

DIRK BERGEMANN  
DEPARTMENT OF ECONOMICS  
YALE UNIVERSITY

## Microeconomic Theory (501b)

### Problem Set 9. Bilateral Trade and Signalling

4/3/08

This problem set is due on Thursday, 4/10/08.

1. **Bilateral Trading.** Consider the following trading game between a buyer and a seller. The buyer has a valuation  $v \sim \mathcal{U}[0, 1]$  and seller has a cost  $c \sim \mathcal{U}[0, 1]$  of producing the product. The valuation and the cost is private information to buyer and seller respectively and the utility function of buyer and seller is  $v - p$  and  $p - c$  if a trade occurs at  $p$  and is 0 and 0 if no trade occurs. Trade is voluntary.

- (a) Describe the ex post efficient trading rule in this environment. Compute the expected net value of the ex post efficient trading rule. (A diagram may help.)
- (b) Consider now the following trading mechanism. Buyer and seller offer a bid and a ask price,  $p_B$  and  $p_S$ , respectively. A trade occurs if  $p_B \geq p_S$  and the trading price is

$$p = \frac{1}{2}(p_B + p_S). \quad (1)$$

Formulate the trading game as a Bayesian game and define the notion of a Bayesian Nash equilibrium for this game.

- (c) Consider now the following fixed price strategy by each player

$$p_B(v) = \begin{cases} 0 & \text{if } v < p \\ p & \text{if } v \geq p \end{cases}$$

and

$$p_S(c) = \begin{cases} p & \text{if } c \geq p \\ 1 & \text{if } c > p \end{cases}$$

for some  $p \in [0, 1]$ . Does this strategy pair for a Bayesian Nash equilibrium for some  $p$ ? For what values of  $p$  does it form a Bayesian Nash equilibrium? Does/do the equilibria lead to the realization of the ex post efficient trading rule? Graphically depict the trading rule/area generated by the fixed price equilibrium strategy.

(d) Now find an equilibrium in linear strategies for buyer and seller with the pricing rule given by (1). Compute the coefficient of the linear trading strategy equilibrium (using the techniques we used for the auction example) and graphically depict the trading rule/area generated by the linear equilibrium strategy.

2. **Gibbons 4.3 and 4.5 (Pooling and Separating Equilibrium)**

3. **Gibbons 4.7 (Hybrid Equilibrium)**

4. **Gibbons 4.14 (Pre-Trial Negotiation)**

5. **Signalling with Many Types.** Consider the signalling model we discussed in class with a modification regarding the ability type.

(a) Suppose there are now a finite number,  $k$ , of ability types

$$a \in \left\{ 1, 1 + \frac{1}{K}, \dots, 1 + \frac{K-1}{K}, 1 + \frac{K}{K} \right\},$$

with associated probabilities  $p_k$ .

- i. Identify the least cost separating equilibrium with the finite number of types. What is the education level provided by each type  $k$ .
  - ii. Can you construct semi-separating equilibria where the types  $k = 1, \dots, l$  pool and all the types  $k = l + 1, \dots, K$  separate from the pool and from each other.
- (b) Now consider the case of a continuum of types  $a \in [1, 2]$  and a continuous density  $f(a)$  about the interval  $[1, 2]$ . Construct the least cost separating equilibrium.

6. **Reading.** Gibbons, Chapter 4. Salanie, Chapter 4.