Opacity and the Optimality of Debt for Liquidity Provision

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History and features of financial (banking) crises

In the last 200 years or so there were several systemic financial crises:

1837, 1857, 1873, 1884, 1893, 1907, 1914, 1930s, and 2007

All these financial crises have debt as a common feature.
History and features of financial (banking) crises

In the last 200 years or so there were several systemic financial crises: 1837, 1857, 1873, 1884, 1893, 1907, 1914, 1930s, and 2007

All these financial crises have debt as a common feature.

Questions

What is so special about debt?

Why didn’t people learn, and repeat “mistake (=crisis)” again and again?
Reoccurrence of history

“The causes of crises are so subtle in nature that any attempt to foretell fluctuations in the financial world might well be considered hazardous.

No one in the opening months of the year thought of doubting the continuance of the rising tide of prosperity which had begun to gather strength the year before.

And yet the closing months of the year saw such a destruction of trade and credit, and the downfall of so many powerful houses, that the financial situation in New York and, indeed, throughout the United States generally, occasioned deep anxiety to the financiers of the world.”
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Swanson (1908, p. 65)
Popular claims and blames

Wildcat banking (Moral hazard of bankers or even fraudulent behavior)

  1900s:  Run away with money (Free banking era)

  2007:  Excessive risk taking (securitization, false rating)
Popular claims and blames

Wildcat banking (Moral hazard of bankers or even fraudulent behavior)

1900s: Run away with money (Free banking era)

2007: Excessive risk taking (securitization, false rating)

Questions

Was moral hazard the main (or even the only) cause of a financial crisis?

Or are there some “more structural” reasons for the occurrence of a financial crisis?
Empirical observations


Devaluation of State Bonds that backed the bank notes (weakening of asset side of the bank’s balance sheet) “caused” bank failures.

Falling asset prices and not wildcat theory was the cause.
Empirical observations


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Banking Panics in free banking era occurred at business cycle peaks.

The information indicated that a recession was imminent, households withdrew money (for consumption smoothing).
“Concisely stated, the problem before us at the present time is to develop a sound banking system in the United States . . . why has the United States not found a satisfactory solution to its banking problems during the long period of agitation and reform that stretches back to 1836?”
“Concisely stated, the problem before us at the present time is to develop a sound banking system in the United States . . . why has the United States not found a satisfactory solution to its banking problems during the long period of agitation and reform that stretches back to 1836?

Perhaps the chief reason is our habit of application - of being content with immediate remedies for pressing problems . . .

To find a permanent solution we must first analyze the fundamental bases of the problem – not merely its superficial inconveniences.”

James (1934, p. 1)
Our paper

**Part 1**: The optimal design of “information insensitive” and “liquid” securities for trading in secondary markets

→ A theory of the creation of “private money” (or collateral assets)

→ Optimality of debt for liquidity provision

**Part 2**: A new theory of (the current) financial crisis
Part 3: A big picture story

- The growth of the global economy in the last fifteen years or so has created enormous wealth.

- How did the financial system react to the economic growth and “store” the huge amount of newly created wealth?

- Endogenous responses: Repo banking (collateralized lending), securitization, high leverage, complex securities, rating agencies

→ Our unified interpretation: it is about “information insensitive” debt
Related Literature

Liquidity provision

Diamond and Dybvig (1983), Gorton and Pennacchi (1990)

→ assume the existence of debt (demand deposit)

Optimal security design (Optimality of debt)


→ Monitoring and control rights

Myers and Majluf (1984), Demarzo and Duffie (1999)

→ Focus on (one-time) trading in primary market
Agency based corporate finance literature

Analyzes how informed issuer designs a security to minimize the adverse consequences of ex ante asymmetric information in the primary market

Our paper

The optimal design of a security for secondary market trading (liquidity provision)

We ask how security design can prevent asymmetric information from arising in the first place.
Agenda

Two date model
  Information and welfare
  Optimal security design

Three date model
  Optimal security design
  Equilibrium Analysis
  Welfare

Discussion
The Two Date Model

An exchange economy with one consumption good, two dates, and two risk neutral agents

\[ U_A = \alpha C_{A1} + C_{A2} \quad \omega_A = (0, X), \]
\[ U_B = C_{B1} + C_{B2} \quad \omega_B = (w, 0) \]

where \( \alpha > 1 \), \( w \) is a constant, and \( X \) is a continuous random variable with distribution \( F(x) \) and positive support on \([x_L, x_H] \).
The Two Date Model

An exchange economy with one consumption good, two dates, and two risk neutral agents

\[ U_A = \alpha C_{A_1} + C_{A_2} \quad \omega_A = (0, X), \]

\[ U_B = C_{B_1} + C_{B_2} \quad \omega_B = (w, 0) \]

where \( \alpha > 1 \), \( w \) is a constant, and \( X \) is a continuous random variable with distribution \( F(x) \) and positive support on \( [x_L, x_H] \).

Information

Ignorance (IG): Agents only know the distribution \( F(x) \)

Perfect Information (PI): Agents know the true \( t=2 \) realization of \( X \) at \( t=1 \).
Information and Welfare

Outside option
No trade, agents consume their endowments.

Ignorance: \( E_{U_A} = E[X] \) \( E_{U_B} = w \)
Perfect Information: \( U_A = x \) \( U_B = w \)
Information and Welfare

Outside option
No trade, agents consume their endowments.

Ignorance: \( EU_A = E[X] \quad EU_B = w \)
Perfect Information: \( U_A = x \quad U_B = w \)

Proposition
For any \( F(x) \), if \( x_L < w \) and \( \alpha w < x_H \), then ex ante the efficient allocation under Ignorance yields a strictly higher welfare than the efficient allocations under Perfect Information.
Intuition (Apple Economy)

\( w = 2 \) and \( X \) is either 1 or 3 with equal probability: \( E[X] = 2 \)

Under Ignorance: Agent A and B trade 2 apples

\[ EU_A(\text{max}) = 2\alpha \quad EU_B = 2 \]
Intuition (Apple Economy)

$w=2$ and $X$ is either 1 or 3 with equal probability: $E[X]=2$

Under Ignorance: Agent A and B trade 2 apples

$EU_A(\text{max})=2\alpha$  \hspace{1cm} $EU_B=2$

Perfect Information

If $x=1$, agents trade 1 apple (agent A can only consume 1 apple at $t=1$)
If $x=3$, agents trade 2 apples (A consumes 2 apples at $t=1$ and 1 apple at $t=2$)

$EU_A(\text{max})=1.5\alpha+0.5$  \hspace{1cm} $EU_B=2$

News is asymmetric: bad news reduces what agent A can buy from agent B, but the effect of good news is bounded by what agent B can sell.
Proof

The set of all (feasible) allocations is given by:

\[
c = \begin{bmatrix}
  c_{A1} & c_{A2} \\
  c_{B1} & c_{B2}
\end{bmatrix}
\]

where

(i) \( c_{ht} \geq 0 \) for all \( h \) and \( t \)

(ii) \( \sum_h c_{h1} \leq w \)

(iii) \( \sum_h c_{h2} \leq x \).
Case: Perfect Information (PI)

Agents know that true realization $x$ (i.e., $X$ is not a random variable) at the time of contracting and trade.

The individual rationality (IR) constraints are given as follows:

Agent A: $\alpha c_{A1} + c_{A2} \geq x$ \hspace{1cm} (IR$_A$)

Agent B: $c_{B1} + c_{B2} \geq w$ \hspace{1cm} (IR$_B$)
In Appendix A we characterize the full set of efficient allocations and the Pareto frontier.

Here: Focus on the allocations that maximize agent A payoff while agent B is getting his reservation (no trade) utility, i.e. choose \((c_{A1}, c_{A2}, c_{B1}, c_{B2})\) to

\[
\text{Max } \alpha c_{A1} + c_{A2} \text{ subject to:}
\]

(i) \(c_{ht} \geq 0\),

(ii) \(\sum_h c_{h1} \leq w\),

(iii) \(\sum_h c_{h2} \leq x\),

(iv) \(c_{B1} + c_{B2} \geq w\)
$U_A = \alpha C_{A1} + C_{A2}$ \quad $\omega_A = (0, x)$ \quad $\alpha > 1$

$U_B = C_{B1} + C_{B2}$ \quad $\omega_B = (w, 0)$

**Case:** $x < w$

$U_B(\omega_B) = w \quad (IR_B)$
\[ U_A = \alpha C_{A1} + C_{A2} \quad \omega_A = (0, x) \quad \alpha > 1 \]

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**Case:** \( x < w \)

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**Case:** \( x < w \)
$U_A = \alpha C_{A1} + C_{A2}$ \hspace{0.5cm} $\omega_A = (0, x)$ \hspace{0.5cm} $\alpha > 1$

$U_B = C_{B1} + C_{B2}$ \hspace{0.5cm} $\omega_B = (w, 0)$

**Case: $x < w$**

$C_A^* = (x, 0), \hspace{0.5cm} C_B^* = (w-x, x)$

$\Rightarrow \hspace{0.5cm} U_A(C_A^*) = \alpha x > U_A(\omega_A) = x$
The solution of

\[
\text{Max } \alpha c_{A1} + c_{A2} \text{ subject to:}
\]

(i) \( c_{ht} \geq 0 \),
(ii) \( \sum_h c_{h1} \leq w \),
(iii) \( \sum_h c_{h2} \leq x \),
(iv) \( c_{B1} + c_{B2} \geq w \)

is given by:

(a) \( x < w \)  \hspace{1cm} (b) \( x \geq w \)

\[
c_A^{\text{max}} = \begin{bmatrix}
x & 0 \\
 w - x & x \\
\end{bmatrix}
\]
Case: $x \geq w$
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Case: $x \geq w$

$C_A^* = (w, x-w), \; C_B^* = (0, w)$,
The solution of

\[
\text{Max } \alpha c_{A1} + c_{A2} \text{ subject to:}
\]

(i) \( c_{ht} \geq 0 \), (ii) \( \sum_h c_{h1} \leq w \), (iii) \( \sum_h c_{h2} \leq x \), (iv) \( c_{B1} + c_{B2} \geq w \)

is given by:

(a) \( x < w \)

\[
c_{A}^{\text{max}} = \begin{bmatrix} x & 0 \\ w - x & x \end{bmatrix}
\]

(b) \( x \geq w \)

\[
c_{A}^{\text{max}} = \begin{bmatrix} w & x - w \\ 0 & w \end{bmatrix}.
\]

Note: \( c = \begin{bmatrix} c_{A1} & c_{A2} \\ c_{B1} & c_{B2} \end{bmatrix} \)
Ex ante (t=0), agent A’s expected utility is given by:

\[ EU^P_A (\text{max}) = \alpha \int_{x_L}^{w} x \cdot f(x) \, dx + \alpha \int_{w}^{x_H} w f(x) \, dx + \int_{x_L}^{x_H} \max[x - w, 0] f(x) \, dx. \]
Ex ante (t=0), agent A’s expected utility is given by:

\[ EU_A^{PI} (\text{max}) = \alpha \int_{x_L}^{w} x \cdot f(x) dx + \alpha \int_{x_L}^{x_H} wf(x) dx + \int_{x_L}^{x_H} \max[x - w, 0] f(x) dx. \]

\[ c_A^{\text{max}} = \begin{bmatrix} x & 0 \\ w - x & x \end{bmatrix} \]

\[ c_A^{\text{max}} = \begin{bmatrix} w & x - w \\ 0 & w \end{bmatrix}. \]

(a) \( x < w \)  
(b) \( x \geq w \)
Case: Ignorance (IG)

The realization of \( x \) is not known at the date of contracting. The allocation that maximizes agent A’ EU subject to \( IR_B \) is given by:

(a) \( E[x] < w \)  
(b) \( E[x] \geq w \)

\[
C_A^{\text{max}} = \begin{bmatrix} E[x] & 0 \\ w - E[x] & x \end{bmatrix}, \quad C_A^{\text{max}} = \begin{bmatrix} w & \max(x - s(x), 0) \\ 0 & s(x) \end{bmatrix},
\]

where \( s(x) \) is a function specifying what agent B gets conditional on the outcome \( x \) and where \( E[s(x)] = w \).
Suppose $E[x] \geq w$, ex ante, agent A’s expected utility is given by:

$$EU_A^{IG} = \alpha \int_{x_L}^w w \cdot f(x)dx + \alpha \int_{x_H} x_H w f(x)dx + \int_{x_L}^{x_H} \max[x - s(x),0] f(x)dx.$$
Welfare comparison

\[
EU_{A}^{PI} (\text{max}) = \alpha \int_{x_L}^{w} x \cdot f(x)dx + \alpha \int_{w}^{x_H} w f(x)dx + \int_{x_L}^{x_H} \max[x - w, 0] f(x)dx
\]

\[
EU_{A}^{IG} (\text{max}) = \alpha \int_{x_L}^{w} w \cdot f(x)dx + \alpha \int_{w}^{x_H} w f(x)dx + \int_{x_L}^{x_H} \max[x - s(x), 0] f(x)dx
\]

Note \( E[s(x)] = w \).

\[ \rightarrow \quad EU_{A}^{IG} (\text{max}) > EU_{A}^{PI} (\text{max}). \]

Remark: Proposition also holds for \( w < E[X] \).
Remark

The allocation that maximizes agent B’ payoff is given by:
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c_A = (x/\alpha, 0) \quad c_B = (w-x/\alpha, x)
Remarks

(1) If $x_L < w$ and $w < x_H$, then $EU_B^{IG} (\text{max}) > EU_B^{PI} (\text{max})$.

(2) Hirschleifer (1971) shows that information destroys the ability to co-insure when agents are risk averse.

(3) In our paper information destroys the ability to transact the optimal amount between risk neutral agents.
**Lemma**
For any $\alpha$ and $w$, if the distribution $F$ is sufficiently risky, then ignorance yields higher welfare than prefect and symmetric information.

**Remark**
If $w < x_L$ or $x_H < w$, both configurations yield the same welfare for any efficient allocations.
General information structures

If agents receive imperfect or partial information \( \{I\} \) about \( X \) and \( E[x \mid I] < w \) for some \( I \), the same logic shows that agents are worse off than having no information. Thus we have:

**Corollary**

If \( E[x \mid I] < w \) and \( \alpha w < E[x \mid I] \) for some \( I \), Ignorance yields higher ex ante welfare than partial and symmetric information (derived from any signal structure).
Security Design (Two Date Model)

- Agents need to trade to implement the efficient allocation.
- Agent A has to issue a security (i.e., a contractual claim on the outcome of X) to agent B in exchange for agent B’s consumption goods, w.
- Agent A makes a take-it-or-leave-it contract offer to agent B.
Security Design (Two Date Model)

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**Assumption I**: \( w < E[X] \).

Otherwise, efficient trade requires selling the whole project.

**Remark**
We show later that the optimality result also holds for \( w \geq [X] \).
Set of contracts

Let $S$ denote the set of all possible securities (contracts), i.e., functions, $s(x)$, which satisfy the resource feasibility (or limited liability) constraint, $s(x) \leq x$.

Any mapping $s: X \rightarrow \mathbb{R}$ with $s(x) \leq x$ is an element of $S$.

Examples

(i) Equity: $s(x) = \beta x$ where $\beta \in (0, 1]$ is the share on the $x$;

(ii) Debt: $s(x) = \min\{x, D\}$ where $D \leq x_H$ is the face value of the debt;

(iii) Step function contract: $y_i$ if $x \in [x_{L}, x_{1}]$, $y_i$ if $x \in [x_{i-1}, x_{i}]$ where $y_i \leq x_i$;

(iv) State contingent securities: $s(x_i) = y_i$ where $y_i \leq x_i$;

(v) Stochastic contracts: $s(x_i) = y_i$ where $y_i: x_i \rightarrow [x_L, x_i]$ with distribution $F_i$. 
Assumption (Information Acquisition)

At t=1, agent can learn about the true realization of X at the cost $\gamma$ (in terms of utility) before they trade.
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At $t=1$, agent can learn about the true realization of $X$ at the cost $\gamma$ (in terms of utility) before they trade.

Why do we or the agents care about information acquisition?
- Information is costly and has no social value (exchange economy)
- Information reduces welfare
- Information acquisition creates potential adverse selection

A motive for security design

Minimize the incentive of private information acquisition
Individual decision problem (equilibrium analysis later)

Suppose a security $s$ with price $p = E[s(x)]$ is “on the table”.

An agent decides whether to buy it or not.

Definition (The value of information):
Suppose the decision of the agent is whether to buy a particular security or not. The value of information of a buy ($B$) transaction is defined as

$$\pi_B = EU(PI) - EU(IG).$$
The value of information for a buyer

Ignorance

Agent B is willing to buy the security since \( E[s(x)] = p \).

\[
EU_B(IG) = \int_Q s(x) \cdot f(x) dx
\]

where \( Q = \{x\} \)
The value of information for a buyer

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Agent B is willing to buy the security since \( E[s(x)]=p \).

\[
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\]

where \( Q=\{x\} \)

Perfect information

Agent B does not buy in states \( Q_< = \{x|s(x) < p\} \) and consumes unspent amount \( p \)

Agent B buys security in states \( Q_> = \{x|s(x) \geq p\} \)

\[
EU_B(PI) = \int_{Q<} p \cdot f(x) dx + \int_{Q>} s(x) f(x) dx
\]
\[ \pi_B = EU(PI) - EU(IG) \]

\[ \iff \pi_B = \left( \int_{Q^<} p \cdot f(x) dx + \int_{Q^>} s(x) \cdot f(x) dx \right) - \int_Q s(x) \cdot f(x) dx \]

\[ \iff \pi_B = \left( \int_{Q^<} p \cdot f(x) dx + \int_{Q^>} s(x) \cdot f(x) dx \right) - \left( \int_{Q^<} s(x) \cdot f(x) dx + \int_{Q^>} s(x) \cdot f(x) dx \right) \]

\[ \iff \pi_B = \int_{Q^<} (p - s(x)) \cdot f(x) dx \]

\[ \iff \pi_B = \int_{x_L}^{x_H} \max[p - s(x), 0] \cdot f(x) dx \]
The value of information for seller of security (agent A) is:

\[ \pi_S = \int_{Q_>} (s(x) - \alpha p) \cdot f(x) dx \]

where \( Q_> = \{ x | s(x) > \alpha p \} \)

For \( \alpha = 1 \):

\[ \pi_S = \int_{x_L}^{x_H} \max[s(x) - p, 0] \cdot f(x) dx \]
Definition (Information-sensitivity of a security)

Suppose an agent can buy either of two securities that have the same expected value and the same price. Security i is said to be less *information-sensitive* than security j for agent h if the value of information for buying security i is lower than the value of information for buying security j, i.e. $\pi_h^i < \pi_h^j$.

Question

Which security $s(x)$ with expected payoff $E[s(x)]=w$ and price $p=E[s(x)]$ is least information sensitive?
Minimization in functional space \{s(x)\} and solution to hold for all F(x).

(i) \[
\min_{\{s\}} \pi_B = \int_{x_L}^{x_H} \max[p - s(x), 0] \cdot f(x)dx
\]

(ii) \[
\min_{\{s\}} \pi_S = \int_{x_L}^{x_H} \max[s(x) - p, 0] \cdot f(x)dx
\]

(iii) \[
\min_{\{s\}} \pi_B + \pi_S = \int_{x_L}^{x_H} |s(x) - p| \cdot f(x)dx
\]

where

\{s\}={s(x) \leq x \text{ and } E[s(x)]=w \text{ and } p=E[s(x)]\} \quad \text{(no restriction on } s(x) \text{ except resource constraint)}
**Theorem**

Assume for all $s$ that $E[s^D(x)] = E[s(x)] = p$ and that $s(x) \leq x$. Debt is a least information-sensitive security.

A *standard debt contract* is given by:

$$
\begin{align*}
    s^D(x) &= D & \text{if } x > D \\
    s^D(x) &= x & \text{if } x \leq D.
\end{align*}
$$

where $D$ is the face value. Given price $p$, $D$ is determined by

$$
\int_{x_L}^{D} xf(x)dx + \int_{D}^{x_H} Df(x)dx = p = w
$$
Intuition

Step 1: Debt is a least information-sensitive security for buyer (agent B).

\[ \pi_B = \int_{x_L}^{x_H} \max[p - s(x), 0] \cdot f(x) \, dx \]

\[ Q_B^D = \{x \mid s^D(x) < p \} \]

\[ Q_B^D \subseteq Q_B^S \]

and \[ p - s^D(x) = p - x \leq p - s(x) \quad \text{for all } x < p \text{ and all } s \in S \]

slope=1
Step 2: Suppose $\alpha=1$ (the case of $\alpha>1$ is considered in Appendix A).

The value of information to agent A is equal to the value of information to agent B.

![Diagram]

Buyer must be compensated for expected loss (blue area) in low states by expected gains (red area) in good states. Read area is expected loss of seller, i.e. he repays $s(x)>p$. Since $p=E[s(x)]$, both areas are the same.
Remark

For $\alpha > 1$, $\pi_B > \pi_A$.

\begin{align*}
    U_A &= \alpha C_{A1} + C_{A2} \\
    U_B &= C_{B1} + C_{B2}
\end{align*}

Implication

- This Theorem states, that in markets where agents want to prevent information acquisition from arising, debt is the optimal trading security.

- If the trading of debt triggers information acquisition, then so does the trade of any other security with the same expected value (or price).
**Theorem (Characterization)**

The set \( \{s\} \) of securities with \( p = E[s(x)] = w \) that has the minimal information sensitivity is given by \( \{s: s(x) = x \text{ for } x \leq w \text{ and } s(x) \geq w \text{ for } x > w\} \).
**Corollary (Equity)**

Equity is strictly more information-sensitive than debt.

**Corollary (Toxic Asset or Equity Residual)**

The security with the maximal information-sensitivity is given by: \( \bar{s}(x) = 0 \) for \( x \in [x_L, d] \) and \( \bar{s}(x) = x \) for \( x \in [d, x_H] \) where \( d \) solves \( \int_d^{x_H} x f(x) dx = w \).

![Diagram](image-url)
Corollary (Maximum Debt Issuance)

Suppose agents can learn about the true $x$ at the cost $\gamma$. The maximum amount of debt that agent A can issue at $t=1$ without triggering information acquisition by agent B is given by $\min\{w, E[x], p\}$ where $p$ solves the following equation:

$$\int_{x_L}^{p} (p - x) f(x) dx = \gamma$$

Remark

This result is needed for the equilibrium analysis.
Optimality of debt under adverse selection

Proposition

Suppose agent B is informed and agent A is uninformed (and cannot acquire information). The optimal contract that agent A offers agent B is a debt contract with face value $d$ and price $d$, where $d$ maximizes $(1 - F(d))d$. 
Intuition

Debt maximizes the probability of trading a given amount $d$

There is no overpayment: $s(x) = d$ for any $x > d$. (With equity $s(x) > d$ for some $x$)
Proof

Step 1

Suppose agent A offers to sell $s^D(x) = \min[x, d]$ for the price $p=d$.

Since agent B knows the true value of $x$, he buys $s^D(x)$ only if $s^D(x) \geq p = d$ (i.e. $x \geq d$)
Proof

Step 1

Suppose agent A offers to sell $s^D(x) = \min[x,d]$ for the price $p=d$.

Since agent B knows the true value of $x$, he buys $s^D(x)$ only if $s^D(x) \geq p = d$ (i.e. $x \geq d$)

The set of states with no trade is $Q^D = \{x|x < d\}$.

The probability of trade is $1 - F(d)$.

If trade occurs agent A repays $d$ to agent B at $t=2$. 
Now, note:

(i) Since $s^D(x) = x$ for $x \leq d$, there exists no other contract $s$ where the sets of states with no trade is smaller than $Q^D$, i.e. $Q^D \subseteq Q^S = \{x | s(x) < d\}$ and $1 - F(d) \geq prob(trade \ under \ contract \ s)$ for all $s \in S$. 
Now, note:

(i) Since \( s^D(x) = x \) for \( x \leq d \), there exists no other contract \( s \) where the sets of states with no trade is smaller than \( Q^D \), i.e. \( Q^D \subseteq Q^S = \{ x \mid s(x) < d \} \) and \( 1 - F(d) \geq \text{prob}(\text{trade under contract } s) \) for all \( s \in S \).

(ii) Consider a contract \( s \) where \( s(x) = x \) for \( x \leq d \) and \( s(x) > d \) for some \( x \). If trade occurs in these states, then agent A repays \( s(x) > d \) to agent B.
Step 2

The expected utility of agent A is

\[ EU_A = \alpha(1 - F(d))d + E[X] - (1 - F(d))d. \]

Agent A chooses d to maximize \( EU_A \) and thus \( (\alpha - 1)(1 - F(d))d \), i.e. \( (1 - F(d))d \).
Remark

Debt is also optimal if agent A is informed and B is uninformed.
Optimality of Debt

Agent A makes a take-it-or-leave-it contract offer.

Now suppose (only) agent B can acquire information at the cost \( \gamma \).

Proposition

For any \( \{F, \alpha, \gamma\} \), debt is optimal, i.e. it maximizes the expected utility of agent A.
Intuition

Agent A has three potential best responses. All cases involve issuing debt.

Strategy I (Down-Sizing of Debt)

Theorem 2 shows that if agent A wants to issue a security with $p = \mathbb{E}[s(x)]$ and avoid information acquisition by agent B, then issuing debt is optimal. Such a contract has a price $p^I$ and a face value $D^I$ such that

$$
\int_{x_L}^{p^I} (p^I - x) f(x) dx = \gamma \\
\int_{x_L}^{D^I} x f(x) dx + \int_{D^I}^{x_H} f(x) dx = p^I,
$$

i.e., the expected payoff is: $E[s^{D^I}] = p^I$. 
However if $\gamma$ is small, then $p^I$ (the amount agent A can consume) is small.

$$\int_{x_L}^{p^I} (p^I - x) f(x) dx = \gamma$$
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$$\int_{x_L}^{p^I} (p^I - x) f(x) dx = \gamma$$

Agent A can potentially consume more if he increases the face value of debt to $D^{II}>D^I$ such that $E[s^{II}(x)]>p^{II}>p^I=E[s^I(x)]$.

By charging a price less than expected payoff, $p^{II}<E[s^D(x)]$, agent B gets some surplus which reduces his incentive to acquire information.
Strategy II (Debt with Surplus Sharing)

Agent A offers debt with a price less than expected payoff, $p^{	ext{II}}=E[s(x)]-(\pi-\gamma)$ so as to bribe agent B not to acquire information by giving him some of the trading gains.
Strategy II (Debt with Surplus Sharing)

Agent A offers debt with a price less than expected payoff, $p_{II}^{II} = E[s(x)] - (\pi - \gamma)$ so as to bribe agent B not to acquire information by giving him some of the trading gains.

Theorem 2 shows that debt minimizes $\pi$ and thus the bribe agent A has to give to agent B.

In this case agent A chooses debt with face value $D_{II}$ to maximize

$$EU = \alpha(E[s^D(x)] - (\pi^D - \gamma)) + E[x] - E[s^D(x)]$$

where $\pi^D$ increases in $E[s^D(x)]$. 
Formally, Contract (II) is debt with face value $D^\Pi$ (and price $p^\Pi$) that maximizes:

\[
\alpha \left( \int_{x_L}^{D} xf(x)dx + \int_{D}^{x_H} Df(x)dx - \left( \pi^D - \gamma \right) \right) + E[X] - \int_{x_L}^{D} xf(x)dx + \int_{D}^{x_H} Df(x)dx
\]

where

\[
\pi^D = \int_{x_L}^{x_H} \max[q^\Pi - x, 0] \cdot f(x)dx.
\]
With strategy II, the bribe agent A has to pay agent B may be large if $\pi$ is large and $\gamma$ is small. Therefore, if $\gamma$ is sufficiently low, then avoiding information acquisition may not be optimal.
Strategy III (Debt with Information Acquisition)

Suppose agent A wants to consume the amount $p^{III}$.

Then debt is optimal since it maximizes the probability of obtaining $p^{III}$.

Facing an informed agent B the set of states with no trade is minimized since $s^D(x) = x$ for $x \leq p^{III}$. 
Strategy III (Debt with Information Acquisition)

Suppose agent A wants to consume the amount $p^{III}$.

Then debt is optimal since it maximizes the probability of obtaining $p^{III}$.

Facing an informed agent B the set of states with no trade is minimized since $s^D(x) = x$ for $x \leq p^{III}$.

Given the “desired” $p^{III}$, the optimal modified debt contract has price $p = p^{III}$ and a face value $p^{III} + \tilde{\gamma}$ such that $\pi(p^{III} + \tilde{\gamma}) = \gamma$,

i.e. when agent B acquires information he just covers his information cost.
Formally, Contract (III) is debt with price $p^{	ext{III}}$, face value $D^{	ext{III}} = p^{	ext{III}} + \tilde{\gamma}$, and an expected payoff $E[s^D(x)] < p^{	ext{III}}$ where:

\[ p^{	ext{III}} \text{ maximizes } \alpha (1 - F(p^{	ext{III}})) p^{	ext{III}} + E[x] - R \]

where $R$ (the expected repayment) is given by:

\[ R = \int_{p^{	ext{III}}}^{p^{	ext{III}} + \tilde{\gamma}} x f(x \mid x \geq p^{	ext{III}}) \, dx - \int_{p^{	ext{III}}}^{x_H} (p^{	ext{III}} + \tilde{\gamma}) \cdot f(x \mid x \geq p^{	ext{III}}) \, dx \quad \text{and} \]

\[ \tilde{\gamma} \text{ solves } \pi(p^{	ext{III}}, \tilde{\gamma}) = \gamma : \int_{p^{	ext{III}}}^{p^{	ext{III}} + \tilde{\gamma}} (x - p^{	ext{III}}) f(x) \, dx + \int_{p^{	ext{III}}}^{x_H} (p^{	ext{III}} + \tilde{\gamma} - p^{	ext{III}}) f(x) \, dx = \gamma. \]
To summarize, agent A has three potential best responses.

\[ EU_A(I) = \alpha p^I + E[x] - p^I \]

\[ EU_A(II) = \alpha p^{II} + E[x] - q^{II} \quad \text{(note } p^I < p^{II} < q^{II}) \]

\[ EU_A(III) = \alpha (1 - F(p^{III})) p^{III} + E[x] - R. \]

Agent A chooses the strategy with the highest expected utility.

In any case, debt is issued.
Remark

The assumption that $w < E[x]$, is not crucial.

If $w \geq E[x]$ and $\gamma$ is high, then selling the whole project or issuing a “degenerated” debt contract with face value $D=x_H$, is optimal.

If $\gamma < \gamma'$, selling the whole project is not optimal, but issuing debt with face value $D<x_H$ is optimal.
A Three Date Model

\[
U_A = \alpha C_{A1} + C_{A2} + C_{A3} \quad \varpi_A = (0,0,X) \\
U_B = C_{B1} + C_{B2} + C_{B3} \quad \varpi_B = (w,0,0) \\
U_C = C_{C1} + C_{C2} + \alpha C_{C3}, \quad \varpi_C = (0,w,0).
\]

Efficient trading
- Agent A trades with B at \( t=1 \).
- Agent B trades with C at \( t=2 \) (this is only possible if B has traded with A).

Assumption
- Agent C enters the economy at \( t=2 \), i.e. agents A and B cannot sign any contract with agent C at \( t=1 \).
- Agent C can only trade with agent B at \( t=2 \).

We could assume \( U_A = \alpha C_{A1} + \frac{1}{\alpha} C_{A2} + C_{A3} \) and \( U_C = C_{C1} + \frac{1}{\alpha} C_{C2} + C_{C3} \).
The set of contracts:

At $t=1$, the set of contracts agent $A$ can issue to agent $B$ is $s \in S = \{s: s(x) \leq x\}$.

At $t=2$ the set of contracts agent $B$ can trade with agent $C$ is given by $\hat{S} = \{\hat{s}: \hat{s}(y) \leq y\}$ where $y = s(x)$ denotes the payoff of the security that agent $B$ has bought from agent $A$. 
Information Acquisition

Only agent B can acquire information about the true value of x.

Public Information

- At $t=1$ the agents’ prior on $X$ is given by the (mixture) distribution $F_m$ with density

$$f_m(x) = \sum_{k=1}^{K} \lambda_k f_k(x) \quad \text{where} \quad \lambda_k \geq 0, \quad \sum_{k=1}^{K} \lambda_k = 1,$$

- $F_k$ has positive support on $[x^k_L, x^k_H]$ where $x^k_H < x^k_{L+1}$ for all $k$.

- At $t=2$, the agents receive a public signal about the “true” distribution of $X$.

- Public signal induces a partitional information structure.
Sequence of Moves and Events

$t=1.1$: Agent A makes a take-it-or-leave-it contract offer of $s(x)$ to agent B.
$t=1.2$: Agents B chooses whether to produce information about the true $x$ at the cost $\gamma$, or not.
$t=1.3$: Agent B accepts the contract or not.

If there is no trade between agents A and B, the game ends. Otherwise:

$t=2.0$: The public signal $F_k$ is observed.

$t=2.1$: Agent C makes a take-it-or-leave-it contract offer $\hat{s}(y)$ to agent B.
$t=2.2$: Agents B chooses whether to produce information about the true $x$ at the cost $\gamma$, or not.
$t=2.3$: Agent B accepts the offer or not.

$t=3$: Agent B redeems $s(x)$ with agent A, and agent C redeems $\hat{s}(y)$ with agent B.
Interpretation 1

- Agent A and C are regional banks and agent B is investment banking (intermediary with special expertise).
- At \( t=1 \) bank A has a shortage of cash while bank B has excess cash to lend out against collateral \( s(x) \).
- At \( t=2 \) bank B has a shortage of cash while bank C has excess cash and wants to store it for usage at \( t=3 \).
- For this purpose bank C wants to buy bank B’s security.
Interpretation 1

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- For this purpose bank C wants to buy bank B’s security.

Interpretation 2

- Agent B is an intermediary, buying a loan from agent A (the originator) and selling it to agent C (the investor).
Proposition (Debt maximizes trading capacity)
Suppose agents cannot acquire private information. Then issuing debt at date 1 maximizes the payoff of agent A as well as the amount of trade between agents B and C, the trading capacity. Debt dominates any contract $s \in S$ in terms of total payoff.
Intuition

- At $t=2$, public signal $k$ changes the market value (price) of the security $s(x)$

$$ p_k = E_k[s(x)] $$

- If there is bad news, i.e. $E_k[s^D(x)] < E_m[s^D(x)] = w$, then the market price of debt (trading capacity) is larger than the one of any other security, since $s^D(x) = x \geq s(x)$ for all $x \leq D$. 
Intuition

- At time $t=2$, public signal $k$ changes the market value (price) of the security $s(x)$
  \[ p_k = E_k[s(x)] \]

- If there is bad news, i.e. $E_k[s^D(x)] < E_m[s^D(x)] = w$, then the market price of debt (trading capacity) is larger than the one of any other security, since $s^D(x) = x \geq s(x)$ for all $x \leq D$.

- If there is good news, i.e. $E_k[s^D(x)] \geq E_m[s^D(x)] = w$, then agent B trades $w$ (efficient amount) by selling a fraction of his debt.

Remark
This result is similar to our previous welfare result (Hirschleifer under risk neutrality).
Information Sensitivity at interim date

\[ \pi(k) = \int_{x_L}^{x_H} \max[p_k - s(x), 0] \cdot f_k(x) \, dx \]

where \( p_k = \int_{x_L}^{x_H} s(x) \cdot f_k(x) \, dx \).

\( \pi \) is typically non-monotonic in \( k \) for any security (even \( k \) is ordered by FOSD).
Information Sensitivity at interim date

\[
\pi(k) = \int_{x_L}^{x_H} \max[p_k - s(x), 0] \cdot f_k(x) \, dx
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where

\[
p_k = \int_{x_L}^{x_H} s(x) \cdot f_k(x) \, dx.
\]

\(\pi\) is typically non-monotonic in \(k\) for any security (even \(k\) is ordered by FOSD).

“Good news” increases price (triangle) but there is less probability mass on the left tail.

“Bad news” decreases price (triangle), but there is more probability mass on the left tail.
Example
\(F_1 \sim u[0, 0.05], F_2 \sim u[0, 0.1], \ldots, F_{60} \sim u[0, 3], F_{61} \sim u[0.05, 3], \ldots, F_{119} \sim u[2.95, 3], \quad \lambda_i = \frac{1}{119}\)

\[\Rightarrow\quad \text{Ex ante } F_m = F_{60}\]

At \(t=1\), if debt with face value \(D=1\) is issued then \(p^D_m = \frac{5}{6}, \quad \pi^D (m) \approx 0.116\)

At \(t=2\): if \(F_k = F_{30} \sim u[0,1.5]\), then \(p^D (k=30) = \frac{2}{3}, \quad \pi^D (k=30) \approx 0.1482\)
**Proposition (Trading at t=2)**

Suppose debt with price w has been issued at t=1. At t=2, if

(a) \( E_k[s^D(x)] \geq E_m[s^D(x)] \), there is efficient trade between B and C.

(b) \( E_k[s^D(x)] < E_m[s^D(x)] \) and \( \pi^D(k) \leq \gamma \), there is efficient trade between B and C.

(c) \( E_k[s^D(x)] < E_m[s^D(x)] \) and \( \pi^D(k) > \gamma \), then as a best response, agent C either chooses:

   (i) Strategy I (Maximum debt write-down)

   (ii) Strategy II (Surplus Sharing)

   (iii) Strategy III (Adverse Selection).
Proposition (Debt Equilibrium)

Consider the economy $\{\alpha, \gamma, w, \{F_i\}_{i=1}^N\}$ and suppose $\gamma \geq \pi^D(m)$. Then there exists an equilibrium with the following properties:

At $t=1$, agent A consumes $w$ by issuing debt to agent B. No information is acquired.

At $t=2$, depending on the revealed distribution $F_k$, the following cases can arise:

(i) There is efficient trade between agents B and C. No information is acquired.

(ii) There is inefficient trade between agents B and C. No information is acquired.

(iii) Agent B produces information and trade occurs with probability less than one.
Example

$F_1 \sim u[0, 0.8]$, $F_2 \sim u[0.8, 1.2]$, $F_3 \sim u[1.2, 2]$ and $\lambda_1 = \lambda_2 = \varepsilon$, and $\lambda_3 = 1 - 2\varepsilon$.

At $t=1$:

\[ f_m = \frac{5\varepsilon}{4} \text{ for } x \in [0, 0.8], \]
\[ f_m = \frac{5\varepsilon}{2} \text{ for } x \in [0.8, 1.2], \]
\[ f_m = \frac{5(1-\varepsilon)}{4} \text{ for } x \in [1.2, 2] \text{ and } f_m = 0 \text{ else}. \]

$\varepsilon = 0.0001$, $w = 1$, $\gamma = 0.001$, and $\alpha = 1.2$

Remark

Ex ante, it is very likely that $F_3 \sim u[1.2, 2]$ is the true distribution.
Equilibrium (example):
Debt with face value \( D \approx 1 \) and price \( p^D_m = 1 \) is issued at \( t=1 \) and \( \pi^D(m) \approx 0 \).
At \( t=2 \),
- If there is good news (i.e., \( F=F_3 \)), there is efficient trade between B and C and agent B consumes \( w=1 \).

- If there is bad news (i.e., \( F=F_2 \)), then the market price of debt drops from 1 to 0.95 and agent C buys a senior tranche of 87.2% of agent B’s debt, i.e. a new debt contract with market price of 0.83 (backed by the original debt)

\[ \rightarrow \text{Haircut of } 13\% = 1 - \frac{0.83}{0.95} \text{ (B and C trade 0.83 instead of 0.95)} \]

- If there is very bad news (i.e., \( F=F_1 \)), then the market price of debt is 0.4 and agent C buys a senior tranche of 10% of agent B’s debt for price of 0.04

\[ \rightarrow \text{Haircut of } 90\% = 1 - \frac{0.04}{0.4} \text{ (B and C trade 0.04 instead of 0.4)} \]
Proposition

The debt equilibrium is a second best outcome, i.e., it is constrained efficient.
A Big Picture Story

Repo Market

- The (sale and repurchase agreement) repo market is a very large, short-term lending market that provides collateralized financing.

- Estimates of the size of the repo market are that it is roughly $8 trillion to $10 trillion. (Gorton (2009))

- The repo market traditionally was confined to U.S. Treasury securities.
  ➔ Shortage of treasuries as collateral for repo banking
  ➔ Global economic growth (China heavily accumulated T-Bills)

- In the last 10 years it has grown to accept a broad range of securitized bonds as collateral.
Securitization

Feature:
- Master trust (Special purpose vehicle, SPV) is bankruptcy remote.
- SPV keeps residual equity.
- Securitization is about the creation of information insensitive debt (tranching instead of selling equity) used as collateral for repo banking

- Creation of debt also creates residual equity (“toxic” asset)
  - Residual equity has maximal $\pi$ and is kept by originator.
  - Trading of residual equity may trigger information acquisition and make debt information sensitive

- Information insensitive collateral is important.

- In the event of default of borrower, the lender is stuck with collateral.

- If lender anticipates difficulty in resale, he may not accept the collateral in the first place (or raise haircut)

- Interestingly, stocks are typically not accepted as collateral in repo market.
The Panic of 2007-2008 was essentially a run on the repo market.

- Repos were not rolled over (i.e. haircuts rose sharply)
- Repo market stopped functioning.
- Many banks faced severe problem of short term refinancing.

→ lead to massive deleveraging

Main Reasons

1. Counterparty risk of big institutions became real (Failure of Lehman)
2. Information insensitive collaterals became information sensitive
   - House prices did not rise
   - ABX index revealed MBS were more risky than perceived previously
History and features of financial (banking) crises

Systemic financial crises (e.g., 1837, 1857, 1873, 1884, 1893, 1907, 1914, 1930s, and 2007) have debt as the common feature.

Why didn’t people learn and repeat “mistake”?

Our theory suggests that the creation of information insensitive debt (demand deposit and repo) is welfare maximizing despite the potential occurrence of a crisis.
Conclusion

Debt is the optimal security for liquidity provision (i.e. very short term trading in secondary markets) because it is least information sensitive, i.e.

- Debt gives rise to the lowest incentive to acquire information
- Debt maximizes trade and minimizes losses of uninformed agents if there is adverse selection
- Debt has lowest price fluctuation when there is public information

→ A theory of the creation of “private money” (or collateral assets)

The economy as a whole has to create enough debt for trade. A financial crisis arises if information insensitive debt becomes information sensitive.

→ Private money loses its acceptability as a vehicle to transfer wealth
Extensions

- The role of banks in internalizing information externality of the production of socially valuable information (bank dominates market finance)

- Portfolio theory when agents care about information sensitivity

- Monetary policy in this economy (is about information)

- Rating agency and the release of coarse information for liquidity provision