1. (30 minutes) A university is considering adopting a policy that would help faculty members who are parents cover the cost of day-care. The policy makers have two goals:

(i) increase the utility of faculty members who are parents;
(ii) increasing the number of research hours (assuming more day-care consumption increases research hours).

A parent faculty member maximizes each pay period his/her utility for day-care ($d$) and a composite of all other consumption goods ($c$) subject to a simple linear budget constraint, and has an income of $m = $1000 per pay period to allocate between $c$ and $d$. The cost of hourly day-care in town is $p_d = $12. The cost of the composite good is $p_c = $1.

The parent support committee proposes to give a lump sum subsidy of $g = $200 dollars per pay period to any parent who wants the support, so his/her income would be $m = $1000 + $g$. However, due to tax rules, a family that receives day-care support $g$ may not ask for a tax deduction on day-care expenses and must pay the full price $p_d = $12 per hour of day-care, while a family that does not receive the support receives a tax deduction so that in effect they face a price $p_d = $10 per hour of day-care.

Faculty members first choose whether to ask for day care support and then choose a level of consumption $c$ and $d$ that maximize their utility subject to the appropriate budget line.

Kim who is a faculty member and a parent has a utility for day care and other goods given by

$$u_K(c, d) = c^3 d.$$  

(a) (5) Describe the budget line analytically and then present a carefully labeled diagram (using all of the above information) for a faculty member who chooses to receive day care support from the university but cannot get tax deduction?

(b) (5) Describe the budget line analytically and then present a carefully labeled diagram (using all of the above information) for a faculty member who chooses not to receive day care support from the university but gets tax deduction instead?

(c) (5) Find analytically Kim’s optimal consumption of day-care and other goods if s/he chooses the university support.
(d) (5) Find analytically Kim’s optimal consumption of day-care and other goods if s/he chooses the tax deduction instead.

(e) (5) Evaluate the indirect utility function of Kim under each option analytically. Would Kim prefer to get the university support for day-care or the tax deduction?

(f) (5) With respect to the goals of the university given by (i) and (ii), does the university achieve its policy goal. Briefly explain why or why not?

2. (45 minutes) Ann and Bob are living in a very small town in Wisconsin. The economy consists of bread, \( b > 0 \), and cheese \( c > 0 \). The utility function of Ann is given by:

\[
u_A(b_A, c_A) = \ln b_A + \ln c_A\]

and of Bob is given by

\[
u_B(b_B, c_B) = \gamma \ln b_B + \ln c_B\]

with \( \gamma > 1 \). The social endowment of the economy is given by \( b = 2 \) and \( c = 1 \).

(a) (5) Give a definition of a feasible allocation \((b_A, c_A, b_B, c_B)\) for this endowment economy.

(b) (5) Draw the Edgeworth box of this endowment economy and label it carefully (axis’, direction, etc.) for the feasible consumption allocation:

\[
\left(b_A = 1, c_A = \frac{3}{4}, b_B = 1, c_B = \frac{1}{4}\right),
\]

and with indifference curves through the feasible allocation defined above for Ann and Bob.

(c) (5) Define the notion of a Pareto efficient allocation for an endowment economy (without referring to marginal rate of substitution.)

(d) (5) Describe graphically the conditions for a Pareto efficient allocation in the Edgeworth box and give the corresponding analytic conditions for a Pareto efficient allocation in this economy.

(e) (10, 5, 5) Ann and Bob have agreed that Ann receives a weight of \( \alpha \) and Bob a weight of \( 1 - \alpha \) in the social welfare function with \( 0 \leq \alpha \leq 1 \) given by:

\[
\alpha u_A(b_A, c_A) + (1 - \alpha) u_B(b_B, c_B)
\]

Derive the Pareto efficient allocation as a function of \( \alpha \) and \( \gamma \) (taking account of the resource constraints). Describe briefly the objective function, the choice variables and the method you use to derive the Pareto efficient allocation. Briefly describe how the Pareto efficient allocation varies in \( \gamma \) for a given \( \alpha \).
Now describe graphically the Pareto efficient allocations in the Edgeworth box as a function of $\alpha$ when $\alpha$ varies from 0 to 1 (for a given $\gamma$, say $\gamma = 2$) and relate the graph to the preferences of the agents.