1. First, we examine a monopolistic firm. The firm faces a market described
by the demand function \( p = A - By \), where \( p \) is the price the firm receives
if it sells quantity \( y \) of output. The firm’s cost function is given by \( C(y) = \frac{1}{2}y^2 \).

(a) Find the profit-maximizing quantity of output and the corresponding
profit for this firm.

(b) Show that if the firm could sell more output at the (constant) equi-
librium price, it would do so. (To do this, take a derivative of the
firm’s payoff under the assumption that price is constant rather than
a function of output, and then evaluate this derivative at the equi-
librium price and quantity from part 1a.)

(c) Let \( p^* \) be the equilibrium price and \( y^* \) the equilibrium quantity, so
that \( p^* = A - By^* \). Now suppose the demand in the market shifts, so
that the new demand curve is \( p = A' - B'y \), with \( A' > A \) and \( B' > B \),
but with it still being the case that \( p^* = A' - B'y^* \). Hence, if the
firm does not change its quantity, its price will also not change. Is
the quantity of output \( y^* \) still optimal, given this new demand curve?
If not, will the firm decide to produce more or less? Formulate your
first-order condition from part 1a in terms of the elasticity of demand,
and use this to explain your result in this part.

2. Now suppose you must design a tax on the monopoly. Let the demand
function be \( p = A - By \).

(a) First suppose you consider a sales tax of 10% on this market. Suppose
that the firm has to pay the tax, as is typically the case with sales
taxes. Hence, if the firm produces quantity \( y \) of output, the price
paid by consumers is \( p = A - By \), but the price received by the firm
is \( .9p = .9(A - By) \), where the .9 appears because the firm gets to
keep only ninety percent of the purchase price, paying the remaining
ten percent to the government in taxes. Find the profit maximizing
quantity of output for the firm, the price paid by consumers, the
price received by the firm, and the firm’s profit. Explain how these
answers compare to those of part 1a. In particular, do consumers
pay more as a result of the tax? Does the price received by the firm
fall? Does the firm’s profit fall?
(b) Instead of a sales tax, the government considers a profits tax. Hence, if the firm chooses quantity of output $y$ and charges price $p = A - By$, the government collects $t( py - \frac{1}{2} y^2 ) = t[( A - By)y - \frac{1}{2} y^2]$ in tax revenue, leaving $(1 - t)( py - \frac{1}{2} y^2 ) = (1 - t)[( A - By)y - \frac{1}{2} y^2]$ as after-tax profit for the firm. Once again, find the firm’s profit-maximizing quantity of output and the resulting price. What effect does the profits tax have on the quantity of output and price? What effect does it have on the firm’s profit? In light of your answers, given that a fixed amount of revenue is to be raised, which tax would consumers prefer—a sales tax or profits tax? Why?

(c) Now suppose the cost function is given by $\frac{1}{2} x^2 + F$, where $F$ is a fixed cost. For example, $F$ may be the cost of conducting environmental impact studies or acquiring the licenses needed to produce. What effect does $F$ have on the firm’s optimal quantity of output, price, and profits? Many communications firms are monopolies because the government gives them exclusive rights to use certain bands of air-wave lengths. Sometimes the government simply gives the firms this exclusive right, while other times it sells the right to the highest bidder. The latter method has been criticized on the grounds that it will drive up the firms’ costs and hence the prices charged to consumers. What do you think of this argument?

3. Now suppose we have a duopoly market, with two firms. We have assumed that firms 1 and 2 produce perfectly homogeneous products, in the sense that they sell their output at a price that depends only on the total amount produced in the market and not on who does the producing. To see how this assumption might be generalized, let the profit functions for the two firms be given by (where $A > 0$, $B > 0$)

$$\pi_1(x_1, x_2) = (A - Bx_1 + Dx_2)x_1 - cx_1$$
$$\pi_2(x_1, x_2) = (A - Bx_2 + Dx_1)x_2 - cx_2.$$ 

(a) Explain what it means for $D$ to be negative, and what it means for $D$ to be positive. (Think about substitutes and complements.) If $D > 0$, we will assume that $B > D$. What is the economic interpretation of this assumption, and what would happen if it did not hold?

(b) Find the equilibrium quantities of $x_1$ and $x_2$ in this market. To do this, find the first-order condition for each firm, and solve the resulting pair of linear equations for $x_1$ and $x_2$.

Reading Assignment: NS Chapter 14, 15