1. **Third Degree Price Discrimination.** Consider a monopolist (say a local movie theatre in Fort Lauderdale) which has two distinct client groups, adults and seniors. The inverse demand for the adults is given by

\[ p(q_A) = a - bq_A, \]

and the inverse demand of retirees is given by

\[ p(q_B) = \frac{a}{3} - \frac{b}{3}q_R. \]

(a) Describe the demand function in the two markets graphically and then compute the demand elasticity in each market.

(b) Compute the demand function \( q(p) \) under the assumption that the movie theater can only offer a single price to both segments of the market. (Hint: at a given price add the demand of the adults and senior market. You need to go from the inverse demand function to the demand function.) Illustrate the aggregate demand function in contrast to the demand functions in each segment. Now compute the optimal price of the movie theatre when it can only offer a single and common price to the market segments. Who goes to the movies and who doesn’t?

(c) Next we allow the movie theatre to offer different prices in each segment and customers cannot misrepresent their identity. What is the optimal price in each one of the markets?

(d) Compare the welfare of the consumers and the revenue of the consumer after the introduction of different prices across different segments. Explain.

2. The duopoly model we have been working with views firms as choosing their quantities of output, with the market then setting a common price for the two firms. Here is an alternative model. Firms 1 and 2 set prices \( p_1 \) and \( p_2 \). If \( p_1 < p_2 \), firm 1 sells quantity \( A - Bp_1 \) and firm 2 sells nothing. Similarly, if \( p_1 > p_2 \), firm 2 sells quantity \( A - Bp_2 \) and firm 1 sells nothing. If \( p_1 = p_2 \), each firm sells half of the quantity \( A - Bp_1 = A - Bp_2 \). Suppose that each firm has constant marginal cost \( c \), so that the cost of producing output \( x_i \) for firm \( i \) is \( C(x_i) = cx_i \).
(a) Find the equilibrium prices $p_1$ and $p_2$ in this market. This is not a job for calculus, because the firms’ payoffs are not differentiable functions of $p_1$ and $p_2$. Instead, try a few combinations of prices, and for each one, ask yourself whether each firm is doing the best it can given the other firm’s price, or whether either firm has an incentive to change its price. You have an equilibrium when each firm is setting a price that maximizes its profits, given the price of the other firm.

(b) How does the outcome you’ve found in [2a] compare to the equilibrium of the quantity-setting model, or to the competitive outcome in this market? Now suppose you are involved in a case before the Justice Department, in which a merger is to be evaluated that will leave a market with only two firms. The concern is that this merger will lead to higher consumer prices. If you represented one of the firms that wanted to merge, which model of the resulting duopoly market would you be inclined to use as the basis for your analysis? Which would you use if you worked for the Justice Department? As an outsider, which do you think is more appropriate?


(a) Find all pure strategy equilibria in the following games, often referred to as “Battles of Sexes”:

\[
\begin{array}{ccc}
\text{Bob} & \text{Ballet} & \text{Soccer} \\
\text{Ann} & 2, 1 & 0, 0 \\
\text{Soccer} & 0, 0 & 1, 2 \\
\end{array}
\]

(b) and “Hawk-Dove”:

\[
\begin{array}{ccc}
\text{Defend} & \text{Attack} \\
\text{Defend} & 3, 3 & 1, 4 \\
\text{Attack} & 4, 1 & 0, 0 \\
\end{array}
\]

4. Voluntary Contribution Games. Consider the voluntary contribution game where for all $i \in I$:

\[
u_i(g_1, \ldots, g_I) = \ln(w - g_i) + \ln\left(\sum_{j=1}^{I} g_j\right).
\]

and each agent has to make a choice as to how much of the public good $g_i$ he wants to contribute and how much to keep for private consumption $w - g_i$, where the wealth level $w$ is the same for all the agents.

(a) Define the situation as a game and define the Nash equilibrium.
(b) Derive the socially optimal allocation in which each agent has the same welfare weight (equal to one)

(c) Derive the unique symmetric Nash equilibrium.

(d) Compare the socially optimal contribution with the equilibrium contribution. What can you say about the relationship as $I \to \infty$.

5. **Reading Assignment:** NS Chapter 8, 14, 15