

Temptation and Taxation

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Abstract

We study optimal taxation when consumers have temptation and self-control problems. Embedding the class of preferences developed by Gul and Pesendorfer into a standard macroeconomic setting, we first prove, in a two-period model, that the optimal policy is to subsidize savings when consumers are tempted by “excessive” impatience. The savings subsidy improves welfare because it makes succumbing to temptation less attractive. We then study an economy with a long but finite horizon which nests as a special case the Phelps-Pollak-Laibson multiple-selves model. We prove that when period utility is logarithmic the optimal savings subsidies increase over time. Moreover, as the horizon grows large, the optimal policy prescribes a constant subsidy, in contrast to the well-known Chamley-Judd result.

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1 Introduction

Experimental and introspective evidence suggests that consumers exhibit preference reversals as time passes. Such evidence has led to the development of models in which consumers have “time-inconsistent preferences” (see Laibson (1997), who builds on earlier work by Strotz (1956) and Phelps and Pollak (1968)). In models with time-inconsistent preferences, a sequence of the consumer’s different “selves”, each valuing consumption streams in a unique way, play a dynamic game. In this game of conflict across selves, one can define Pareto frontiers among selves and discuss non-cooperative equilibria of the dynamic game relative to this frontier. Consequently, policy proposals by an outside authority, such as the government, do not, in general, lead to unambiguous recommendations without deciding how to assign welfare weights to the different selves.

In contrast, Gul and Pesendorfer (2001, 2004, 2005) develop an alternative, axiomatic, approach to modelling preference reversals. This approach does not necessitate splitting up the consumer into multiple selves. To address reversals, Gul and Pesendorfer formalize the ideas of temptation and self-control: they define preferences over consumption sets rather than over consumption sequences and then discuss temptation and self-control in terms of preferences over these sets. The axiomatization delivers a representation theorem with utility over consumption sets expressed in terms of two utility functions, one describing *commitment utility*, which gives the ranking that the consumer uses to compare consumption bundles, as opposed to consumption sets, and the other *temptation utility*, which plays a key role in determining how actual consumption choices depart from what commitment utility would dictate. A parameter, γ , regulates how strongly temptation utility influences consumption choices; $\gamma = 0$ delivers standard utility, where consumers act without commitment/self-control problems.

Using the Gul-Pesendorfer model, it is straightforward to ask normative questions. The purpose of this paper is to examine optimal tax policy with their model. In particular, we look at “Ramsey taxation”, i.e., we consider whether and how linear tax-transfer schemes can be used to improve consumer welfare. The restriction to linear schemes is arbitrary, and more general schemes would be more helpful, but linear schemes are of particular interest since most policy discussions are cast in terms of proportional taxes.¹ The question here, then, is how different “distortion rates” affect the welfare of consumers who suffer from

¹In Section 2.5.2 we discuss the “command” optimum in which the government chooses *for* the consumer, thereby eliminating temptation and achieving the best possible outcome for the consumer. The government could implement this policy by confiscating the consumer’s entire resources and then reducing his choice set to a singleton.

temptation and self-control problems.

We look at a two-period model with general preferences, except that we specialize temptation utility to reflect impatience, since this is the object of our study. In the two-period model, the consumption set faced by an agent is the usual triangle, and taxes alter the precise nature of the triangle. We first analyze “partial equilibrium”, where we let the government use a tax-transfer scheme that—for the consumer’s actual choice—uses up no net resources and thus is self-financing. For example, the government can make consumption in period 1 more expensive relative to consumption in period 2 by subsidizing period-2 consumption, and to the extent the consumer responds by buying more consumption in period 2 than his endowment, the government must use a lump-sum tax in period 1 to balance its budget. We show that, in general, taxation can improve welfare, and that a temptation towards impatience calls for subsidizing consumption in period 2. We also examine the size of these subsidies.

Subsidizing period-2 consumption improves welfare because it makes temptation less attractive. To see this, note that in Gul and Pesendorfer’s framework the consumer’s actual choice (in the two-period model) maximizes the sum of the commitment and temptation utilities. This choice represents a compromise between maximizing commitment utility and minimizing the cost of exerting self-control; the cost of self-control increases to the extent that the consumer’s actual choice deviates from succumbing completely to the temptation to consume more today. The consumer’s *welfare*, therefore, is the sum of the commitment and temptation utilities, *less* the temptation utility evaluated at the most tempting choice. Because of the envelope theorem, a small increase (from zero) in the subsidy for period-2 consumption has no effect on the sum of the commitment and temptation utilities. But this increase reduces the value of succumbing to temptation: the consumer receives a smaller subsidy if he gives in to temptation (because in that case he consumes more today and less tomorrow), but the (lump-sum) tax that the consumer pays to finance the subsidy remains unchanged (because it depends on the consumer’s *actual* choice).

We then consider general-equilibrium effects, which are important for two reasons. First, in an endowment economy, tax policy is not useful at all. In this case, the consumption allocation cannot be altered, and for it to be supported in equilibrium by a triangle budget set, the slope of this set, net of taxes, must be unaffected by policy. Thus, pre-tax prices adjust to undo fully the tax wedge. Second, we show that if there is intertemporal production (we consider the standard neoclassical investment technology), government policy again has a role to play, by altering equilibrium investment. In particular, using a representative-agent equilibrium model, we show that the partial-equilibrium result remains intact: it is optimal

to subsidize investment. The intuition underlying this result is the same as in the partial-equilibrium model, though general-equilibrium effects on prices reduce the size of the optimal subsidy.

Are our results special to the two-period model? In a setting with standard preferences and a choice between distorting either investment or labor supply, for example, Chamley (1986) and Judd (1985) show that, in the long run, the government should not distort investment. In our model, labor supply is inelastic, but, nonetheless, is it possible that in the long run, investment should not be distorted/subsidized? We show, with a simple example, that the answer is, in general, no. We extend the two-period model to a T -period model in a way guided by the applied macroeconomic literature, which uses time-additive, stationary utility. Thus, we let commitment utility take the standard form and allow temptation utility simply to have a different “current” (or short-run) discount factor than commitment utility, reflecting impatience. This means that temptation utility reflects “quasi-geometric” discounting of the future, which amounts to the assumption that nothing can be tempting other than changing current consumption relative to future consumption. Quasi-geometric temptation also nests two cases of special interest, one being the time-inconsistent preferences considered by Laibson (the case $\gamma = \infty$, where the consumer succumbs to temptation) and the other the special case in Gul and Pesendorfer (2004), where temptation utility puts zero weight on the future (thus the consumer is tempted to consume his entire wealth today). Then, specializing further to logarithmic period utility, we fully solve for the laissez-faire and optimal outcomes. We find that optimal investment subsidy rates increase over time in any finite-horizon model. More importantly, as T approaches infinity, the optimum calls for a constant subsidy rate on investment, in contrast to the Chamley-Judd result.

Section 2 looks at the two-period model and Section 3 looks at the T -period model. Section 4 concludes. Proofs of all propositions are gathered in an appendix.²

2 The two-period model

For illustrating temptation and self-control problems in the savings context, a two-period model captures much of the essence, and in this section we provide some general results for this setting. In Section 3 we then examine some aspects of the further dynamics that appear in models with more periods.

²We omit the proof of Proposition 4 for brevity but it is available upon request from the authors.

2.1 Preferences

A typical consumer in the economy values consumption today (c_1) and tomorrow (c_2). Specifically, the consumer has Gul-Pesendorfer preferences represented by two functions $u(c_1, c_2)$ and $v(c_1, c_2)$. The decision problem of a typical consumer, then, is:

$$\max_{c_1, c_2} \{u(c_1, c_2) + v(c_1, c_2)\} - \max_{\tilde{c}_1, \tilde{c}_2} v(\tilde{c}_1, \tilde{c}_2)$$

subject to a budget constraint that we will specify below. We make three assumptions.

Assumption 1 $u(c_1, c_2)$ and $v(c_1, c_2)$ are twice continuously differentiable.

Assumption 2 $\frac{u_1(c_1, c_2)}{u_2(c_1, c_2)} < \frac{v_1(c_1, c_2)}{v_2(c_1, c_2)}$ for all c_1 and c_2 .

Assumption 3 $u_1, u_2, v_1, v_2 > 0$; $u_{11}, u_{22}, v_{11}, v_{22} < 0$; and $u_{12}, v_{12} \geq 0$.

Assumption 2 specializes to the case where temptation utility is tilted towards current consumption more than is commitment utility.

2.2 Budget constraints

Each consumer is endowed with k_1 units of capital at the beginning of the first period and with one unit of labor in each period. Consumers rent these factors at given prices. Let r_1 (r_2) and w_1 (w_2) be the gross return on savings and wage rate in the first (second) period, respectively, and P be the price vector defined as $P = (r_1, r_2, w_1, w_2)$. We will specify the determination of prices in the following subsections. Given these prices, the consumer's budget constraint is described by the set

$$B(k_1, P) \equiv \{(c_1, c_2) : \exists k_2 : c_1 = r_1 k_1 + w_1 - k_2 \text{ and } c_2 = r_2 k_2 + w_2\},$$

where k_2 is the consumer's asset holding at the beginning of period 2 (i.e., his savings in period 1).

Inserting the definitions of the functions u and v into (2.1) and combining terms, a typical consumer's decision problem is:

$$\max_{(c_1, c_2) \in B(k_1, P)} \{u(c_1, c_2) + v(c_1, c_2)\} - \max_{(\tilde{c}_1, \tilde{c}_2) \in B(k_1, P)} v(\tilde{c}_1, \tilde{c}_2).$$

In this two-period problem, the “temptation” part of the problem (i.e., the second maximization problem in the objective function) plays no role in determining the consumer’s actions in period 1. The temptation part of the problem does, however, affect the consumer’s welfare, as we discuss below in Section 2.4.

Letting $\bar{u} = u + v$, the consumer’s intertemporal first-order condition is

$$\frac{\bar{u}_1(c_1, c_2)}{\bar{u}_2(c_1, c_2)} = \frac{u_1(c_1, c_2) + v_1(c_1, c_2)}{u_2(c_1, c_2) + v_2(c_1, c_2)} = r_2.$$

It is straightforward to see that the intertemporal consumption allocation (which, in effect, maximizes $u + v$) represents a compromise between maximizing u and maximizing v . In contrast, the allocation that maximizes the temptation utility is

$$\frac{v_1(\tilde{c}_1, \tilde{c}_2)}{v_2(\tilde{c}_1, \tilde{c}_2)} = r_2.$$

Assumptions 2 and 3 imply that $\tilde{c}_1 > c_1$ and $\tilde{c}_2 < c_2$.

2.3 Government policy

We will examine the effects of proportional taxes and subsidies. Thus, let there be a lump-sum transfer s and a proportional tax τ_i on investment in the first period. The consumer’s budget set, then, is:

$$B_\tau(k_1, P) \equiv \{(c_1, c_2) : \exists k_2 : c_1 = r_1 k_1 + w_1 + s - (1 + \tau_i)k_2 \text{ and } c_2 = r_2 k_2 + w_2\},$$

where k_2 is the consumer’s asset holding at the beginning of period 2 (i.e., his savings in period 1). We assume that the government balances its budget in each period. Since the government has no exogenous expenditures to finance, its budget constraint reads: $s = \tau_i \bar{k}_2$, where \bar{k}_2 is the representative agent’s savings in period 1.

The government’s objective is to choose the tax rate and transfer so that an individual’s welfare is maximized in equilibrium (subject to the government’s budget constraint). With a change in taxes, individuals are induced to behave differently, but in addition temptation changes because taxation changes the shape of the budget sets. It is thus not a priori clear how taxes influence equilibrium utility.

2.4 Partial equilibrium

In this section, we examine the effects of proportional taxes and subsidies for a fixed price vector $P = (r_1, r_2, w_1, w_2)$. The following proposition states that it is optimal to subsidize savings in this case.

Proposition 1 *In the partial-equilibrium two-period model, the optimal investment tax τ_i^{PE} is negative.*

As becomes clear in the proof of the proposition, the optimal investment subsidy is positive if the representative consumer's actual saving is greater than what would have been chosen had he succumbed to temptation. This can be explained intuitively. Consider increasing the investment subsidy from $\tau_i = 0$. The marginal effect of this increase on $u + v$ is zero in the two-period model since the consumer is choosing his saving optimally. However, the marginal effect of this increase on temptation utility v is negative. To see this, consider the fact that the government sets tax rates so as to balance the budget based on equilibrium behavior: in equilibrium, the consumer pays a lump-sum tax equal to the amount of the investment subsidy received. When the investment subsidy is positive, a consumer who deviates to save less would thus not receive as large an investment subsidy, while paying the same tax. Therefore, the temptation is now less attractive: the effect on temptation utility v of increasing the investment subsidy marginally is negative.

With optimal taxation, thus, the consumer is induced to save more, so that his intertemporal consumption allocation is tilted more towards the future than in the absence of taxation. At the same time, the change in the slope of the consumer's budget constraint reduces (other things equal) the temptation faced by the consumer. The net result is to increase the consumer's welfare.

2.5 General equilibrium

There are important differences between the cases with and without intertemporal production.

2.5.1 An endowment economy

In an endowment economy, prices adjust so that consumers choose to hold the endowment at all points in time. The proportional tax τ_i , therefore, cannot influence (realized) consumption. Furthermore, taxes are not useful for decreasing the disutility of self-control either.

In equilibrium, the slope of the budget line at the endowment point is given from preferences (where commitment utility and temptation utility both matter). Because equilibrium consumption cannot change in response to taxes, this slope does not change: the slope is determined by the net-of-tax return on savings, and any change in tax rates simply changes the before-tax return. Thus, taxes do not influence the choice set of consumers: whatever temptations consumers face, they cannot be influenced by a proportional tax.³

2.5.2 An economy with intertemporal production

In this section, we examine the effects of proportional taxes in a production-economy. Let f be a standard neoclassical aggregate production function and let there be standard geometric depreciation at rate d . The main difference between the partial-equilibrium analysis and the present analysis is that prices are given by

$$r_1 = r(\bar{k}_1) = 1 + f'(\bar{k}_1) - d \text{ and } w_1 = w(\bar{k}_1) = f(\bar{k}_1) - f'(\bar{k}_1)\bar{k}_1$$

and

$$r_2 = r(\bar{k}_2) = 1 + f'(\bar{k}_2) - d \text{ and } w_2 = w(\bar{k}_2) = f(\bar{k}_2) - f'(\bar{k}_2)\bar{k}_2 ,$$

so that tax policy can influence prices through an impact on investment. Proposition 2 states that the government can improve an individual's welfare by imposing a negative tax (i.e., a subsidy) on investment.

Proposition 2 *In the two-period production economy, the optimal investment tax τ_i^{GE} is negative.*

Propositions 1 and 2 establish that investment subsidies improve welfare, but they are silent on the sizes of both the optimal subsidies and the welfare improvements that accompany them. A proper quantitative analysis, however, requires both extending the model to include more time periods and finding a reasonable way to calibrate its key parameters (especially the parameters governing preferences). Krusell, Kuruşçu, and Smith (2009) tackles these tasks.

Nonetheless, there is a qualitative question of theoretical interest: how does optimal policy, and in particular the implied saving behavior, compare to that dictated by commitment

³Nonlinear taxes, of course, would change the consumer's equilibrium choice set, and would therefore affect the disutility of self-control.

utility (i.e., that which would be chosen if the consumer—or the government, via general non-linear taxation—had access to commitment)? Actual choices in this model are informed by both commitment and temptation utility, i.e., they end up “in between” what commitment utility and temptation utility would dictate. Here we specialize utility functions to show that there is no presumption that in general optimal saving lies in between too. In particular, optimal saving can actually prescribe less consumption today than would commitment utility.

We specialize with functional-form assumptions that fit the applied macroeconomic literature: we assume

$$\begin{aligned} u(c_1, c_2) &= \frac{c_1^{1-\sigma}}{1-\sigma} + \delta \frac{c_2^{1-\sigma}}{1-\sigma}, \text{ and} \\ v(c_1, c_2) &= \gamma \left\{ \frac{c_1^{1-\sigma}}{1-\sigma} + \delta \beta \frac{c_2^{1-\sigma}}{1-\sigma} \right\}, \end{aligned}$$

so that $\beta < 1$ regulates temptation impatience relative to commitment impatience. In this case, the consumer’s problem can be written as

$$\max_{(c_1, c_2) \in B_r(k_1, P)} (1 + \gamma) \frac{c_1^{1-\sigma}}{1-\sigma} + \delta (1 + \beta\gamma) \frac{c_2^{1-\sigma}}{1-\sigma} - \gamma \max_{(\tilde{c}_1, \tilde{c}_2) \in B_r(k_1, P)} \left\{ \frac{\tilde{c}_1^{1-\sigma}}{1-\sigma} + \delta \beta \frac{\tilde{c}_2^{1-\sigma}}{1-\sigma} \right\}.$$

In the expression above, $\gamma \left\{ \frac{c_1^{1-\sigma}}{1-\sigma} + \delta \beta \frac{c_2^{1-\sigma}}{1-\sigma} - \left[\frac{\tilde{c}_1^{1-\sigma}}{1-\sigma} + \delta \beta \frac{\tilde{c}_2^{1-\sigma}}{1-\sigma} \right] \right\}$ is the cost of self-control. As we show in Section 3, where we extend the model to T periods, under these functional-form assumptions the Gul-Pesendorfer model of temptation and self-control nests the multiple-selves model. In addition, the tractability provided by these assumptions allows us to obtain some additional analytical results.

We consider first the case of logarithmic (period) utility. The following proposition gives explicit solutions for both laissez-faire and optimal saving.

Proposition 3 *In the two-period model with logarithmic utility, laissez-faire savings are a fraction $\frac{\delta(1+\beta\gamma)}{1+\gamma+\delta(1+\beta\gamma)}$ of present-value wealth. The optimal investment tax is $\tau_i^* = \frac{1+\beta\gamma}{1+\gamma} - 1 < 0$, and the associated savings fraction is $\frac{\delta}{1+\delta}$.*

To obtain the savings rate under commitment, set $\gamma = 0$ (or $\beta = 1$) in the expression for the laissez-faire savings rate; to obtain the savings rate when succumbing to temptation, set $\gamma = \infty$. The laissez-faire saving rate, then, lies in between these two extremes. Moreover, the optimal savings rate is identical to the laissez-faire savings rate under commitment. This result holds because, in the special case of logarithmic utility, the ratio of

temptation consumption to actual consumption is a constant that depends only on preference parameters. This fact implies, in turn, that the cost of self-control depends only on preference parameters and, in particular, does not depend either on prices or taxes. Changes in the subsidy, consequently, leave the cost of self-control unchanged when utility is logarithmic, and the government in effect chooses the optimal subsidy rate simply to maximize commitment utility.

With logarithmic utility, the competitive equilibrium allocation with optimal taxation coincides with the command (or commitment) outcome, that is, the allocation that obtains when the government chooses *for* the consumer by restricting his consumption set to a singleton (or when the consumer does not suffer from self-control problems). Specifically, the command (or commitment) allocation maximizes welfare using the commitment utility function with $\beta = 1$. But welfare is higher under the command outcome than under the competitive equilibrium allocation with optimal taxation because the consumer does not incur a self-control cost when his choice set is a singleton.

Proposition 3 shows that logarithmic utility leads to more than a marginal distortion: the prescription is to distort all the way to the commitment policy. Is this case a bound on the size of the distortion, or does optimal policy sometimes prescribe a distortion that is strong enough to go beyond commitment utility? The following proposition answers this question affirmatively, simply by providing an analysis in the neighborhood of logarithmic utility.

Proposition 4 *Let $\tau_i(\sigma)$ be the optimal investment subsidy for a given σ . Then $\frac{d\tau_i(\sigma)}{d\sigma}|_{\sigma=1} < 0$ if and only if: $\beta \log \left(\frac{\beta(1+\gamma+\delta(1+\beta\gamma))}{(1+\beta\gamma)(1+\delta\beta)} \right) - \log \left(\frac{1+\gamma+\delta(1+\beta\gamma)}{(1+\gamma)(1+\delta\beta)} \right) + \frac{(1-\beta)(1+\delta)}{1+\gamma+\delta(1+\beta\gamma)} > 0$.*

Thus, given that $\tau_i(1) = \tau_i^* = (1 + \beta\gamma)/(1 + \gamma) - 1$, the optimal subsidy will be larger than the commitment subsidy for $\sigma > 1$ (provided that the rather lengthy condition in the proposition is satisfied).⁴ That is, when $\sigma > 1$, the competitive equilibrium allocation under optimal taxation is tilted *less* towards current consumption than is the commitment allocation, and when $\sigma < 1$ it is tilted *more* towards current consumption.

The intuition for this result is the following. When σ is higher the elasticity of intertemporal substitution (EIS) is lower. When this elasticity is low, actual consumption does not respond much to changes in the effective gross interest rate $(1 + \tau_i)^{-1} r(\bar{k}_2)$. Since actual

⁴We have not been able to find parameter values such that this lengthy condition is violated, and it is straightforward to prove that at least near $\beta = 0$ it is always satisfied.

consumption does not change much with a higher subsidy, actual utility $u + v$ is not affected much either. On the other hand, when the government increases the subsidy to saving, the consumer’s lifetime income under temptation behavior becomes lower since, by following temptation, the consumer ends up not receiving the subsidy despite paying a lump-sum tax. Thus, it is optimal to set a higher subsidy the lower is the EIS. In the extreme case in which actual consumption does not respond to changes in the effective gross return at all, it is optimal to increase the subsidy as much as possible because, in that case, a subsidy lowers temptation utility while not affecting actual utility at all. On the other hand, when utility is linear, the consumption allocation is very responsive to the subsidy and as a result prices completely offset the effect of the subsidy. Since, in this case, the effective gross interest rate cannot be influenced by government policy, tax policy is completely ineffective.

3 The T -period model

Does the prescription that investment should be subsidized extend to a longer-horizon model? The well-known Chamley-Judd result might suggest that it does not (for an excellent exposition and discussion of this result, see Atkeson, Chari, and Kehoe (1999)). A general analysis of this question is beyond the scope of the current paper, but it is relatively straightforward to construct a counterexample to show that no general result like the Chamley-Judd one holds for this model. The purpose of the present section is thus to provide analysis of a simple and interpretable special case in which investment subsidies indeed are called for not only in the short run but also in the long run.

To this end, we now extend the model above to have T periods. This extension requires us to specialize preferences, again along the lines of what seems useful for applied macroeconomic modeling and for comparisons with the Chamley-Judd literature. In particular, we use “quasi-geometric temptation”, which we show nests the Laibson model for general constant-relative-risk-aversion preferences (it is the $\gamma = \infty$ case). Our demonstration that investment should be subsidized in the long run relies on specializing further to logarithmic utility, since this assumption permits an explicit solution both for laissez-faire outcomes and optimal outcomes.

3.1 Quasi-geometric temptation

Consider a T -period (periods $0, 1, \dots, T$) production economy where taxes and transfers are allowed to be different across periods. The agent makes his decision taking as given the

aggregate prices as functions of the aggregate capital \bar{k} , the law of motion for aggregate capital $\bar{k}' = G_t(\bar{k})$, and the sequence of transfers and taxes. The problem of the price-taking agent in period t , using recursive notation (a prime symbolizes next-period values), is given by

$$U_t(k, \bar{k}) = \max_{c, k'} [u(c) + \delta U_{t+1}(k', G_t(\bar{k})) + V_t(k', G_t(\bar{k})) - \max_{\tilde{c}, \tilde{k}'} V_t(\tilde{k}', G_t(\bar{k}))],$$

where the temptation function is *quasi-geometric*:

$$V_t(k, \bar{k}) = \gamma[u(c) + \beta \delta U_{t+1}(k', G_t(\bar{k}))],$$

with a budget constraint (which applies for both actual and temptation choices) given by

$$c + (1 + \tau_{it})k' = r(\bar{k})k + w(\bar{k}) + s_t.$$

The investment subsidy τ_{it} is allowed to depend on time and the lump-sum transfer s_t varies with τ_{it} and \bar{k} so as to ensure that the government's budget balances. The consumer's actual savings are determined by a "realized" decision rule $k' = g_t(k, \bar{k})$; similarly, savings when succumbing to temptation are determined by a "temptation" decision rule $\tilde{k}' = \tilde{g}_t(k, \bar{k})$.

Definition 1 *A time- t recursive competitive equilibrium for this economy consists of a pair of decision rules $g_t(k, \bar{k})$ and $\tilde{g}_t(k, \bar{k})$, a pair of value functions $U_t(k, \bar{k})$ and $V_t(k, \bar{k})$, pricing functions $r(\bar{k})$ and $w(\bar{k})$, and a law of motion for aggregate capital $G_t(\bar{k})$, such that*

1. *given $U_t(k, \bar{k})$ and $V_t(k, \bar{k})$, $g_t(k, \bar{k})$ solves the maximization problem above and $\tilde{g}_t(k, \bar{k})$ maximizes $V_t(k, \bar{k})$.*
2. *prices are given by $r(\bar{k}) = 1 - d + f'(\bar{k})$ and $w(\bar{k}) = f(\bar{k}) - f'(\bar{k})\bar{k}$;*
3. *the law of motion for aggregate capital is consistent with the individual decision rule, i.e., $g_t(\bar{k}, \bar{k}) = G_t(\bar{k})$; and*
4. *the government budget balances in each period: $s_t = \tau_{it}G_t(\bar{k})$.*

We require the government to run a balanced budget in this definition, but this requirement is not restrictive, because a Ricardian-equivalence result obtains straightforwardly in this environment.⁵

⁵With borrowing constraints, Ricardian equivalence might fail to hold in the model of temptation and self-control even if the timing of taxes does not influence actual consumption choices or equilibrium interest rates. In particular, borrowing constraints could still affect welfare if they restrict the temptation choice but not the actual choice.

3.2 Generalized Euler equations

Solving for equilibrium requires finding two decision rules, one for actual savings decisions and one for temptation savings decisions. It is straightforward to derive a pair of “generalized Euler equations” (GEEs) that determine these two decision rules. These GEEs will prove useful for interpreting the policy results in the T -period model. The GEE for the actual choice is:

$$u'(c_t) = \delta \frac{1 + \beta\gamma}{1 + \gamma} \frac{r(\bar{k}_{t+1})}{1 + \tau_{it}} \{(1 + \gamma)u'(c_{t+1}) - \gamma u'(\tilde{c}_{t+1})\},$$

where c_t and c_{t+1} are the actual consumption levels in periods t and $t + 1$ and \tilde{c}_{t+1} is temptation consumption in period $t + 1$. The GEE for the temptation choice is

$$u'(\tilde{c}_t) = \delta\beta \frac{r(\bar{k}_{t+1})}{1 + \tau_{it}} \{(1 + \gamma)u'(\tilde{c}_{t+1}) - \gamma u'(\tilde{c}_{t+1})\},$$

where \tilde{c}_t and \tilde{c}_{t+1} are the consumption levels in periods t and $t + 1$ in the hypothetical case that the consumer succumbs to temptation today and \tilde{c}_{t+1} is temptation consumption in period $t + 1$ given that the consumer succumbs today.⁶

The GEEs differ from standard Euler equations in two ways. First, the discount factors are smaller than the discount factor, δ , for commitment utility (the discount factor in the GEE for actual consumption is between δ and the discount factor, $\beta\delta$, for temptation utility). Second, there is an additional term, $\gamma(u'_{t+1} - \tilde{u}'_{t+1})$, on the right-hand side of the GEEs. This term is positive because utility is strictly concave and temptation consumption exceeds actual consumption (assuming impatience). Thus, relative to the standard consumption-savings model, there is an additional benefit to saving here.

3.3 Characterization

In this section we specialize preferences to cases that are of particular interest from the perspective of the macroeconomics literature. These will then be used in the subsequent section, where we study optimal policy in the T -period model.

We look first at (period) utility functions with a constant elasticity of intertemporal substitution, i.e., $u(c) = c^{1-\sigma}/(1 - \sigma)$, for $\sigma > 0$ (or logarithmic utility if $\sigma = 1$). For this case, our model nests the Laibson formulation. In particular, Proposition 5 shows that (given prices) as $\gamma \rightarrow \infty$ the consumer’s value function converges to the one under commitment utility but evaluated at temptation consumption.

⁶Be aware, then, that \tilde{c}_{t+1} has two different meanings in the two GEEs.

Proposition 5 *Given a law of motion for aggregate capital, $\bar{k}' = G_t(\bar{k})$, and a sequence of taxes and transfers, as $\gamma \rightarrow \infty$, the Gul-Pesendorfer (GP) model converges to the Laibson model, i.e., the value functions and consumption choices of the consumer in the GP setting are given by*

$$\begin{aligned}
 U_t(k, \bar{k}) &= \frac{c^{1-\sigma}}{1-\sigma} + \delta U_{t+1}(k', G_t(\bar{k})), \\
 \text{where } (c, k') &= \arg \max \left(\frac{c^{1-\sigma}}{1-\sigma} + \delta \beta U_{t+1}(k', G_t(\bar{k})) \right) \\
 &\text{s.t. } c + (1 + \tau_{it})k' = r(\bar{k})k + w(\bar{k}) + s_t.
 \end{aligned}$$

This limit offers a resolution to the problem of which of the consumer's "selves" to use when assessing welfare in the multiple-selves model. Specifically, in this limit the consumer succumbs completely to temptation, but he evaluates welfare by discounting using the discount factors in commitment utility. For the case of logarithmic utility, we obtain a similar result regardless of the extent to which the consumer succumbs to temptation (i.e., for *any* value of γ).

Proposition 6 *Given a law of motion for aggregate capital, $\bar{k}' = G_t(\bar{k})$, and a sequence of taxes and transfers, when $u(c) = \log(c)$, the value function and consumption choices of the agent are given by*

$$\begin{aligned}
 U_t(k, \bar{k}) &= \log(c) + \delta U_{t+1}(k', G_t(\bar{k})) \\
 \text{where } (c, k') &= \arg \max \left[(1 + \gamma) \log(c) + \delta(1 + \beta\gamma) U_{t+1}(k', G_t(\bar{k})) \right] \\
 &\text{s.t. } c + (1 + \tau_{it})k' = r(\bar{k})k + w(\bar{k}) + s_t.
 \end{aligned}$$

This result holds because, as in the two-period model with logarithmic utility, both actual and temptation consumption are proportional (at any point in time) to lifetime income, with the constant of proportionality depending only on preference parameters (and not on prices or taxes). Thus, the ratio of actual to temptation consumption depends only on preference parameters at any point in time. As in the two-period model, then, the self-control cost depends only on preference parameters and does not vary either with prices or policy.

3.4 Optimal policy

In this section, we study optimal taxes under commitment with logarithmic preferences allowing any values for β and γ , thus nesting the Laibson setting as well as the setting

proposed in Gul and Pesendorfer (2004), where $\beta = 0$. As in the previous sections, the government's objective is to maximize time-0 lifetime utility of the representative agent. Proposition 6 shows that the welfare of the representative agent at time 0 is:

$$U_0(\bar{k}_0, \bar{k}_0) = \text{a constant} + \sum_{t=0}^T \delta^t u(c_t).$$

The government's goal is to maximize this welfare function subject to the aggregate resource constraint:

$$c_t + \bar{k}_{t+1} - (1 - d) \bar{k}_t = f(\bar{k}_t).$$

The welfare-maximizing consumption allocation, therefore, must satisfy the following first-order condition at every point in time:

$$\frac{u'(c_t)}{u'(c_{t+1})} = \delta r(\bar{k}_{t+1}).$$

As in the two-period model with logarithmic utility, then, the government's optimal policy replicates the commitment allocation.

To find the tax policy that generates the commitment allocation as a competitive equilibrium outcome, it is straightforward to use the optimality conditions of a typical (competitive) consumer to find the sequence of tax rates that induce him to choose it (see the proof of Proposition 7 for details). Proposition 7 gives the optimal sequence of subsidies to investment.

Proposition 7 *The optimal subsidy at time t is given by*

$$\tau_{it} = \begin{cases} \frac{\gamma(\beta-1)}{1+\gamma} & \text{for } t = T - 1 \\ \frac{\gamma(\beta-1)}{1+\gamma+\delta(1+\delta+\dots+\delta^{T-2-t})(1+\beta\gamma)} & \text{for } t < T - 1. \end{cases}$$

The optimal investment subsidies are all positive because, when the self-control cost is independent of prices and policies (as it is under logarithmic utility), the government's objective reduces to maximizing the commitment utility function. Thus, the optimal government policy is to replicate the commitment savings rate, and since the savings rate is lower in competitive equilibrium than under commitment, the optimal policy is to subsidize savings.

The optimal subsidies are also increasing as an individual individual comes closer to period T : $\tau_{i,T-2} < \dots < \tau_{i1} < \tau_{i0} < 0$. To see why, examine the GEE at time t :

$$\frac{c_{t+1}}{c_t} = \frac{\delta(1+\beta\gamma)r(\bar{k}_{t+1})}{1+\gamma} \frac{1}{1+\tau_{it}} \left[1 + \gamma \left(1 - \frac{c_{t+1}}{\tilde{c}_{t+1}} \right) \right].$$

The ratio of actual consumption to temptation consumption grows larger t increases, so the term in square brackets on the right-hand side of the (rearranged) GEE is larger at earlier than at later dates.⁷ As a result, the right-hand side of the Euler equation is closer to the right-hand side of the commitment Euler equation at early dates. Replicating the right-hand side of the commitment Euler equation, therefore, requires a smaller subsidy at earlier dates.

An immediate implication of Proposition 7 is

Corollary 1 *As $T \rightarrow \infty$, the optimal subsidy at any fixed t converges to:*

$$\tau_i = \frac{\gamma(\beta - 1)}{1 + \gamma + \frac{\delta}{1-\delta}(1 + \beta\gamma)}.$$

Thus, the celebrated Chamley-Judd result that investment (alternatively, capital income) should be undistorted in the long run does not apply in this model. For any finite horizon, the optimal subsidy rate will in fact increase over time, and for the infinite horizon case the optimal subsidy rate is time-invariant.⁸

4 Conclusions and remarks

The present paper makes clear that when consumers suffer from temptation and self-control problems, linear tax schedules can improve consumers' welfare, even though such schedules are not very powerful tools for restricting consumers' choice sets. The direction of the change is the expected one—when temptation is characterized by “excessive impatience”, optimal policy is to subsidize savings. Such subsidies improve welfare because they make succumbing to temptation less attractive. This result obtains in partial equilibrium and

⁷To see why ratio of actual to temptation consumption decreases with age, note that consumption at time t is given by

$$c_t = \frac{1}{1 + \frac{\delta(1+\beta\gamma)}{1+\gamma}m_t}Y_t$$

and temptation consumption is given by

$$\tilde{c}_t = \frac{1}{1 + \delta\beta m_t}Y_t,$$

where Y_t is lifetime income at time t and $m_t = 1 + \delta + \dots + \delta^{T-t-2}$. The m_t s decrease over time, so c_t/\tilde{c}_t increases over time.

⁸A similar result holds when u has constant elasticity of intertemporal substitution different from one and the consumer succumbs to temptation (i.e. the case with $\gamma = \infty$); details can be provided from the authors upon request. In this case, the self-control cost is zero, so that, as under logarithmic utility, this cost does not depend either on prices or policy. The government's objective, then, is to replicate the commitment savings rates, which requires subsidizing investment since savings rates are too low in competitive equilibrium.

in general equilibrium, so long as the general-equilibrium economy involves intertemporal production. The general-equilibrium effects dampen the optimal distortion relative to the partial-equilibrium case; in an endowment economy, the general-equilibrium effects on prices completely offset the partial-equilibrium welfare gains and the linear tax policy is powerless. It is also possible that the optimal subsidy is large enough that the implied consumer saving is higher than it would have been had the consumer had full commitment (and, thus, neither temptation nor self-control would be an issue). Clearly, in this case, the large subsidy to saving is optimal because linear taxes cannot alone deal completely with the temptation problem, so that the high subsidy to saving instead helps to make temptation less costly for the consumer.

The results from the two-period model are robust in that multi-period settings also involve distortions. We look at a case of particular interest from the perspective of the applied macroeconomic literature and show that distortions are optimal also in the long run.

How large are the welfare gains from optimal linear policy? This question is quantitative and requires a quantitative answer. Such an answer has to build on a calibration of our “new” parameters governing the nature and strength of the temptation and self-control problems. Also, from the results in this paper it is clear that the precise nature of the technology is important as well—since endowment economies leave no room at all for linear taxes to improve on allocations. In Krusell, Kuruşçu, and Smith (2009) we solve a calibrated macroeconomic model of the kind considered here in order to answer the quantitative question.

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5 Appendix

Proof of Proposition 1: Set the derivative of prices with respect to policy equal to zero in the proof of Proposition 2 to obtain the proof of this proposition.

Proof of Proposition 2: Letting $\bar{u}(c_1, c_2) = u(c_1, c_2) + v(c_1, c_2)$, the first-order conditions for the competitive consumer’s maximization problem are given by

$$(1 + \tau_i)\bar{u}_1(c_1, c_2) = r(\bar{k}_2)\bar{u}_2(c_1, c_2) \text{ and } (1 + \tau_i)v_1(\tilde{c}_1, \tilde{c}_2) = r(\bar{k}_2)v_2(\tilde{c}_1, \tilde{c}_2),$$

where

$$c_1 = r(\bar{k}_1)k_1 + w(\bar{k}_1) + s - (1 + \tau_i)k_2 \text{ and } c_2 = r(\bar{k}_2)k_2 + w(\bar{k}_2).$$

$$\tilde{c}_1 = r(\bar{k}_1)k_1 + w(\bar{k}_1) + s - (1 + \tau_i)\tilde{k}_2 \text{ and } \tilde{c}_2 = r(\bar{k}_2)\tilde{k}_2 + w(\bar{k}_2).$$

Using the government budget constraint, $s = \tau_i\bar{k}_2$, the value function of the representative agent is given by

$$U(\bar{k}_1, \bar{k}_1, \tau_i) = \bar{u}(r(\bar{k}_1)\bar{k}_1 + w(\bar{k}_1) + s - (1 + \tau_i)\bar{k}_2, r(\bar{k}_2)k_2 + w(\bar{k}_2)) \\ - v(r(\bar{k}_1)\bar{k}_1 + w(\bar{k}_1) + s - (1 + \tau_i)\tilde{k}_2, r(\bar{k}_2)\tilde{k}_2 + w(\bar{k}_2)).$$

Differentiating the value function with respect to τ_i and using the consumer's first-order conditions, we obtain

$$\frac{dU}{d\tau_i} = \bar{u}_1(c_1, c_2)\tau_i \frac{d\bar{k}_2}{d\tau_i} - v_1(\tilde{c}_1, \tilde{c}_2)\{\bar{k}_2 - \bar{\bar{k}}_2 + \tau_i \frac{d\bar{k}_2}{d\tau_i}\} - v_2(\tilde{c}_1, \tilde{c}_2)r'(\bar{k}_2)(\bar{\bar{k}}_2 - \bar{k}_2) \frac{d\bar{k}_2}{d\tau_i},$$

where we have also used the fact that $w'(\bar{k}_2) + r'(\bar{k}_2)\bar{k}_2 = 0$ and $w'(\bar{k}_2) + r'(\bar{k}_2)\bar{\bar{k}}_2 = r'(\bar{k}_2)(\bar{\bar{k}}_2 - \bar{k}_2)$. Setting the derivative to zero, we obtain the following equation that determines the optimal τ_i :

$$\tau_i = \frac{v_1(\tilde{c}_1, \tilde{c}_2) \left\{ 1 - \widetilde{MRS} \frac{dr(\bar{k}_2)}{d\tau_i} \right\} (\bar{k}_2 - \bar{\bar{k}}_2)}{[\bar{u}_1(c_1, c_2) - v_1(\tilde{c}_1, \tilde{c}_2)] \frac{d\bar{k}_2}{d\tau_i}},$$

where $\widetilde{MRS} = \frac{v_2(\tilde{c}_1, \tilde{c}_2)}{v_1(\tilde{c}_1, \tilde{c}_2)}$.

Using the first order conditions for the consumer, and assumptions 2 and 3, it is easy to see that $c_1 < \tilde{c}_1$ and $c_2 > \tilde{c}_2$ and that $\bar{u}_1(c_1, c_2) - v_1(\tilde{c}_1, \tilde{c}_2) = u_1(c_1, c_2) + v_1(c_1, c_2) - v_1(\tilde{c}_1, \tilde{c}_2) > 0$. We also have $\bar{k}_2 - \bar{\bar{k}}_2 > 0$. Moreover, taking the derivative of the first-order condition for the actual choice with respect to τ_i , we can show that $\frac{d\bar{k}_2}{d\tau_i} < 0$. In partial equilibrium, $1 - \widetilde{MRS} \frac{dr(\bar{k}_2)}{d\tau_i} = 1$ since prices are constant. Given this, it is obvious that the optimal tax is negative in partial equilibrium. For the general equilibrium case, we need to show that $1 - \widetilde{MRS} \frac{dr(\bar{k}_2)}{d\tau_i} > 0$, which implies that the optimal tax is negative, i.e., $\tau_i < 0$. In equilibrium,

$$r(\bar{k}_2) \times MRS = r(\bar{k}_2) \times \widetilde{MRS} = 1 + \tau_i,$$

where $MRS = \frac{\bar{u}_2(c_1, c_2)}{\bar{u}_1(c_1, c_2)}$. Therefore, it is enough to show that $1 - MRS \frac{dr(\bar{k}_2)}{d\tau_i} > 0$. Taking the derivative of $r(\bar{k}_2) \times MRS = 1 + \tau_i$ with respect to τ_i , we obtain $\frac{dMRS}{d\tau_i} r(\bar{k}_2) + MRS \frac{dr(\bar{k}_2)}{d\tau_i} = 1$. Given that $MRS = \frac{\bar{u}_2(c_1, c_2)}{\bar{u}_1(c_1, c_2)}$ and $\frac{d\bar{k}_2}{d\tau_i} < 0$, it is then clear that $\frac{dMRS}{d\tau_i} > 0$.

Proof of Proposition 3: See the proof to Proposition 7, which studies a T -period economy with logarithmic utility.

Proof of Proposition 4: This proof is omitted for brevity but is available upon request from the authors.

Proof of Proposition 5: Let Y_t be the lifetime income from period t on which is given by $r_t k_t + w_t + \frac{w_{t+1}}{r_{t+1}} + \dots$ for a given price sequence $\{(r_t, w_t)\}_{t=0}^T$. (For simplicity, we assume that taxes and transfers are zero in the proof of the proposition, but it is straightforward to adapt the proof to allow for non-zero taxes.) The budget constraint of the agent at time t

in terms of consumption in that period and the lifetime income from next period on is given by

$$c_t + \frac{Y_{t+1}}{r_{t+1}} = Y_t.$$

To prove this proposition, we show that the optimization problem of the consumer in period t takes the following form:

$$\begin{aligned} U_t(Y_t) &= \max_{Y_{t+1}} \frac{1+\gamma}{1-\sigma} \left(Y_t - \frac{Y_{t+1}}{r_{t+1}} \right)^{1-\sigma} + \delta(1+\beta\gamma)U_t(Y_{t+1}) \\ &\quad - \gamma \left\{ \max_{\tilde{Y}_{t+1}} \frac{1}{1-\sigma} \left(Y_t - \frac{\tilde{Y}_{t+1}}{r_{t+1}} \right)^{1-\sigma} + \delta\beta U_t(\tilde{Y}_{t+1}) \right\} \end{aligned}$$

where $U_t(Y_t)$ is given by

$$U_t(Y_t) = b_t \frac{Y_t^{1-\sigma}}{1-\sigma}.$$

b_t is a constant that depends on utility parameters, prices, and time period t . In addition, starting from the last period, we show that, as $\gamma \rightarrow \infty$, $Y_t \rightarrow \tilde{Y}_t$, and that the value function of the consumer for each t is given by

$$U_t(Y_t) = \frac{1}{1-\sigma} \left(Y_t - \frac{\tilde{Y}_{t+1}}{r_{t+1}} \right)^{1-\sigma} + \delta U_t(\tilde{Y}_{t+1}).$$

In the expression above and below, we simply notation by omitting the dependence of the temptation maximizer on the state.

We start with period $T-1$. Note that $b_T = 1$. Thus, the value function in the last period of life is

$$U_T(Y_T) = \frac{Y_T^{1-\sigma}}{1-\sigma}.$$

Given the last period value function, the problem of the agent in period $T-1$ is given by

$$\begin{aligned} U_{T-1}(Y_{T-1}) &= \max_{Y_T} \frac{1+\gamma}{1-\sigma} \left(Y_{T-1} - \frac{Y_T}{r_T} \right)^{1-\sigma} + \frac{\delta(1+\beta\gamma)}{1-\sigma} Y_T^{1-\sigma} \\ &\quad - \gamma \left\{ \max_{\tilde{Y}_T} \frac{1}{1-\sigma} \left(Y_{T-1} - \frac{\tilde{Y}_T}{r_T} \right)^{1-\sigma} + \frac{\delta\beta}{1-\sigma} \tilde{Y}_T^{1-\sigma} \right\}. \end{aligned}$$

The optimal decision rules for the actual and temptation solutions, respectively, are given by

$$c_{T-1} = \frac{1}{1 + \left(\frac{\delta(1+\beta\gamma)}{1+\gamma} \right)^{1/\sigma} r_T^{(1-\sigma)/\sigma}} Y_{T-1} \quad \text{and} \quad Y_T = \frac{\left(\frac{\delta(1+\beta\gamma)}{1+\gamma} \right)^{1/\sigma} r_T^{1/\sigma}}{1 + \left(\frac{\delta(1+\beta\gamma)}{1+\gamma} \right)^{1/\sigma} r_T^{(1-\sigma)/\sigma}} Y_{T-1}$$

and

$$\tilde{c}_{T-1} = \frac{1}{1 + (\delta\beta)^{1/\sigma} r_T^{(1-\sigma)/\sigma}} Y_{T-1} \text{ and } \tilde{Y}_T = \frac{(\delta\beta)^{1/\sigma} r_T^{1/\sigma}}{1 + (\delta\beta)^{1/\sigma} r_T^{(1-\sigma)/\sigma}} Y_{T-1}.$$

We show that as $\gamma \rightarrow \infty$, the value function converges to the commitment value function while the actual decision of the agent converges to the temptation decision rule. From the expressions above, it should be clear that $c_{T-1} \rightarrow \tilde{c}_{T-1}$ and $Y_T \rightarrow \tilde{Y}_T$. Next we show that

$$U_{T-1}(Y_{T-1}) = \frac{\tilde{c}_{T-1}^{1-\sigma}}{1-\sigma} + \delta \frac{\tilde{Y}_T^{1-\sigma}}{1-\sigma}.$$

To show this, we need to show that $\lim_{\gamma \rightarrow \infty} \gamma(c_{T-1}^{1-\sigma} + \delta\beta Y_T^{1-\sigma} - \tilde{c}_{T-1}^{1-\sigma} - \delta\beta \tilde{Y}_T^{1-\sigma}) = 0$. As $\gamma \rightarrow \infty$, $c_{T-1}^{1-\sigma} + \delta\beta Y_T^{1-\sigma} - \tilde{c}_{T-1}^{1-\sigma} - \delta\beta \tilde{Y}_T^{1-\sigma} \rightarrow 0$. Thus, we need to apply L'Hopital rule. For this purpose, first use the optimal decision rules to obtain

$$\lim_{\gamma \rightarrow \infty} \gamma \left(\frac{1 + \delta\beta \left(\frac{\delta(1+\beta\gamma)}{1+\gamma} \right)^{(1-\sigma)/\sigma} r_T^{(1-\sigma)/\sigma}}{\left(1 + \left(\frac{\delta(1+\beta\gamma)}{1+\gamma} \right)^{1/\sigma} r_T^{(1-\sigma)/\sigma} \right)^{1-\sigma}} - 1 \right). \quad (1)$$

Applying L'Hopital by letting $\tilde{\gamma} = 1/\gamma$ and $\tilde{\gamma} \rightarrow 0$, it is easy to show that the limit above converges to zero. Thus,

$$\lim_{\gamma \rightarrow \infty} U_{T-1}(Y_{T-1}) = \frac{\tilde{c}_{T-1}^{1-\sigma}}{1-\sigma} + \delta \frac{\tilde{Y}_T^{1-\sigma}}{1-\sigma}.$$

The problem of the agent in period t is exactly the same except that expression (1) contains b_{t+1} as follows:

$$\lim_{\gamma \rightarrow \infty} \gamma \left(\frac{1 + \delta\beta b_{t+1} \left(\frac{\delta(1+\beta\gamma)b_{t+1}}{1+\gamma} \right)^{(1-\sigma)/\sigma} r_T^{(1-\sigma)/\sigma}}{\left(1 + \left(\frac{\delta(1+\beta\gamma)b_{t+1}}{1+\gamma} \right)^{1/\sigma} r_T^{(1-\sigma)/\sigma} \right)^{1-\sigma}} - 1 \right), \quad (2)$$

where $b_T = 1$ and b_t is given recursively as

$$b_t = (1 + \gamma) \left(1 + \left(\frac{\delta(1 + \beta\gamma)b_{t+1}}{1 + \gamma} \right)^{1/\sigma} r_{t+1}^{(1-\sigma)/\sigma} \right)^\sigma - \gamma \left(1 + (\delta\beta b_{t+1})^{1/\sigma} r_{t+1}^{(1-\sigma)/\sigma} \right)^\sigma.$$

Using this equation and the fact that $b_T = 1$, we can show that $\lim_{\tilde{\gamma} \rightarrow 0} \frac{db_{t+1}}{d\tilde{\gamma}} = 0$ for all t , which implies that the expression in (2) converges to zero as $\tilde{\gamma} \rightarrow 0$. Thus,

$$U_t(Y_t) = \frac{\tilde{c}_t^{1-\sigma}}{1-\sigma} + \delta b_{t+1} \frac{\tilde{Y}_{t+1}^{1-\sigma}}{1-\sigma} = \frac{\tilde{c}_t^{1-\sigma}}{1-\sigma} + \delta U_{t+1}(\tilde{Y}_{t+1}).$$

Proof of Proposition 6: Simply let $\gamma \rightarrow \infty$ in the proof of Proposition 7.

Proof of Proposition 7:

Problem at time $T - 1$:

The consumer's problem reads

$$\max_{c_{T-1}, c_T} (1 + \gamma) \log(c_{T-1}) + \delta(1 + \beta\gamma) \log(c_T) - \gamma \max_{\tilde{c}_{T-1}, \tilde{c}_T} \log(\tilde{c}_{T-1}) + \delta\beta \log(\tilde{c}_T)$$

subject to the budget constraints

$$c_{T-1} + (1 + \tau_{i,T-1})k_T = r(\bar{k}_{T-1})k_T + w(k_T) + s_T \text{ and } c_T = Y_T = r(\bar{k}_T)k_T + w(\bar{k}_T).$$

The rest-of-lifetime budget constraint is thus

$$c_{T-1} + c_T \frac{1 + \tau_{i,T-1}}{r(\bar{k}_T)} = r(\bar{k}_{T-1})k_{T-1} + w(\bar{k}_{T-1}) + s_{T-1} + w(\bar{k}_T) \frac{1 + \tau_{i,T-1}}{r(\bar{k}_T)} = Y_{T-1}.$$

The first-order condition is $\frac{1}{c_{T-1}} = \frac{\delta(1+\beta\gamma)}{1+\gamma} \frac{r(\bar{k}_T)}{1+\tau_{i,T-1}} \frac{1}{c_T}$. Inserting c_T into the rest-of-lifetime budget constraint, we obtain

$$c_{T-1} = \frac{1 + \gamma}{1 + \gamma + \delta(1 + \beta\gamma)} Y_{T-1} \text{ and } c_T = \frac{\delta(1 + \beta\gamma)}{1 + \gamma + \delta(1 + \beta\gamma)} \frac{r(\bar{k}_T)}{1 + \tau_{i,T-1}} Y_{T-1}.$$

This implies

$$\tilde{c}_{T-1} = \frac{1}{1 + \delta\beta} Y_{T-1} \text{ and } \tilde{c}_T = \frac{\delta\beta}{1 + \delta\beta} \frac{r(\bar{k}_T)}{1 + \tau_{i,T-1}} Y_{T-1}.$$

Notice that the c and the \tilde{c} are constant multiples of each other. As a result, the value function becomes

$$U_{T-1}(k_{T-1}, \bar{k}_{T-1}, \tau) = \log(c_{T-1}) + \delta \log(c_T) + \text{a constant}.$$

Setting $k_{T-1} = \bar{k}_{T-1}$, the government's problem reduces to maximizing

$$U_{T-1}(\bar{k}_{T-1}, \bar{k}_{T-1}, \tau) = \log(c_{T-1}) + \delta \log(c_T)$$

subject to economy's resource constraints in both periods:

$$c_{T-1} + \bar{k}_T = (1 - d)\bar{k}_{T-1} + f(\bar{k}_{T-1}) \text{ and } c_T = (1 - d)\bar{k}_T + f(\bar{k}_T).$$

Thus, the optimal allocation must satisfy

$$\frac{1}{c_{T-1}} = \delta r(\bar{k}_T) \frac{1}{c_T}.$$

The government implements these allocations by setting the tax rates to replicate these allocations in competitive equilibrium. That is, the government chooses $\tau_{i,T-1} = \frac{\gamma(\beta-1)}{1+\gamma}$.

Now rewrite the value function in period $T-1$ to be used in the problem of the consumer in period $T-2$ by inserting the consumption allocations as functions of Y_{T-1} . This delivers

$$U_{T-1}(k_{T-1}, \bar{k}_{T-1}, \tau) = (1 + \delta) \log(Y_{T-1}) + \delta \log(r(\bar{k}_T)/(1 + \tau_{i,T-1})) + \text{a constant.}$$

Problem at time $T-2$: Using the $T-2$ budget constraint and the rest-of-lifetime budget constraint at time $T-1$ for the consumer, we obtain the rest-of-lifetime budget constraint at time $T-2$ as

$$\begin{aligned} c_{T-2} + \frac{1 + \tau_{i,T-2}}{r(\bar{k}_{T-1})} Y_{T-1} &= Y_{T-2} \\ &= r(\bar{k}_{T-2})k_{T-2} + w(\bar{k}_{T-2}) + s_{T-2} + \frac{w(\bar{k}_{T-1}) + s_{T-1}}{r(\bar{k}_{T-1})} (1 + \tau_{i,T-2}) + \frac{w(\bar{k}_T)}{r(\bar{k}_{T-1})r(\bar{k}_T)} (1 + \tau_{i,T-2})(1 + \tau_{i,T-1}). \end{aligned}$$

The objective of the government is to maximize

$$\begin{aligned} \max_{c_{T-2}, Y_{T-1}} (1 + \gamma) \log(c_{T-2}) + \delta(1 + \beta\gamma) \left[(1 + \delta) \log(Y_{T-1}) + \delta \log\left(\frac{r(\bar{k}_T)}{1 + \tau_{i,T-1}}\right) + \text{a constant} \right] \\ - \gamma \max_{\tilde{c}_{T-2}, \tilde{Y}_{T-1}} \log(\tilde{c}_{T-2}) + \delta\beta \left[(1 + \delta) \log(\tilde{Y}_{T-1}) + \delta \log\left(\frac{r(\bar{k}_T)}{1 + \tau_{i,T-1}}\right) + \text{a constant.} \right] \end{aligned}$$

The first-order condition is

$$\frac{1}{c_{T-2}} = \frac{\delta(1 + \delta)(1 + \beta\gamma)}{1 + \gamma} \frac{r(\bar{k}_{T-1})}{1 + \tau_{i,T-2}} \frac{1}{Y_{T-1}}.$$

Using the budget constraint, we obtain

$$c_{T-2} = \frac{1 + \gamma}{1 + \gamma + \delta(1 + \delta)(1 + \beta\gamma)} Y_{T-2} \quad \text{and} \quad Y_{T-1} = \frac{\delta(1 + \delta)(1 + \beta\gamma)}{1 + \gamma + \delta(1 + \delta)(1 + \beta\gamma)} \frac{r(\bar{k}_{T-1})}{1 + \tau_{i,T-2}} Y_{T-2}.$$

Inserting Y_{T-1} in terms of c_{T-1} into the consumer's problem, we obtain the following Euler equation.

$$\frac{1}{c_{T-2}} = \frac{\delta(1 + \delta)(1 + \beta\gamma)}{1 + \gamma + \delta(1 + \beta\gamma)} \frac{r(\bar{k}_{T-1})}{1 + \tau_{i,T-2}} \frac{1}{c_{T-1}}.$$

The temptation allocations are given by

$$\tilde{c}_{T-2} = \frac{1}{1 + \delta\beta(1 + \delta)} Y_{T-2} \quad \text{and} \quad \tilde{Y}_{T-1} = \frac{\delta\beta(1 + \delta)}{1 + \delta\beta(1 + \delta)} \frac{r(\bar{k}_{T-1})}{1 + \tau_{i,T-2}} Y_{T-2}.$$

The objective function of the government is

$$U_{T-2}(k_{T-2}, \bar{k}_{T-2}, \tau_i) = \log(c_{T-2}) + \delta(1 + \delta) \log(Y_{T-1}) + \delta^2 \log\left(\frac{r(\bar{k}_T)}{1 + \tau_{i,T-1}}\right) + \text{a constant.}$$

Since c_{T-1} is a multiple of Y_{T-1} and c_T is a multiple of $\left(\frac{r(\bar{k}_T)}{1 + \tau_{i,T-1}}\right) Y_{T-1}$, inserting those we obtain

$$U_{T-2}(k_{T-2}, \bar{k}_{T-2}, \tau_i) = \log(c_{T-2}) + \delta \log(c_{T-1}) + \delta^2 \log(c_T) + \text{a constant.}$$

Then, the optimal consumption allocation for the government's problem is given by the first-order condition

$$\frac{1}{c_{T-2}} = \delta r(\bar{k}_{T-1}) \frac{1}{c_{T-1}}.$$

To implement this allocation, the government sets

$$\frac{\delta(1 + \delta)(1 + \beta\gamma)}{1 + \gamma + \delta(1 + \beta\gamma)} \frac{1}{1 + \tau_{i,T-2}} = \delta,$$

which delivers $\tau_{i,T-2} = \frac{\gamma(\beta-1)}{1 + \gamma + \delta(1 + \beta\gamma)}$.

Problem at $T - 3$: The first-order condition for the consumer is

$$\frac{1}{c_{T-3}} = \frac{\delta(1 + \delta + \delta^2)(1 + \beta\gamma)}{1 + \gamma} \frac{r(\bar{k}_{T-2})}{1 + \tau_{i,T-3}} \frac{1}{Y_{T-2}} = \frac{\delta(1 + \delta + \delta^2)(1 + \beta\gamma)}{1 + \gamma + \delta(1 + \delta)(1 + \beta\gamma)} \frac{r(\bar{k}_{T-2})}{1 + \tau_{i,T-3}} \frac{1}{c_{T-2}}.$$

Then, the government sets s_{T-3} so that

$$\frac{\delta(1 + \delta + \delta^2)(1 + \beta\gamma)}{1 + \gamma + \delta(1 + \delta)(1 + \beta\gamma)} \frac{1}{1 + \tau_{i,T-3}} = \delta.$$

This gives $\tau_{i,T-3} = \frac{\gamma(\beta-1)}{1 + \gamma + \delta(1 + \delta)(1 + \beta\gamma)}$.

Problem at $T - j$: Continuing the procedure backwards, we can show that

$$\tau_{i,T-j} = \frac{\gamma(\beta-1)}{1 + \gamma + \delta(1 + \delta + \dots + \delta^{j-2})(1 + \beta\gamma)}.$$