1. **Nonlinear Pricing.** Consider the problem of an optimal menu when $v(\theta, q) = \theta \sqrt{q}$ and $c(q) = q$ and the distribution is given by the uniform distribution on the unit interval.

   (a) Compute the revenue maximizing direct mechanism, which associates to every reported type $\theta$ a pair $(q(\theta), t(\theta))$ of quantities $q(\theta)$ and prices $t(\theta)$, or $\theta \mapsto (q(\theta), t(\theta))$.

   (b) Translate the direct mechanism into an indirect mechanism, in particular to a nonlinear pricing mechanism $(q, t(q))$ which associates to every $q$ a $t(q)$, or $q \mapsto t(q)$.

   (c) What can you say about $t(q)/q$, i.e. the price per quantity as the quantity increases.

2. **Competition with Nonlinear Pricing.** Suppose now that there are a large (infinite) number of sellers who can offer nonlinear pricing contracts $(q, t(q))$ to the customer. The contracts are offered simultaneously and each firm seeks to maximize the expected revenue from its offering. Describe the symmetric equilibrium contract and compare it with the nonlinear pricing result with the monopolist analyzed in (1).

3. **Nonlinear Pricing.** Now generalize the analysis to $\theta v(q)$ with $v'(q) > 0$ and $v''(q) < 0$ and $F(\theta)$, maintain the linear cost model and assume that

   $$\theta - \frac{1 - F(\theta)}{f(\theta)}$$

   is increasing in $\theta$.

   (a) Compute the revenue maximizing direct mechanism.

   (b) Establish that $t(q)/q$ is decreasing in $q$.
(c) Establish that the revenue maximizing direct mechanism could also be implemented by a menu of two part-tariffs, \((T(\theta), p(\theta))\), where \(T(\theta)\) is the fixed fee and \(p(\theta)\) is the price per unit (i.e., if a customer chooses a particular two-part tariff then he has to pay \(T(\theta)\) independent of the quantity he purchase, but can then buy as many units as he likes at the price \(p(\theta)\) per unit. (The concavity of \(t(q)\) might be useful.)

4. **Insurance with Homogeneous Agents.** Consider an economy with a continuum of identical agents \(i\) with \(i \in [0, 1]\) Each one has an endowment of \(w\) with probability \(1 - \pi\) and an endowment \(w - L\) with probability \(\pi\). The risk is idiosyncratic to each agent. \(L > 0\) can be interpreted as the loss from an accident. The von Neumann Morgenstern utility function is \(u'(\cdot) > 0\) and \(u''(\cdot) < 0\). (Hint: For question 4 - 6, a graphical description of the consumer behavior in terms of indifference curves and budget lines across the two states, accident and no accident are very informative and illustrative.)

(a) Consider an egalitarian social welfare function which maximizes the sum (or the integral of the agents) with equal weight to each of the agents. Describe the pareto optimal allocation, using the fact that with a large number (a continuum of agents) a fraction \(\pi\) of the agents always have an accident.

(b) Consider now an insurance company which offers an actuarially fair insurance such that it charges a premium \(\pi\) for every $1 compensation in case of an accident. The premium has to be paid independent of the occurrence of an accident or not. Formulate the decision of the representative household in terms of quantity of insurance purchase given the price \(p = \pi\) of insurance and describe the nature of the optimal household solution. What happens if the price \(p\) increases beyond \(\pi\)?

5. **Insurance with Heterogeneous Agents.** Consider an economy as in (4), but now assume that there are two different kind of consumers. A fraction \(\alpha\) of consumers behaves carefully and a fraction \(1 - \alpha\) of consumers behaves risky, and correspondingly the two groups have different accident rates, \(0 < \pi_c < \pi_r < 1\).

(a) Now analyze the behavior of the two different households for an common insurance premium \(p\). How does the purchase behavior vary as \(p\) varies in the interval \([\pi_c, \pi_r]\).

(b) For an arbitrary price \(p \in [\pi_c, \pi_r]\) and given the optimal purchase behavior of the consumers, does the insurance company receives zero-net profit in expectations?

(c) Does there exists a price \(p \in [\pi_c, \pi_r]\) such that given the optimal purchase behavior of the consumers, the insurance company receives zero-net profit in expectations?
6. **Competition in Contracts.** We now allow for a richer set of insurance contracts (beyond linear price insurance contracts) to be offered by the insurance companies, namely price-quantity contracts. Consider a large number of competing insurance companies who can each offer one or multiple insurance contracts for a given premium and a given coverage level. Each contract \( j \) is restricted to ask for a premium \( P_j \) for a net coverage \( C_j \). In other words, the unit price of contract \( j \) is \( p_j = P_j / (P_j + C_j) \) for a quantity of coverage \( q_j = (P_j + C_j) \) and the net coverage is \((1 - p_j) q_j = C_j\).

(a) We now look for a perfect bayesian equilibrium of this competition game in which the competing firms simultaneously make offers to the customers, and then given the offers the customers accept or reject at most one insurance contract. Since there are only two types in the economy, we can restrict attention to pooling and separating contracts in equilibrium.

   i. First argue that there can be no pooling equilibrium in this competitive insurance market.

   ii. Describe the nature of a contract equilibrium which displays separation across types. What can you say about the insurance coverage offered in these contracts to different types of the agents.

   iii. As a function of \( \alpha \), the fraction of low risk agents, what can you say about the existence of perfect Bayesian equilibrium in this contract game.

7. Compare the nature of the competitive market and its impact for the social efficiency of the market outcome in the model of nonlinear pricing and the insurance model. Are there any differences between the two markets and if so, what explains the nature of the differences.

**Reading.** MWG Chapter 13, Gibbons, Chapter 4. Salanie, Chapter 4. The linear cost function and concave cost function is analyzed in the model by (Maskin and Riley 1984). The linear utility and convex cost function is analyzed in (Mussa and Rosen 1978). The question 5 - 7 essentially cover the material presented in (Rothschild and Stiglitz 1977) and (Wilson 1977) as described in (Laffont 1989), Chapter 8.

**References**


