1. Choice and Demand. Consider the general budget line
\[ p_x x + p_y y = m. \]
For the following utility functions find the demand for goods \( x \) and \( y \) as a function of \( p_x, p_y \) and \( m \). For the Cobb-Douglas utility function, solve the problem using both, the method of substitution and the method of Lagrange. For the Leontief utility function, we saw in class that we can only employ the method of substitution.

(a) Cobb-Douglas utility:
\[ u(x, y) = x^a y^b. \]
where \( a, b > 0; \)
(b) “Leontief Utility”:
\[ u(x, y) = \min\{ax, by\}; \]
where \( a, b > 0; \)

2. Budget and Demand. The federal government provides grants to state and local governments to assist in funding all sorts of activities, like police protection and education. Two particular types of grants that the federal government uses are:

- Non-Matching Grants: the federal government provides the state government with a quantity of income, which the state can use for any purpose it likes;
- Matching Grants: for every dollar the state spends on the provision of some good or service the federal government provides the state government with an additional dollar to be spent on that good.

You are a high-ranking official at the U.S. Department of Education, and have been given the task of allocating grant money to the Connecticut state government to improve its public school education. The governor of Connecticut, whom we assume has preferences that reflect the preferences of her constituents, receives utility from the allocation of Connecticut income between two commodities:
• $x =$ education
• $y =$ composite bundle of all other private consumption,
where "$y$" reflects income of Connecticut’s residents that the residents spend themselves. The governor’s preferences over $x$ and $y$ are given by the following utility function:

$$u(x, y) = 2\sqrt{x} + 2\sqrt{y}$$

The income of the state is $m = $100 million. The price of a unit of composite consumption ($p_y$) is $1$, while the price of a unit of education ($p_x$) is $2$.

(a) Graphically describe:
   i. the governor’s preferences for education and private consumption and
   ii. the state’s budget constraint.

(b) Find the optimal provision of education that the governor would choose in the absence of any federal grant. You may assume an interior solution and that preferences are convex (which they are).

(c) What is the governor’s MRS at $(x, y) = (10, 80)$, where magnitudes are expressed in millions of dollars? Interpret. Why is this not an optimal choice of education and private consumption.

(d) Suppose that you, as a high-ranking federal government official, offer Connecticut a non-matching grant of $50 million.
   i. Graphically describe the effect of this grant on the state’s budget constraint.
   ii. Calculate Connecticut’s optimal level of education provision with this grant. What level of utility does Connecticut’s governor reach with the grant?

(e) Repeat part (d) under the premise that you offer Connecticut a dollar-for-dollar education matching grant.

Reading Assignment: NS Chapter 2,4 and 5.