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Microeconomic Theory (501b)

Problem Set 8. Bayesian Games

3/27/08

This problem set is due on Thursday, 4/3/08.

1. **First Price Auction with Private Values.** Consider a first-price sealed-bid auction of an object between two risk-neutral bidders. Each bidder i (for $i = 1, 2$) simultaneously submits a bid $b_i \geq 0$. The bidder who submits the highest bid receives the object and pays his bid; both bidders win with equal probability in case they submit the same bid. Before the auction takes place, each bidder i privately observes the realization of a random variable t_i that is drawn independently from a uniform distribution over the interval $[0, 1]$. The actual valuation of the object to bidder i is equal to $t_i + 0.5$. Therefore, the payoff of bidder i is given by

$$u_i = \begin{cases} t_i + 0.5 - b_i & \text{if } b_i > b_j \\ \frac{1}{2}(t_i + 0.5 - b_i) & \text{if } b_i = b_j \\ 0 & \text{if } b_i < b_j \end{cases}.$$

   (a) Derive the symmetric linear Bayesian Nash equilibrium for this game (i.e., each bidder uses an equilibrium strategy of the form $b_i = \alpha t_i + \beta$).

   (b) What is the conditionally expected payoff of bidder i with type t_i in this equilibrium?

2. **First Price Auction with Common Values.** Consider the first-price auction in problem (2), except that the actual valuation of the object to bidder i is now equal to $t_i + t_j$ $(j \neq i)$ and therefore the payoff of bidder i now becomes

$$u_i = \begin{cases} t_i + t_j - b_i & \text{if } b_i > b_j \\ \frac{1}{2}(t_i + t_j - b_i) & \text{if } b_i = b_j \\ 0 & \text{if } b_i < b_j \end{cases}.$$

   Notice that, given his own private type t_i, the expected value of the object is $t_i + 0.5$ (i.e., the same as that in problem (2)).

   (a) Derive the symmetric linear Bayesian Nash equilibrium for this game.

(b) Compare the equilibrium bid for any given type t_i in this problem to that in problem (2). Interpret your result.

3. **Ex Post Equilibrium.**

   (a) Define a notion of ex post Bayesian Nash equilibrium for a finite game (i.e. a finite set of pure actions and a finite number of players), similar to the ex ante and the interim version of the Bayesian Nash equilibrium we did in class, but appropriate for the notion of ex post (information revelation).

   (b) Which relationships between the ex ante and the interim notion can you establish?

   (c) Consider now the first price and second price auction for the model developed in (1). Does the first price auction have an ex post equilibrium? Does the second price auction have an ex post equilibrium?

4. **MWG 23.E.6 (Bilateral Trade).**

5. **Purification.** Consider the battle of the sexes game:

	Opera	Baseball
Opera	2,1	0,0
Baseball	0,0	1,2

   (a) Compute the pure and mixed strategy equilibria of this complete information game.

   (b) Consider now a perturbed version of the game where

	Opera	Baseball
Opera	$2+\delta\varepsilon_1, 1+\delta\varepsilon_2$	$0+\delta\varepsilon_1, 0$
Baseball	$0, 0+\delta\varepsilon_2$	$1, 2$

   where $\varepsilon_i \sim U[-1,1]$ and $0 < \delta < 1$ and ε_i is private information to agent i. Show that as $\delta \to 0$, *all* equilibria of the complete information game can be obtained as pure strategy limits of the Bayesian game with private information.

Reading. Gibbons, Chapter 3. MWG 8.E