Microeconomic Theory (501b)

Problem Set 11. Mechanism Design

4/16/09

This problem set is due on Thursday, 4/23/09.


2. **Payoff Equivalence.** In class we stated a version of a payoff equivalence result in the case of nonlinear pricing. The full statement is an equivalence result given below.

   **Theorem.** The direct mechanism \( y(\theta) = (q(\theta), t(\theta)) \) is incentive compatible if and only if:
   
   (i) the truth-telling utility is described by:
   
   \[
   U(\theta) - U(0) = \int_{0}^{\theta} q(s) ds,
   \]

   (ii) \( q(s) \) is nondecreasing.

   Now establish that (i) and (ii) imply incentive compatibility, i.e. the part that we did not establish in class.

3. **Revelation Principle.** In class we stated the revelation principle for a single agent. Now, state and proof the revelation principle for many agents with

   (a) for pure strategy equilibrium in dominant strategies;
   (b) for pure strategy Bayesian Nash equilibrium;
   (c) what, if any are differences in the proof of the revelation principle for dominant and Bayesian Nash equilibrium.

4. **Bilateral Trading.** Suppose there is a *continuum* of buyers and sellers (with quasilinear preferences). Each seller initially has one unit of indivisible good and each buyer initially has none. A seller’s valuation for consumption of the good is \( \theta_1 \in [\underline{\theta}_1, \overline{\theta}_1] \), which is independently and identically drawn from distribution \( \Phi_1(\cdot) \) with associated strictly positive density \( \phi_1(\cdot) \). A buyer’s valuation from consumption of the good is \( \theta_2 \in [\underline{\theta}_2, \overline{\theta}_2] \), which is independently and identically drawn from distribution \( \Phi_2(\cdot) \) with association strictly positive density \( \phi_2(\cdot) \).
(a) Characterize the trading rule in an ex post efficient social choice function. Which buyers and sellers end up with a unit of the good?

(b) Exhibit a social choice function that has the trading rule you identified in (a), is Bayesian incentive compatible, and is the individually rational. [Hint: Think of a "competitive" mechanism.] Conclude that the inefficiency identified in the Myerson-Satterthwaite theorem goes away as the number of buyers and sellers grows large.

5. **Adverse Selection.** Consider the binary adverse selection model with nonlinear pricing that we studied in class. In class, we assumed that the interim (or ex post as they are identical with one agent) participation constraints have to hold. Now suppose that the participation constraint has only to hold ex ante, i.e. in expectation the agent is at least receiving zero expected surplus. Do the analysis under this weaker version of the participation constraint and analyze how the optimal contract with ex-ante participation constraint differs from the one subject to the interim participation constraint.

6. **Single Unit Auction.** Suppose the valuation of agent \(i = 1, 2\), and \(j \neq i\), for the object is given by

\[
u_i(\theta_i, \theta_j) = \theta_i + \gamma \theta_j
\]

with \(0 < \gamma < 1\). The type \(\theta_i\) is private information of agent \(i\) and as the valuation of the object by agent \(i\) also depends on the type of his competitor \(j\), we are in a world of interdependent rather than private values.

(a) Find a transfer rule \(t^*\) such that truth-telling is an ex post equilibrium in the direct revelation game and such that the efficient allocation is realized and such that the transfer of each agent only depends on the announcement of the other agent and the allocation decision, but not on the announcement of agent \(i\).

(b) Given the transfer rule, is truth-telling also an equilibrium in dominant strategies?

**Reading.** MWG Chapter 23; Salanie, Chapter 5; Stole Section 3.