Economics 501, Microeconomics, Spring 2008

Homework 10
Due April 17, 2008

10.1 Informed Principal

- We found in class that there are pooling and separating equilibria of the game when the firm offers contracts to the worker after observing education. We look to see whether such contracts are equilibria when the worker first makes an offer.

- If \( p < \frac{1}{2} \) then there are no pooling equilibria: The firm would accept the least cost separating contract since they would believe only the high type would get education\(^1\). This results in utility \( 2 - \frac{1}{2} > 1 + p \) (the highest utility in any pooling equilibrium) for the high type worker so they must offer it instead. Given that the high type does not offer the pooling contract the firm must realize that the low ability worker is the only one to offer pooling. If \( p \geq \frac{1}{2} \) then the least cost pooling outcome is an equilibrium since the high type has no incentive to offer the separating contract in this case. In fact, any pooling equilibrium that gives the high type worker at least 1.5 is a PBE.

- The least-cost separating equilibrium is always an equilibrium, supported by assuming only the low type offer any other wage structure, and so only accepting a pooling contract that pays 1. This survives the intuitive criterion since offering pooling is not equilibrium dominated for any type. This is true for any \( p \). Other separating contracts are not equilibria because the high type worker could offer the least-cost separating contract instead.

10.2 Insurance with Homogenous Agents

- (a) Suppose there is a transfer of \( \tau \) to agents who have an accident and \( t \) to agents who don’t. The the utility of society is

\[
\pi u(w - L + \tau) + (1 - \pi) u(w + t)
\]

\(^1\)To correct a mistake on the previous solutions: even if the firm believed that only the low type offered the separating contract they would still accept it since they would believe that everyone would get education 0, and they would only have to pay the worker the lower wage.
By budget balancedness we must have that
\[ \pi \tau + (1 - \pi) t \leq 0 \]
since the utility function is increasing this must bind so the objective is to choose \( \tau \) to maximize
\[ \pi u(w - L + \tau) + (1 - \pi) u(w - \frac{\pi}{1 - \pi} \tau) \]
which occurs when
\[ \tau - L = -\frac{\pi}{1 - \pi} \tau \]
and so \( \tau = (1 - \pi)L \), which corresponds to full insurance.

(b) Suppose the price of a dollar of insurance is \( p \). Then by buying \( D \) units of insurance the utility of the household is
\[ \pi u(w - L - pD + D) + (1 - \pi) u(w - pD) \]
The household wishes to choose the \( D \) that maximizes their utility. Taking FOC we get that the optimal level of \( D \), given \( p \) is determined by
\[ \pi u'(w - L - pD^* + D^*)(1 - p) - (1 - \pi) u'(w - pD^*)p = 0. \]
Suppose \( p = \pi \). Then this condition reduces to
\[ \pi(1 - \pi)(u'(w - L - pD^* + D^*) - u'(w - pD^*)) = 0 \]
and since \( u \) is strictly concave, this implies
\[ w - L - pD^* + D^* = w - pD^* \]
and so
\[ D^* = L. \]
Now suppose \( p > \pi \). Then the FOC implies that
\[ 1 < \frac{p(1 - \pi)}{\pi(1 - p)} = \frac{u'(w - L - pD^* + D^*)}{u'(w - pD^*)} \]
which implies that
\[ w - L - pD^* + D^* < w - pD^* \]
and so
\[ D^* < L. \]
10.3 Insurance with Heterogeneous Agents

- (a) From our answers to question 2 we can see that for \( i = c, r \)

\[
\frac{p(1 - \pi_i)}{\pi_i(1 - p)} = \frac{u'(w - L - pD_i^* + D_i^*)}{u'(w - pD_i^*)}
\]

So \( D_i(p) \) is decreasing in \( p \), with

\[
D_r(p) = \begin{cases} 
L, p = \pi_r \\
> L, p < \pi_r
\end{cases}
\]

\[
D_c(p) = \begin{cases} 
L, p = \pi_c \\
< L, p > \pi_c
\end{cases}
\]

- (b) Note, that by the implicit function theorem \( D_i(p) \) is continuous function of \( p \). The profits of the firm from offering price \( p \) is then

\[
\gamma(p) = \alpha(p - \pi_c)D_c(p) + (1 - \alpha)(p - \pi_r)D_r(p)
\]

which is a continuous function of \( p \). Note that

\[
\gamma(\pi_c) = (1 - \alpha)(\pi_c - \pi_r)D_r(\pi_c) < 0
\]

so it is not necessarily true that the firm would earn 0 profits in expectations.

- (c) Notice that \( \gamma(p) \) is continuous with \( \gamma(\pi_c) < 0 \) and \( \gamma(\pi_r) = \alpha(\pi_r - \pi_c)D_c(\pi_r) \geq 0 \) so there must be a \( p \in [\pi_c, \pi_r] \) with \( \gamma(p) = 0 \).

10.4 Competition in Contracts

- (i) Suppose there is a pooling contract. In order for the firm to earn 0 profits it must be that

\[
p = \frac{P}{P + C} = \alpha \pi_c + (1 - \alpha) \pi_r.
\]

In order for no firm to be able to enter it must be that \( P + C \) is the amount of insurance the cautious consumer would purchase at price \( p \) (if it was lower a firm could offer more insurance at price slightly above \( p \) and all consumers would buy, if it was higher could offer less insurance at the same rate and only the low risk consumers would
buy resulting in positive profits). The risky consumers would prefer more insurance at this rate and so their indifference curve is flatter, so it must be possible to find a price arbitrarily close to \( p \) and a lower level of coverage which the cautious consumer would buy but the risky consumer would not. A firm could offer this contract and earn positive profits.

• (ii) Suppose we have a separating contract: \((P_c, C_c), (P_r, C_r)\). Consider the contract for the risky agents first. Since we are in a competitive environment we must have that the firms earn 0 profits, so

\[
p_r = \frac{P_r}{P_r + C_r} = \pi_r
\]

We must also have \( P_r + C_r = L \) since otherwise a firm could offer contract \( L \) at per unit price slightly above \( \pi_r \) (since the agent would strictly prefer to more insurance at \( p_r = \pi_r \)). So we must have that \((P_r, C_r) = (\pi_r L, (1 - \pi_r)L)\). This results in utility to the risky agent of

\[
u(w - \pi_r L).
\]

Now consider the contract to the cautious agent. In order to get separation it must be that the utility from choosing \((P_c, C_c)\) is no higher then from choosing \((P_r, C_r)\). That is,

\[
\pi_r u(w - \pi_r L) \geq \pi_r u(w - L + C_c) + (1 - \pi_r) u(w - P_c)
\]

Notice three things:

1. Zero profit condition guarantees that \( \frac{P_c}{P_c + C_c} = \pi_c \) so \( P_c = \frac{\pi_c C_c}{1 - \pi_c} \).
2. Must have \( P_c + C_c < L \) or the risky agents would select it.
3. The constraint must bind since otherwise a firm could offer slightly more insurance at a slightly higher price and earn positive profits (since the risky still would not buy).

Notice that if \( \pi_r \) is much larger than \( \pi_c \) the insurance is far away from the full insurance level. Importantly, the separating equilibrium does not depend on \( \alpha \).

• (iii) Suppose the fraction of risky consumers is small. As the fraction of risky goes to 0 the price of the pooling contract must approach \( \pi_c \) and so the utility to the cautious consumer must go to the utility from
full insurance at the actuarilly fair rate. So if the fraction of risky customers is small the cautious consumers would prefer the pooling contract to the separating contract. So a firm could offer a pooling contract, at a price slightly above the actuarilly fair price, and earn positive profits. So the separating equilibrium breaks down. But we have already established that there cannot be a pooling equilibrium so there cannot exist an equilibrium when the fraction of risky consumers is small.