium interest rate is approximately 4% (a standard number in the existing literature). In general, we can choose β to match the observed real interest rate.

Given the specification of the utility function, $\bar{\theta}$ is simply a scaling parameter: without loss of generality, we set $\bar{\theta} = 1$. The capital-output ratio in our economy is given by

$$\frac{\lambda K}{\lambda Y} = \frac{K}{Y} = \frac{K}{\bar{\theta}} = K,$$

where as in Section 3, Y is output per entrepreneur. To roughly match the capital-output ratio in the U.S. economy, therefore, we set $K = 2.65$.

We let $\delta = 0.1$, implying a quarterly depreciation rate of approximately 2.5% as in many existing macroeconomic studies. This choice for δ implies that the steady-state investment-to-output ratio is $(\lambda\delta K)/(\lambda Y) = 0.265$.

We set $\pi = 0.5$, in which case σ can be interpreted as the standard deviation of an entrepreneur's output (moreover, when $\bar{\theta} = 1$, σ can be interpreted as the coefficient of variation of an entrepreneur's output). The value of σ can be calibrated in at least three ways. First, at any point in time, σ is the cross-sectional standard deviation of output across firms (entrepreneurial projects). Second, for any given firm, σ is the standard deviation of its output across time. Consequently, microeconomic evidence concerning cross-sectional variation of output across firms or temporal variation of output for a given firm could be used to calibrate σ . Finally, as we show below, changes in the value of σ have a dramatic effect on the rate at which consumption inequality within a cohort of entrepreneurs increases as the cohort ages. Thus one could use evidence on consumption inequality (see, for example, Deaton and Paxson, 1994) to calibrate σ .

The final three parameters for which we must choose parameters are L (amount of labor per project), η (the weight on leisure in the utility function), and $\bar{h}$ (hours of work supplied by an entrepreneur in each period). We choose L and η by imposing two conditions on the model economy: one, average hours of work per worker (i.e., H) is equal to one-third; two, labor's share of income is equal to 0.64 (this is a typical number from existing macroeconomic studies). We interpret entrepreneurial consumption (i.e. C_E) as labor income, so that labor's share of income is given by

$$\frac{(1 - \lambda)\omega H + \lambda C_E}{\lambda Y} = \frac{\lambda\omega L + \lambda C_E}{\lambda Y} = \frac{\omega L + C_E}{Y}.$$

Note that when $H = \frac{1}{3}$, the expected amount of time that a new worker spends working over the course of his lifetime is equal to one-third of his lifetime time endowment. Accordingly, we set $\bar{h}$ equal to one-third as well.

Appendix A describes the numerical methods that we use to compute equilibria.

Table 1

	$\sigma = 0.5$	$\sigma = 0.25$	$\sigma = 0.1$	$\sigma = 0$
r	0.03998	0.03998	0.03997	0.03997
ω	0.3597	0.3608	0.3617	0.3622
λ	0.19238	0.19245	0.19251	0.19255
B/K	0.0717	0.0724	0.0734	0.0744
w_0	-37.670	-37.644	-37.601	-37.577
H	0.3335	0.3336	0.3338	0.3339
C_W	0.1434	0.1438	0.1441	0.1443
C_E	0.1331	0.1316	0.1305	0.1299
$\text{Var}(\log(c_E))$ (age 0)	0.0012	0.0008	0.0004	0
$\text{Var}(\log(c_E))$ (age 30)	0.129	0.069	0.024	0

Notes: σ is the standard deviation of an entrepreneur's output, r is the interest rate, ω is the wage rate, λ is the fraction of entrepreneurs, B/K is the fraction of aggregate capital that is held by financial intermediaries (recall that aggregate assets held by financial intermediaries are given by λB and aggregate capital is given by λK), w_0 is the expected lifetime utility of a new individual,¹² H is hours of work per worker, C_W is consumption per worker, C_E is consumption per entrepreneur, and $\text{Var}(\log(c_E))$ is the cross-sectional variance of the logarithm of consumption within a cohort of entrepreneurs.

4.2. Results

In our baseline model, we set $\sigma = 0.25$. In this case, we must set $L = 1.4$ and $\eta = 1.67$ in order to ensure that hours per worker is equal to one-third and labor's share of income is equal to 0.64. We also compute the equilibrium of our economy for three additional values of σ : 0, 0.1, and 0.25. When $\sigma = 0$, an entrepreneur's output is constant over time, thereby eliminating problems of moral hazard. Alternatively, one could imagine that an entrepreneur's output does vary over time but can be freely observed by the financial intermediary. In this case, the optimal dynamic contract implements the same allocation that obtains when $\sigma = 0$. We refer to this allocation, which features full risk-sharing, as the first-best allocation. Table 1 summarizes the equilibrium values of key endogenous variables for each of the four parameterizations of the model that we consider.

Table 1 contains several important results. First, financial intermediaries hold positive amounts of capital in equilibrium: depending on the value of σ , roughly 7% of the aggregate capital stock is held by financial intermediaries. As we show in Appendix A, a sufficient condition for $B > 0$ is that the payments to the financial intermediary of a typical entrepreneur decline, on average, as the entrepreneur ages. In our calibrated economies, we find that the average payments to financial intermediaries of a cohort of entrepreneurs do decline over time. Fig. 1 illustrates this finding for the case $\sigma = 0.25$.¹³ For the case $\sigma = 0.25$, Fig. 2 graphs the payment schedule $m(w, \theta)$ for each value of θ . Note that $m(w, \theta)$ is a decreasing function of w for each θ . Fig. 3 graphs the law of motion for an entrepreneur's promised utility.

¹²To be precise, the numbers in the row labelled ' w_0 ' are actually equal to w_0 minus a typical entrepreneur's expected lifetime utility of leisure (which is a constant in our model).

¹³The horizontal line in Fig. 1 is the value of the resources that the financial intermediary commits to the entrepreneur at each stage of the entrepreneur's life.

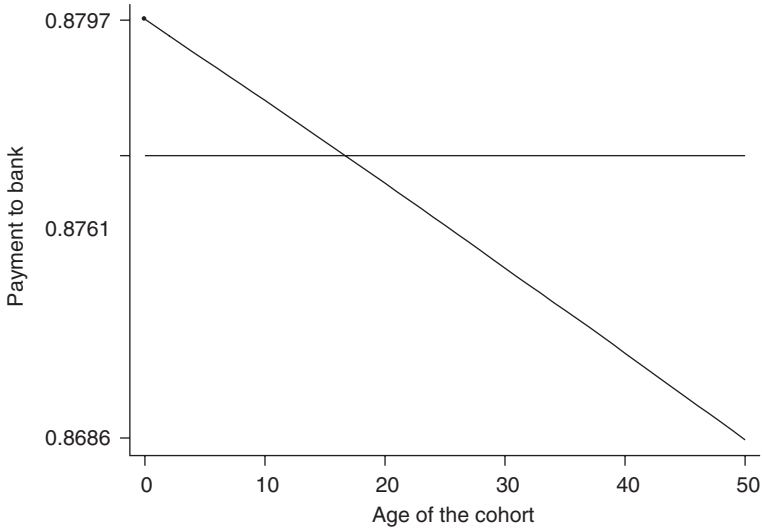


Fig. 1. Average payments of a cohort of entrepreneurs (sigma = 0.25).

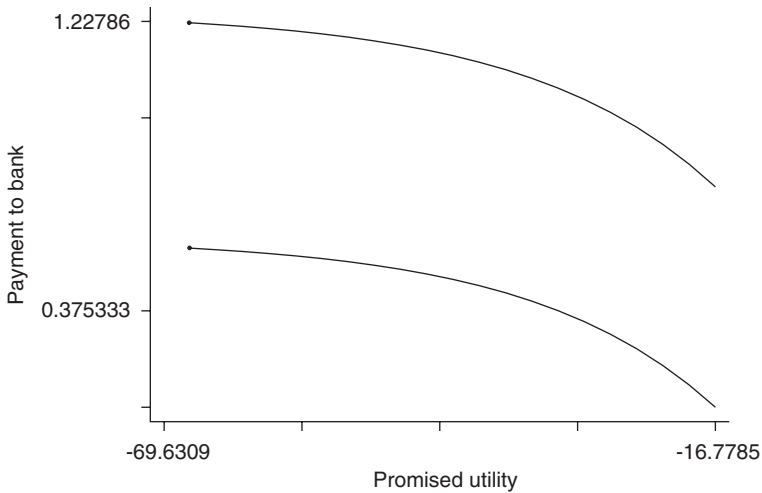


Fig. 2. Entrepreneur's payment to bank (sigma = 0.25) (upper line = high output, lower line = low output).

As we discuss in more detail in Appendix A, there are several factors that contribute to the determination of the size and sign of B , assets that are held by financial intermediaries in equilibrium. First, there is the “interest rate effect” through which the entrepreneur’s promised utility is pushed up over time because $R > \beta^{-1}$; other things equal, the interest rate effect implies that financial intermediaries should hold positive amounts of assets, since, in this case, a typical entrepreneur’s payments to the financial intermediary tend to fall over time. Second, there is an “incentive effect” which implies that the entrepreneur’s promised utility falls over time when $R = \beta^{-1}$, in which case (other things equal) financial

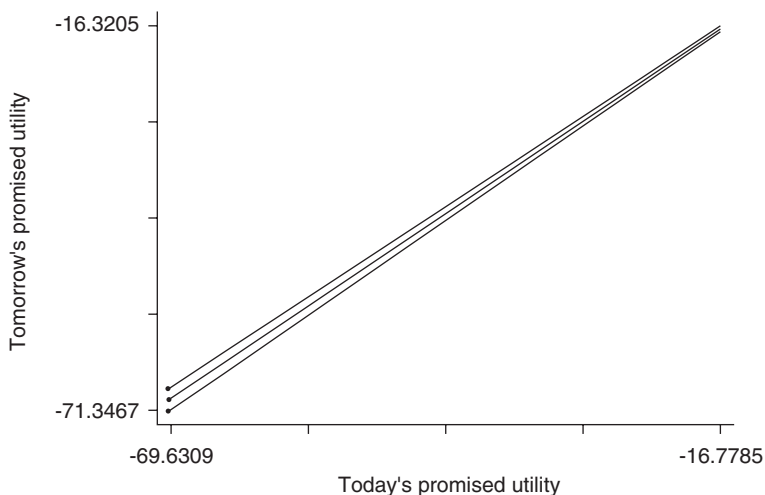


Fig. 3. Law of motion for promised utility ($\sigma = 0.25$) (upper line = high output, lower line = low output).

intermediaries would hold negative amounts of assets. The net impact of the two effects on the amount of assets that financial intermediaries hold can be evaluated quantitatively. We find that the interest rate effect dominates for low values of σ but that the incentive effect dominates for higher values of σ (in our experiments, the incentive effect dominates when $\sigma = 0.5$, but the interest effect dominates for the other values of σ that we examined). Fig. 3, which graphs the law of motion for promised utility for the case $\sigma = 0.25$, summarizes the net quantitative effect of the two offsetting mechanisms. Notice that promised utility rises when the entrepreneur reports a high level of output and falls when the entrepreneur reports a low level of output (the middle line in Fig. 3 is the 45° line). Although it is difficult to discern in the graph, the average of the two decision rules (one for each value of θ) lies above the 45° line. In other words, the equilibrium law of motion $W(\theta, w)$ implies that the stochastic process for the entrepreneur's promised utility is a submartingale. Thus promised utility tends to rise over time. For the case $\sigma = 0.5$, the counterpart of Fig. 3 is qualitatively similar, but the average of the two decision rules lies *below* the 45° line, and the entrepreneur's promised utility is a supermartingale.

A third and final mechanism is at work in our economy: the “concavity effect”. This mechanism tends to push the average payments of a cohort of entrepreneurs down over time. In particular, as shown in Fig. 2, the payment schedule $m(\theta, w)$ is a *concave* function of w for each value of θ . Thus, as the promised utility levels of a cohort of entrepreneurs spread out over time, Jensen's inequality implies that average payments to financial intermediaries of this cohort will fall over time even if the law of motion for promised utility is an exact random walk with no drift. As discussed above, promised utility levels in fact tend to drift (either up or down) over time. When promised utility levels drift up over time (leading average payments from a cohort of entrepreneurs to financial intermediaries to fall over time), the third mechanism provides another force that tends to push payments down over time. When promised utility levels drift down over time (leading average payments to financial intermediaries to rise over time), the third mechanism counteracts the effects of this downward drift. Quantitatively, we find that, when $\sigma = 0.5$, although the

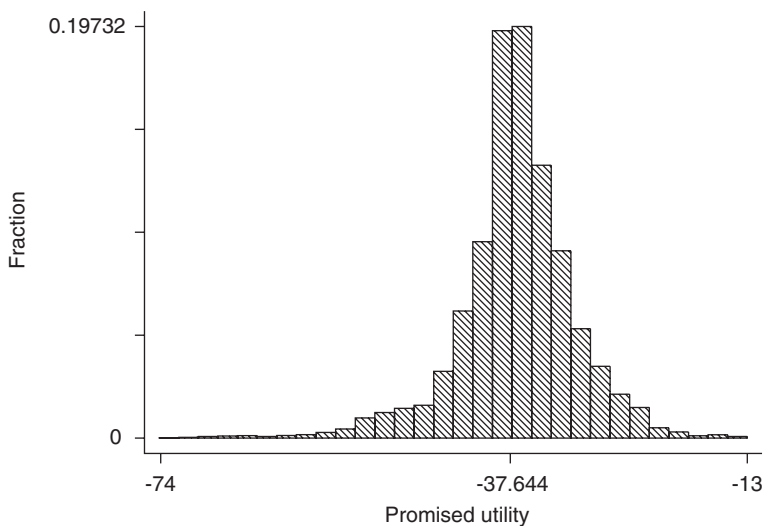


Fig. 4. Equilibrium distribution of promised utilities ($\sigma = 0.25$).

entrepreneur's promised utility drifts down over time, the third mechanism (due to the concavity of the function m) dominates, so that the average payments of a cohort of entrepreneurs again fall over time.

Fig. 4 displays (an approximation to) the equilibrium distribution of promised utilities Γ for the case $\sigma = 0.25$.¹⁴ The distribution has a large variance, reflecting the fact that the promised utility of unlucky entrepreneurs (those with streaks of low output) falls over time, while the promised utility of lucky entrepreneurs (those with streaks of high output) rises over time. As discussed above, however, promised utility tends to rise on average when $\sigma = 0.25$. This fact is reflected in the mean of Γ , which is slightly higher than the initial promised utility w_0 of a new entrepreneur.

A second important finding contained in Table 1 is that private information has very small effects on aggregate economic activity, even when σ is large. Although increases in σ tend to depress aggregate output and aggregate capital (λ falls as σ increases), the quantitative effects are tiny. The effects on the welfare of a newly born individual are somewhat larger: when $\sigma = 0.25$, for example, a new individual is worse off, relative to the case $\sigma = 0$, by the equivalent of 0.4% of per period consumption. Nonetheless, the welfare effects of private information in an economy with dynamic contracting are small in an absolute sense. In other words, the optimal provision of intertemporal incentives by means of the dynamic contract is very effective at mitigating the effects of private information.¹⁵ Even when σ is large, the equilibrium allocation is close to a first-best allocation (i.e., one that would obtain in the absence of private information).

A third important finding that emerges from Tables 1 and 2 concerns heterogeneity in consumption within a cohort of entrepreneurs. Deaton and Paxson (1994) document that

¹⁴Fig. 4 is a histogram based on the first 10,000 simulated values for promised utility; see Appendix A for details concerning this simulation.

¹⁵Khan and Ravikumar (2001) make a similar point.

Table 2

	$\psi = 0.01$	$\psi = 0.05$	$\psi = 0.1$
r	0.03999	0.04005	0.04014
ω	0.3608	0.3607	0.3606
λ	0.1923	0.1916	0.1906
B/K	0.073	0.076	0.079
w_0	-37.645	-37.648	-37.652
H	0.3333	0.3318	0.3298
C_W	0.1437	0.1429	0.1420
C_E	0.1316	0.1319	0.1322

Notes: See Table 1.

in U.S. data the variance of the logarithm of consumption within a cohort of individuals increases dramatically as the cohort ages (from 0.25 at age 25 to 0.47 at age 55). Table 1 shows that our model economy exhibits the same pattern: for example, when $\sigma = 0.5$, the variance of log consumption within a cohort of entrepreneurs increases from roughly zero at birth (since entrepreneurs are ex ante identical) to 0.13 at age 30. Although this increase is not as large as the one observed in the data, it nonetheless indicates that the optimal provision of intertemporal incentives could be one force explaining increasing dispersion of consumption over time within a cohort of ex ante identical individuals.¹⁶ Fig. 5 graphs the cross-sectional variance of log consumption against the age of the cohort for the case $\sigma = 0.25$. This figure shows that the cross-sectional variance increases linearly with the cohort's age, a shape which is roughly consistent with the facts documented in Deaton and Paxson (1994).

As a final point, note that since dynamic contracts are very effective in an environment in which financial intermediaries cannot observe an entrepreneur's output, it is clear that the effects of introducing CSV (i.e., allowing financial intermediaries to observe an entrepreneur's output by paying a fixed cost) would be small.

4.3. Static vs. dynamic contracting

How do dynamic contracts compare to static contracts in mitigating the effects of private information in equilibrium? This section answers this question by studying the allocations that obtain when financial intermediaries are restricted to use static contracts, that is, contracts that reflect one-shot interactions between the financial intermediary and the firm.

One problem that arises here is that under the current information structure, it can be trivial to make dynamic contracts dramatically more efficient than static contracts. In particular, static contracts require that the entrepreneur's payments be constant across the states of the entrepreneur's output in order to enforce truth-telling. This implies that the static contract provides no risk-sharing. Moreover, holding constant the mean and dispersion of returns on the entrepreneur's project, equilibrium aggregate output under static contracts is zero if there is a positive probability of a zero return for the

¹⁶See Storesletten et al. (2004) for an alternative explanation.

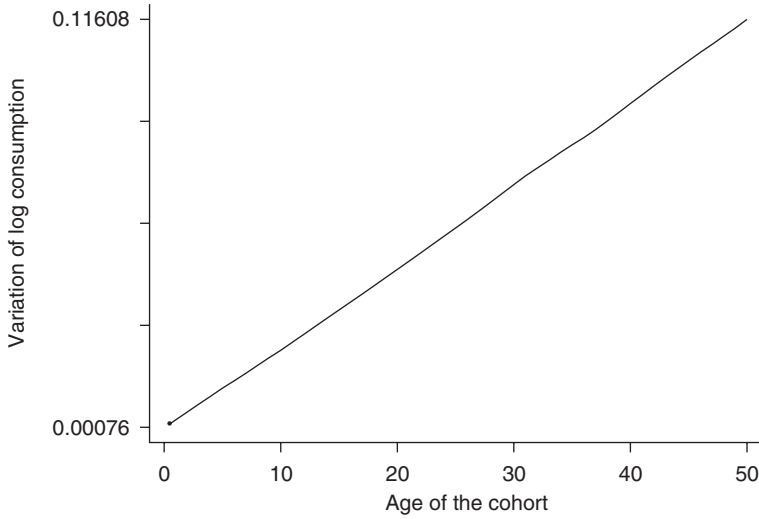


Fig. 5. Cross-sectional variance of log consumption ($\sigma = 0.25$).

entrepreneur. To get around this difficulty, we modify the information structure of our model by assuming that there is a CSV technology: at a cost $a > 0$, the intermediary can monitor the entrepreneur's report of θ . This CSV technology allows (full) risk-sharing to be feasible at a cost, and it also allows lending and borrowing to take place even if there is a positive probability of a zero return for the entrepreneur's project.

Under static contracting, the entrepreneur's expected utility w is constant over time. Because there are only two output states in our calibrated economy, it is straightforward to show that the financial intermediaries' optimal monitoring strategy is either to monitor in the state of low output or not to monitor at all. This implies that, given a level of promised utility w , the expected value of the financial intermediary is equal to

$$v(w) = (1 - R^{-1}\Delta)^{-1} \max[v_M(w), v_N(w)], \quad (21)$$

where $v_M(w)$ is the expected value of a (one-period) contract if the intermediary monitors the entrepreneur and $v_N(w)$ is the value of a (one-period) contract if the intermediary does not monitor the entrepreneur. Specifically, letting m_i denote the entrepreneur's payment in state i ,

$$v_M(w) = \max_{m_L, m_H} (\pi m_L + (1 - \pi)m_H - \pi a),$$

subject to:

$$0 \leq m_i < \theta_i, \quad i = L, H,$$

and

$$\pi U(\theta_L - m_L) + (1 - \pi)U(\theta_H - m_H) = (1 - \beta\Delta)w.$$

In addition, $v_N(w) = (1 - R^{-1}\Delta)^{-1}m^*$, where m^* satisfies

$$\pi U(\theta_L - m^*) + (1 - \pi)U(\theta_H - m^*) = (1 - \beta\Delta)w.$$

Notice that given concavity of U , the optimal contract with monitoring provides the entrepreneur with full risk-sharing: $\theta_L - m_L = \theta_H - m_H$. The optimal contract without monitoring, on the other hand, provides the entrepreneur with no risk-sharing.

Because the expected utility of an entrepreneur is constant in an equilibrium with static contracts, entrepreneurs' total deposits B at the financial intermediary are equal to 0. Formally, an equilibrium with static contracts consists of prices r and ω , an initial promised utility w_0 , a fraction of entrepreneurs λ , and a value function v for the intermediary such that: one, newly born consumers are indifferent between becoming a worker and becoming an entrepreneur (see Eq. (10)); the intermediary behaves optimally (see Eq. (21)); the labor market clears (see Eq. (11)); the asset market clears (see Eq. (19) with $B = 0$); and the intermediary earns zero profits:

$$v(\omega_0) = \frac{(r + \delta)K + \omega L + A}{1 - R^{-1}A},$$

where A denotes the expected cost of monitoring ($A = \pi a$ if the intermediary monitors and equals 0 otherwise).

Before comparing the allocations that obtain under static and dynamic contracting, we note that in the environment with dynamic contracting, we do not allow intermediaries to have access to the CSV technology.¹⁷ Consequently, the comparisons that we make below between static and dynamic contracts should be regarded as lower bounds on comparisons between static and dynamic contracts when a CSV technology is available in both settings.

We solve for the critical value of the cost of monitoring a for which the welfare of a newly born individual (i.e., w_0) is the same under both static and dynamic contracting. This critical value depends on σ , the coefficient of variation of an entrepreneur's output. For $\sigma = 0$, there exists no such value: as reported in Table 1, $w_0 = -37.577$ under dynamic contracting, but $w_0 = -37.585$ under static contracting. This is equivalent to a per period consumption loss of 0.05%. The reason for this loss is that even when there is no private information, the dynamic contract, unlike the static contract, allows an entrepreneur's promised utility to grow deterministically over time. As we discuss in Appendix A, this growth is optimal in an environment in which the equilibrium gross interest rate satisfies $R > \beta^{-1}$.

For large enough values of σ , however, there do exist positive values of a for which w_0 is the same in both contracting environments. For $\sigma = 0.1, 0.25$, and 0.5 , these values of a are 0.0011, 0.0041, and 0.0059, respectively.¹⁸ Recall that an entrepreneur's average output is equal to 1. For $\sigma = 0.1$, therefore, newly born individuals are better off (worse off) in the environment with dynamic contracting if the monitoring cost is more than (less than) roughly 0.1% of output. For $\sigma = 0.5$, newly born individuals are better off (worse off) with dynamic contracts if the monitoring cost is more than (less than) roughly 0.6% of output. Because entrepreneurs are monitored only in the low state, aggregate monitoring costs as a fraction of aggregate output are half as large as these percentages. Given the stark nature of the model, it is difficult to calibrate the monitoring cost. Nonetheless, the clear lesson

¹⁷Solving for an equilibrium in which intermediaries are allowed to use dynamic contracts and have access to CSV is difficult and beyond the scope of this paper.

¹⁸For large enough values of a , the intermediary chooses not to monitor in equilibrium. For the values of a considered here, the intermediary does choose to monitor.

from these results is that dynamic contracting is more effective than static contracting at mitigating the effects of private information unless monitoring costs are very small.¹⁹

5. A policy experiment

The purpose of this section is to show that the quantitative framework we have constructed provides a vehicle for conducting various types of policy experiments. As an example, in this section, we analyze the effects of requiring financial intermediaries to hold a fraction of their deposits as reserves. These reserves cannot be used to finance projects and thus do not earn interest; they are essentially stored goods.²⁰ In this case, the market-clearing condition in the asset market is

$$\lambda K = (1 - \lambda)(1 - \psi)D(r, \omega) + \lambda B, \quad (22)$$

where ψ is the reserve ratio.

For the case $\sigma = 0.25$, Table 2 summarizes the equilibrium effects of three different reserve ratios (1%, 5%, and 10%).

Table 2 (which should be compared to the second column of Table 1) shows that the cost of holding 10% of total deposits as required reserves is roughly 1% of aggregate output (compare the level of λ in the third column of Table 2 to that in the second column of Table 1). Notice, however, the economy's total capital stock, which is equal to $\lambda K + (1 - \lambda)\psi D(r, \omega)K$, increases. For instance, when $\psi = 0.1$, $\lambda K + (1 - \lambda)\psi D(r, \omega) = 0.210K$, a roughly 9% increase over $0.192K$, the total capital stock when $\psi = 0$.

Our model is also suitable for other policy experiments. For example, another natural policy experiment is to evaluate the impact of public policy (e.g., capital taxation) on people's choices of their occupations.

6. Conclusion and extensions

This paper develops and implements a general equilibrium model with private information in which borrowers and lenders enter into long-term credit relationships. We calibrate the model to U.S. aggregate data and use the model to analyze the impact of informational frictions on the behavior of the macroeconomic aggregates and on the welfare of individuals.

A key assumption in our model is that there are no aggregate risks, so that the macroeconomic aggregates do not vary over time. An important extension to our model would be to introduce aggregate uncertainty in the form of common shocks to the productive opportunities of entrepreneurs. This extension would allow us to examine how financial intermediation affects the propagation of aggregate shocks. Introducing aggregate uncertainty is a challenging computational problem since agents in the economy then need to keep track of the dynamic behavior of the distribution of asset holdings across

¹⁹We also find that, relative to dynamic contracting, static contracting lowers the equilibrium fraction λ of entrepreneurs, thereby reducing equilibrium output (which is proportional to λ in our environment). Moreover, this reduction in output becomes larger as the monitoring cost increases.

²⁰Huybens and Smith (1998) and Ennis (2001) also study the implications of reserve requirements for macroeconomic variables. The loan contracts that they incorporate in their models are one-period contracts, and their models are not calibrated to the U.S. economy.

workers and of the distribution of promised utilities across entrepreneurs. This is the subject of ongoing research.²¹

Appendix A

In this appendix, we discuss in detail the determinants of the assets held by the financial intermediaries. As a starting point, it is useful to notice that optimal dynamic contracting is not necessary for the existence of the assets held by financial intermediaries. Assets held by financial intermediaries can arise in our model as an equilibrium phenomenon even when full optimality of the long-term contract is *not* assumed.

To see this, let $\sigma = \{m_t(h^t)\}$ be any incentive compatible contract that the financial intermediary and the entrepreneurs wish to enter into (this contract need not be an optimal one). Here h^t is the history of reported outputs of a t -period-old entrepreneur up to and including period t and $m_t(h^t)$ is the payment from the entrepreneur to the financial intermediary in period t conditional on t . Let H^t denote the set of all possible histories of a t -period-old entrepreneur. Then the per capita net payments from a cohort of t -period-old entrepreneurs to the financial intermediary can be written

$$X_t \equiv \int_{H^t} [m_t(h^t) - E] dF_t(h^t),$$

where F_t is the distribution of history h^t over H^t . As in Section 3, let $B_0 = 0$ and define B_t for $t \geq 1$ recursively using Eq. (17). Then, as in Section 3, B is equal to $\sum_{t=0}^{\infty} B_t$.

If B is well-defined (i.e., if B is finite), then it must be the case that $\lim_{t \rightarrow \infty} B_t = 0$. Proposition 1 shows that B_t diverges as t goes to infinity if the zero-profit condition for new contracts (see Eq. (12)) is not satisfied. In other words, if the value of a new contract is not zero, then B is not well-defined.

Proposition 1. *If the zero-profit condition for new contracts does not hold, then B_t diverges as t goes to infinity.*

The proofs of the propositions in this appendix are relegated to the end of the appendix.

Although the zero-profit condition for new contracts is a necessary condition for B to be well-defined, it is not a sufficient condition, as illustrated by the following example. Suppose that the contract σ is such that

$$X_t = \frac{1 - R^{-1}\Delta}{1 - R^{-1}\gamma\Delta} - \gamma^t,$$

where $1 < \gamma < R\Delta^{-1}$. It is straightforward to verify that the zero-profit condition (30) is satisfied in this case. Using Eq. (17), one can show that, for $t \geq 0$,

$$B_t = \frac{R^{-1}(1 - \Delta)}{1 - R^{-1}\gamma\Delta} ((\gamma\Delta)^t - \Delta^t).$$

Thus B_t goes to zero as t goes to infinity if and only if $0 < \gamma\Delta < 1$. If, on the other hand, $\Delta^{-1} < \gamma < R\Delta^{-1}$, then the zero-profit condition is satisfied and yet B_t diverges so that B is not well-defined.

²¹In particular, we are currently exploring whether the “approximate aggregation” findings of Krusell and Smith (1998) extend to the economic environment in this paper.

Proposition 2 shows that if an entrepreneur's consumption space is bounded and the zero-profit condition for new contracts holds, then B is finite.²²

Proposition 2. *If new contracts earn zero profits and if an entrepreneur's consumption space C is bounded, then B is finite.*

If the contract is such that X_t is constant for all t , then the zero-profit condition (30) implies that $X_t = 0$ for all t and thus $B_t = 0$ for all t . For example, if the long-term contract is just a sequence of identical one-period contracts (as in the model of Carlstrom and Fuerst, 1997), then $B_t = 0$ for all t and $B = 0$. Conversely, if $B_t = 0$ for all t , then by applying Eq. (17) repeatedly it is easy to show that $X_t = 0$ for all t . In general, however, B_t is not constant at zero in equilibrium. In fact, B_t can be positive, negative, or zero, depending critically on the sequence $\{X_t\}$, i.e., on the profile of the per capita net payments that a cohort of entrepreneurs makes to the financial intermediary as the cohort ages.

Suppose, for example, that the equilibrium contract σ is such that $\{X_t\}$ is a strictly decreasing sequence. Then the zero-profit condition (30) implies that there exists $T > 0$ such that $X_t \geq 0$ for $t \leq T$ and $X_t < 0$ for $t > T$. As a result, one can show that there exists $S > T$ such that $\{B_t\}_{t=0}^S$ is an increasing sequence and $\{B_t\}_{t=S}^\infty$ is a decreasing sequence (whose limit is zero provided C is bounded). Moreover, $B_t > 0$ for all $t \geq 1$, implying that $B > 0$.²³

Conversely, suppose that $\{X_t\}$ is a strictly increasing sequence. In this case, there exists $T > 0$ such that $X_t \leq 0$ for $t \leq T$ and $X_t > 0$ for $t > T$. As a result, there exists $S > T$ such that $\{B_t\}_{t=0}^S$ is a decreasing sequence and $\{B_t\}_{t=S}^\infty$ is an increasing sequence (whose limit is zero provided C is bounded), from which it follows that $B_t < 0$ for all $t \geq 1$ and $B < 0$.

A.1. The case of CARA utility

To gain further insights into the determinants of B , the assets that financial intermediaries hold in the equilibrium of our model, it is instructive to study an example for which we can explicitly characterize the structure of the optimal contract. Specifically, we will consider the case of exponential utility $U(c, \tilde{l}) = -\exp(-c - \tilde{l})$ (for analytical convenience, we allow consumption to be negative; for simplicity, we set $\tilde{l} = 0$). This example allows us to obtain several analytical results which show how several different mechanisms interact to determine the size of B . In Section 5, we use numerical methods to study a calibrated economy in which individuals have constant relative risk aversion (CRRA) utility functions. The mechanisms that we discuss in detail in this section for the case of exponential utility continue to play key roles when individuals have CRRA utility.

Under exponential utility, the value function for the problem of optimal dynamic contracting takes the form $v(w) = A_0 + A_1 \log(-w)$, where A_0 and A_1 are constants, and the optimal contract takes the following form: $m(\theta_i, w) = \log(m_i^*) + \log(-w)$ and

²²Note, however, that boundedness of the consumption space is not a necessary condition for B to be well-defined, as illustrated by the preceding example when $\gamma < \Delta^{-1}$.

²³To see that $B_t > 0$ for $t \geq 1$, suppose that there exists $t^* > 0$ such that $B_{t^*} < 0$. It must be the case that $t^* > S$, since $B_0 = 0$ and $\{B_t\}_{t=0}^S$ is an increasing sequence. In addition, since $\lim_{t \rightarrow \infty} B_t = 0$, there exists $s > t^*$ such that $B_s > B_{t^*}$. But this is a contradiction, since $\{B_t\}_{t=S}^\infty$ is a decreasing sequence.

$W(\theta_i, w) = w_i^* w$, where (m_i^*, w_i^*) solves the following optimization problem:

$$\max_{(m_i, w_i)_{i=1}^N} \sum_{i=1}^N \pi_i \left[\log m_i + \frac{R^{-1} \Delta}{1 - R^{-1} \Delta} \log w_i \right] \quad (23)$$

subject to:

$$m_i > 0, \quad w_i > 0, \quad \forall i \quad (24)$$

$$\exp(-\theta_i) m_i + \beta \Delta w_i \leq \exp(-\theta_j) m_j + \beta \Delta w_j, \quad \forall \theta_i, \theta_j, \quad (25)$$

$$\sum_{i=1}^N \pi_i [\exp(-\theta_i) m_i + \beta \Delta w_i] = 1. \quad (26)$$

The constant $A_0 = 1/(1 - \Delta R^{-1}) [\sum_{i=1}^n \pi_i \log(m_i^*) + \Delta R^{-1}/(1 - R^{-1} \Delta) \sum_{i=1}^n \pi_i \log(w_i^*)]$ and the constant $A_1 = 1/(1 - R^{-1} \Delta)$.

Let $\{w_t\}$ denote the stochastic process of an entrepreneur's expected utility. Under exponential utility, this process can be described as follows: if $t = 0$, then $w_t = w_0$; if $t \geq 1$, then $w_t = w_i^* w_{t-1}$ if $\theta_t = \theta_i$. Moreover, $\Pr\{w_t = w_{i_t}^* w_{i_{t-1}}^* \cdots w_{i_1}^* w_0\} = \pi_{i_t} \pi_{i_{t-1}} \cdots \pi_{i_1}$ for all t and all $(i_t, i_{t-1}, \dots, i_1) \in \tilde{N}^t$, where $\tilde{N} = \{1, 2, \dots, N\}$. These facts imply that $\int \log(-w) d\Gamma_t(w) = t \sum \pi_i \log(w_i^*) + \log(-w_0)$.²⁴ We use this result below in order to characterize the behavior of the sequence of per capita net payments $\{X_t\}$.

Note that $\bar{m}(w) \equiv \sum \pi_i m(\theta_i, w) = \sum \pi_i \log(m_i^*) + \log(-w)$. Hence,

$$\begin{aligned} X_t &\equiv \int (\bar{m}(w) - E) d\Gamma_t(w) \\ &= \sum \pi_i \log(m_i^*) - E + \int \log(-w) d\Gamma_t(w) \\ &= \sum \pi_i \log(m_i^*) - E + \log(-w_0) + t \sum \pi_i \log(w_i^*). \end{aligned} \quad (27)$$

Clearly, X_t is either monotonically increasing or monotonically decreasing in t , depending on the sign of $\sum \pi_i \log(w_i^*)$. It follows that $B < 0$ if $\sum \pi_i \log(w_i^*) > 0$, $B > 0$ if $\sum \pi_i \log(w_i^*) < 0$, and $B = 0$ if $\sum \pi_i \log(w_i^*) = 0$. Moreover, given Eq. (27) and the zero-profit condition (12), one can show that

$$B = -\frac{\Delta}{(1 - \Delta)(R - \Delta)} \sum \pi_i \log(w_i^*).$$

²⁴To see this, first note that $\int \log(-w) d\Gamma_t(w) = \log(-w_0) + Z_t$, where $Z_t \equiv \sum (\pi_{i_t} \pi_{i_{t-1}} \cdots \pi_{i_1}) \log(w_{i_t}^* w_{i_{t-1}}^* \cdots w_{i_1}^*)$ (the summation is over $(i_t, i_{t-1}, \dots, i_1) \in \tilde{N}^t$). Next, we show by induction that $Z_t = t \sum \pi_i \log(w_i^*)$ holds for all $t \geq 1$. It clearly holds for $t = 1$. Let $t \geq 1$ and suppose that $Z_t = t \sum \pi_i \log(w_i^*)$. Then

$$\begin{aligned} Z_{t+1} &= \sum_{(i_t, \dots, i_1)} (\pi_{i_t} \pi_{i_{t-1}} \cdots \pi_{i_1}) \sum_{i_{t+1}} \pi_{i_{t+1}} [\log(w_{i_t}^* w_{i_{t-1}}^* \cdots w_{i_1}^*) + \log(w_{i_{t+1}}^*)] \\ &= \sum_{(i_t, \dots, i_1)} (\pi_{i_t} \pi_{i_{t-1}} \cdots \pi_{i_1}) \log(w_{i_t}^* w_{i_{t-1}}^* \cdots w_{i_1}^*) + \sum_{(i_t, \dots, i_1)} (\pi_{i_t} \pi_{i_{t-1}} \cdots \pi_{i_1}) \sum_{i_{t+1}} \pi_{i_{t+1}} \log(w_{i_{t+1}}^*) \\ &= Z_t + \sum \pi_i \log(w_i^*) \\ &= (t+1) \sum \pi_i \log(w_i^*). \end{aligned}$$

Note that aggregate amount of assets held by financial intermediaries B exists even though the optimal contract (payment scheme) is not bounded (so that the conditions of Proposition 2 are not satisfied).

With exponential utility, then, the shape of the sequence $\{X_t\}$, and hence the size of B , depends critically on the sign of $\sum \pi_i \log(w_i^*)$. What determines the sign of $\sum \pi_i \log(w_i^*)$? There are several factors. First, because of private information, full intertemporal risk-sharing is not achievable, and the optimal dynamic contract implies that on average the entrepreneur borrows against his future income. Consequently, relative to full risk-sharing, on average the entrepreneur's expected utility falls over time. In particular, when $R = \beta^{-1}$, $\sum \pi_i W(\theta_i, w) < w$, as shown by Green (1987), Thomas and Worrall (1990), and Atkeson and Lucas (1992). We call this tendency for promised utility to fall over time (when $R = \beta^{-1}$) the “incentive effect”. Since an entrepreneur's payment to the financial intermediary increases as promised utility falls, the incentive effect, other things equal, causes X_t to increase over time.

Second, recall that the optimal provision of intertemporal incentives implies that the distribution of promised utilities of a cohort of entrepreneurs spreads out over time. The optimal payment schedule $m(\theta_i, w)$ is a *concave* function of w for each value of θ_i , so Jensen's inequality implies that X_t decreases as t increases even if the law of motion for promised utility is an exact random walk (with no drift). Specifically, if $\sum \pi_i w_i^* = 1$ (or $\sum \pi_i W(\theta_i, w) = w$), then $\sum \pi_i \log(w_i^*) < 0$. We call this the “concavity effect”.²⁵

Lastly, a key parameter in the determination of the assets held by financial intermediaries is R , the rate of interest at which financial intermediaries discount future cash flows. As R increases, the financial intermediary becomes less patient, in which case it is optimal for the financial intermediary to receive payments from the entrepreneur earlier. That is, a higher R causes X_t to tend to decrease over time. Conversely, a lower R causes X_t to tend to increase over time. We call this the “interest rate effect”. Formally, we have

Proposition 3. *In the case of CARA, $\sum \pi_i \log(w_i^*)$ is decreasing in R , and there exists $\bar{R} > \Delta$ such that $B \geq 0$ if and only if $R \geq \bar{R}$.*

Finally, it is useful to note that under complete information with exponential utility, the optimal contract takes the form: $W(w) = (R\beta)^{-1}w$ and $m(w) = \bar{\theta} + \log(1 - R^{-1}) + \log(-w)$. In this case, the incentive effect and the concavity effect play no role in determining the sequence $\{X_t\}$. Instead, the interest rate effect determines whether $\{X_t\}$ is an increasing, decreasing, or constant sequence. Specifically, $\{X_t\}$ is a strictly decreasing (increasing) sequence when $R > \beta^{-1}$ ($R < \beta^{-1}$); when $R = \beta^{-1}$, $\{X_t\}$ is a constant sequence. So $B > 0$ if $R > \beta^{-1}$, $B < 0$ if $R < \beta^{-1}$, and $B = 0$ if $R = \beta^{-1}$.

To sum up, this appendix shows that the size and sign of the assets held by financial intermediaries depends critically on how the per capita net payments to the financial intermediary of a cohort of entrepreneurs change as the cohort ages. In an example with exponential utility, we show that this payment schedule is either strictly increasing, strictly decreasing, or constant. We also show that, in general, the amount of assets that financial

²⁵A static contracting example sheds light on why the payment schedule is concave in promised utility. Suppose that there is no uncertainty and that the entrepreneur has a constant endowment θ . The contract requires the entrepreneur to make payment m to the financial intermediary; in addition, the contract delivers to the entrepreneur his reservation utility w . In other words, the contract must satisfy: $u(\theta - m) = w$, where u is the entrepreneur's concave utility function. Then $m = \theta - u^{-1}(w)$, which is a concave function of w . Adding uncertainty to this setup does not change the results.

intermediaries hold in equilibrium is positive (negative) when the payment schedule is decreasing (increasing) and is zero when the payment schedule is constant. In addition, we show how three mechanisms—the incentive effect, the interest rate effect, and the concavity effect—interact to determine the shape of the payment schedule. Finally, we show which features of the optimal contract are critical for understanding each of these effects. Although we study in the next section an economy with CRRA utility, all of the insights gleaned from the example with exponential utility also apply to this economy.

A.2. Proofs of Propositions

Proof of Proposition 1. Iterate on Eq. (17) to obtain the following expression for B_{t+1} (for $t \geq 0$):

$$B_{t+1} = R^t(1 - \Delta) \sum_{i=0}^t (R^{-1}\Delta)^i X_i. \quad (28)$$

Ignoring the term $1 - \Delta$, B_{t+1} is the product of two terms, the first of which (R^t) goes to infinity (since $R > 1$). Consequently, B_t diverges as t goes to infinity if

$$\lim_{t \rightarrow \infty} \sum_{i=0}^t (R^{-1}\Delta)^i X_i \neq 0. \quad (29)$$

Now note that the zero-profit condition (see Eq. (12)) can be written in terms of the X_t 's as follows:

$$\sum_{i=0}^{\infty} (R^{-1}\Delta)^i X_i = 0. \quad (30)$$

Thus the left-hand side of Eq. (29) is simply the zero-profit condition. $\square$

Proof of Proposition 2. The zero-profit condition (30) can be written

$$\sum_{i=0}^t (R^{-1}\Delta)^i X_i + \sum_{i=t+1}^{\infty} (R^{-1}\Delta)^i X_i = 0,$$

from which it follows (using Eq. (28)) that

$$\begin{aligned} B_{t+1} &= -(1 - \Delta)R^t \sum_{i=t+1}^{\infty} (R^{-1}\Delta)^i X_i \\ &= -(1 - \Delta)R^{-1}\Delta^{t+1} \sum_{i=0}^{\infty} \Delta^i X_{t+1+i}. \end{aligned}$$

Since C is bounded, the sequence $\{X_t\}$ is bounded, implying in turn that the sum $\sum_{i=0}^{\infty} \Delta^i X_{t+1+i}$ is bounded (since $0 < \Delta < 1$). Defining $Z_t \equiv \sum_{i=0}^{\infty} \Delta^i X_{t+1+i}$, we can now write:

$$B = \sum_{t=0}^{\infty} B_t = -(1 - \Delta)R^{-1} \sum_{t=0}^{\infty} \Delta^t Z_t.$$

Since the sequence $\{Z_t\}$ is bounded and since $0 < \Delta < 1$, B is finite. $\square$

Proof of Proposition 3. Let $\alpha = R^{-1}\Delta/(1 - R^{-1}\Delta)$. To show that $\sum \pi_i \log(w_i^*)$ is decreasing in R we need only show that $\sum \pi_i \log(w_i^*)$ is increasing in α . Let $X(\alpha) = \sum \pi_i \log(m_i^*)$, and $Y(\alpha) = \sum \pi_i \log(w_i^*)$, where X and Y are written as functions of α to emphasize that they depend on α . Let

$$\Omega \equiv \left\{ (X, Y) : X = \sum \pi_i \log(m_i), Y = \sum \pi_i \log(w_i), (m_i, w_i) \text{ s.t. (24), (25), (26)} \right\}.$$

Note that Ω is not a function of α . Then for all α , $(X(\alpha), Y(\alpha))$ solves the following optimization problem:

$$\begin{aligned} \max \quad & X + \alpha Y, \\ \text{s.t.} \quad & (X, Y) \in \Omega. \end{aligned} \quad (31)$$

To prove the proposition we need only show that $Y(\alpha_2) \geq Y(\alpha_1)$ for all α_1 and α_2 satisfying $\alpha_2 > \alpha_1 > 0$. Suppose $Y(\alpha_2) < Y(\alpha_1)$. Now either $X(\alpha_2) + \alpha_1 Y(\alpha_2) > X(\alpha_1) + \alpha_1 Y(\alpha_1)$ or $X(\alpha_2) + \alpha_1 Y(\alpha_2) < X(\alpha_1) + \alpha_1 Y(\alpha_1)$ must hold. Suppose the first inequality holds. Then clearly $(X(\alpha_1), Y(\alpha_1))$ is not a solution to the programming problem (31) at $\alpha = \alpha_1$, a contradiction. Thus, to prove the proposition, we need only show that the second inequality cannot hold either. Suppose the second inequality holds. Then

$$Y(\alpha_1) - Y(\alpha_2) \geq \frac{X(\alpha_2) - X(\alpha_1)}{\alpha_1},$$

which, together with $Y(\alpha_2) < Y(\alpha_1)$ and $\alpha_2 > \alpha_1$, in turn implies

$$Y(\alpha_1) - Y(\alpha_2) > \frac{X(\alpha_2) - X(\alpha_1)}{\alpha_2},$$

or equivalently,

$$X(\alpha_2) + \alpha_2 Y(\alpha_2) < X(\alpha_1) + \alpha_2 Y(\alpha_1).$$

This contradicts the assumption that $(X(\alpha_2), Y(\alpha_2))$ is a solution to the programming problem (31). To complete the proof, notice that $\sum \pi_i \log(w_i^*)$ is positive for R sufficiently close to Δ , and it is negative if R is sufficiently high. $\square$

Appendix B

In this appendix, we describe the numerical algorithm that we use to compute equilibria. The steady-state equilibrium can be computed by finding the prices r and ω that solve the pair of equations $F_1(r, \omega) = 0$ and $F_2(r, \omega) = 0$. The function F_1 corresponds to the zero-profit condition (12), while the function F_2 corresponds to market-clearing in the asset market (see Eq. (19)). To evaluate these functions, we proceed as follows:

1. Choose r and ω .
2. Solve the worker's problem (2) and calculate D and H according to Eqs. (3) and (4).
3. Solve the dynamic contracting problem (5).
4. Use the market-clearing condition in the labor market (see Eq. (11)) to solve for λ :

$$\lambda = \frac{H}{H + L}. \quad (32)$$

5. Use Monte Carlo simulation to compute an approximation to M , as defined in Eq. (15). Specifically, use a (pseudo)random number generator to generate a sequence $\{\theta_t\}_{t=0}^N$ and a sequence $\{u_t\}_{t=0}^N$, where the u_t 's are independent and identically distributed and each u_t is uniformly distributed on the $[0, 1]$ interval.²⁶ Use these simulated sequences to generate a simulated sequence $\{w_t\}_{t=0}^N$, where w_t is promised utility in period t . In particular, proceed as follows:
- (a) Set $w_0 = V_w(0; r, \omega)$ and $t = 0$.
 - (b) If $u_t \leq 1 - \Delta$, then the entrepreneur dies at the beginning of next period and is replaced by a new entrepreneur. In this case, set $w_{t+1} = w_0$. Otherwise, set $w_{t+1} = W(\theta_t, w_t)$.
 - (c) Increment t . Iterate on steps (b)–(d) until $t = N$.

Given $\{w_t\}_{t=0}^N$ and $\{\theta_t\}_{t=0}^N$, the approximation to M is computed as follows:

$$\hat{M} = N^{-1} \sum_{t=0}^N m(\theta_t, w_t).$$

6. Evaluate $F_1(r, \omega)$ and $F_2(r, \omega)$, defined as follows:

$$F_1(r, \omega) = v(V_w(0; r, \omega); r) - (1 - R^{-1}\Delta)^{-1}E, \quad (33)$$

$$F_2(r, \omega) = \lambda K - (1 - \lambda)D - r^{-1}\lambda(E - \hat{M}), \quad (34)$$

where λ is given by Eq. (32) and $E = (r + \delta)K + \omega L$.²⁷

We use Newton's method with numerical (one-sided) derivatives to find the values of r and ω which set the functions F_1 and F_2 to zero.

Given our functional form assumptions for the utility function, the consumer's problem can be solved using a guess-and-verify approach (in this case, the value function takes the form $V_w(d) = a_0 + a_1 \log(d + a_2)$, where a_0 , a_1 , and a_2 can be expressed as functions of the structural parameters β , Δ , R , and ω). The optimal decision rules for the worker's problem are then given by

$$d' = f(d; r, \omega) = \frac{\omega(R\beta - 1)}{R - \Delta} + R\beta d$$

and

$$h = g(d; r, \omega) = \frac{\gamma R + (1 - \gamma)\Delta R\beta - \Delta}{R - \Delta} - \frac{R(1 - \gamma)(1 - \beta\Delta)}{\omega}d,$$

where $\gamma = 1/(1 + \eta)$.

Note that we allow workers to choose negative hours. If we were to impose the constraint $h \geq 0$, then we would be forced to solve the worker's problem numerically. In our calibrated equilibrium, very few (roughly 0.5%) of workers choose negative hours, so allowing negative hours is not important from a quantitative standpoint. In this sense, little would be gained by incurring the cost of using a numerical algorithm.

²⁶In practice, we use antithetic variates when conducting the Monte Carlo simulations. For our problem, this approach increases numerical accuracy without increasing computational cost.

²⁷Note that we have emphasized the dependence of v on r by introducing r as an explicit argument of v .

Alternatively, one can interpret leisure in the worker's utility function as, say, house-cleaning services.²⁸ The worker can either provide these services himself when he is not working in the market or he can purchase these services in the market at wage ω . Under this interpretation, the utility function takes the form $u(c, 1 - h_m + h_c)$, where h_m is the amount of time that worker spends in market activities (so that $1 - h_m$ is the amount of house-cleaning services that he provides himself) and h_c is the amount of house-cleaning services that he purchases in the market. The worker's budget constraint then takes the form: $c + \Delta d' + wh_c = Rd + wh_m$, or $c + \Delta d' = Rd + w(h_m - h_c)$. Now define $h = h_m - h_c$, so that the utility function becomes $u(c, 1 - h)$ and the budget constraint becomes $c + \Delta d' = Rd + wh$, as in the original formulation of the problem. If $h \geq 0$, then there is an indeterminacy in the choice of h_m and h_c , but if $h < 0$, then it must be the case that $h_m = 0$ and $h_c > 0$ (since market hours h_m cannot be negative).

Using the formulas for the worker's decision rules, one can derive analytical expressions for deposits per worker and hours per worker (recall from Section 2.1 that $d_t(r, \omega)$ is the deposits of a worker who is t periods old):

$$D(r, \omega) = (1 - \Delta) \sum_{t=0}^{\infty} \Delta^t d_t(r, \omega) = \frac{\omega \Delta (R\beta - 1)}{(1 - \Delta R\beta)(R - \Delta)},$$

$$H(r) = (1 - \Delta) \sum_{t=0}^{\infty} \Delta^t g(d_t; r, \omega)$$

$$= \frac{\gamma R + (1 - \gamma) \Delta R\beta - \Delta}{R - \Delta} - \frac{R(1 - \gamma)(1 - \beta \Delta) \Delta (R\beta - 1)}{(1 - \Delta R\beta)(R - \Delta)}.$$

Note that, under our functional form assumptions on the worker's utility function, the total amount of labor supplied by workers does not depend on the wage ω .

We compute the solution to the dynamic contracting problem using numerical methods similar to those described in the Appendix to [Krusell and Smith \(1998\)](#).²⁹ When implementing these methods, we take advantage of the fact that one of the incentive (truth-telling) constraints (in particular, the one associated with the low state) never binds, while the other incentive constraint (in particular, the one associated with the high state) always binds. In the simulation of promised utilities described above, we use 500,000 antithetic pairs (i.e., in terms of computation time, $N = 1,000,000$).

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²⁸We thank Per Krusell for suggesting this interpretation.

²⁹When $\sigma = 0$, the dynamic contracting problem has an analytical (closed-form) solution. To find this solution, we use a guess-and-verify approach with the guess for the value function being $v(w) = b_0 + b_1 \exp(b_2 w)$.

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