Economics 121b: Intermediate Microeconomics

Problem Set 8: Monopoly and Duopoly

3/24/10

This problem set is due 3/31.

1. (a) The firm maximizes profit \( \max_y \pi = py - C(y) \).

\[
\pi = (A - By)y - \frac{1}{2}y^2
\]

\[
\frac{d\pi}{dy} = A - 2By - y = 0
\]

\[
y = \frac{A}{2B + 1}
\]

The price is thus \( p = A - By = \frac{AB + A}{2B + 1} \) and the profit is \( \pi = \frac{A^2}{2(2B + 1)} \).

(b) Taking price as constant,

\[
\frac{d\pi}{dy} = p - y = A - By - y = \frac{AB}{2B + 1} > 0
\]

Therefore the firm wants to increase quantity to increase profit.

(c) The first order condition can be written as:

\[
\frac{d\pi}{dy} = p + y \frac{dp}{dy} - \frac{dC}{dy} = 0
\]

\[
p(1 + \frac{y \frac{dp}{dy}}{p \frac{dy}{dy}}) - \frac{dC}{dy} = 0
\]

\[
p(1 - \frac{1}{e}) - \frac{dC}{dy} = 0
\]

where \( e \) is defined as the absolute value of the elasticity of demand. A monopoly firm always produces at the elastic portion of the demand curve and sets \( p(1 - 1/e) = MC \). Writing \( y = \frac{A}{B} - \frac{p}{B} \), the elasticity of the demand curve is \( e = -\frac{dy}{dp} \frac{p}{y} = \frac{1}{B} \frac{p}{y} \). At the new demand curve and at the old equilibrium quantity, \( p^* = A' - B'y^* \), so
\[ e' = \frac{1}{B} \frac{p^*}{y^*} \]

The latter term is the elasticity of the old demand curve at the equilibrium quantity. From the equation \( p(1 - 1/e) = MC \), we know that if \( e \) decreases, \( p \) increases. So the monopolist increases price and decreases quantity.

2. (a) We can again maximize the firm’s profit, now subject to the sales tax. We have \( \max_y 0.9py - C(y) \), where \( p \) is the price they charge the consumer.

\[
\pi = 0.9(A - By)y - \frac{1}{2}y^2
\]

\[
\frac{d\pi}{dy} = 0.9A - 1.8By - y = 0
\]

\[
y = \frac{0.9A}{1.8B + 1}
\]

Note that this quantity is smaller than the equilibrium quantity in [9.1]. With this quantity, consumer pays \( p = (A - By) = \frac{0.9AB + A}{1.8B + 1} \), the firm receives \( p_F = 0.9p = \frac{0.81AB + 0.9A}{1.8B + 1} \), and its profit is \( \pi = \frac{0.495A^2}{1.8B + 1} \). The consumer pays more, but the firm receives less and earn less profit. In other words, the tax is borne by both the consumer and the firm.

(b) Notice that the maximization problems of \( \max_y \pi \) and \( \max_y (1 - t)\pi \) yield the same solution as long as \( t \) is independent of \( y \). The output produced by the firm and the price it charges are unaffected by a profits tax, while the firms’ after-tax profit decreases. The consumer thus prefers to have the profits tax.

(c) Again, the maximization problems of \( \max_y \pi \) and \( \max_y \pi - F \) yield the same solution as long as \( F \) is independent of \( y \). The firm chooses the same monopoly quantity and price. The only difference is that its profit is decreased by \( F \). As a result, charging a firm a fixed cost, such as charging it fee in return for the right to serve a market, will have no effect on the price charged in that market.

3. (a) When \( D \) is negative, firm 2’s product has a negative effect on firm 1’s price and profit. This means that consumers consider the two products as substitutes. When \( D \) is positive, firm 2’s product has a positive effect on firm 1’s price and profit. In this case, consumers consider the two products to be complements. More of product 2 makes product 1 look scarce and hence increases its price, since the two goods are complements.
The economic interpretation of $B > D$ is that the direct effect of $x_1$ on the price of product 1 should be more than the effect of $x_2$. If $D > B$, the firms could charge arbitrarily high prices by jointly producing arbitrarily large amounts of output, and there would be no upper bound on their profits.

(b) The first order condition for firm 2’s maximization problem is given by

$$\frac{d\pi_2(x_1, x_2)}{dx_2} = A + Dx_1 - 2Bx_2 - c = 0$$

Firm 2’s best response is given by

$$x_2 = \frac{A + Dx_1 - c}{2B}$$

Symmetrically, firm 1’s best response is given by

$$x_1 = \frac{A + Dx_2 - c}{2B}$$

Solving for the two response functions, the equilibrium quantities are

$$x_1^* = \frac{A - c}{2B - D}$$
$$x_2^* = \frac{A - c}{2B - D}$$