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Microeconomic Theory (501b)

Problem Set 7. Bayesian Games

3/6/08

This problem set is due on Thursday, 3/27/08.

1. **A Simple Bayesian Game.** Consider the following Bayesian game.

   - Nature determines whether the payoffs are as in Game 1 (the game on the left) or Game 2, each game being equally likely.
   - Player 1 (the row player) learns whether nature has drawn Game 1 or Game 2, but player 2 does not.
   - Player 1 chooses either T or B; player 2 simultaneously chooses either L or R.
   - Payoffs are given by the game drawn by nature.

	L	R
T	$1,1$	$0,0$
B	$0,0$	$0,0$

   Game 1

	L	R
T	$0,0$	$0,0$
B	$0,0$	$2,2$

   Game 2

   (a) Draw the extensive-form game tree of the above game.

   (b) Write down the set of pure strategies for each player.

   (c) Find all the pure-strategy Bayesian Nash equilibria.

2. Consider the following game of public good provision with private costs $c_i \geq 0$:

	C	D
Contribute	$1-c_1, 1-c_2$	$1-c_1, 1$
Don't Contribute	$1, 1-c_2$	$0,0$

   where the cost c_i is i.i.d distributed with a uniform density on $[0,2]$.

   (a) Define the notion of a mixed strategy in this Bayesian game and define the notion of a Bayesian equilibrium for this game.

   (b) Compute a Bayesian Nash equilibrium of this game in pure strategies.

(c) Consider now a general, continuous and strictly increasing distribution of i.i.d. costs given by $F(c_i)$ with $F(0) = 0$. Show that there always exists a pure strategy equilibrium where $s_i(c_i) = C$ for all $c_i \leq c_i^*$ and $s_i(c_i) = D$ for all $c_i > c_i^*$ for some c_i^*.

3. Consider the battle of the sexes game:

	Opera	Baseball
Opera	2,1	0,0
Baseball	0,0	1,2

(a) Compute the pure and mixed strategy equilibria of this complete information game.

(b) Consider now a perturbed version of the game where

	Opera	Baseball
Opera	$2 + \delta\varepsilon_1, 1 + \delta\varepsilon_2$	$0 + \delta\varepsilon_1, 0$
Baseball	$0, 0 + \delta\varepsilon_2$	$1, 2$

where $\varepsilon_i \sim U[-1, 1]$ and $0 < \delta < 1$ and ε_i is private information to agent i. Show that as $\delta \to 0$, *all* equilibria of the complete information game can be obtained as pure strategy limits of the Bayesian game with private information.

4. This question asks you to show the equivalence ex ante and interim definitions of (Bayesian) Nash equilibrium in static incomplete information games with a finite number of types. Consider a game consisting of a set of players $1, ..., I$; each player has a finite set of actions A_i; each player has a type $t_i \in T_i$. The profile of types $t \equiv (t_1, ..., t_I)$ is chosen according to a probability distribution p with $p(t) > 0$ for all $t \in T$. Payoff function for player i is a function $g_i : A \times T \to \mathbb{R}$. An incomplete information game strategy for player i is a function $s_i : T_i \to A_i$. Write S_i for the set of such strategies, let $S = S_1 \times .. \times S_I$, and write $s(t) = (s_1(t_1), .., s_I(t_I))$.
Show that strategy profile s^* is an ex ante Bayesian Nash equilibrium of this game if and only if it is an interim Bayesian Nash equilibrium.

5. Consider Akerlof's model as we presented it in class. We referred to v_S and v_B as the marginal willingness to pay of buyer and seller, which were commonly known. We assumed that the quality of the car, θ, was unobservable and private information to the seller. Suppose now that v_S is also private information to the seller and is distributed uniformly on the interval $[\bar{v}_S - \varepsilon, \bar{v}_S + \varepsilon]$ for some $\varepsilon > 0$. (Hint: A graphical and verbal discussion for this question is sufficient. The problem was suggested by one of your classmates, but the calculus turns out to be more complex than I had thought initially.)

(a) Derive necessary and sufficient conditions such that there is positive trade in some equilibrium. (Hint: It might be helpful to depict the joint distribution of ε and θ in a two-dimensional graph.)

(b) How does the introduction of private information about the willingness to pay of the seller affect the conditions. Does the probability of trade decrease or increase with ε.

Reading MWG: 8E. G: Chapter 3

References