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Final Exam
Economics 501b
Microeconomic Theory

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This is a **closed-book** exam. The exam lasts for **180** minutes. Please write clearly and legibly. Be especially careful in the definition of the game, the payoff function and the equilibrium notions. The allocated points are also a good indicator for your time budget. Please record the answer for each question in a separate bluebook.

1. **(40)** Consider two firms (players 1 and 2) who are working on a joint project and a bank (player 3) who is a potential investor in the project. First, the entrepreneurs simultaneously decide whether to devote high or low effort to research on the project. They then make a presentation to the bank. If both firms choose high effort in their preliminary research, then the presentation goes well, and otherwise it goes poorly. The bank only observes the outcome of the presentation and not the firms' effort levels. After the presentation, the bank decides whether to invest in the project. Each firm receives a payoff of 5 if the bank invests and 0 otherwise. In addition, choosing high effort costs a firm 1, while choosing low effort is free. Investing costs the bank 2 and brings a return of 3 for each firm who chose high effort (i.e., a return of 6 if both chose high effort, 3 if only one did, and 0 if neither did). If the bank does not invest, his payoff is \$0. All players are risk neutral.

   (a) Draw an extensive form representation of this game.

   (b) Find all perfect Bayesian equilibria in which all players choose pure strategies.

   (c) Find all sequential equilibria in which all players choose pure strategies. Be precise about the difference between a perfect Bayesian and sequential equilibrium.

2. **(20)** Suppose that two players repeatedly play the following normal form game:

	T	M	B
T	$3,3$	$2,1$	$-2,4$
M	$1,2$	$1,1$	$-1,3$
B	$4,-2$	$3,-1$	$0,0$

Suppose this game is infinitely repeated with a common discount factor $\delta \in (0,1)$, and that each player plays the following strategy:
Begin play in stage I. Stage I: If T has always been played, play T. Otherwise, begin stage II. Stage II: If M has always been played since stage II began, play M; otherwise, begin Stage III. Stage III: Play B.

   (a) Present the automaton representation of these strategies. What is the payoff from playing these strategies?

   (b) For what values of the discount factor δ is this strategy profile a subgame-perfect equilibrium? What is the highest discount factor for which this outcome can be supported as the outcome of a subgame-perfect equilibrium? Substantiate your answer carefully, and be precise.

3. **(30)** Suppose an entrepreneur owns an assets which can take n different values, $a_i \in \{a_1, a_2, ..., a_n\}$ where, $a_1 < a_2 < \cdots < a_n$. The entrepreneur knows the value of the asset. He would like to sell the asset but outside investors only have a uniform probability distribution over all possible values a_i .The outside investors are always paying the expected value of the asset given their information. The agent can choose to *truthfully disclose* the value of the asset. (To be sure, he can either disclose the true value or not disclosing anything; in particular if he chooses to disclose, he cannot misrepresent the value of the asset.)
The game proceeds in three stages. First the entrepreneur chooses a disclosure policy, second the entrepreneur ask a price for the asset, third the outside investor accepts if the price is equal or below the expected value of the asset.

   (a) Define a strategy for entrepreneur and investor in this game. Define a Perfect Bayesian Equilibrium of this game.

   (b) Suppose there is no cost to disclosure. Derive a perfect Bayesian equilibrium of this game.

   (c) Suppose now that there is a positive cost of disclosure, say $K > 0$. Suppose further that $a_k - a_{k-1} > a_{k-1} - a_{k-2}$ for all k. How does the perfect Bayesian equilibrium strategy for the entrepreneur change with an increase in K. Present your arguments clearly.

4. **(40)** A firm run by a risk-neutral manager has assets in place whose value can be 0 or 1 in the future, and a new investment project whose gross value can also be 0 or 1; the start-up cost of the new project is 0.5.
   Denote by a_i and n_i the respective success probabilities for the assets in place and the new project in the state of nature i. Suppose that there are only two states of nature, $i = G, B$, each occurring with probability 0.5. In state G , the good state, we have $a_G > a_B \geq 0$, and $n_G > n_B \geq 0$. The value of the firm is $v = a_i + n_i$. (The probability of each asset to have a positive payoff are thus perfectly correlated.) We introduce private information by assuming that the manager observes the true state of nature before making the investment decision, but that outside investors do not observe the underlying state the firm is in. Assume that the firm is initially fully owned by its manager. Suppose, however, that the new investment project has to be fully funded externally by a risk-neutral investor.

   (a) Describe the socially efficient investment policy.

   (b) Next consider the signalling game the entrepreneur plays when he is constrained to raising capital in the form of a new equity issue (a share s of the gross value of the assets and hence an expected gross value $s \cdot (a_i + n_i)$). The firm faces the simple decision problem with two actions: 'issue new equity and invest in the new project' or 'do not issue'. The equity has to be issued for the entire firm value $v = a_i + n_i$ rather than the new project exclusively. Suppose further that $n_G > n_B > 0.5$.
   The game occurs in two stages. First the firm does or does not offer an equity share s and, then, second the investor accepts or rejects the financing proposal. The investor is assumed to accept the proposal if and only if the net utility, expected returns minus invested funds is nonnegative (i.e. the market for funds is competitive).

      i. Define the notion of a separating and a pooling perfect Bayesian equilibrium for this game.
      ii. Derive a pooling equilibrium of the game. Is it socially efficient?
      iii. Derive a separating equilibrium of the game. Is it socially efficient?
      iv. For what values of a_i and n_i are the equilibria above unique and when do they coexist?
      v. Define the Cho-Kreps intuitive criterion for this game.
      vi. Do the equilibria, provided they exist, satisfy the Cho-Kreps intuitive criterion?

5. **(50)** Consider the simple case of an indivisible public project that has value S for consumers. A single firm (monopolist) can realize the project. Its cost function is

$$c = c(e, \beta) = \beta - e, \tag{1}$$

where β is a known efficiency parameter and e is the managers' effort. If the firm exerts effort level e, it decreases the (monetary) cost of the project by e and incurs a disutility (in monetary units)of $\psi(e)$. This disutility displays $\psi', \psi'' > 0$, and satisfies $\psi(0) = 0$ and $\lim_{e \to \beta} \psi(e) = +\infty$. The firm's utility level is:

$$U = t - (\beta - e) - \psi(e)$$

The "reservation utility" of the firm is normalized to 0. Let $\lambda > 0$ denote the shadow cost of public funds. That is taxation inflicts a disutility $\$(1 + \lambda)$ on taxpayers in order to levy \$1 for the state. The net surplus of consumers/taxpayers if the project is realized is $S - (1 + \lambda)t$.

(a) Assume first that cost and in particular effort is observable by the regulator. Describe the optimal solution $\{e^*, t^*(e^*)\}$ for a utilitarian regulator, whose ex-post social welfare can be described by

$$S + U - (1 + \lambda)t$$

who has to respect the participation constraint by the firm. Briefly describe the intuition of your result.

(b) Show that the optimal solution can be implemented through fixed price contract such that $t^*(e) = t^*$ for all e provided that the project is realized.

(c) Suppose now that the firm could either be efficient β_l or inefficient β_h with $\beta_l < \beta_h$. The prior probability of each type is given by p_l and p_h. The regulator only observes the realized cost c as defined in (1) and can make a transfer t to the firm. However, he does not observe β and e separately. A contract based on the observables t and c specifies a transfer-cost pair for each type of firm, namely $\{t(\beta_l), c(\beta_l)\}$ for type β_l and $\{t(\beta_h), c(\beta_h)\}$ for type β_h. Define the optimization program for the regulator who wants to maximize social welfare and would like to make separate offers to low and high cost types of the firm.

(d) Derive the effort levels under the optimal regulation scheme.

(e) What can you say about efficiency and information rent for the firms.