Question 1. Consider an economy with two consumers, Alice and Bob, and two goods, $x_1$ and $x_2$. Alice has utility function $U^A(x^A_1, x^A_2) = x^A_1 x^A_2$ and Bob has utility function $U^B(x^B_1, x^B_2) = (x^B_1)^2 x^B_2$. Alice is endowed with one unit of good 1 and Bob with one unit of good 2.

1.1 Define “Pareto efficient.” Find the set of Pareto efficient allocations for this economy.

1.2 Now suppose Alice is endowed with one unit of good one and Bob is endowed with one unit of time. A firm produces good 2 according to the production function $y_2 = 3L^B$, where $L^B$ is the quantity of labor Bob sells to the firm. Time does not enter Bob’s utility function, and so Bob is willing to sell the entire unit of time to the firm. Define “competitive equilibrium.” Find the competitive equilibrium of this economy. Is the outcome Pareto efficient? Explain why or why not.

1.3 Suppose you would like to increase Alice’s utility to some number $U^A$ that is larger than the utility she receives in the competitive-equilibrium of [1.2]. Explain how reallocating the initial endowment between Alice and Bob could lead to a competitive equilibrium in which Alice gets utility level $U^A$. Be as precise as you can, if possible identifying (but not solving) the equations that would identify the new endowment.

1.4 Now return to the economy of [1.2] and suppose the government must raise an amount of tax revenue $T$. What kind of tax would you advocate the government adopt, and why?

1.5 Return now to the economy of part [1.1], in which there is no time, no firm, and no production. Now assume that Alice’s utility function is given by $U^A(x^A_1, x^A_2) = x^A_1 x^A_2 - x^B_1$. Hence, for some reason, Bob’s consumption of good 1 reduces Alice’s utility. Find the competitive equilibrium of this economy. (The answer to this part of this question should be immediate, and requires no calculations.) Is this equilibrium efficient? Explain why or why not.

1.6 Continuing with the economy of part [1.4], suppose that we introduce a new market, in which Bob must buy from Alice the right to consume good $x_2$. Find the competitive equilibrium of this economy. Is it Pareto efficient? Why or why not?
**Question 2.** Consider two firms, 1 and 2, who produce in a duopoly market. Firm 1 chooses its quantity of output $x_1$ and firm 2 chooses $x_2$. Given these outputs, the market price is given by $p = A - B(x_1 + x_2)$. There are no costs of production.

1. Write the profit functions for firms 1 and 2. Find the optimal quantity of output $x_2$ for firm 2, as a function of $x_1$ (i.e., find firm 2’s best response function).

2. Find the equilibrium quantities of output for firms 1 and 2 in this market. Now suppose the firms 1 and 2 could agree that for every unit of output produced by firm 1, the firm must pay a tax $t_1$ to firm 2, and that firm 2 must similarly pay a tax $t_2$ to firm 1 for every unit of output produce by firm 2. Explain how these taxes can increase the profits of both firms. Assuming that $t_1 = t_2$, find the tax levels that maximize the firms’ profits.

3. Now suppose that firm 1 chooses $x_1$, which firm 2 observes and then chooses $x_2$. Using your calculation of firm 2’s best response function in question [2.1], write firm 1’s payoff solely as a function of $x_1$. Find the optimal quantity of output for firm 1. How does it compare to the output firm 1 would produce if it was a monopoly in this market?

**Question 3.** "A perfectly competitive firm has no control over price, while a monopoly can set its price. A monopoly will accordingly pass a profits tax on to its consumers, while a perfectly competitive firm will not do so.” Assess this statement. In particular:

1. Consider the monopoly first. Derive conditions identifying its profit-maximizing price and output without a profits tax (i.e., a tax on what we ordinarily call excess profits), and analogous conditions with a profits tax. Using these, compare the prices paid by the consumer with and without the profits tax.

2. Now consider a competitive firm. Assume that the firm has a constant-returns-to-scale production function, or, equivalently, has constant costs (and that every other firm in the market is identical). Identify the conditions for profit maximization for this firm, with and without the profits tax. Using these, compare the prices paid by the consumer with and without the profits tax.

3. How would your answer change if the tax if levied on normal rather than excess profits?