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Microeconomic Theory (501b)

Problem Set 10. Signalling and Insurance Markets

4/10/08

This problem set is due on Thursday, 4/17/08.

1. **Informed Principal.** Consider the problem we discussed at the end of the class on Tuesday. Suppose the timing is given as follows: (i) nature determines the ability of the worker, (ii) the worker offers a menu of wages as a function of the education level to the firm: $w : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ (iii) the firm decides to accept or reject the menu of wages, (iv) the worker chooses an education level and (v) the worker receives a wage $w(e)$ according to the menu agreed earlier. Derive the perfect bayesian equilibrium of this game. Is it unique? How does it change in p? (This is a simple version of the informed principal model with common values of (Maskin & Tirole 1992).)

2. **Insurance with Homogeneous Agents.** Consider an economy with a continuum of identical agents i with $i \in [0,1]$ Each one has an endowment of w with probability $1-\pi$ and an endowment $w-L$ with probability π. The risk is idiosyncratic to each agent. $L > 0$ can be interpreted as the loss from an accident. The von Neumann Morgenstern utility function is $u'(\cdot) > 0$ and $u''(\cdot) < 0$. (Hint: For question 2 - 4, a graphical description of the consumer behavior in terms of indifference curves and budget lines across the two states, accident and no accident are very informative and illustrative.)

   (a) Consider an egalitarian social welfare function which maximizes the sum (or the integral of the agents) with equal weight to each of the agents. Describe the pareto optimal allocation, using the fact that with a large number (a continuum of agents) a fraction π of the agents always have an accident.

   (b) Consider now an insurance company which offers an actuarially fair insurance such that it charges a premium π for every \$1 compensation in case of an accident. The premium has to be paid independent of the occurrence of an accident or not. Formulate the decision of the representative household in terms of quantity of insurance purchase given the price $p = \pi$ of insurance and describe the nature of the optimal household solution. What happens if the price p increases beyond π?

3. **Insurance with Heterogeneous Agents.** Consider an economy as in (2), but now assume that there are two different kind of consumers. A fraction α of consumers behaves carefully and a fraction $1-\alpha$ of consumers behaves risky, and correspondingly the two groups have different accident rates, $0 < \pi_c < \pi_r < 1$.

   (a) Now analyze the behavior of the two different households for an common insurance premium p. How does the purchase behavior vary as p varies in the interval $[\pi_c, \pi_r]$.

   (b) For an arbitrary price $p \in [\pi_c, \pi_r]$ and given the optimal purchase behavior of the consumers, does the insurance company receives zero-net profit in expectations?

   (c) Does there exists a price $p \in [\pi_c, \pi_r]$ such that given the optimal purchase behavior of the consumers, the insurance company receives zero-net profit in expectations?

4. **Competition in Contracts.** We now allow for a richer set of insurance contracts (beyond linear price insurance contracts) to be offered by the insurance companies, namely price-quantity contracts. Consider a large number of competing insurance companies who can each offer one or multiple insurance contracts for a given premium and a given coverage level. Each contract j is restricted to ask for a premium P_j for a net coverage C_j. In other words, the unit price of contract j is $p_j = P_j / (P_j + C_j)$ for a quantity of coverage $q_j = (P_j + C_j)$ and the net coverage is $(1 - p_j)\, q_j = C_j$.

   (a) We now look for a perfect bayesian equilibrium of this competition game in which the competing firms simultaneously make offers to the customers, and then given the offers the customers accept or reject at most one insurance contract. Since there are only two types in the economy, we can restrict attention to pooling and separating contracts in equilibrium.

      i. First argue that there can be no pooling equilibrium in this competitive insurance market.

      ii. Describe the nature of a contract equilibrium which displays separation across types. What can you say about the insurance coverage offered in these contracts to different types of the agents.

      iii. As a function of α, the fraction of low risk agents, what can you say about the existence of perfect Bayesian equilibrium in this contract game.

Reading. MWG: 13. G: 4. The question 2 - 4 essentially cover the material presented in (Rothschild & Stiglitz 1977) and (Wilson 1977).

References

Maskin, E. & J. Tirole. 1992. “The Principal-Agent Relationship with an Informed Principal II: Common Values.” *Econometrica* 60:1–42.

Rothschild, M. & J. Stiglitz. 1977. “Equilibrium in Competitive Insurance Markets.” *Quarterly Journal of Economics* 90:629–649.

Wilson, C.A. 1977. “A Model of Insurance Markets with Incomplete Information.” *Journal of Economic Theory* 16:167–207.