Final Exam
Economics 501b
Microeconomic Theory
May 2008

This is a closed-book exam. The exam lasts for 180 minutes. Please write clearly and legibly. Be especially careful in the definition of the game, the payoff function and the equilibrium notions. The allocated points are also a good indicator for your time budget. Please record the answer for each question in a separate bluebook.
1. Consider two firms (players 1 and 2) who are working on a joint project and a bank (player 3) who is a potential investor in the project. First, the entrepreneurs simultaneously decide whether to devote high or low effort to research on the project. They then make a presentation to the bank. If both firms choose high effort in their preliminary research, then the presentation goes well, and otherwise it goes poorly. The bank only observes the outcome of the presentation and not the firms’ effort levels. After the presentation, the bank decides whether to invest in the project. Each firm receives a payoff of 5 if the bank invests and 0 otherwise. In addition, choosing high effort costs a firm 1, while choosing low effort is free. Investing costs the bank 2 and brings a return of 3 for each firm who chose high effort (i.e., a return of 6 if both chose high effort, 3 if only one did, and 0 if neither did). If the bank does not invest, his payoff is $0. All players are risk neutral.

(a) Draw an extensive form representation of this game.

The extensive form has two key features. First, player 2 does not observe the choice of player 1 and therefore there is an information set connecting the two possible decision nodes of player 2. Second, player 3 only observes the choices of the two players if they both choose $H$, therefore there is an information set connecting the three nodes associated with choices $HL$, $LH$ and $LL$.

(b) Find all perfect Bayesian equilibria in which all players choose pure strategies.

There are two PBE in this game. Let $G$ denote the information set associated with a good outcome of the presentation and $B$ the information set associated with a bad outcome and let $\mu$ be the belief that player 3 assigns to both 1 and 2 having played $L$ if the outcome of the presentation is bad. One equilibrium is, $\{LL,(I,NI),\mu = 1\}$, where $(I,NI)$ indicates $I$ played at $G$ and $NI$ is played at $B$. Note that $\mu$ is determined in this equilibrium as it is on the equilibrium path. Also note that we are not free to specify the action at $G$ because it is a singleton information set and sequential rationality requires that player 3 choose $I$ at this information set.

A second equilibrium is $\{HH,(I,NI),\mu \geq \frac{1}{3}\}$. We are free to assigned the belief $\mu$ in this equilibrium and the specified bound ensures that player 3 will choose $NI$ at information set $B$. If $\mu < \frac{1}{3}$, then player 3 will choose $I$ at info set $B$ and there is a profitable deviation for players 1 and 2.

There are no other PBE as we might expect due to the symmetry of player 1 and player 2.
(c) Find all sequential equilibria in which all players choose pure strategies. Be precise about the difference between a perfect Bayesian and sequential equilibria.

The set of SE is a subset of the set of PBE and consequently we need only consider the two PBE above and determine if they are SE.

The equilibrium \{LL, (I, NI), \mu = 1\} is clearly sequential as all beliefs are determined by Baye’s rule in the PBE.

The equilibrium \{HH, (I, NI), \mu \geq \frac{1}{3}\}, however, is not sequential. To see this let \phi be the probability player 3 assigns to the actions HL at info set B and let \epsilon be the probability with which player 1 player L. Then it must be the case that \phi is of order 1 - \epsilon while 1 - \phi is of order \epsilon. Therefore the belief \phi = 1 in the limit and we conclude that \mu = 0. As discussed above \mu = 0 implies that player 3 will play I and therefore the equilibrium is not sequential.

2. Suppose that two players repeatedly play the following normal form game:

\begin{align*}
L & C & R \\
T & 3,3 & 2,1 & -2,4 \\
M & 1,2 & 1,1 & -1,3 \\
B & 4,-2 & 3,-1 & 0,0
\end{align*}

Suppose this game is infinitely repeated, and that each player plays the following strategy: Begin play in stage I. Stage I: If T has always been played, play T. Otherwise, begin stage II. Stage II: If M has always been played since stage II began, play M; otherwise, begin Stage III. Stage III: Play B.

(a) Present the automaton representation of these strategies. What is the payoff from playing these strategies?

Nearly everyone got the Automaton correct. However, note that the automaton should represent the actions of both players and as such the transitions should reflect the actions of both player, rather than just one.

The value of the automaton is obviously \(V(I) = 3, V(II) = 1, V(III) = 0\).

(b) For what values of the discount factor is this strategy profile a subgame-perfect equilibrium?

We require that the incentives constrained hold at every state in the automaton. The incentive at at \(W(III)\) is trivial as \(BB\) is a nash equilibrium, which leaves states \(W(I)\) and \(W(II)\) to check.

For \(W(I)\) we require

\[ V(I) = 3 \geq 4(1 - \delta) + \delta \Rightarrow \delta \geq \frac{1}{3}. \]
For $W(II)$ we require
\[ V(II) = 1 \geq 3(1 - \delta) \Rightarrow \delta \geq \frac{2}{3}. \]

Therefore for $\delta \in \left[\frac{2}{3}, 1\right]$ this is a SPE.

(30) Suppose an entrepreneur owns an assets which can take $n$ different values, $a_i \in \{a_1, a_2, ..., a_n\}$ where, $a_1 < a_2 < ... < a_n$. The entrepreneur knows the value of the asset. He would like to sell the asset but outside investors only have a uniform probability distribution over all possible values $a_i$. The outside investors are always paying the expected value of the asset given their information. The agent can choose to **truthfully disclose** the value of the asset.

(a) Define a strategy for entrepreneur and investor in this game. Define a Perfect Bayesian Equilibrium of this game.

Let $N$ be the action “not disclose” and $D$ be the action “disclose”. A strategy for the entrepreneur is a mapping
\[ S_E : \{a_1, \ldots, a_n\} \rightarrow \{D, N\} \times \mathbb{R}^+. \]

The strategy assigns to each type a choice of whether to disclose or not to disclose and a price in $\mathbb{R}^+$.

Let $A$ be the action “accept ” and $NA$ the action “not accept”. A strategy for the investor is a mapping
\[ S_I : \{a_1, \ldots, a_n\} \cup \{N\} \times \mathbb{R}^+ \rightarrow \{A, NA\}. \]

The strategy associates with each possible disclosure and price, or non disclose and price combination a choice of either $A$ or $NA$.

A perfect Bayesian equilibrium is a triple \{\(S_I, S_E, \{\mu(a_i|\cdot)\}_{i=1}^n\)\} where $\mu(a_i|\cdot) : \{a_1, \ldots, a_n\} \cup \{N\} \rightarrow [0, 1]$ are the beliefs of the entrepreneur such that:

i. $\sum_{i=1}^n \mu(a_i|x) = 1, \forall x \in \{a_1, \ldots, a_n\} \cup N$;

ii. $\mu(a_i|x) \geq 0 \forall a_i$ and $\forall x \in \{a_1, \ldots, a_n\} \cup N$;

iii. Beliefs are derived from Baye’s rule where possible (i.e. $\mu(a_i|a_i) = 1$, $\mu(a_i|N) = \frac{1}{|C|}$ where $C$ is the set of types that do not disclose and $|C|$ is the number of elements in $C$);

iv. $u_E(S_E(a_i), S_E(S_I(a_i))) \geq u_E(S'_E(a_i), S_E(S'_I(a_i)))$ for all mappings $S_I$ and for all types $a_i$; and

v. $\sum_{i=1}^n u_E(S_I(x, p), a_i, p)\mu(a_i|x) \geq \sum_{i=1}^n u_E(S'_I(x, p), a_i, p)\mu(a_i|x)$ for all $x \in \{a_1, \ldots, a_n\} \cup \{N\}$, all $p \in \mathbb{R}^+$ and all mappings $S_I$.

(b) Suppose there is no cost to disclosure. Derive a perfect Bayesian equilibrium of this game.

First, note that the investor will accept only if $p \leq \mathbb{E}(a|x)$ where $\mathbb{E}$ is the expectation under equilibrium beliefs and $x \in \{a_1, \ldots, a_n\} \cup \{N\}$
is the action taken by the entrepreneur. Second note that \( \mathbb{E}(a|x) \) is known by the entrepreneur in equilibrium. Therefore we conclude that \( p = \mathbb{E}(a|x) \) in any equilibrium where \( x \) is the action taken.

Next, define set \( C \) such that \( i \in C \) implies that in equilibrium agent \( i \) chooses \( N \) then Baye’s rule implies that \( \mathbb{E}(a|N) = \frac{1}{|C|} \sum_{i \in C} a_i \) where \( |C| \) is the number of elements in set \( C \).

With this as a set up, note that \( \frac{1}{|C|} \sum_{i \in C} a_i < a_n \) for any set \( C \), and that \( \mathbb{E}(a|a_n) = a_n \). Therefore, by disclosing individual \( n \) gets price \( a_n \) and utility 0, but by not disclosing \( a_n \) receives strictly less than 0. Therefore, we conclude that \( a_n \) discloses. Similar logic then implies that \( a_{n-1} \) discloses and then that all types of entrepreneurs (with the possible exception of \( a_1 \)) disclose.

Therefore, we have a PBE given by:

\[
S_I(a_i) = (a_i, a_i)
\]

\[
S_E(a_i, p) = \begin{cases} A & p \leq a_i \\ NA & p > a_i \end{cases}
\]

\[
S_E(N, p) = \begin{cases} A & p \leq a_1 \\ NA & p > a_1 \end{cases}
\]

and

\[
\mu(a_i|x) = \begin{cases} 1 & x = a_i \\ 0 & x \neq a_i \end{cases}
\]

for \( i > 1 \) and

\[
\mu(a_1|x) = \begin{cases} 1 & \text{if } x = a_1 \text{ or } x = N \\ 0 & \text{otherwise} \end{cases}
\]

Note that you must specify the off equilibrium beliefs and also that there is another PBE very similar to this one in which \( S_E(a_1) = (N, a_1) \).

(c) Suppose now that there is a positive cost of disclosure, say \( K > 0 \). Suppose further that \( a_k - a_{k-1} > a_{k-1} - a_{k-2} \) for all \( k \). Derive now the general properties of the perfect Bayesian equilibrium for (general) \( K > 0 \). Present your arguments clearly.

First, note that the logic above no longer applies. Second, note that the potential benefit to disclosure is increasing in \( i \). Therefore we look for an equilibrium in which types \( a_i \) with \( i < k \) do not disclose and types \( a_i \) with \( i \geq k \) do disclose. The incentive constraints for this equilibrium are

\[
a_k - K \geq \frac{1}{k-1} \sum_{i=1}^{k-1} a_i
\]
and

\[ a_{k-1} - K \leq \frac{1}{k-1} \sum_{i=1}^{k-1} a_i. \]

The first constraint says that type \( a_k \) prefers to disclose, while the second constrained implies that individual \( k-1 \) does not wish to disclose. Further note that the restriction \( a_k - a_{k-1} > a_{k-1} - a_{k-2} \) implies that these two constraints are sufficient to ensure that no type has an incentive to deviate.

Therefore the equilibrium divides types in to two groups, disclosers and non disclosers. In general as \( K \) increases the group of disclosers becomes smaller and that if \( K \) is very large, no type discloses. On the other hand if \( K \) is very small we return to the equilibrium in part (b).

3. (40) A firm run by a risk-neutral manager has assets in place whose value can be 0 or 1 in the future, and a new investment project whose gross value can also be 0 or 1; the start-up cost of the new project is 0.5. Denote by \( a_i \) and \( n_i \) the respective success probabilities for the assets in place and the new project in state of nature \( i \). Suppose that there are only two states of nature, \( i = G, B \), each occurring with probability 0.5. In state \( G \), the good state, we have \( a_G > a_B \), and \( n_G > n_B > 0.5 \). The value of the firm is \( v = a_i + n_i \). We introduce private information by assuming that the manager observes the true state of nature before making the investment decision, but that outside investors do not observe the underlying state the firm is in. Assume that the firm is initially fully owned by its manager. Suppose, however, that the new investment project has to be fully funded externally by a risk-neutral investor.

(a) Describe the socially efficient investment policy.

Answer:

(10) Since \( n_i > \frac{1}{2} \) the cost of the new project it is optimal for the project to be undertaken in both states.

(b) Next consider the signalling game the entrepreneur plays when he is constrained to raising capital in the form of a new equity issue. The firm faces a very simple signalling problem with only two actions: ‘issue new equity and invest in the new project’ and ‘do not issue’. The equity has to be issued for the entire firm value \( v = a_i + n_i \) rather than the new project exclusively.

i. Define the notion of a separating and a pooling perfect Bayesian equilibrium for this game.

Answer:

(5) A strategy for the for the firm is \( s : \{G, B\} \rightarrow \{\emptyset \cup [0,1]\} \) where \( \emptyset \) reflects the decision to not issue equity.

A strategy for the investor is \( I : [0,1] \rightarrow \{A, R\} \).
For each \( s \) that could be proposed there is an induced belief of the Investor \( P(i = G|s) = p(s) \in [0, 1] \).

Denote by \( u_i(s) \) the utility to type \( i \) from proposing \( s \) given that the Investor is playing strategy \( I \).

We have a pooling equilibrium if
\[
\begin{align*}
  s(G) &= s(B) = s^* \in \{\emptyset \cup [0, 1]\} \\
  I(s) &= A \text{ iff } s(p(s)(a_G + n_G) + (1 − p(s))(a_B + n_B)) ≥ \frac{1}{2} \\
  p(s^*) &= \frac{1}{2} \\
  u_i(s^*) &≥ u_i(s) \text{ for all } s \in \{\emptyset \cup [0, 1]\}, i \in \{G, B\}.
\end{align*}
\]

We have a separating equilibrium if
\[
\begin{align*}
  s(G) &≠ s(B) \in \{\emptyset \cup [0, 1]\} \\
  I(s) &= A \text{ iff } s(p(s)(a_G + n_G) + (1 − p(s))(a_B + n_B)) ≥ \frac{1}{2} \\
  p(s(G)) &= 1, p(s(B)) = 0 \\
  u_i(s(i)) &≥ u_i(s) \text{ for all } s \in \{\emptyset \cup [0, 1]\}, i \in \{G, B\}.
\end{align*}
\]

ii. Derive a pooling equilibrium of the game. Is it socially efficient?

Answer:

(5) Consider a pooling equilibrium where both firms issue equity \( s^* \). In order for the Investor to be willing to accept the equity issue it must be that
\[
s^*(\frac{a_G + n_G}{2} + \frac{a_B + n_B}{2}) ≥ \frac{1}{2}
\]

If we specify beliefs
\[
p(i − G|s) = \frac{1}{2} \text{ for all } s < s^*
\]

then issuing equity lower than \( s^* \) would be rejected so is equivalent to not issuing.

Since we are looking only for a pooling equilibrium we can consider the case where this holds, so
\[
s^* = \frac{1}{a_G + n_G + a_B + n_B}
\]

In order for both types of firm to be willing to issue equity it must be that
\[
(1 − s^*)(a_i + n_i) ≥ a_i \text{ for } i = B, G
\]

Notice that
\[
(1−s^*)(a_B+n_B) > \left(1 - \frac{1}{2(a_B + n_B)}\right)(a_B+n_B) = a_B+n_B-\frac{1}{2} > a_B
\]
So we have a pooling equilibrium if

\[(1 - s^*)(a_G + n_G) \geq a_G.\]

Since both firms issue equity and invest this is socially efficient.

iii. Derive a separating equilibrium of the game. Is it socially efficient?

Answer:
First notice that we cannot have an equilibrium where the firm does not offer equity in the bad state, since an offer of

\[s' = \frac{1}{2(a_B + n_B)}\]

would always be accepted by the Investor for any beliefs and would result in positive payoff to the firm in the bad state, as we have demonstrated above.

Next we can notice that it would not be possible to have different type firms issue different levels of equity because then the type that issued higher \( s \) would like to mimic the type that issued lower \( s \). So the only possibility is for only the bad type to issue equity \( s_B \) and the good type not to issue. Since only the bad firm issues equity it must be that

\[s_B^*(a_B + n_B) \geq \frac{1}{2}\]

We can specify beliefs of

\[p(i - G|0) = 0 \text{ for all } s < s_B^*\]

and ensure that any offer of less than \( s_B^* \) would be rejected by the Investor.

And so we know the firm will then choose to issue

\[s_B^* = \frac{1}{2(a_B + n_B)},\]

and we know that

\[(1 - \frac{1}{2(a_B + n_B)})(a_B + n_B) = a_B + n_B - \frac{1}{2} > a_B\]

so the firm is willing to issue equity in the bad state. In the good state, by issuing equity level \( s_B \) the payoff will be

\[(1 - \frac{1}{2(a_B + n_B)})(a_G + n_G)\]

and it must not be optimal for the firm to issue equity in the good state. So we have a separating equilibrium provided

\[(1 - \frac{1}{2(a_B + n_B)})(a_G + n_G) \leq a_G.\]
The separating equilibrium is obviously not optimal given that the new investment projects are not always implemented. In fact, the best projects \((n_G)\) are precisely the ones not to be realized.

iv. For what values of \(a_i\) and \(n_i\) are the equilibria above unique and when do they co-exist?
Answer:
(5) We know that a pooling equilibrium exists iff
\[
(1 - \frac{1}{a_G + n_G + a_B + n_B})(a_G + n_G) \geq a_G
\]
And a separating equilibrium exists if
\[
(1 - \frac{1}{2(a_B + n_B)})(a_G + n_G) \leq a_G
\]
So we have that they co-exist if
\[
(1 - \frac{1}{2(a_B + n_B)}) \leq \frac{a_G}{a_G + n_G} \leq (1 - \frac{1}{a_G + n_G + a_B + n_B}).
\]
When
\[
\frac{a_G}{a_G + n_G} > 1 - \frac{1}{a_G + n_G + a_B + n_B}
\]
only the separating equilibrium exists.

v. Define the Cho-Kreps intuitive criterion for this game.
Answer:
(5) Fix an equilibrium. Denote the payoff to type \(i\) as \(u^*_i\), \(i = G, B\). Suppose there is an equity issue \(s\) such that, the payoff to type \(i \in \{G, B\}\) from offering it is less than \(u^*_i\) for any beliefs (and subsequent acceptance/rejection rule) the Investor could have. Then we say that \(s\) is equilibrium dominated for type \(i\). The intuitive criterion says that if \(s\) is equilibrium dominated for type \(i\) but not for type \(j \neq i\) then the belief of the Investor after seeing \(s\) offered must be
\[
P(i|s) = 0.
\]

vi. Do the equilibria, provided they exist, satisfy the Cho-Kreps intuitive criterion?
Answer:
(5) Both the pooling and the separating equilibrium survive the intuitive criterion.
First notice that beliefs for
\[
s < s^*_G = \frac{1}{2(a_G + n_G)}
\]
are irrelevant for the Investor’s decision: no matter the beliefs about the state conditional on the share offered they should reject.

Consider the pooling first. Shares $s > s^*$ are equilibrium dominated for both firm, while $s \in [s_G, s^*)$ are not equilibrium dominated for either firm. So the Intuitive Criterion places no restriction on the relevant beliefs after any size of equity issue. Now consider the separating equilibrium. In order for it to be an equilibrium we need the Investor to reject if $s < s^*_B$. Since $s \in [s^*_G, s^*_B)$ is not equilibrium dominated for $i = B$ we are free to place enough weight on state $B$ for such an equity issue to be rejected. So we have that both the pooling and separating equilibria are consistent with the Intuitive Criterion.

4. (50) Consider the simple case of an indivisible public project that has value $S$ for consumers. A single firm (monopolist) can realize the project. Its cost function is

$$c = c(e, \beta) = \beta - e, \quad (1)$$

where $\beta$ is a known efficiency parameter and $e$ is the managers’ effort. If the firm exerts effort level $e$, it decreases the (monetary) cost of the project by $e$ and incurs a disutility (in monetary units) of $\psi(e)$. This disutility displays $\psi', \psi'' > 0$, and satisfies $\psi(0) = 0$ and $\lim_{e \to 0} \psi(e) = +\infty$. The firm’s utility level is:

$$U = t - (\beta - e) - \psi(e)$$

The “reservation utility” of the firm is normalized to 0. Let $\lambda > 0$ denote the shadow cost of public funds. That is taxation inflicts a disutility $(1 + \lambda)$ on taxpayers in order to levy $\$1$ for the state. The net surplus of consumers/taxpayers if the project is realized is $S - (1 + \lambda)t$.

(a) Assume first that cost and in particular effort is observable by the regulator. Describe the optimal solution $\{e^*, t^*(e^*)\}$ for a utilitarian regulator, whose ex-post social welfare can be described by

$$S + U - \lambda t$$

who has to respect the participation constraint by the firm. Briefly describe the intuition of your result.

Answer:
(10) The problem is

$$\max_{e,t} \{S - \lambda t - (\beta - e) - \psi(e)\}$$

subject to

$$t - (\beta - e) - \psi(e) \geq 0 \quad (2)$$
Hence
\[ t = (\beta - e) + \psi(e) \]
and hence the problem reduces to
\[ \max_e \left\{ S - (1 + \lambda) ((\beta - e) + \psi(e)) \right\} \]
or
\[ \psi'(e^*) = 1 \quad \text{(3)} \]
and
\[ t(e^*) = (\beta - e^*) + \psi(e^*) . \]

As there is a social cost to providing the transfers, the transfers are determinate even though we started with quasilinear utilities, nonetheless the efficient effort choice is unaffected by the cost of funds \( \lambda \).

(b) Show that the optimal solution can be implemented through fixed price contract such that \( t^*(e) = t^* \) for all \( e \) provided that the project is realized.

Answer:
(10) Suppose the planner offers
\[ t^* = \begin{cases} t(e^*) = (\beta - e^*) + \psi(e^*) & \text{if the project is implemented} \\ 0, & \text{otherwise} \end{cases} \]
so the transfer to the firm is independent of the effort level. We showed in part (a) that the firm’s participation constraint is satisfied. Given that the firm is willing to produce they then choose \( e \) to maximize
\[ U = t^* - (\beta - e) + \psi(e) \]
which results in FOC
\[ \psi'(e) = 1 \]
and the first best solution is realized.

(c) Suppose now that the firm could either be efficient \( \beta_l \) or inefficient \( \beta_h \) with \( \beta_l < \beta_h \). The prior probability of each type is given by \( p_l \) and \( p_h \). The regulator only observes the realized cost \( c \) as defined in (1) and can make a transfer \( t \) to the firm. However, he does not observe \( \beta \) and \( e \) separately. A contract based on the observables \( t \) and \( c \) specifies a transfer-cost pair for each type of firm, namely \( \{t(\beta_l), c(\beta_l)\} \) for type \( \beta_l \) and \( \{t(\beta_h), c(\beta_h)\} \) for type \( \beta_h \). Define the optimization program for the regulator who wants to maximize social welfare and would like to make separate offers to low and high cost types of the firm.

Answer:
(15) Define \( p = p_t \). The planner offers \( \{(t_t, c_l), (t_h, c_h)\} \). Since
\[
c_i = \beta_i - e_i
\]
choosing \( c_i \) also pins down \( e_i \). Since the \( c \) is observable the decision for the firm is to choose \( e_i \) and realize the project with cost \( c_i \) or \( e_j - \beta_j + \beta_i \) and realize the project with cost \( c_j \).

The problem is now
\[
\max \{S - (1 - p) \left( \lambda t_h + (\beta_h - e_h) + \psi(e_h) \right) - p \left( \lambda t_l + (\beta_l - e_l) + \psi(e_l) \right) \}
\]
subject to the participation constraints:
\[
t_i - (\beta_i - e_i) - \psi(e_i) \geq 0
\]
and incentive constraints:
\[
t_i - c_i - \psi(e_i) \geq t_j - c_j - \psi(e_j + \beta_i - \beta_j).
\]

The most common mistake was in the incentive constraints where replicating the cost \( c_j \) meant to adjust the effort provided to \( e_j + \beta_i - \beta_j \), where the differential comes from the fact that different types have different fixed costs, and this is what introduces the Spence-Mirrlees condition (indirectly) into the cost function. Most people wrote the IC constraints as replicating \( e_j \).

(d) Derive the effort levels under the optimal regulation scheme.

Answer:
(15) As \( \beta_i - \beta_h < 0 \), it follows that the low cost firm may have an incentive to pretend to be a high cost firm, as it would then have to produce a lower effort \( e \), namely by \( \beta_l - \beta_h \). So the IR constraint for the low cost firm cannot bind
\[
t_l - c_l - \psi(e_l) \geq t_h - c_h - \psi(e_h + \beta_l - \beta_h) > t_h - c_h - \psi(e_h) \geq 0.
\]

Since the IR constraint doesn’t bind the IC constraint must. Since one IR constraint must bind, we know it must be the IR constraint of the \( \beta_h \) types bind. We can then derive the optimal effort levels subject to the participation constraint of the high cost type and incentive constraint of the low cost type binding, which yields:
\[
\hat{t}_h = (\beta_h - e_h) + \psi(e_h)
\]
and
\[
\hat{t}_l = \hat{t}_h - e_h - \psi(e_h + \beta_l - \beta_h) + (e_l + \psi(e_l))
\]
Inserting this in the program of the social planner and we get
\[
\max \left\{ \begin{array}{l}
S - (1 - p) (1 + \lambda) ((\beta_h - e_h) + \psi(e_h)) \\
-p (\lambda (\psi(e_h) - \psi(e_h + \beta_l - \beta_h)) + (1 + \lambda) ((\beta_l - e_l) + \psi(e_l)))
\end{array} \right\}
\]
and taking first order conditions we get

\[ 1 = \psi'(e_l) \Rightarrow \hat{e}_l = e_l^*, \]

and

\[ 0 = -(1 - p)(1 + \lambda)(-1 + \psi'(e_h)) - p(\lambda(\psi'(e_h) - \psi'(e_h + \beta_l - \beta_h))) \]

or

\[ \psi'(\hat{e}_h) = 1 - \frac{p\lambda(\psi'(e_h + \beta_l - \beta_h) + 1)}{(1 + \lambda)(1 - p) + p\lambda} < 1 \]

and thus we find underprovision of effort by the high cost company.

(e) What can you say about efficiency and information rent for the firms.

Answer:

(5) The more efficient (\( \beta_L \)) firms choose the optimal \( e_L \). They earn positive informational rent since they could pretend to be the high cost firm. In order to lower the informational rent to type \( L \), \( e_h \) is distorted downwards so that the cost of the high cost firm is higher making it more difficult for the low cost firm to mimic the high cost.