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Microeconomic Theory (521b)

Problem Set 1. The Email Game

2/28/08

This problem set is due next Thursday, 3/6/08.

1. **The Electronic Mail Game.** Consider the following email game due to (Rubinstein 1989). Each of two players has to choose one of the actions A or B. With probability $p < \frac{1}{2}$ the game in which the players are involved is G_β; with probability $1-p$ it is G_α. In both G_α and G_β it is mutually beneficial for the players to choose the same action, but the action that is best depends on the game: in G_α the outcome (A, A) is best, while in game G_β the outcome (B, B) is best. The payoffs are shown in the figure below, where $L > M > 1$. Which is the true game is known initially only to player 1:

G_α		A	B
	A	M, M	$1, -L$
	B	$-L, 1$	$0, 0$

and

G_β		A	B
	A	$0, 0$	$1, -L$
	B	$-L, 1$	M, M

   (a) Assume first that player 2 cannot obtain any information about the true game. Find the Bayesian Nash equilibrium. What is the expected payoff for each player in this equilibrium?

   (b) Assume next that player 1 can communicate with player 2 in such a way that the game becomes common knowledge between the 2 players. Show that the following is a Bayesian Nash equilibrium: each chooses A if the true game is G_α and chooses B otherwise.

   (c) Suppose now the players can communicate, but the means that is open to them does not allow the game to become common knowledge. Specifically, the players are restricted to communicate via computers under the following protocol. If the game is G_β, then player 1's computer *automatically* sends a message to player 2's computer; if the game is G_α then no message is sent. If a computer receives a message then it *automatically* sends a confirmation; this is so not only for the original message but also for the confirmation, the confirmation of the confirmation, and so on. The protocol is designed to send confirmations because the technology has the property that there is a small probability $\epsilon > 0$ that any given message does not arrive at its intended destination. If a message does not arrive then the communication stops. At the end of the communication phase each player's screen displays the number of messages T_i that his machine has sent.

i. Formulate the above situation as a Bayesian game.

ii. Define a strategy for each player.

iii. Prove the following statement by mathematical induction: *There is a unique Bayesian Nash equilibrium, in which both players always choose A.* The steps below will guide you through the proof.

- Argue first that player 1 will choose A if $T_1 = 0$. Argue next that player 2 will choose A if $T_2 = 0$ by comparing his minimum payoff from choosing A against his maximum payoff from choosing B.
- Assume each player plays A if $T_i = k$. Argue that each will continue to play A if $T_i = k + 1$. A crucial step involves showing the following posterior beliefs and evaluating the expected payoff from choosing each action:

$$\Pr\left[T_2 = k | T_1 = k+1\right] = \frac{\epsilon}{\epsilon + (1-\epsilon)\cdot\epsilon} > \frac{1}{2};$$
$$\Pr\left[T_1 = k+1 | T_2 = k+1\right] = \frac{\epsilon}{\epsilon + (1-\epsilon)\cdot\epsilon} > \frac{1}{2}.$$

The material of the first lecture is covered in a very nice survey article, (Morris 2002) and related material appears in (Morris and Shin 1997).

References

Morris, S. (2002): "Coordination, Communication and Common Knowledge: A Perspective on the Electronic Mail Game," *Oxford Review of Economic Policy*, 18, 433–445.

Morris, S., and H. Shin (1997): "Approximate Common Knowledge and Co-Ordination: Recent Lessons from Game Theory," *Journal of Logic, Language and Information*, 6, 171–190.

Rubinstein, A. (1989): "The Electronic Mail Game: Strategic Behavior under 'Almost Common Knowledge'," *American Economic Review*, 79, 385–391.