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Microeconomic Theory (501b)

Problem Set 11. Adverse Selection and Mechanism Design

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This problem set is due on Thursday, 4/24/08.

1. Consider the following problem with I agents and a social planner. The utility of each agent i is given by

$$v_i(x, t_i, \theta_i) = u_i(x, \theta_i) - t_i.$$

The utility of the social planner is

$$v_0(x, t) = u_0(x) + \sum_{i=1}^{I} t_i,$$

where $x \in \mathbb{R}_+^n, \theta_i \in \mathbb{R}_+$ and $u_i(x, \theta_i) \geq 0$ and $u_0(x) \leq 0, u_0(0) = 0$ and $u_i(x, \theta_i)$ are continuous in x and θ_i. The paramter θ_i determines the preferences of agent i and is for this question considered complete information.

   (a) Define the egalitarian social welfare function (i.e. equal weights to each member of the society) including the agents and the planner. Describe the properties of the socially efficient allocation associated with the egalitarian social welfare function.

   (b) Consider the generalization of the egalitarian social welfare function to a utilitarian social welfare function with constant and positive, but not necessarily equal weights to each member of the society. Describe the properties of the socially efficient allocation associated with a general utilitarian social welfare function.

2. Consider now the same environment as in question 1, but let θ_i be private information of agent i and let $f^* : \Theta \to X \subset \mathbb{R}_+$ be the social choice function which maximizes the egalitarian social welfare function for every type profile θ. Consider now a *direct* mechanism in which agent is asked to report his type and following the reports $\theta' = (\theta'_1, ..., \theta'_i, ..., \theta'_I)$, thruthful or not, the social planner follows an outcome function $g : \Theta \to X \times \mathbb{R}^I$, where the monetary transfer are given by

$$t_i^G(\theta') = h_i(\theta'_{-i}) - \sum_{j \neq i} u_j(f^*(\theta'), \theta'_j). \quad (1)$$

(The notation θ_i' refers to the report of the agent i in contrast to the true type θ_i), where $h_i(\theta_{-i})$ is some function which only depends on the reports of the other agents. The outcome function

$$g(\theta') = \left(f^*(\theta'), \left(t_i^G(\theta)\right)_{i=1}^I\right)$$

is referred to as the Groves mechanism.

(a) Show that a truthful reporting strategy for every agent: $s_i^* : \Theta_i \to \Theta_i$ with $s_i^*(\theta_i) = \theta_i$ present a weakly dominant strategy for every agent i. Start by defining carefully the incentive constraints for every agent i.

(b) Given an arbitrary Groves mechanism, can we satisfy the ex post participation constraint of every agent i, i.e. the requirement that every agent receives a nonnegative net utility from participating in the mechanism.

3. Consider now a special version of the Groves mechanism, referred to as the Pivotal mechanism, where

$$h_i(\theta'_{-i}) = \sum_{j \neq i} u_j\left(f^*_{-i}(\theta'), \theta'_j\right), \tag{2}$$

where $f^*_{-i}(\theta')$ is the social choice function which maximizes the egalitarian social welfare function of a society consisting of everybody excluding agent i.

(a) Does a truthful reporting strategy for every agent: $s_i^* : \Theta_i \to \Theta_i$ with $s_i^*(\theta_i) = \theta_i$ still present a weakly dominant strategy for every agent i in this specific mechanism.

(b) Given the Pivot mechanism, can we satisfy the ex post participation constraint of every agent i, i.e. the requirement that every agent receives a nonnegative net utility from participating in the mechanism given his type and the type announcement of all other agents.

(c) Show that in the truthtelling equilibrium every agent receives as his net utility his marginal contribution to the social welfare.

(d) Does the Pivot mechanism lead to a weak "no budget deficit" condition of the form

$$\sum_{i=1}^I t_i(\theta') \geq 0$$

and/or a stronger "no budget deficit" condition of the form:

$$\sum_{i=1}^I t_i(\theta') + u_0(f^*(\theta')) \geq 0\,.$$

4. Consider now a special allocation problem in which the social planner has n units of the object to allocate at constant marginal cost which we normalize to zero. Each agent has a unit demand with a valuation of θ_i for a single unit and $n < I$. Derive the allocation rule and the transfer pricing rule implied by the Pivot mechanism above for $n = 1$ and for $n > 1$.

5. **Revised.** For the binary type model with monopolistic price discrimination, suppose that the seller is restricted to use a two part tariff rather than a general nonlinear pricing rule, i.e.

$$t(q) = F + pq$$

where F is fixed fee and independent of q and p is the linear price for quality. The utility function of the consumer is given by

$$u(\theta_i, q) = \theta_i v(q)$$

with $v'(q) > 0$ and $v''(q) < 0$. In fact we simplify further and let

$$v(q) = \frac{1 - (1-q)^2}{2}.$$

   (a) Derive the optimal (F, p) for the monopolist as a function of the primitives of the model.

   (b) How does the solution of the two-part tariff compare with the non-linear pricing rule in terms of

      i. social efficiency,

      ii. revenues for the monopolist.

6. Local and Global Constraints with Spence Mirrlees Preferences.

   (a) Exercise 2.1 in (Salanie 1997).

   (b) Show that you can extend the argument to all preferences satisfying Spence-Mirrlees preferences, which are characterized by (supposing differentiability)

$$\frac{\partial u(\theta, q)}{\partial q} > 0 \text{ and } \frac{\partial^2 u(\theta, q)}{\partial q \partial \theta} > 0.$$

   is sufficient to consider the case with finitely many types.

   (c) Consider the finite model for the product case

$$u(\theta, q) = \theta \cdot q$$

   and solve the optimal quality provision problem of the seller.

i. As a first intermediate step describe the net utility that each type gets after you replaced the transfers by appealing to the appropriate incentive and participation constraint

ii. As a second intermediate step describe the net revenue that the seller gets from the provision of quality q_k to type θ_k.

Reading MWG: 14, 23, S: 2.

References

Salanie, B. 1997. *The Economics of Contracts.* Cambridge: MIT Press.