

# Final Exam Economics 501b Microeconomic Theory

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- This is a **closed-book** exam. The exam lasts for **180** minutes. Please write clearly and legibly. Be especially careful in the definition of the game, the payoff function and the equilibrium notions. The allocated points are also a good indicator for your time budget. Please record the answer for each question in a separate bluebook.

1. **(45)** Consider the following model of Cournot competition with differentiated products and  $I$  firms. The price for the product of firm  $i$  is given by

$$p_i(q) = a - q_i - \gamma \sum_{j \neq i} q_j$$

with  $0 < \gamma < 1$ . The marginal cost of production is equal to zero.

1. Define the notion of iterative elimination of strictly dominated strategies for this game.
  2. Find a Nash equilibrium of the Cournot game.
  3. Is the Nash equilibrium unique?
  4. Find the set of strategies which survive iterative elimination of strictly dominated strategies. Does your answer depend on the value of  $\gamma$  and the number of firms  $I$ ? If so, how? (A graphical illustration might be helpful.)
2. **(45)** Consider the following card game between two players, Ann (A) and Bob (B). Each player has four cards: one, two, three, and a joker. Ann wins the game if there is either a match of jokers (two jokers played) or a mismatch of *numbered* cards (one, two for example). In all other circumstances, Bob wins the game. Ann and Bob have to simultaneously choose a card, and the winner receives \$1 from the loser according to the above rules of winner determination.
1. Define and carefully write down the normal form of this game.
  2. Completely describe the set of pure strategy equilibria.
  3. Completely describe the set of all strictly mixed strategy equilibria. Does the game favor either Ann or Bob?

3. **(45)** Consider a second price auction with two bidders and an entrance fee  $f$ . The valuation for each bidder is uniformly distributed in  $[0, 1]$ . The timing is as follows. First the seller offers a second price auction with an entrance fee  $f > 0$  which is a fee that the bidders have to pay if they participate in the auction, independently of the outcome of the auction. Second, given the auction format, the bidders have to simultaneously decide whether or not to participate, and if so, how much to bid. If they don't bid at all (or equivalently bid zero) then they don't have to pay the fee but will also never get the object.
1. Describe the payoff function of each bidder as a function of an arbitrary pair of bids.
  2. Define the notion of a strategy and the notion of a Bayesian Nash equilibrium for the auction game with the entry fee. Describe the expected payoff of the auctioneer as a function of the bidding strategies and the reserve price.
  3. Does the imposition of an entrance fee change the bidding strategy of the agents?
  4. Compute the symmetric Bayesian Nash equilibrium.
  5. Compute the seller's expected revenue from the auction with an arbitrary entrance fee  $f > 0$ .
  6. Which entrance fee maximizes the expected revenue of the auctioneer? Is the resulting allocation ex-post efficient (assume the seller's valuation of the good is zero)?
4. **(45)** Consider a firm that can invest an amount  $I$  in a project generating a high observable cash flow  $C > 0$  with probability  $\theta$  and 0 otherwise. This probability depends on the firm's efficiency (type)  $0 < \theta_l < \theta_h < 1$ . Let  $\Pr(\theta = \theta_l) = \alpha$ . The firm needs to raise  $I$  from external investors who do not observe the value of  $\theta$ . Assume that  $\theta_l C - I > 0$ . The external investors are perfectly competitive.
1. Suppose that firms can only promise to repay an amount  $R$  chosen by the firm (with  $0 \leq R \leq C$ ) when cash flow is  $C$  and 0 otherwise.
    1. For this game carefully define the notion of Perfect Bayesian Equilibrium.
    2. Can you find a separating PBE? In other words, can a good firm signal its type?
  2. Suppose now that the firm also has the possibility of pledging some assets as a collateral for the loan. Should a "default" occur (the firm being unable to repay  $R$ ), an asset of value  $K$  to the firm is transferred to the creditor whose valuation is  $xK$  with  $0 < x < 1$ . The size of the collateral  $K$  is a choice variable of the entrepreneur.
    1. For this extended game, define the notion of a Perfect Bayesian Equilibrium.
    2. Give a necessary and sufficient condition for the least cost separating equilibrium to exist. How does it depend on  $\alpha$  and  $x$ ?