Economics 121b: Intermediate Microeconomics
Final Exam

1. (a) See Figure 1. The graphs set $\beta = \frac{1}{2}$.

(b) The analytic condition for an efficient allocation is the equality of the agents’ MRS at that allocation, or

$$MRS_A(\omega - x) = MRS_B(x),$$

where $x$ is the efficient allocation and $\omega$ is the vector of endowments. With the given utility functions and endowments this condition becomes

$$\frac{1}{\beta} = \frac{x_{B2}}{\beta x_{B1}}$$

or

$$x_{B1} = x_{B2} \quad (1)$$

and (from the total consumption = total endowment condition)

$$x_{A1} = 150 - x_{B1} \quad (2)$$

and

$$x_{A2} = 200 - x_{B2} = 200 - x_{B1} \quad (3)$$

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Combining equations (2) and (3) yields
\[ x_{A2} = x_{A1} + 50. \] (4)

Hence, efficient allocations satisfy equations (1) and (4), which are the same due to the construction of the Edgeworth box. See Figure 2 for a graphical description.

(c) To calculate the competitive equilibrium, we maximize each agent’s utility subject to the respective budget constraint. For Ann (normalizing \( p_1 = 1 \)):
\[
\max_{x_{A1}, x_{A2}} x_{A1} + \beta x_{A2} \quad \text{s.t.} \quad x_{A1} + p_2 x_{A2} = 100
\]

and for Bob:
\[
\max_{x_{B1}, x_{B2}} \ln (x_{B1}) + \beta \ln (x_{A2}) \quad \text{s.t.} \quad x_{B1} + p_2 x_{B2} = 50 + p_2 200.
\]

Since Ann’s utility function is linear, her optimal consumption is
\[
(x_{A1}^*, x_{A2}^*) = \begin{cases} (100, 0) & \text{if } \beta < p_2 \\ \left(c, \frac{100-c}{p_2}\right), c \in (0, 100) & \text{if } \beta = p_2 \\ \left(0, \frac{100}{p_2}\right) & \text{if } \beta > p_2 \end{cases}
\] (5)

Taking the ratio of the two FOCs for Bob (i.e., calculating the MRS), we get
\[
\frac{x_{B2}}{\beta x_{B1}} = \frac{1}{p_2}.
\]
Plugging this into Bob’s budget constraint yields

\[
(x_{B1}^*, x_{B2}^*) = \left( \frac{50 + 200p_2}{1 + \beta}, \frac{\beta(50 + 200p_2)}{(1 + \beta)p_2} \right).
\] (6)

Now we have to determine which of the three cases in (5) is consistent with Bob’s demand function (6). If \( x_{A1}^* = 100 \), it follows that \( x_{B1}^* = 50 \) so that, from (6), \( \beta = \frac{200}{50} p_2 \), which contradicts \( \beta < p_2 \). Similarly, if \( x_{A1}^* = 0 \), \( x_{B1}^* = 150 \) and \( \beta = \frac{200p_2 - 100}{150} < \frac{200\beta - 100}{150} \iff \beta > 2 \), which is inconsistent with \( \beta < 1 \). Hence, the only possibility is \( \beta = p_2 \), which leads to the competitive equilibrium

\[
(x_{A1}^*, x_{A2}^*, x_{B1}^*, x_{B2}^*) = \left( \frac{100 - 50\beta}{1 + \beta}, \frac{150}{1 + \beta}, \frac{50 + 200\beta}{1 + \beta}, \frac{50 + 200\beta}{1 + \beta} \right).
\]

[Calculation of equilibrium consumption was not asked.]

(d) The relative price between consumption today and consumption tomorrow is defined by

\[
\frac{p_1}{p_2} = \frac{1}{p_2}
\]

This can be interpreted as the interest rate because the interest rate measures the cost of future consumption in terms of present consumption. From \( p_2 = \beta \) we get

\[
r = \frac{1 - \beta}{\beta}.
\]

(e) See Figure 2.

2. (a)

\[
\frac{\partial x_i}{\partial p_j} = \kappa \text{ for } i \neq j
\] (7)

If \( \kappa \) is positive, then as \( p_j \) increases \( x_i \) increases. This makes \( x_1 \) and \( x_2 \) substitutes. On the other hand, if \( \kappa \) is negative, then as \( p_j \) increases \( x_i \) decreases. This makes \( x_1 \) and \( x_2 \) complements.

(b) The strategy of firm \( i \) is a function \( f : p_i \rightarrow p_j \). Alternately, the strategy of firm \( i \) can be denoted by \( p_i(p_j) \). The Nash equilibrium for the game is given by a pair \( (p_1^*, p_2^*) \) such that:

\[
\pi_1(p_1^*, p_2^*) \geq \pi_1(p_1, p_2^*) \forall p_1
\] (8)

and

\[
\pi_2(p_1^*, p_2^*) \geq \pi_2(p_1^*, p_2) \forall p_2
\] (9)

(c) The profit maximization problem for firm 1 is:

\[
\max_{p_1} \pi_1(p_1, p_2) = p_1 x_1(p_1, p_2) - c_1 x_1(p_1, p_2)
\] (10)

\[
= p_1 (1 - \beta p_1 + \kappa p_2)
\] (11)
Taking the first order conditions we get:

\[
\frac{\partial \pi_1(p_1, p_2)}{\partial p_1} = 0
\] (12)

\[
1 - 2\beta p_1 + \kappa p_2 = 0
\] (13)

Rearranging this gives the best response function for firm 1 as:

\[
p_1 = \frac{1 + \kappa p_2}{2\beta}
\] (14)

Similarly, we get the best response function for firm 2 as:

\[
p_2 = \frac{1 + \kappa p_1}{2\beta}
\] (15)

Solving these two equations simultaneously gives:

\[
p_1^* = \frac{1}{2\beta - \kappa}
\] (16)

Similarly we get:

\[
p_2^* = \frac{1}{2\beta - \kappa}
\] (17)

(d) Note that firm 2’s best response function stays the same as before. That is,

\[
p_2 = \frac{1 + \kappa p_1}{2\beta}
\] (18)

Since firm 1 is the leader, when makes it’s decision it takes firm 2’s best response into account.

\[
\max_{p_1} \pi_1(p_1, p_2) = p_1 x_1(p_1, p_2) - c_1 x_1(p_1, p_2)
\] (19)

\[
= p_1 (1 - \beta p_1 + \kappa p_2(p_1))
\] (20)

\[
= p_1 (1 - \beta p_1 + \kappa \frac{1 + \kappa p_1}{2\beta})
\] (21)

Taking first order conditions gives us:

\[
p_1^* = \frac{2\beta + \kappa}{2(2\beta^2 - \kappa^2)}
\] (22)

Plugging this into the best response of player 2 gives:

\[
p_2^* = \frac{4\beta^2 + 2\beta\kappa - \kappa^2}{2(2\beta^2 - \kappa^2)}
\] (23)
3. (a)  
i. The zero expected profit condition for each type of consumer, \( i = l, h \)
is
\[
p_i(\alpha_i q_i - q_i) + (1 - p_i)\alpha_i q_i = 0,
\]
(24)
where \( p_h = \frac{2}{3} \) and \( p_l = \frac{1}{3} \) are the probabilities of having a loss, \( q_i \) is the amount of insurance of consumer of type \( i \) demands, and \( \alpha_i q_i \) is the premium the insurer charges a type \( i \) consumer. Equation (24) implies
\[
\alpha_h = \frac{2}{3} \quad \text{and} \quad \alpha_l = \frac{1}{3}.
\]

ii. Each consumer type maximizes
\[
\max_{q_i} p_i \sqrt{20 - 10 - p_i q_i + q_i + (1 - p_i) \sqrt{20 - p_i q_i}}.
\]
The FOC is
\[
\frac{p_i(1 - p_i)}{2 \sqrt{20 - 10 - p_i q_i + q_i}} = \frac{(1 - p_i)p_i}{2 \sqrt{20 - p_i q_i}},
\]
which implies
\[
q_i^* = 10
\]
for both consumer types. Hence, both types buy full insurance.

(b)  
i. The high risk type individuals will try to impersonate the low risk type individuals.

ii. The separating equilibrium will consist of premium and coverage pairs for both types. Let’s denote this pair by \( (p_l, q_l) \) for the low risk type and \( (p_h, q_h) \) for the high risk type. The premium charges is the same as part (a) as it comes from the condition that the firm earns zero expected profit. So
\[
p_l = \frac{q_l}{3} \quad \text{and} \quad p_h = \frac{2q_h}{3}
\]
(25)
Since the industry is perfectly competitive the high risk type will be offered full insurance as in part (a). Thus
\[
q_h = 10
\]
(26)
Furthermore, it must be the case that the high risk type is not able to impersonate the low risk type. That is, incentive compatibility must bind for the high risk type.
\[
\sqrt{(20 - p_h)} = \frac{2}{3} \sqrt{(10 + q_l - p_l)} + \frac{1}{3} \sqrt{(20 - p_i)}
\]
(27)
Plugging in \( p_l = \frac{q_l}{3}, p_h = \frac{2q_h}{3} \) and \( q_h = 10 \) gives:
\[
q_l = 0.9267
\]
(28)
Hence the menu of choices offered by the insurance company in order to get a separating equilibrium is \( (0.3089, 0.9267) \) for the low risk type and \( (20/3, 10) \) for the high risk type.
A graphical depiction of the equilibrium is given below.
Figure 3: Question 3(b)ii