This problem set is not due but rather constitutes practice on the material presented in class during this week. A solution to the problem set will be posted on Thursday, 4/30/09. Enjoy your summer!

1. Consider the regular moral hazard model with a risk-neutral principal and a risk averse agent. The agent can choose between two effort levels, $a_i \in \{a, \overline{a}\}$ with associated cost $c_i \in \{c, \overline{c}\} = \{0, c\}$, with $c > 0$. Each action generates stochastically one of two possible profit levels, $x_i \in \{x, \overline{x}\}$ with $p(x | a) > p(x | \overline{a})$. The utility function of the agent is

$$u(w, c_i) = \ln w - c_i$$

The value of the outside option is normalized to 0. (Risk-neutrality of the principal implies that his payoff function is $x - w$.)

(a) Carefully describe the principal-agent problem when the principal wishes to implement the high effort level $\overline{a}$.

(b) Solve explicitly for the optimal wage schedule to be offered to the agent which implements the high effort level $\overline{a}$.

(c) (Renegotiation 1) Consider now the following extension to the moral hazard problem. After the principal has offered an (arbitrary) wage schedule and the agent has chosen and performed an (arbitrary) effort level, but before $x$ is revealed, the principal has the possibility to offer a new contract to the agent. The agent can either accept or reject the new offer. If he accepts the new contract, then it replaces the old contract, if he rejects the new contract, then the old one remains in place. Show that there is no subgame perfect equilibrium of the game where $\Pr(a = \overline{a}) = 1$. (Hint: Consider the optimal contract after $a$ has been chosen but before $x$ has been realized.)

(d) (Renegotiation 2) Consider now the following modification to the renegotiation problem above. Suppose now that the agent can make the new proposal and the principal can either accept or reject the new offer. Suppose further that the principal can observe the action at the time of the new proposal but that the contract can only depend on $x$ and not on $a$. The timing is otherwise unchanged. Derive the subgame perfect equilibrium of the principal-agent problem. What can you say about the efficiency of the arrangement.
2. (Moral Hazard in Teams, (Holmstrom 1982)) Consider the following moral hazard problem with many agents. Suppose output is one-dimensional, deterministic and concave in $a_i$ and depends on the effort of the $n$ agents in the team:

$$x = x(a_1, a_2, ..., a_n).$$

Each agent $i$ has convex effort cost $c_i(a_i)$. Each agent observe his effort and the joint output $x$. A contract among the agents is a set of wages \(\{w_i(x)\}_{i=1}^{n}\), which depend only on the publicly observable output $x$. The set of wages have to be budget balanced, or

$$\sum_{i=1}^{n} w_i(x) = x$$

for all $x$. (Think of the team as a cooperative or partnership). The utility function of each agent is $w_i(x) - c_i(a_i)$.

(a) Describe the first-best allocation policy $a^*$.

(b) Suppose (without loss of generality) that the team is restricted to using differentiable wages, or $w'_i(x)$ exists for all $x$ and $i$. Show that there is no wage schedule which allows the team to realize the first best policy.

(c) Next, introduce an $n + 1$th, who does not deliver any effort to the team, but can be entitled to transfers (the “principal” or “budget-breaker”). Show that you can now design a wage schedule, not necessarily differentiable, such that

$$\sum_{i=1}^{n+1} w_i(x) = x$$

for all $x$. In fact, you can design the contract such that even

$$\sum_{i=1}^{n} w_i(x) = x$$

holds on the equilibrium path, but not off the equilibrium path.

3. (First Order Stochastic Dominance versus Monotone Likelihood Ratio ((Milgrom 1981))).

(a) Define monotone likelihood ratio and first-order stochastic dominance.

(b) Show that with two outcomes, the two notion are equivalent.

(c) Give an example to show that the equivalence does not hold in general. Show that the monotone likelihood ratio property implies first order stochastic dominance.
4. (Moving Support.) Consider a standard agency model with a risk and effort averse agent. The stochastic output of his action is given by

\[ x = a + \varepsilon \]

where the support of \( \varepsilon \) is given by \([-k, +k]\), where \( k > 0 \). Suppose for simplicity that \( \varepsilon \) is uniform on this interval.

(a) Show that with this specification of the production technology, the agency problem can be resolved at the first-best: \( a^* \) and \( w^* \).

(b) Can you give a general condition in terms of the likelihood ratios when the first-best can be achieved.

5. Reading MWG: 14 S: Chapter 5.

References
