What Elasticity of the Matching Function is consistent with U.S. Aggregate Labor Market Data?

Björn Brügemann*

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Abstract

The elasticity of the matching function is a key parameter of the search and matching model. There is disagreement which value of this elasticity is appropriate in the context of studying the cyclical behavior of the U.S. labor market. Shimer (2005) obtains an estimate of 0.28 by regressing the job finding probability on the vacancy-unemployment ratio. Mortensen and Nagypál (2007) infer a value of 0.54 from the slope of the Beveridge curve. Both approaches rely on assumptions that are problematic. After addressing these problems, both approaches yield estimates between 0.37 and 0.46.

*Department of Economics, Yale University, 28 Hillhouse Avenue, CT 06511. E-mail: bjoern.bruegemann@yale.edu.

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1 Environment

Time is continuous. There is a unit mass of ex ante identical infinitely lived workers participating in the labor market. At a point in time one of these workers is either employed or unemployed. There are no transitions between participation and non-participation. The process by which unemployed workers find jobs is modeled as a matching technology. Specifically, the flow of newly employed workers is given by $m(u, v)$ where u is the number of unemployed workers, v is the number of vacancies, and m is a constant returns to scale matching function. Throughout it is assumed that the functional form of the matching function is Cobb-Douglas

$$m(u, v) = \mu u^{1-\eta} v^{\eta}.$$

Define the vacancy-unemployment ratio $\theta \equiv \frac{v}{u}$. The arrival rate of jobs from the perspective of a worker is $\frac{m(u,v)}{u}$ and can be written as a function of the v-u ratio θ:

$$f(\theta) \equiv m(1, \theta) = \mu\theta^{\eta}. \tag{1}$$

This arrival rate is referred to as the *job finding rate*.

Employed workers exit from employment into unemployment with time varying arrival rate x_t. This arrival rate is referred to as the *employment exit rate*. There is no on-the-job search.

Table 1: Standard Deviations and Correlations

		$\tilde{u}$	$\tilde{e}$	$\tilde{v}$	$\tilde{\theta}$	$\tilde{f}$	$\tilde{F}$	$\tilde{x}$	$\tilde{X}$
St. dev.		0.190	0.014	0.202	0.382	0.162	0.118	0.075	0.076
	$\tilde{u}$	1	−0.735	−0.894	−0.972	−0.954	−0.949	0.709	0.709
	$\tilde{e}$		1	0.800	0.789	0.775	0.784	−0.397	−00.397
	$\tilde{v}$			1	0.975	0.893	0.897	−0.684	−0.684
Corr.	$\tilde{\theta}$				1	0.948	0.948	−0.715	−0.715
	$\tilde{f}$					1	0.999	−0.584	−0.584
	$\tilde{F}$						1	−0.574	−0.574
	$\tilde{x}$							1	1
	$\tilde{X}$								1

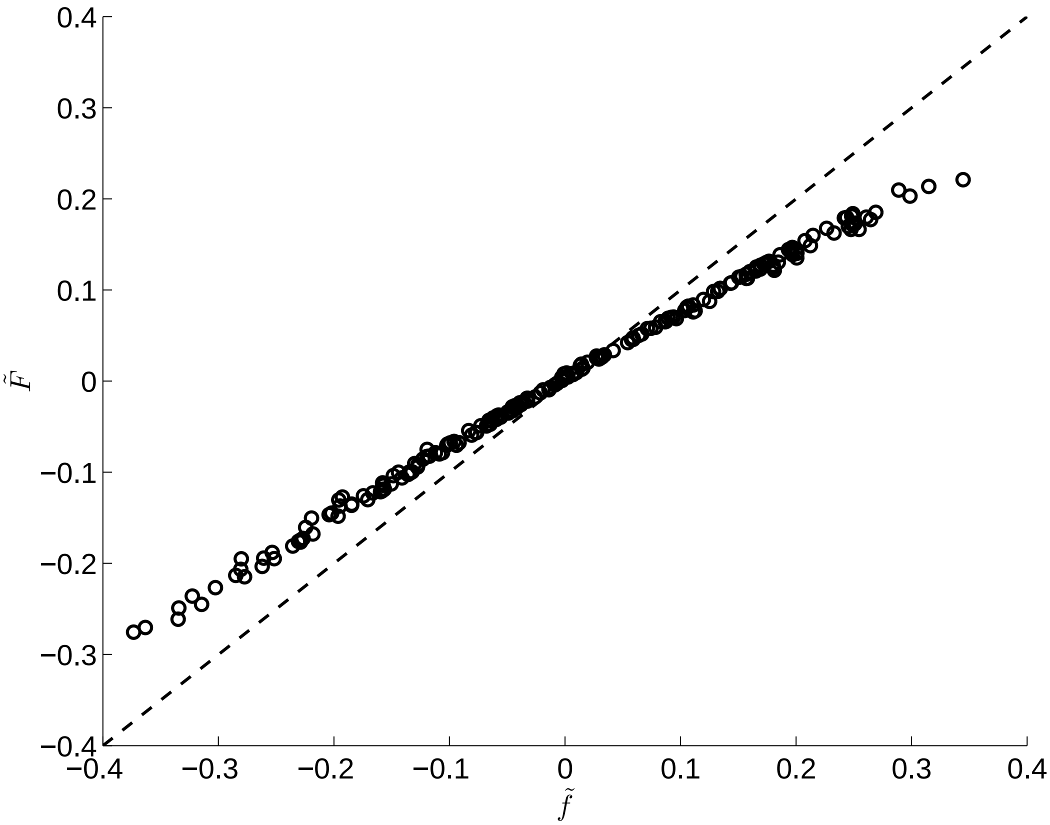
2 U.S. Aggregate Labor Market Data

In this section I briefly summarize the data used by Shimer. Mortensen and Nagypál rely on the same data. The raw data consists of four monthly time series from January 1951 to January 2004. The first three are constructed by the BLS from the CPS: the level of unemployment u_t, the level of civilian employment e_t, and the number of workers unemployed for 0 to 4 weeks u_t^s. The fourth is the Conference Board help-wanted advertising index v_t, used as a proxy for vacancies.

Define the *job finding probability* F_t as the probability that a worker unemployed at time t does not remain unemployed throughout until time $t+1$. If the job finding rate is constant at f_t between t and $t+1$, then under the assumptions of Section 1 the job finding probability is given by $F_t = 1 - e^{-f_t}$. Similarly, define the *employment exit probability* X_t as the probability of a worker employed at time t exiting into unemployment between t and $t+1$. Again, if the employment exit rate is constant at x_t during this time interval, then $X_t = 1 - e^{-x_t}$. Shimer (2007) shows that under the assumptions in Section 1, the job finding probability can be written in terms of unemployment and short-term unemployment

$$F_t = 1 - \frac{u_{t+1} - u_{t+1}^s}{u_t},$$

Figure 1: Job Finding Rate $\tilde{f}$ vs. Job Finding Probability $\tilde{F}$.



while the employment exit probability is very well approximated by

$$X_t = \frac{u^s_{t+1}}{e_t \left(1 - \frac{1}{2}F_t\right)}.$$

Having measured the two probabilities, measures of the corresponding rates are obtained through the relationships $f_t = -\log(1 - F_t)$ and $x_t = -\log(1 - X_t)$.

For the following analysis all time series are transformed as follows. First, quarterly series are constructed as averages of monthly values. Second, logarithms are taken. Finally, the series are detrendend using a Hodrick-Prescott filter with smoothing parameter 10^5. If y is the original monthly series, let $\tilde{y}$ denote the time series resulting from these transformation.

Table 2: Shimer's approach

	(1)	(2)
left hand side	$\tilde{F}$	$\tilde{f}$
right hand side	$\tilde{\theta}$	$\tilde{\theta}$
direct regression	0.292 (0.007)	0.401 (0.009)
reverse regression	0.325 (0.008)	0.447 (0.010)
R^2	0.899	0.898

NOTE.– Standard errors in parentheses.

3 Shimer's (2005) approach.

Equation (1) implies[1]

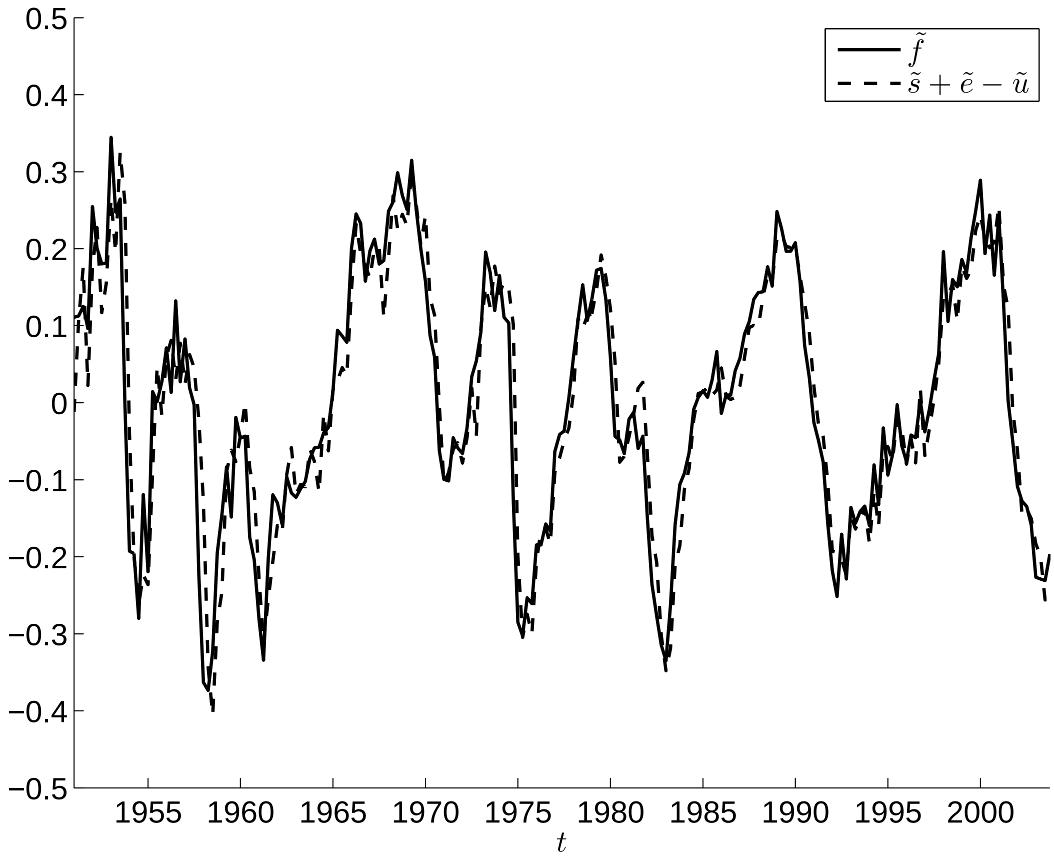
$$\tilde{f}_t = \eta \tilde{\theta}_t.$$

Shimer's approach is to estimate η from this relationship. He implements this approach using $\tilde{F}_t$ as a proxy for $\tilde{f}_t$, and obtains an estimate of 0.28. Replicating this regression, I obtain the estimate 0.29. Using $\tilde{F}_t$ as a proxy for $\tilde{f}_t$ is not innocuous, however. Figure 1 plots $\tilde{F}$ against $\tilde{f}$. The relationship is approximately linear over the relevant range, so the correlation is near one. Importantly, the relative standard deviation is 0.73. Another way to see that F is less variable than f is to compute the elasticity of F with respect to f, given by $\frac{e^{-f} f}{1-e^{-f}}$. Evaluating this expression at the mean of f, which is 0.61, this computation also yields 0.73. Intuitively, at low levels of the job finding rate f, doubling this rate also roughly doubles the probability of finding a job. But if the level of f is high, then the worker is already very likely to find a job, so doubling the job finding rate can no longer double the job finding probability.

This discussion implies that regressing $\tilde{f}_t$ on θ_t will yield a coefficient of $\frac{0.29}{0.73} = 0.4$ where 0.29 is the coefficient from regressing $\tilde{F}$ on $\tilde{\theta}$, and 0.73 is the relative standard deviation of $\tilde{F}$ and $\tilde{f}$. The regression results are collected in Table 2. Column (1) replicates Shimer's regression of $\tilde{F}$ on $\tilde{\theta}$, while column (2) shows the results from regressing $\tilde{f}$ on $\tilde{\theta}$. *A priori* it is not clear whether it is better to estimate η by regressing $\tilde{f}$ on $\tilde{\theta}$ or by running the

[1]This does not hold exactly due to monthly averaging, but this does not matter in practice.

Figure 2: Time Series of $\tilde{f}$ and $\tilde{x} + \tilde{e} - \tilde{u}$.



reverse regression of $\tilde{\theta}$ on $\tilde{f}$, so in Table 2 I report both. For the corrected version of the approach, direct and reverse regression together suggest the range $[0.4, 0.45]$ for η.

4 Mortensen and Nagypál's approach

If the job finding rate and the employment exit rate remain constant at f and x, respectively, then the unemployment rate $\frac{u}{u+e}$ will converge to $\frac{x}{x+f}$. Mortensen and Nagypál's approach is based on the following observation made by Shimer (2007): due to the large empirical values of f and x, adjustment to $\frac{x}{x+f}$ is very rapid, with the consequence that $\frac{x}{x+f}$ provides an excellent approximation of $\frac{u}{u+e}$. Taking this relationship with equality $\frac{u}{u+e} = \frac{x}{x+f}$ and substituting the matching function yields

$$\frac{u}{u+e} = \frac{x}{x + \mu \left(\frac{v}{u}\right)^{\eta}}. \tag{2}$$

Table 3: Mortensen and Nagypál Approach

	(1)	(2)
left hand side	$\tilde{e}-\tilde{u}$	$\tilde{x}+\tilde{e}-\tilde{u}$
right hand side	$\tilde{\theta}$	$\tilde{\theta}$
direct regression	0.513 (0.008)	0.370 (0.012)
reverse regression	0.539 (0.006)	0.458 (0.015)
R^2	0.952	0.808

NOTE.– Standard errors in parentheses.

This formula suggests that η can be estimated using data on u, e, v and x. Moreover, Mortensen and Nagypál discuss estimation of the matching function elasticity in the context of a model with a constant employment exit rate. Under the restriction of a constant employment exit rate, equation (2) permits estimation of η using data on u, e and v alone. Proceeding in this way, they arrive at an estimate of 0.544.

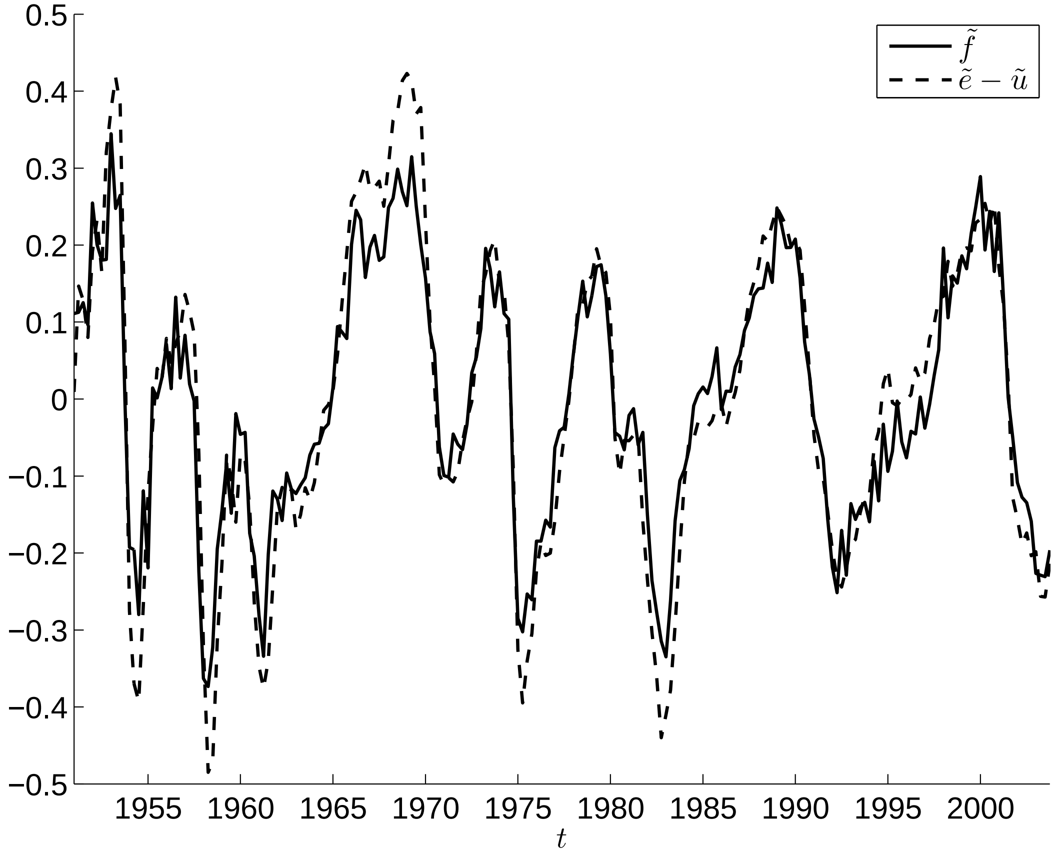
Notice that this approach requires stronger assumptions than Shimer's approach. Specifically, Shimer's approach is valid whether or not the employment exit rate is constant, and it does not rely on adjustment to be sufficiently fast. If both of these assumptions are correct, then one would expect both approaches to yield the same answer. The fact that they yield different answers suggests that one of these assumptions must be violated. I will now show that the difference in estimates is due to variation in the employment exit rate.

To clarify the connection between the two approaches, notice that $\frac{u}{u+e}=\frac{x}{x+f}$ can be rewritten as $f=x\frac{e}{u}$ or in terms of transformed variables

$$\tilde{f}=\tilde{x}+\tilde{e}-\tilde{u}. \tag{3}$$

One way of thinking about Mortensen and Nagypál's approach is that they follow Shimer's approach, but using a different measure of the job finding rate. Equation (3) suggests $\tilde{x}+\tilde{e}-\tilde{u}$ as a measure of the job finding rate. Figure 2 shows the time paths of $\tilde{f}$ and $\tilde{x}+\tilde{e}-\tilde{u}$. They track each other closely with a correlation of 0.942, which of course is just a restatement of Shimer's observation alluded to above. Their standard deviations are roughly equal at 0.162 and 0.158, respectively. This is not the measure which Mortensen

Figure 3: Time Series of $\tilde{f}$ and $\tilde{e} - \tilde{u}$.



and Nagypál use, however, since they also impose a constant employment exit rate. So implicitly they use $\tilde{e} - \tilde{u}$. Figure 3 compares the time paths of $\tilde{f}$ and $\tilde{e} - \tilde{u}$. If the latter is interpreted as a proxy of the job finding rate, then it overstates the increase in this rate in booms, since the entire decrease in unemployment is attributed to an increase in the job finding rate and none to a decline in the employment exit rate. Column (1) of Table 3 repeats the regressions of Table 2, but with $\tilde{e} - \tilde{u}$ in place of $\tilde{f}$. Mortensen and Nagypál implicitly run the reverse regression, and my replication of this regression yields 0.539 compared to their estimate of 0.544. The corresponding direct regression yields 0.513. Ignoring fluctuations in the employment exit rate is not innocuous, however, as is clear from contrasting Figures 2 and 3. Column (2) of Table 3 shows how the regression results change if fluctuations in the employment exit rate are taken into account, that is if $\tilde{x} + \tilde{e} - \tilde{u}$ rather than $\tilde{e} - \tilde{u}$ is used. The results are similar to those obtained in Column (2) of Table 2 from the corrected version of Shimer's approach, although with $[0.37, 0.46]$ the range implied by direct and reverse regressions is somewhat wider. Thus both approaches yield very similar results once their respective problems have been addressed.

References

Mortensen, D. T. and Nagypál, É. (2007). 'More on Unemployment and Vacancy Fluctuations.' *Review of Economic Dynamics*, vol. 10, pp. 327–347.

Shimer, R. (2005). 'The Cyclical Behavior of Equilibrium Unemployment and Vacancies.' *American Economic Review*, vol. 95(1), pp. 25–49.

Shimer, R. (2007). 'Reassessing the Ins and Outs of Unemployment.' NBER Working Paper 13421.