1 Question 8.1: First Price Auction w/ Private Values

8.1(a)
First, notice that given \( b_j(t_j) \), bidding \( b_i \) gives player \( i \) of type \( t_i \) an expected payoff of

\[
E_{b_j(t_j)}[\pi_i(b_i, t_i)] = (t_i + 0.5 - b_i) Pr(b_i > b_j(t_j)) + \frac{1}{2}(t_i + 0.5 - b_i) Pr(b_i = b_j(t_j)).
\]

Assume for \( b_j = \alpha t_j + \beta \forall j = 1, 2 \), then we have for player \( i \neq j \),

\[
E_{b_j(t_j)}[\pi_i(b_i, t_i)] = (t_i + 0.5 - b_i) Pr(b_i > \alpha t_j + \beta) + \frac{1}{2}(t_i + 0.5 - b_i) Pr(b_i = \alpha t_j + \beta) \\
= (t_i + 0.5 - b_i) \frac{b_i - \beta}{\alpha} + \frac{1}{2}(t_i + 0.5 - b_i) Pr(t_j = \frac{b_i - \beta}{\alpha}) \\
= (t_i + 0.5 - b_i) \frac{b_i - \beta}{\alpha} + \frac{1}{2}(t_i + 0.5 - b_i) (0) = (t_i + 0.5 - b_i) \frac{b_i - \beta}{\alpha}.
\]

F.O.C. yields

\[-\frac{b_i - \beta}{\alpha} + \frac{1}{\alpha}(t_i + 0.5 - b_i) = 0 \]

\[\Rightarrow -\frac{b_i - \beta}{\alpha} + \frac{1}{\alpha} t_i = -\frac{b_i - \beta}{\alpha} + \frac{1}{\alpha} t_i = 0 \Rightarrow b_i = \frac{1}{\alpha} (0.5 + \beta) + \frac{1}{\alpha} t_i.
\]

Matching to the initial guess, we have:

\[
\frac{1}{\alpha} (0.5 + \beta) + \frac{1}{\alpha} t_i = \beta + \alpha t_i \\
\Rightarrow \left\{ \begin{array}{l}
\alpha = 0.5 \\
\frac{1}{\alpha} (0.5 + \beta) = \beta
\end{array} \right\} \Rightarrow \left\{ \begin{array}{l}
\alpha = 0.5 \\
\beta = 0.5
\end{array} \right\}.
\]

8.1(b)

\[
E_{b_j(t_j)}[\pi_i(b_i = 0.5 t_i + 0.5, t_i)] = (t_i + 0.5 - b_i) \frac{b_i - 0.5}{0.5} |_{b_i = 0.5 + 0.5t_i} = \frac{1}{2} t_i^2.
\]

2 Question 8.2: First Price Auction w/ Common Values

8.2(a)
Assume \( b_j = \alpha t_j + \beta, \forall j = 1, 2 \). Assume further that \( \alpha \neq 0 \) (since we’ll need to divide by \( \alpha \) later). Then, for \( i \neq j \), we have,

\[
E_{b_j(t_j)}[\pi_i(b_i, t_i)] = \int_0^{b_i-\beta} \frac{\alpha}{2} (t_i + t_j - b_i) dt_j + \int_{\alpha b_i-\beta}^{b_i-\beta} \frac{1}{2} (t_i + t_j - b_i) dt_j + \int_{b_i-\beta}^{1} 0 dt_j \\
= \int_0^{b_i-\beta} (t_i + t_j - b_i) dt_j = (t_i - b_i) \frac{b_i - \beta}{\alpha} + \frac{1}{2} \left( \frac{b_i - \beta}{\alpha} \right)^2.
\]

FOC w.r.t \( b_i \) yields:
\[
- \frac{b_i - \beta}{\alpha} + \frac{1}{\alpha} (t_i - b_i) + \frac{b_i - \beta}{\alpha} = 0
\]

\[
\Rightarrow \alpha(t_i + \beta - 2b_i) + b_i - \beta = 0
\]

\[
\Rightarrow b_i = \frac{\beta - \alpha \beta - \alpha t_i}{1 - 2\alpha} = \frac{\beta - \alpha \beta}{1 - 2\alpha} + \frac{-\alpha}{1 - 2\alpha} t_i.
\]

Matching to the initial guess, we have:

\[
\frac{\beta - \alpha \beta}{1 - 2\alpha} + \frac{-\alpha}{1 - 2\alpha} t_i = \beta + \alpha t_i,
\]

or equivalently

\[
\frac{\beta - \alpha \beta}{1 - 2\alpha} = \beta \quad \text{and} \quad \frac{-\alpha}{1 - 2\alpha} = \alpha.
\]

Solving gives

\[
\begin{cases} 
  \alpha = 1 \\
  \beta = 0
\end{cases}
\]
or

\[
\begin{cases} 
  \alpha = 0 \\
  \beta \in R
\end{cases}
\]
(rejected, since \( \alpha \neq 0 \)).

Hence, \( b_i(t_i) = t_i, \forall i = 1, 2 \).

8.2(b)

The equilibrium bid in this case is \( t_i \), which is lower than the private value equilibrium bid \( \frac{1}{2} t_i + \frac{1}{2} \). This is because given that \( i \) wins the object, her expected payoff from the object in the private value auction is simply \( t_i + \frac{1}{2} t_i \), whereas her expected payoff from the object in the common value auction is \( t_i + \frac{1}{2} t_i \leq t_i + \frac{1}{2} \). Hence, bidders in private value auction are willing to bid more. This is an example of the "winner's curse" in common value auctions: winning the object reveals negative information about the values the other players place on the object, so in equilibrium the bidders hedge their bids down.

3 Question 8.3: First Price Auctions [hard]

We solve the "is there a pure BNE" part of (a) first. And then solve (a) and (c) [generalization of (a)] together.

8.3(a) [second part: is there pure BNE?]

Let's first consider the question if there is a pure strategy BNE first. Suppose there exists one of the following general form:

<table>
<thead>
<tr>
<th>Player I</th>
<th>high</th>
<th>low</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player II</td>
<td>( b^I_1 )</td>
<td>( b^I_2 )</td>
</tr>
</tbody>
</table>

We will show (1) \( b^I_1 = b^II_1 = v_i \); (2) Given that, \( \exists (b^I_1, b^II_1) \) making the strategy profile above a pure BNE. Facing player \( I \)’s equilibrium strategy, player \( II \) of type \( l \) by bidding \( b \), gives an expected utility of

\[
U_{II} (b, v_i) = \alpha I \{ b > b^I_1 \} [v_i - b] + \alpha I \{ b = b^I_1 \} \frac{[v_i - b]}{2}
\]

\[
+ (1 - \alpha) I \{ b > b^I_2 \} [v_i - b] + (1 - \alpha) I \{ b = b^I_2 \} \frac{[v_i - b]}{2}.
\]

Consider \( b^II_1 \) which maximizes \( U_{II} (b, v_i) \). First, it is clear that \( b^II_1 \notin (\max \{ b^I_1, b^II_1 \}, \infty) \) [if \( b^II_1 \in (\max \{ b^I_1, b^II_1 \}, \infty) \), always does better.] Next \( b^II_1 \notin (\min \{ b^I_1, b^II_1 \}, \max \{ b^I_1, b^II_1 \}) \) for a similar reason. We consider the following remaining possibilities:
We now look for a symmetric equilibrium strategy for both the low and high types. 8.3(a) and (c)
equilibrium. So we’ve established that there is no pure BNE in this game. 

1. \( b^I_l \in [0, \min \{b^I_l, b^I_h\}] \). This results in a payoff of 0. In this case, we must have \( \min \{b^I_l, b^I_h\} \geq v_l \), otherwise by bidding \( b^I_l = \min \{b^I_l, b^I_h\} < v_l \), player II can win a positive payoff with positive probability. But if \( \{b^I_l, b^I_h\} \geq v_l \), this means player I of the low type bids \( b^I_l \geq v_l \), which results in a non-positive expected payoff, given that player II bids \( b^I_l < \min \{b^I_l, b^I_h\} \). Then \( b^I_l \) cannot be the low type player I’s equilibrium strategy because bidding \( b^I_l - \varepsilon \) strictly increases his expected payoff. Hence \( b^I_l \not\in [0, \min \{b^I_l, b^I_h\}] \) in equilibrium.

2. \( b^I_l = \min \{b^I_l, b^I_h\} \). This cannot be an equilibrium if \( b^I_l = \min \{b^I_l, b^I_h\} > v_l \) since player II would prefer to lose the bid to get 0 than to incur a loss at \( b^I_l = \min \{b^I_l, b^I_h\} \). On the other hand, it cannot be an equilibrium if \( b^I_l = \min \{b^I_l, b^I_h\} < v_l \), because then player II would bid \( \min \{b^I_l, b^I_h\} + \varepsilon \), which would result in a change in her payoff that is proportional to

\[
[v_l - \min \{b^I_l, b^I_h\} - \varepsilon] - \frac{[v_l - \min \{b^I_l, b^I_h\}] - 2}{2} > 0, \text{ for } \varepsilon > 0 \text{ small.}
\]

Hence, we are left with the case \( b^I_l = \min \{b^I_l, b^I_h\} = v_l \).

3. \( b^I_l = \max \{b^I_l, b^I_h\} \). Similarly, \( b^I_l = \max \{b^I_l, b^I_h\} \) cannot be an equilibrium if \( \max \{b^I_l, b^I_h\} \neq v_l \).

Hence, we only need to consider \( b^I_l = \max \{b^I_l, b^I_h\} = v_l \).

In conclusion, from cases 2 and 3, we know that for any pure strategy BNE, we must have \( b^I_l = v_l \). And by symmetry, we conclude that \( b^I_l = b^I_l = v_l \).

Now consider the high type of player II. If \( b^I_h < v_h \), then the high-type player 2’s BR is either \( b^I_h + \varepsilon \) or \( v_l + \varepsilon \), neither of which can be an equilibrium. So for \( b^I_l \) to be an equilibrium strategy, we must have either \( b^I_h \geq v_h \). But then, the high-type player II’s best response would be to bid \( v_l + \varepsilon \), which is not an equilibrium. So we’ve established that there is no pure BNE in this game.

8.3(a) and (c)

We now look for a symmetric equilibrium strategy for both the low and high types.

First, notice that any symmetric equilibrium strategy will result in \( b(v_l) \leq v_i \) (with probability 1), \( \forall i \in \{l, h\} \) (for otherwise, some player will incur negative expected payoff.)

Assume first (to be verified later) that the high type will not bid below \( v_l \) with positive probability.

- Consider first the low type bidder. Suppose the symmetric equilibrium strategy is to bid strictly below \( v_l \) with positive probability, associated with cumulative distribution function \( F \) over \( [z_l, z_h] \). Given that all his opponents adopt such a strategy (if they are high type, they will bid above \( v_l \) by assumption), then a low type by bidding \( x \) obtains a payoff of

\[
(1 - \alpha)^{I-1} [F(x)]^{I-1} (v_l - x).
\]

If he also bids according to \( F \), then the bidder gets an expected payoff of

\[
\int_{z_l}^{z_h} (1 - \alpha)^{I-1} [F(x)]^{I-1} (v_l - x) dF(x). \quad (1)
\]

If he deviates to play \( \tilde{F}(x) \) such that \( \tilde{F}(x) = \begin{cases} 
0, x < z_l + \varepsilon \\
F(z_l + \varepsilon), x = z_l + \varepsilon \\
F(x), x > z_l + \varepsilon
\end{cases} \), he will obtain

\[
\int_{z_l}^{z_h} (1 - \alpha)^{I-1} [F(x)]^{I-1} (v_l - x) d\tilde{F}(x) \quad (2)
\]

\[
= (1 - \alpha)^{I-1} F(\varepsilon + z_l) [v_l - z_l - \varepsilon] + \int_{\varepsilon + z_l}^{z_h} (1 - \alpha)^{I-1} [F(x)]^{I-1} (v_l - x) dF(x). \quad (3)
\]
Now subtracting (2) from (1) yields
\[
(1 - \alpha)^{l-1} \left\{ F(\varepsilon + z_l) [v_l - z_l - \varepsilon] + \int_{z_l + \varepsilon}^{z_h} [F(x)]^{l-1} (v_l - x) dF(x) - \int_{z_l}^{z_l + \varepsilon} [F(x)]^{l-1} (v_l - x) dF(x) \right\} \\
= (1 - \alpha)^{l-1} \left\{ F(\varepsilon + z_l) [v_l - z_l - \varepsilon] - \int_{z_l}^{z_l + \varepsilon} [F(x)]^{l-1} (v_l - x) dF(x) \right\} \\
= (1 - \alpha)^{l-1} \int_{z_l}^{z_l + \varepsilon} (v_l - z_l - \varepsilon) - (F(x))^{l-1} (v_l - x) dF(x) \\
> 0, \text{ for } \varepsilon > 0 \text{ sufficiently small.}
\]

This is because when \( \varepsilon \) is close to 0, \((v_l - z_l - \varepsilon) - (F(x))^{l-1} (v_l - x) > 0 \forall x \in [z_l, z_l + \varepsilon] \), as \([F(x)]^{l-1} < 1 \). In other words, we have demonstrated the existence of a \( \tilde{F}(x) \neq F(x) \) that does strictly better than \( F \), which is a contradiction. Hence, we conclude that a symmetric equilibrium strategy for the low type must make him bid \( b(v_l) = v_l \) (with probability 1).

**Now consider the high-type bidder.** Since we know for two players there is no pure strategy BNE, we consider mixed strategy BNE for the high type with continuous (so there is no mass point and thus tie occurs with probability 0) cumulative distribution function \( H(b) \) over some support \([b, \tilde{b}] \subseteq [v_l, v_h]\).

Now \( b = v_l \), for if \( b > v_l \), then bidding between \( b \) and \( v_l \) has the same chance of winning as \( b \), but incurs less loss. Bidding \( b = v_l + \varepsilon \) for \( \varepsilon > 0 \) infinitesimally small gives the high type an expected payoff that converges to \((1 - \alpha)^{l-1} (v_h - v_l) \) as \( \varepsilon \to 0^+ \). As for the upper bound \( \tilde{b} \), it must give an expected payoff of \( v_h - \tilde{b} \), since bidding \( \tilde{b} \) surely wins. Hence we must have
\[
(1 - \alpha)^{l-1} (v_h - v_l) = v_h - \tilde{b} \text{ or } \tilde{b} = v_h - (1 - \alpha)^{l-1} (v_h - v_l) < v_h. \tag{4}
\]

Moreover, because we assume \( H(b) \) is continuous over \([v_l, \tilde{b}] \), we must have \( \forall b \in (v_l, \tilde{b}) \).

\[
(1 - \alpha)^{l-1} (v_h - v_l) = (v_h - b) \left\{ (1 - \alpha)^{l-1} + \sum_{k=1}^{l-1} C_l^{l-1} \alpha^k (1 - \alpha)^{l-1-k} H^k(b) \right\}.
\]

Dividing each side by \((1 - \alpha)^{l-1}\) yields
\[
(v_h - v_l) = (v_h - b) \left[ 1 + \sum_{k=1}^{l-1} C_l^{l-1} \left( \frac{\alpha H(b)}{1 - \alpha} \right)^k \right]. \tag{5}
\]

Now we know
\[
\left( 1 + \frac{\alpha H(b)}{1 - \alpha} \right)^{l-1} = \sum_{k=0}^{l-1} C_l^{l-1} \left( \frac{\alpha H(b)}{1 - \alpha} \right)^k = \sum_{k=1}^{l-1} C_l^{l-1} \left( \frac{\alpha H(b)}{1 - \alpha} \right)^k + 1
\]

so (5) becomes
\[
(v_h - v_l) = (v_h - b) \left[ \left( 1 + \frac{\alpha H(b)}{1 - \alpha} \right)^{l-1} \right]
\]

which is equivalent to
\[
H(b) = \frac{1 - \alpha}{\alpha} \left\{ \left[ \frac{v_h - v_l}{v_h - b} \right]^{l+1} - 1 \right\}.
\]

Finally, we verify that \{low type bidding \( v_l \) and high type bidding according to \( H(b) \) for \( b \in [v_l, \tilde{b}] \} \) is indeed a BNE. The high type is indifferent by construction, over the support \([v_l, \tilde{b}]\). Moreover, she does not want to deviate to play anything outside the support because

1. Any \( b < v_l \) gives a payoff of zero, which is smaller than the payoff \((1 - \alpha)^{l-1} (v_h - v_l) > 0\) obtained when using the mixed strategy;


2. Any \( b \in (\bar{b}, \infty) \) wins the auction for sure but gives a payoff of \( v_h - b < v_h - \bar{b} = (1 - \alpha)^{I-1}(v_h - v_l) \) by (4).

And since the low type gives a best response \( b(v_l) = v_l \), we have verified that the above strategy profile is indeed a symmetric BNE.

Obviously, if \( I = 2 \), we have the following as a symmetric BNE:
low type bidding \( v_l \); and high type bidding according to
\[
H(b) = \frac{1 - \alpha}{\alpha} \frac{b - v_l}{v_h - \bar{b}}, \forall b \in [v_l, v_l + \alpha (v_h - v_l)].
\]

### 8.3(b) Revenue Equivalence for \( I=2 \)

Consider the Second Price Auction first. The equilibrium in dominant strategies is, as usual, \( b(v_l) = v_l \) for both types. The expected revenue is
\[
R_{(SPA)} = (1 - \alpha)^2 v_l + 2\alpha(1 - \alpha)v_l + \alpha^2 v_h,
\]
where the first term is the probability of having two bidders with low type times the corresponding payment \( v_l \); the second term reflects the cases where one bidder is low type and the other is a high type times the corresponding second price payment \( v_l \), and so on.

In the First Price Auction, the expected revenue is
\[
R_{(FPA)} = (1 - \alpha)^2 v_l + 2\alpha(1 - \alpha)E(b_h) + \alpha^2 E(\max\{b_1, b_2\}),
\]
where \( E(b_h) \) is the expected bid of the high type when she mixes (since that is the expected payment when one bidder is a low type and the other is a high type); and \( E(\max\{b_1, b_2\}) \) is the expected payment when the two bidders are of a high type).

Clearly, \( R_{(SPA)} = R_{(FPA)} \) iff
\[
2\alpha(1 - \alpha)v_l + \alpha^2 v_h = 2\alpha(1 - \alpha)E(b_h) + \alpha^2 E(\max\{b_1, b_2\}).
\]

Now,
\[
E(b_h) = \int_{v_l}^{\bar{b}} b.dH(b) = \left[bH(b) - v_lH(v_l)\right] - \int_{v_l}^{\bar{b}} H(b)db, \text{ int. by parts}
\]
\[
= \bar{b} - \int_{v_l}^{\bar{b}} \frac{1 - \alpha}{\alpha} \frac{b - v_l}{v_h - \bar{b}}db.
\]
On the other hand,

\[
E(\max\{b_1, b_2\}) = 2 \int_{v_l}^{\tilde{b}} \int_{b'}^{\tilde{b}} b.dH(b')dH(b), \text{ where the factor of 2 comes from symmetry}
\]

\[
= 2 \int_{v_l}^{\tilde{b}} \left[ \tilde{b} - b' H(b') - \int_{b'}^{\tilde{b}} H(b)db \right] dH(b'), \text{ int. by parts}
\]

\[
= 2\tilde{b} - 2 \int_{v_l}^{\tilde{b}} \left[ b'H(b') + \int_{b'}^{\tilde{b}} H(b)db \right] dH(b')
\]

\[
= 2\tilde{b} - 2 \left\{ b'H(b') H(b') + H(b') \int_{b'}^{\tilde{b}} H(b)db \right\}_{b' = \tilde{b}} - \int_{v_l}^{\tilde{b}} \left[ b'h'(b') + H(b') - H(b') \right] H(b') db'
\]

\[
= 2\tilde{b} - 2 \left( \tilde{b} - \int_{v_l}^{\tilde{b}} H^2(b') db' \right) = 2 \int_{v_l}^{\tilde{b}} b'H(b') dH(b') = 2 \int_{v_l}^{\tilde{b}} b' \frac{1}{2} dH^2(b')
\]

\[
= \left[ b'H^2(b') \right]_{b' = v_l} - \int_{v_l}^{\tilde{b}} H^2(b') db', \text{ int. by parts}
\]

Putting in the expressions for \(E(b_h)\) and \(E(\max\{b_1, b_2\})\) above, we get

\[
2\alpha(1 - \alpha)E(b_h) + \alpha^2 E(\max\{b_1, b_2\})
\]

\[
= 2\alpha(1 - \alpha)\tilde{b} + \alpha^2 \tilde{b} - 2(1 - \alpha)^2 \int_{v_l}^{\tilde{b}} b - v_l \frac{v_l}{v_h - b} db - (1 - \alpha)^2 \int_{v_l}^{\tilde{b}} \left( \frac{b - v_l}{v_h - b} \right)^2 db
\]

\[
= 2\alpha(1 - \alpha)\tilde{b} + \alpha^2 \tilde{b} - (1 - \alpha)^2 \int_{v_l}^{\tilde{b}} \left[ 1 + 2 \left( \frac{b - v_l}{v_h - b} \right) + \left( \frac{b - v_l}{v_h - b} \right)^2 \right] db + (1 - \alpha)^2 \int_{v_l}^{\tilde{b}} 1 db
\]

\[
= 2\alpha(1 - \alpha)\tilde{b} + \alpha^2 \tilde{b} + (1 - \alpha)^2 (\tilde{b} - v_l) - (1 - \alpha)^2 \int_{v_l}^{\tilde{b}} \left[ 1 + 2 \left( \frac{b - v_l}{v_h - b} \right) + \left( \frac{b - v_l}{v_h - b} \right)^2 \right] db
\]

\[
= \tilde{b} - (1 - \alpha)^2 v_l - (1 - \alpha)^2 \int_{v_l}^{\tilde{b}} \left[ 1 + \frac{b - v_l}{v_h - b} \right]^2 db
\]

\[
= \tilde{b} - (1 - \alpha)^2 v_l - (1 - \alpha)^2 (v_h - v_l) \left[ (v_h - b)^{-1} \right]_{v_l}^{\tilde{b}}, \text{ after integration}
\]

\[
= \tilde{b} - (1 - \alpha)^2 v_l - (1 - \alpha)^2 (v_h - v_l) \left[ (v_h - \tilde{b})^{-1} - (v_h - v_l)^{-1} \right]
\]

Substituting in \(\tilde{b} = v_l + \alpha (v_h - v_l)\) yields

\[
2\alpha(1 - \alpha)E(b_h) + \alpha^2 E(\max\{b_1, b_2\})
\]

\[
= v_l + \alpha (v_h - v_l) - (1 - \alpha)^2 v_l - (1 - \alpha)^2 [(v_h - v_l) - \alpha (v_h - v_l)]^{-1} - (v_h - v_l)^{-1}
\]

\[
= v_l + (2\alpha - 1) (v_h - v_l) + (1 - \alpha)^2 (v_h - 2v_l)
\]

\[
= [1 - 2\alpha + 1 - 2(1 - \alpha)^2] v_l + (2\alpha - 1 + 1 - 2\alpha + \alpha^2) v_h
\]

\[
= 2\alpha (1 - \alpha) v_l + \alpha^2 v_h.
\]

This establishes the condition for revenue equivalence.

\textbf{8.3(c) } I > 2

See above.
4 Question 8.4: All Pay Auction

8.4(a) and (b)

We do (a) and (b) together. Let there be \( I \geq 2 \) bidders with uniformly distributed valuations. Remember that in this incomplete information game, a player’s type is her valuation and a strategy is a function mapping valuations into bids.

Look for a symmetric equilibrium in which players use a strictly increasing (identical) bidding function \( b_i(v_i) = b(v) \) \( \forall i \). If each player \( j \neq i \) follows strategy \( b(v_j) \), then player \( i \)'s expected payoff is given by:

\[
E[u_i(v_i, b_i)] = v_i \prod_{j \neq i} \Pr(b_i > b(v_j)) - b_i = v_i \prod_{j \neq i} \Pr(b^{-1}(b_i) > v_j) - b_i
\]

\[
= v_i F^{I-1}(b^{-1}(b_i)) - b_i = v_i \left[ b^{-1}(b_i) \right]^{I-1} - b_i.
\]

Differentiating with respect to \( b_i \) one obtains

\[
v_i (I-1) \left[ b^{-1}(b_i) \right]^{I-2} \frac{1}{b'(b^{-1}(b_i))} = 1.
\]

Imposing symmetry with \( b_i = b(v_i) \) yields,

\[
v_i (I-1) \left[ v_i \right]^{I-2} \frac{1}{b'(v_i)} = 1.
\]

\[
\Rightarrow \quad b'(v_i) = (I-1) \left[ v_i \right]^{I-1}
\]

\[
\Rightarrow \quad b(v_i) = \frac{I-1}{I} v_i^I + C.
\]

Now if \( v_i = \min v = 0 \), player \( i \) will surely lose. He'll therefore want to bid the lowest possible as he has to pay anyway. Hence, \( b(v_i = 0) = 0 \) \( \Rightarrow C = 0 \).

Therefore, we conclude that

\[
b(v_i) = \frac{I-1}{I} v_i^I, \quad \forall i \in \{1, 2, 3, ..., I\}.\]

This gives the solution to (b). For (a), we can verify easily that when \( I = 2 \), the bidding function is indeed quadratic in \( v_i \) with

\[
b^{I=2}(v_i) = \frac{1}{2} v_i^2, \quad \forall i \in \{1, 2\}.
\]