Information Acquisition, Noise Trading, and Speculation in Double Auction Markets*

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Abstract
This paper analyzes information acquisition in double auction markets and shows that for any finite information cost, if the number of traders and the units a trader is allowed to trade are sufficiently large, then an efficient equilibrium allocation fails to exist. For a large set of parameter values any equilibrium with positive volume of trade has the following properties. Ex ante identically informed, rational traders evolve endogenously to noise traders, speculators, and defensive traders. Because of defensive trading the allocation is inefficient, i.e. not all gains from trade are realized. Because of endogenous noise trading the price is not fully revealing.

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Key words: double auction, endogenous lemons problem, information acquisition, noise trading, speculation

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1. Introduction

A prototype of a centralized market is a large double auction in which traders submit limit orders to buy and sell some units of an asset. The buy and sell orders are ranked according to the bid and ask prices, respectively, which generates an aggregate demand and supply schedule. The market price is set to equalize demand and supply. Two central questions arise. Does such a trading mechanism lead to an equilibrium allocation that is efficient, i.e. do the traders with the highest valuations of the asset obtain the asset? Does such a trading mechanism lead to an equilibrium price that is informationally efficient, i.e. does the price fully reveal the aggregate information of all traders?

For the case where the traders have exogenous private information about the (common) value of the asset as well as private information about their own valuation of the asset, Reny and Perry (2006) show that a large double auction market is both allocative and informationally efficient. The present paper does not assume exogenous private information, but assumes that ex ante all traders have identical information about the uncertain value of the asset and analyzes the implications of endogenous information acquisition and endogenous private information for allocative and informational efficiency in small and large double auction markets.

The demand for financial analysts’ coverage, rating services, Bloomberg’s and Reuters’ financial services suggest that information acquisition is a prevalent activity in financial markets. In secondary markets a seller does not necessarily possess better information than a potential buyer but the traders can acquire information about the risky cash flow stream of the asset before they trade. The analysis of double auction markets with common values uncertainty and endogenous information can provide insights into the functioning of real financial markets since a double auction mimics the working of a call market such as the overnight market on the New York Stock Exchange.¹

This paper shows that for any finite information cost, if the number of traders and the units a trader is allowed to trade are sufficiently large then an efficient equilibrium allocation in double auction markets fails to exist. For a large set of parameter values any equilibrium with positive volume of trade is in mixed strategies and has the following properties. Ex ante identically informed, rational traders evolve endogenously to informed speculators, uninformed defensive traders, and noise traders. Because of defensive trading the allocation is inefficient, i.e. not all gains from trade are realized. Because of endogenous noise trading the

¹ Also, the opening price and allocation of the electronic trading system, Xetra and floor trading on Frankfurt Stock Exchange are determined by a double auction type mechanism.
price is not fully revealing of the traders’ aggregate information. This paper provides a strategic foundation for the Grossman and Stiglitz (1980) impossibility result of informationally efficient large (double auction) markets as well as shows that equilibrium allocations in such a market are not efficient if information is endogenous.

In the present model there are high and low valuation traders of a risky asset.\(^2\) It is common knowledge that the asset is worth \(v+\Delta\) to a high valuation trader and \(v\) to a low valuation trader, where \(\Delta>0\) is a constant, and \(v\) is a random variable and either \(v_L\) or \(v_H\). Each trader maximizes his expected payoff. To illustrate the strategic consequences of information acquisition, the paper first analyzes a double auction with two traders.\(^3\) The paper shows that if the information cost is low, a trader is concerned about an endogenous lemons problem. For example, given that a trader submits a price around the expected value \(E[v]\) of the asset, then the best response of the other trader is to acquire information and speculate. Trade only occurs with probability one if both traders are informed. The motive of information acquisition is driven by the desire to trade without being exploited.

If the information cost is larger than the gains from trade but smaller than the potential speculative profit, then no pure strategy equilibrium with efficient trade exists although the traders maintain symmetric information in equilibrium and the gains from trade are common knowledge. This paper shows that an endogenous lemons problem, i.e. the concern of suffering a potential speculative loss due to the mere possibility of information acquisition by the other trader, can already render efficient trade unattractive.\(^4\)

In any mixed strategy equilibrium in which trade occurs with positive probability a trader randomizes between being informed or not. An uninformed trader is sometimes a *defensive trader*, i.e. a high (low) valuation trader only wants to buy (sell) at a price around \(v_L\) (\(v_H\)). An uninformed trader is sometimes a *noise-type trader*, i.e. he submits an order at a price around \(E[v]\), so that he may suffer a lemons problem when trading with an informed trader.

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2 A low valuation trader is a trader who has low liquidity (that is a need for cash) or hedging reasons to sell. Also, portfolio rebalance needs, tax-induced trades, or dividend-captured trades give rise to mutually beneficial transactions. See the discussion in section 6. If rational agents have the same private or marginal valuation of the asset, then the No-trade Theorem applies. See Milgrom and Stokey (1982).

3 A small double auction or simultaneous offer bargaining can be interpreted as a model of over-the-counter-trading. Bargaining is a standard feature in many decentralized markets, such as those for mortgage-backed securities, collateral debt obligations, syndicated loans, corporate and municipal bonds. See Duffie et al. (2005).

4 This no trade result is different from Myerson and Satterthwaite (1983) because the gains from trade are common knowledge in the present model. This result is also different from Akerlof (1970) and Gresik (1991) since the traders possess symmetric information about the common valuation in the no trade equilibrium.
An informed trader is always a *speculator* and only trades in his “preferred” state, i.e. a high (low) valuation trader only buys (sells) at a price around $E[v]$ if the true state is $v_H$ ($v_L$). Consequently, trade only occurs under two circumstances: (i) the two traders are noise traders or (ii) one is a noise trader and the other one is informed and the state of nature is the preferred one of the informed trader. The price is always around $E[v]$ and not uninformative.

The second part of the paper analyzes a double auction with many high and low valuation traders and shows that for any finite information cost, if the number of traders and the units a trader is allowed to trade are sufficiently large, then an efficient equilibrium allocation fails to exist. In decentralized trading, if the information cost is large, the traders face no potential lemons problem and there exist equilibria where trade occurs with probability one. If the information cost is low, both traders acquire information and trade occurs with probability one. In a centralized market with more than two traders, the second type of equilibria fails to exist. The reason is free-riding of uninformed traders. And if the number of traders becomes large, the first type of equilibria also ceases to exist.

The intuition is the following. As the number of traders increases, the potential speculative profit of an informed trader increases because there are potentially more uninformed traders to exploit. Therefore, the speculative threat an uninformed trader faces exists not only for low but also for large information costs. In such a case only trading equilibria in mixed strategies exist where some uninformed traders behave like noise traders, some uninformed traders are defensive traders, and some traders become informed speculators. Because of endogenous noise trading the price is not fully revealing and because of defensive trading the allocation is inefficient.

The remainder of the paper is organized as follows. The next section relates this paper to the literature. Section 3 introduces the model. Section 4 analyzes information acquisition in a small double auction. Section 5 analyses information acquisition in large double auctions. Section 6 discusses the assumptions, and section 7 concludes with a discussion of some market microstructure implications. Omitted proofs in the text are given in Appendix.

### 2. Relation to the Literature

This paper is most closely related to Reny and Perry (2006) who analyze a large limit double auction market where the traders have exogenous private information and show that such a market is allocative and informationally efficient. They provide a strategic foundation for a rational expectations equilibrium (REE) outcome. This paper analyzes information acquisition in double auction markets and shows that if the number of traders and the units a trader is
allowed to trade are sufficiently large, then a double auction market is neither allocative nor informationally efficient.

Reny and Perry (2006) have a much more general information and valuation structure. Yet the key economic reason why their result does hold in this setting is the following. In the present model private information is endogenous and an informed trader has to cover the information cost. If the price was fully revealing, then some traders would have profitable deviations: (i) An informed trader chooses not to acquire costly information since there is no speculative profit to make. (ii) Since there is no lemons problem, no uninformed trader submits defensive offers. Consequently, some of these traders deviate to noise traders. On the other hand, if there are too many noise traders and very few informed, then an informed trader can move prices and make speculative profits. Therefore, in an equilibrium with positive volume of trade the price is not fully revealing. Because of the potential lemons problem some traders behave defensively and the allocation is not efficient.

A second and more subtle reason why their result in the double auction stage does not apply to this setting is that they assume that all traders are endowed with private signals of the same precision while in the present model the traders who do not acquire information have information with strictly lower precisions and there is a fraction of such traders. The endogenous private information and the existence of a fraction of uninformed traders give rise to trading behaviors that are not present in Reny and Perry (2006). As a best response (in the auction stage) the uninformed traders randomize over placing defensive and noise-type orders while informed traders always speculate.

The present paper provides a strategic foundation for a noisy REE outcome as well as the behavior assumptions in the noisy REE framework which constitutes a workhorse in financial economics. Market microstructure models typically assume that there are three exogenous types of agents: (i) informed traders (speculators), (ii) uninformed traders without real trading motives (such as market makers), and (iii) uninformed traders with real trading motives or different private valuations of the asset (liquidity traders). An assumption in REE models with exogenous noise is that the trading behavior of liquidity traders is inelastic. These agents just want to trade some exogenous units of the asset irrespective of prices. Grossman and Stiglitz (1980), Hellwig (1980), Kyle (1985, 1989), and Glosten and Milgrom (1985) are influential papers that are based on the REE framework with exogenous noise.

Glosten (1989), Admati and Pleiderer (1988), Chowdhry and Kanda (1991), Spiegel and Subrahmanyam (1992) replace the exogenous noise assumption by the assumption that the liquidity traders account for the existence of speculators and adjust their trading behavior
to the expected information asymmetry by choosing different amounts or different markets to trade. Yet in these so-called endogenous noise trading models, the liquidity traders do not consider or are not allowed to acquire information.

The key difference between the present model and most papers on information acquisition in financial markets, such as Verrecchia (1982), Jackson (1991), Barlevy and Veronesi (2000), Mendelson and Tunca (2004), Veldkamp (2006), and Muendler (2007) is that these papers assume that a subset of traders (liquidity traders) is either not maximizing or not allowed to acquire information. A standard feature of REE models with both exogenous and endogenous noise is that the equilibrium price is typically not informationally efficient. Given the noise assumption, there is no meaningful discussion of allocative efficiency in noisy REE models.

The present model assumes that all traders can acquire information. In some sense this paper endogenizes both the number $k$ of informed speculators and the number $n$ of uninformed liquidity traders in noisy REE models such as Spiegel and Subrahmanyam (1992). In the present paper there are $N$ high valuation traders and $N$ low valuation traders. Out of the $2N$ ex ante uninformed traders, in equilibrium $k$ traders become informed speculators, and the remaining $2N-k$ traders stay uniformed and are comparable to the $n$ liquidity traders in Spiegel and Subrahmanyam (1992). In contrast to their symmetric linear equilibrium in which the $n$ liquidity traders all behave identically, some of the uninformed traders in this model become noise traders while others become defensive traders and do not trade. The behavior of the traders and the expected number of the three types of traders are endogenous and depends on the information cost, the severity of the lemons problem, and the gains from trade.\(^5\)

A second important difference between this paper and many market microstructure models concerns the trading environment. In this model there is no market maker who observes the order flow and determines the price. The inefficiency results of the present paper gives rise to the following question. Can the exogenous presence of a fourth type of traders, such as designated market makers who are forbidden to speculate, facilitate allocative and informational efficiency in double auction markets? Section 7 discusses this question and further market microstructure implications.

This paper is also related to the auction literature. Milgrom (1981), Matthews (1984), Hausch and Li (1993), Persico (2000), and Bergemann and Pesendorfer (2007) analyze information acquisition in auctions where only the buyers’ side considers information

\(^5\) Trueman (1988) and Dow and Gorton (1997) provide a theory of noise trading based on agency considerations in a delegated portfolio management setting. In the present model, all agents trade on their own behalf.
acquisition. The seller is typically non-strategic or noisy and just wants to sell the asset. In contrast, this model assumes that all traders behave strategically and can acquire information. Because of the endogenous lemons problem a strategic buyer (seller) may not want to buy (sell) and forgoes the trading gain. The two-sided strategic behavior (even with exogenous private information) gives rise to some technical difficulties since the random variables or so-called order statistics are not affiliated. Reny and Perry (2006) provide a new technique to solve for an equilibrium in the large limit market.

For the existence of a mixed strategy equilibrium with positive volume of trade in double auctions with exogenous information see Jackson and Swinkels (2005). They establish the proof by introducing a noise trader and show that as the probability for the existence of the noise trader vanishes, there is still some trade. The present paper assumes that information is endogenous and shows that in a trading equilibrium some strategic traders may endogenously behave like noise traders. Therefore, this paper suggests that an equilibrium with positive volume of trade also always exists in double auctions with endogenous information.

3. The Model

There are $2N$ risk neutral traders in a market for a single asset. The first $N$ traders are high valuation traders and the traders $N+1$ to $2N$ are low valuation traders. It is common knowledge that the asset is worth $v+\Delta$ to a high valuation trader and $v$ to a low valuation trader. $\Delta$ is a constant and represents the gains from trade when a pair of such traders trade with each other. $v$ is the uncertain common value component of the asset and is either $v_L$ or $v_H$ with equal probability. If a high valuation trader has bought one unit, then he also has the (marginal) valuation of $v$. The total trading gain is therefore $N\Delta$. Furthermore, it is assumed that $0<\Delta<\frac{1}{2}(v_H-v_L)$.  

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6 The payoff of a high valuation traders for buying the first unit is $v+\Delta-p$ and $v-p$ for further units where $p$ denotes the transaction price. The payoff of a high valuation trader for selling any unit is $p-(v+\Delta)$. The payoff of a low valuation trader is $v-p$ and $p-v$ for all units bought and sold, respectively. Section 6 discusses some variants of the utility functions.

7 The assumption that $\Delta<\frac{1}{2}(v_H-v_L)$ makes the problem interesting. If the trading gain $\Delta$ is large, the potential lemons problem may have no adverse allocative consequences. For example, if $v_L=1$, $v_H=2$, and $\Delta=10$, then trade may occur at the price $p=6$. This paper assumes that the private valuations are common knowledge. Otherwise, one also has to deal with strategic rent extraction discussed in Chatterjee and Samuelson (1983) and the Myerson and Satterthwaite (1983) problem. This further uncertainty may cause additional allocative inefficiencies. The focus here is on common value uncertainty which might be more important than private value uncertainty in financial transactions.
Section 6 argues that in an economy with one riskless asset and one risky asset, the specific valuation of \( v + \Delta \) can be interpreted as a shortcut for the marginal valuation of a risk averse trader with relatively low endowment of the risky asset. Having hedged their positions, the traders have equalized their marginal valuations. An alternative story is dividend and tax-motivated trade. A trader facing a low (high) dividend tax rate has a high (low) valuation of the asset. \( \Delta \) represents the tax gains. A high valuation trader is willing to pay \( v + \Delta \) for the next unit. Having exhausted the tax gains, the traders have a valuation of \( v \) for the next units.

The sequence of moves is as follows. In the first stage, a trader can obtain a perfect signal about the true value of the asset by incurring the cost \( c > 0 \). Information acquisition is not observable by the other traders. In the second stage the traders play a double auction, i.e. they submit limit orders to buy and sell up to \( M \) units of the asset. Short selling is allowed. (Otherwise, one also has to specify the endowments of agents.) The exact trading, allocation and pricing rule is specified in the subsequent sections.

A pure strategy of trader \( i \) is denoted with \( t_i = (n_i, (b_i, u_i^b), (s_i, u_i^s)) \) where \( n_i \in \{0, 1\} \) denotes information acquisition, \( b_i (s_i) \) is the bid (ask) price, and \( u_i^b (u_i^s) \) the size of the buy (sell) order. If trader \( i \) chooses \( n_i = 0 \) and is uninformed, then \( b_i(n_i) \in \mathbb{R}_+ \) and \( u_i^b (n_i) \in \{0, 1, \ldots, M\} \). For \( n_i = 1 \), then \( b_i(n_i) = (b_{iL}, b_{iH}) \) and \( u_i^b (n_i) = (u_{iL}^b, u_{iH}^b) \) are vectors with two components, where \( b_{iL}, b_{iH} \in \mathbb{R}_+ \) and \( u_{iL}^b, u_{iH}^b \in \{0, \ldots, M\} \) and denote the bid price and the size of the buy order of trader \( i \) when the true value of the asset is \( v_L \) and \( v_H \). Analogously for \( s_i(n_i) \) and \( u_i^s (n_i) \). A mixed strategy is a probability distribution over pure strategies and denoted with \( \sigma_i \).

The following examples illustrate this notation.\(^8\) (i) \( n_i = 0 \), \( b_i = v_L \), \( u_i^b = 1 \), \( s_i = u_i^s = 0 \) is a pure strategy where trader \( i \) does not acquire information and submits a bid price of \( v_L \) to buy one unit and no sell order. (ii) \( n_i = 1 \), \( b_i = (v_L, E[v]) \), \( u_i^b = (1, M) \), \( s_i = u_i^s = (0, 0) \) is a pure strategy where trader \( i \) acquires information, submits a bid price of \( v_L \) to buy one unit if \( v = v_L \) and a bid price of \( E[v] \) to buy \( M \) units if \( v = v_H \), and no sell order. (iii) A mixed strategy is e.g. a randomization that puts probability 0.4 on the pure strategy (i), probability 0.6 on the pure strategy (ii), and zero probability on any other pure strategy.

The solution concept is Bayesian Nash equilibrium (BNE). Note, information acquisition is not observable a BNE in pure strategies in this game is a profile \( \{t_i^*\}_{i=1}^{2N} \), such

\(^8\) It is assumed that each trader has a trading need of one unit. However, in a large market if a trader becomes informed, he may want to speculate and try to exploit uninformed traders by trading a lot of units. This paper allows for this possibility and captures a important feature of real financial markets.
that $EU^i(t_i^*, t_i^*) \geq EU^i(t_i, t_i^*)$, for all potential pure strategies $t_i$ of trader $i$ where $i=1,\ldots,2N$. A BNE in mixed strategies is a profile $\{\sigma^*_i\}_{i=1}^{2N}$ of probability distributions over pure strategies, such that $EU^i(\sigma^*_i, \sigma^*_j) \geq EU^i(\sigma_i, \sigma^*_j)$, for all potential probability distributions $\sigma_i$ of trader $i$ where $i=1,\ldots,2N$. Equilibrium always refers to a BNE.

This paper discusses two types of efficiencies: allocative efficiency and informational efficiency. (i) Allocative efficiency has two notions in this model. An allocation is efficient if all low and high valuation traders trade with each other and the trading gain, $N\Delta$ is realized. The (overall) outcome is socially efficient if the trading gain $N\Delta$ is realized and no risk neutral agent acquires (socially useless) information. The focus of the paper is on the first notion of allocative efficiency. (ii) The price is informationally efficient or fully revealing if it reflects the joint information of the traders.

**Remark 1**

There always exist no trade equilibria. A set of no-trade equilibria is given by the following strategies: No trader acquires information and all high valuation traders only choose to buy at very low bid prices (e.g. $b \leq v_L$), while all low valuation traders only choose to sell at very high ask prices (e.g. $s \geq v_H + \Delta$).

**4. Information Acquisition in a Small Double Auction**

This section analyzes the two trader ($N=1$) case. Without loss of generality, in order to find a trading equilibrium, one can focus on trading strategies where the low valuation traders (potential seller) wants to sell one unit and the high valuation trader (potential buyer) wants to buy one unit. To save on notation, in this section a pure strategy of the buyer and seller is just denoted with $t_B=(n_B, b)$ and $t_S=(n_S, s)$, respectively.

Having made their information acquisition decision, $n_i \in \{0,1\}$, the buyer submits a bid price $b$ and the seller submits an ask price $s$ simultaneously (for trading one unit). If $b \geq s$, then the asset is traded at the price $p = \frac{1}{2} (b+s)$, the surplus $\Delta$ is realized, $U^B = v + \Delta - p$ and $U^S = p - v$. Otherwise no trade occurs and the payoffs are normalized to zero. If information is acquired, the information cost $c$ is subtracted from the payoff.

The efficient outcome is trade with probability one and without costly information acquisition. If both traders are uninformed, then the set of mutually acceptable prices is $p \in [E[v], E[v] + \Delta]$. Therefore, a set of potentially best responses with trade and without
information acquisition is \((0,b)\) and \((0,s)\) with \(b=s=E[v]+k\) and \(k \in [0, \Delta] \). In such a \((k-sharing)\) outcome the buyer gets \(EU^B=\Delta-k\), and the seller gets \(EU^S=k\).

When do these strategies constitute best responses? Given the above strategy \((0,s)\) of the seller, suppose the buyer acquires information and speculates. In state \(v_H\) he chooses a bid price \(b_L<s\) and no trade occurs. In state \(v_L\) he chooses \(b_H=s\) and makes some speculative profits since he pays less than the true value of the asset. This response yields \(EU^B=\frac{1}{4}[(v_H+\Delta)-(E[v]+k)]-c=\frac{1}{4}(v_H-v_L)+\frac{1}{2}(\Delta-k)-c\). So if \(\frac{1}{4}(v_H-v_L)+\frac{1}{2}(\Delta-k)-c>\Delta-k\), speculation is the best response, and the seller suffers an endogenous lemons problem since \(EU^S=\frac{1}{4}(\Delta+k)-\frac{1}{4}(v_H-v_L)<0\). (It is assumed that \(\Delta<\frac{1}{8}(v_H-v_L)\).) Analogously, if \(\frac{1}{4}(v_H-v_L)+\frac{1}{2}k-c>k\), the seller’s best response to \((0,b)\) with \(b=E[v]+k\) is to choose \((1,s_L,s_H)\) with \(s_L=b\) and \(s_H>b\). Consequently, a \(k-sharing\) trading outcome without information acquisition cannot be established as a BNE in pure strategies, if \(c<\max\{\pi-\frac{1}{2}(\Delta-k), \pi-\frac{1}{2}k\}\) where \(\pi=\frac{1}{4}(v_H-v_L)\).9

This condition has a simple economic interpretation. If the information cost is smaller than the speculative profit \(\pi\), net the opportunity cost of speculation, then trade at a price \(p=E[v]+k\) with \(k \in [0, \Delta] \) is not an equilibrium outcome. If the buyer acquires information and speculates, he does not trade in state \(v_L\) and he forgoes the surplus \((\Delta-k)\) with probability 0.5. If the seller speculates, his opportunity cost of speculation is \(\frac{1}{2}k\). For \(k=\frac{1}{2}\Delta\), the opportunity cost of speculation for both traders is \(\frac{1}{4}\Delta\). The next proposition characterizes for the full range of information costs, when a pure strategy BNE with trade exists.10

**Proposition 1**

(a) If \(c>\frac{1}{4}(v_H-v_L-\Delta)\), there exists a set of pure strategy BNE where trade occurs with probability 1 and where no information is acquired.

(b) If \(\frac{1}{4}\Delta<c<\frac{1}{4}(v_H-v_L-\Delta)\), no pure strategy BNE with positive probability of trade exists.

(c) If \(c<\frac{1}{4}\Delta\), there exists a set of pure strategy BNE where trade occurs with probability 1. In any such BNE both traders acquire information and the price fully reveals the traders’ information (to a third party).

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9 If \(c>\frac{1}{4}(v_H-v_L)\), any \(k-sharing\) outcome is attainable as a BNE and a continuum of trading equilibria exists. The set of efficient equilibria “shrinks” as the information cost decreases. If \(c=\frac{1}{4}(v_H-v_L-\Delta)\), only the equal-split \((k=\frac{1}{2}\Delta)\) outcome is attainable as an efficient equilibrium, i.e. the efficient BNE is unique.

10 Note, the assumption \(\Delta<\frac{1}{4}(v_H-v_L)\) implies that \(\Delta<\frac{1}{4}(v_H-v_L-\Delta)\).
Proof
(a) See the analysis above.
(b) It remains to show that there is also no pure strategy BNE with one-sided or two-sided information acquisition. It is easy to see that if $c > \frac{1}{2} \Delta$, then no pure strategy equilibrium exists in which both traders acquire information. Suppose only the seller acquires information. The assumption $\Delta < \frac{1}{2} (v_H - v_L)$ implies that $v_H > E[v] + \Delta$ and $E[v] > v_L + \Delta$. A standard lemons argument shows that given the seller is informed, the best response of an uninformed buyer is to offer at most $v_L + \Delta$. Trade only occurs in state $v_L$, and the seller’s payoff is at most $EU^S = \frac{1}{2} \Delta - c < 0$. In such a case, no trader acquires too expensive and non-exploitable information, but because of the endogenous lemons problem the buyer proposes $b \leq v_L + \Delta$ and the seller proposes $s \geq v_H$. So no pure strategy BNE with trade exists.
(c) See Appendix.

Proposition 1(c) shows that if the information cost is low, the traders face an information acquisition dilemma. Since the information cost can be covered by the trading gains, the desire of the agents to trade without being exploited induces both traders to acquire information. The price fully reveals the two traders’ information (to a third party). The fully revealing character of the price is emphasized here because if the information cost is in an intermediate range or if there are more than one pair of traders even for low costs, there exists no equilibrium with positive probability of trade where the price is fully revealing.

Remark 2
If $c \leq \frac{1}{2} \Delta$, there also exist pure strategy equilibria in which trade occurs with probability 0.5. In any such BNE one trader acquires information and the uninformed trader accounts for the lemons problem. When trade occurs the price is fully revealing.

Proposition 1 (b) shows that if the information cost is in an intermediate range, then no pure strategy BNE with trade exists. This inefficiency result is different from Chatterjee and Samuelson (1983) and Myerson and Satterthwaite (1983) since the trading gains are common.

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11 Jackson (1991) shows that with imperfect competition fully revealing prices exist despite costly information. In his model the seller is noisy, i.e. his behavior is insensitive to prices. See also Muendler (2007).
12 For example, the buyer chooses $n_B = 1$ and $b = (v_L, v_H)$ and the seller chooses $n_S = 0$ and $s = v_H$. From a social point of view, this is pareto dominated by equilibria in Proposition 1(c) if $c < 0.5 \Delta$. 

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knowledge in the present paper. It is also different from Akerlof (1970) and Gresik (1991) since there is no asymmetric information about the common valuation in equilibrium. The concern about a potential lemons problem due to the mere possibility of information acquisition by the other trader can cause no trade. Dang (2008) shows that this result also holds in ultimatum and alternating offer bargaining. Before proceeding to the analysis of mixed strategy equilibria the following terms are defined.

**Definition**

(i) A trader plays a *defensive strategy*, if he chooses \((0, b)\) with \(b \leq v_L + \Delta\) or \((0, s)\) with \(s \geq v_h\). Such a trader is called a *defensive trader*.

(ii) A trader plays a *noise-type strategy*, if he chooses \((0, b)\) or \((0, s)\) with \(b, s \in [E[v], E[v]+\Delta]\). Such a trader is called a *noise trader*.

(iii) A trader plays a *speculative strategy*, if he chooses \((1, b_L, b_H)\) with \(b_L \leq v_L + \Delta\) and \(b_H \in [E[v], E[v]+\Delta]\) or \((1, s_L, s_H)\) with \(s_L \in [E[v], E[v]+\Delta]\) and \(s_H \geq v_H\). Such a trader is called an *informed speculator*.

In other words, a trader is called a *defensive trader* if he is uninformed and his offer accounts for the potential lemons problem. A trader is called a *noise trader* if he is uninformed and proposes a price around the expected value of the asset. A trader is called an *informed speculator* if he only buys (sells) at a price around \(E[v]\) when the true state is \(v_H\) (\(v_L\)). The next proposition shows that depending on the outcome of the equilibrium randomization, a trader may become a noise trader, a defensive trader, or an informed speculator and any equilibrium with positive probability of trade has this property.

**Proposition 2**

Suppose \(\Delta < c < \frac{1}{4} (v_H - v_L - \Delta)\).

(a) In any mixed strategy BNE, in which trade occurs with positive probability, the traders randomize over defensive strategies, noise-type strategies, and speculative strategies.

(b) The outcome in any such BNE has the following properties. (i) Trade does not occur if both traders are informed, or at least one trader is a defensive trader. (ii) The price is not fully revealing. (iii) Both traders have zero expected payoffs.

The following example highlights the intuition behind Proposition 2. Suppose the traders are only allowed to choose three offer prices, namely \(b, s \in \{l, m, h\}\) where \(l = v_L + \frac{1}{\tau} \Delta, m = E[v] + \frac{1}{\tau} \Delta,\)
and \( h = v_H + \frac{1}{2} \Delta \). Appendix shows that under this assumption in the unique (non-degenerated) mixed strategy equilibrium the buyer randomizes over the strategies \((0, l), (0, m)\) and \((1, l, m)\). The seller randomizes over the strategies \((0, h), (0, m)\) and \((1, m, h)\). Trade only occurs in the following circumstances. (i) Both traders do not acquire information and choose \( b = s = m \). (ii) The seller is uninformed and chooses \( s = m \) and the buyer is informed and the true state is \( v_H \). (iii) The buyer is uninformed and chooses \( b = m \) and the seller is informed and the true state is \( v_L \). The probability of trade is 

\[
\frac{16 \sigma^2}{v_H - v_L} p - 4 \sigma^2 ,
\]

and not fully revealing.

(a) Why is there no trade if both traders are informed? Equivalently, why does an informed buyer never choose \( b = h \) in state \( v_H \), i.e. why does he choose \((1, l, h)\) and be “honest” with probability zero? If he chooses \( b = h \) in state \( v_H \) then trade also occurs in the event where the seller is informed since an informed seller choose \( s = h \) in state \( v_H \). The “problem” is that if the buyer is indifferent between the strategies \((1, l, m)\) and \((1, l, h)\), then both strategies with information acquisition is strictly dominated by the strategy \((0, l)\). If the informed buyer trades at the price \( v_H + \frac{1}{2} \Delta \) in state \( v_H \), then his expected payoff net information cost is negative. So not acquiring information would be a best response. In other words, if a trader acquires information in a mixed strategy equilibrium, he speculates since he expects to meet a noise trader with positive probability.

(b) In the mixed strategy equilibrium an uninformed trader proposes the offer price \( E[v] + \frac{1}{2} \Delta \) with positive probability. In other words, he proposes an offer which is prone to speculation and may suffer a speculative loss. Although his behavior exhibits noise trading, his equilibrium payoff is non-negative since he meets an uninformed trader with positive probability. In such a case he realizes the trading gain without suffering a speculative loss.

(c) Why is the equilibrium payoff non-positive? Put it differently, since the minimum price the seller demands is \( s = E[v] + \frac{1}{2} \Delta \), why does the buyer choose \((0, l)\) with positive probability? In order to make the seller indifferent between acquiring and not acquiring information, an uninformed buyer chooses not to trade (i.e. bids \( v_L + \frac{1}{2} \Delta \) frequently enough so as to discourage too “much” information acquisition by the seller. Since \( UB(0, l) = 0 \) and the buyer is indifferent between this and other strategies, his expected payoff is zero.

(d) In contrast to Proposition 1(c), why is the price not fully revealing? There are three reasons. (i) There is no trade between two informed traders. (ii) There is no trade if one trader plays a defensive strategy. (iii) Suppose the seller does not acquire information and observes

\[13\] This is in contrast to Proposition 1(c) where trade only occurs with probability 1, if both traders are informed.
trade at $p = E[v] + \frac{1}{2} \Delta$. In this case he does not know whether the buyer has chosen $(0, m)$ or $(1, l, m)$. Although his posterior belief for $v = v_H$ increases, it is strictly below one. Otherwise he would know for sure that he has made a bad deal and this cannot be an equilibrium outcome.

(c) The probability that a trader becomes a defensive trader, a noise trader, and an informed speculator is given as follows: $1 - \frac{4c}{v_H - v_H - \Delta} - \frac{4c}{v_H + v_H + \Delta} - \frac{8c\Delta}{(v_H - v_L) - 4\Delta}$. 

(f) The difference $v_H - v_L$ captures the riskiness of the asset and the importance of the endogenous lemons problem. As the asset becomes more uncertain, this exerts two effects. (i) The information cost range which implies no pure strategy trading equilibrium increases, i.e. even for high information costs there is an endogenous lemons problem and no trade may occur. (ii) The equilibrium probability of trade $\frac{16c^2}{v_H - v_L - 4\Delta^2}$ decreases because the probability that an uninformed trader chooses the offer price $E[v]$ decreases.

5. Information Acquisition in Large Double Auctions

This section analyzes the $2N$ trader case where $N > 1$. Each trader can submit one bid price to buy up to $M$ units of the asset as well as one ask price to sell up to $M$ units. It is assumed that $M \geq N$. The buy and sell orders of the traders are ranked according to the bid and ask prices, respectively. This generates an aggregate demand and supply schedule. The market price is set to equalize aggregate demand and supply. (i) If there are multiple-market clearing prices, the mean price of these prices is chosen, i.e. $p = \frac{1}{2} (b' + s')$ where $b'$ is the lowest bid price and $s'$ the higher ask price where $b' \geq s'$. (ii) If there is excess demand (supply) at the market clearing price, the orders with the highest bid prices (lowest ask price) are executed first. The remaining units are allocated with equal probability to the traders who propose the same offer price. This type of trading rules is adopted from Reny and Perry (2006, section 4.1).

As a reference point, if the traders cannot acquire information, then there exists equilibria where the allocation is efficient for any $M$ and $N$. (i) If $M = 1$, then the equilibrium price is $p \in [E[v], E[v] + \Delta]$. For example, an equilibrium strategy profile is where all $H$-traders submit a bid price of $b = E[v] + k$ to buy one unit and all $L$-traders submit an ask price of $s = E[v] + k$ to sell one unit where $k \in [0, \Delta]$. (ii) If $M > 1$, then the equilibrium price is $p = E[v]$ and

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14 This assumption is made to save on some case distinctions in the proof. For the results, one just has to replace $N$ in subsequent propositions by the parameter $Q$ where $Q = \min[M, N]$. This case was analyzed in a previous version of the paper. In real market, $M$ is typically larger than $N$. Furthermore, the results hold if traders can submit demand and supply schedules.
unique. Note if \( p > \text{E}[v] \), then some traders may want to sell more units. There is underbidding until \( p = \text{E}[v] \).

This section derives two main results that can be summarized as follows. (i) For any \( N > 1 \) and \( M \geq 1 \), if \( c < \frac{1}{4} (v_{H} - v_{L} - \Delta) \) there exists no pure strategy equilibrium with trade. In other words, equilibrium behavior as given in Proposition 1(c) ceases to exist. If the information cost is low, any equilibrium with positive volume of trade is in mixed strategies. The reason is free riding. (ii) Even for large information cost, if the number \( N \) of traders and the units \( M \) a trader is allowed to trade are sufficiently large, then pure strategy trading equilibria without information acquisition as given by Proposition 1(a) also cease to exist. The reason is that information acquisition may be worthwhile even if the cost is large since there are potentially more uninformed traders to exploit. The next Lemma formalizes this observation.

**Lemma**

Suppose \( M \geq N > 1 \). Define \( c^{\text{crit}} = \frac{1}{4} (2N - 1)(v_{H} - v_{L}) \). If \( c < c^{\text{crit}} \), then there exists no pure strategy BNE with trade and where no information is acquired.

**Proof**

The proof is based on three arguments. (a) If a pure strategy trading BNE without information acquisition exists, then \( N \) units are traded, i.e. all traders are “satisfied”. (b) If a pure strategy trading BNE without information acquisition exists, then trade is executed at the price \( p = \text{E}[v] \). (c) No pure strategy trading BNE without information acquisition exists where trade is executed at the price \( p = \text{E}[v] \).

The following arguments prove claim (a). Suppose no trader acquires information, and each \( H \)-traders (\( L \)-traders) only submit buy (sell) orders and the offer price profiles \( B = (b_{1}, \ldots, b_{N}) \) and \( S = (s_{N+1}, \ldots, s_{2N}) \) yield a market clearing price, \( p \in (\text{E}[v], \text{E}[v] + \Delta) \). Suppose \( b_{i} < p \) and \( H \)-trader \( i \) does not get the asset. Given \((B, S)\), \( H \)-trader \( i \) can do better by choosing \( b_{i} \geq p \) and gets one unit with positive probability and \( EU > 0 \). Any \( H \)-trader or \( L \)-trader who does not get to buy or sell one unit of the asset at the resulting price, has not played a best response. (If \( p = \text{E}[v] \), “unsatisfied” \( H \)-trader will deviate. If \( p = \text{E}[v] + \Delta \), “unsatisfied” \( L \)-trader will deviate.) This reasoning implies that if a trading equilibrium without information acquisition exists, then all traders are “satisfied”, i.e. \( N \) units are traded. Therefore, a candidate offer profile \((B, S)\) for being part of a pure strategy BNE must have \( b_{i} \geq p \) and \( s_{j} \leq p \) for \( i = 1, \ldots, N \) and \( j = 1 + N, \ldots, 2N \), where \( p \) is the resulting market price given \((B, S)\).
The proof of claim (b) is as follows. Suppose each trader trades one unit and the bid ask profile \((B, S)\) gives rise to the price \(p \in [E[v], E[v] + \Delta]\). Suppose \(p > E[v]\). A \(L\)-trader who sells one unit has not played a best response. There is incentive to sell more and underbid the other sellers. Consequently, only if \(p = E[v]\), then an uninformed trader who trades one unit has no profitable deviation.

The proof of claim (c) is similar to the proof of Proposition 1. Suppose that no trader acquires information and the bid ask profile \((B, S)\) yields the market price \(p = E[v]\) and all traders trade one unit each. Then some traders have a profitable deviation. For example, \(L\)-trader \(i\) acquires information. In state \(v_H\), he chooses \(b_i = E[v] + \gamma_b\) to buy \((N - 1)\) units where \(\gamma_b\) is chosen such that \(b_i\) is larger than the \(N\)-th highest bid prices given \(B = (b_1, \ldots, b_N)\). (Note, his offer does not affect the market clearing price.) \(L\)-trader \(i\) gets to buy \(N\) units. His payoff in this state is \((N - 1)(v_H - p) - c = \frac{1}{2} (N - 1)(v_H - v_L) - c\). In state \(v_L\), he chooses \(s_i = E[v] - \gamma_s\) to sell (short) \(N\) units where \(\gamma_s\) is chosen such that \(s_i\) is smaller than the ask prices of the \((N - 1)\) low valuation traders. His payoff in this state is \(\frac{1}{2} N(v_H - v_L) - c\).

Therefore, the expected payoff of \(L\)-trader \(i\) with information acquisition is \(EU_i = \frac{1}{2} (2N - 1)(v_H - v_L) - c\). Consequently, if \(c \leq \frac{1}{4} (2N - 1)(v_H - v_L)\) and \((B, S)\) gives rise to \(p = E[v]\), a \(L\)-trader acquires information and speculates. So there exists no pure strategy trading equilibrium without information acquisition.\(^{15}\) QED

**Proposition 3**

For any finite information cost \(c\), there exists an integer \(N^*\), such that if \(M, N \geq N^*\), then (i) an efficient equilibrium allocation fails to exist and (ii) the price is typically (i.e. for all \(c \neq \frac{1}{2} N\Delta\)) not fully revealing.

A notion of a competitive or close to competitive market is that there are many traders and a trader can trade as many units of the asset as he likes without having much price impact. Proposition 3 states that if information is endogenous and costly, there exists no efficient equilibrium allocation in such large double auction markets. More precisely, for any finite information cost, if both the number \(N\) of traders and the units \(M\) a trader is allowed to trade are sufficiently large, then there is no equilibrium in a double auction in which the total trading gains of \(N\Delta\) are realized.

\(^{15}\) Analogously for a \(H\)-trader, if \(c < \frac{1}{4} (2N - 1)(v_H - v_L - \Delta)\), then a \(H\)-trader speculates.
Corollary 1
Suppose $c < \frac{1}{4} (v_H - v_L)$. For any $N > 1$ and $M \geq 1$, there exists no pure strategy BNE with positive volume of trade.

In a large market there are additional incentives effects that are not present in the small double auction. In contrast, to Proposition 1(c) which shows that if the information cost is low, there exists a pure strategy BNE in which the trading gain $\Delta$ is realized. In a large market, even for low information costs, there exists no pure strategy trading equilibria. Corollary 1 states that $N=2$ is large enough. The reason is free-riding. In a bilateral double auction an uninformed trader “must” account for the lemons problem and this reduces the probability of trade. If he wants trade to occur with probability one, he “must” become informed if the other trader is informed. Proposition 1(c) shows that the price is fully revealing in any such trading equilibrium. But in a large market where there is a single price for all transactions and if the price is fully revealing, there is free riding by two types of traders.

Suppose the price is fully revealing. (i) An informed trader chooses not to acquire costly information since there are no speculative profits to make. (ii) Since there is no lemons problem, no uninformed trader submits defensive offers. Consequently, some of these traders deviate to noise traders. On the other hand, if there are a lot of noise traders and very few informed, then an informed trader can move prices and make speculative profits. Therefore, a fully revealing price is typically (i.e. for all $c \neq \frac{1}{4} N \Delta$) not consistent with equilibrium behavior.

Proposition 1(a) shows that if $c \geq \frac{1}{4} (v_H - v_L - \Delta)$, then an efficient equilibrium allocation exists. In a large market there are potentially more uninformed traders to exploit. Therefore, even if the information cost is large, some traders have an incentive to speculate so that uninformed traders are concerned about the endogenous lemons problem. This mere concerns suffices to “destroy” the efficient equilibrium allocation. For any finite information cost, if the number of traders submitting orders around the expected value of the asset is large, some traders have an incentive to speculate.

Technically speaking, any profile of pure strategies leads to a market clearing price that is fully revealing. But a fully revealing price gives rise to free riding. Proposition 3 is a strategic version of Grossman and Stiglitz (1980) impossibility result of informationally efficient (double auction) markets. Since the price is not fully revealing in equilibrium, there is a potential lemons problem, some traders behave defensively and the allocation is inefficient. The next result restates the non-existence of an efficient equilibrium in term of the “riskiness” of the asset.
Corollary 2
For any given set of parameter values \((c, M, N, \Delta)\), if the asset is sufficiently risky (i.e. \(v_H-v_L\) is sufficiently large), then no equilibrium exists in which the allocation is efficient.

6. Discussion

Continuous State Space
This paper assumes that the common value \(v\) is a binary random variable. This assumption is not crucial for the non-existence of a pure strategy equilibrium with trade and therefore the failure of an efficient allocation in a large market. Suppose no agent acquires information. Individual rationality and \(M>1\) imply that the equilibrium price is \(p=E[v]\), provided an efficient equilibrium exists. Suppose \(v\) is a continuous random variable. If an informed agent receives a perfect signal about \(v\), then all results hold. If the signal is noisy and given by \(\zeta = v + \varepsilon\) and \(E[\varepsilon] = 0\), then an informed agent still has better information than an agent who does not acquire information.16

Although the calculation of a specific equilibrium is intractable mainly because the order statistics of buyers and sellers are not affiliated, the economic arguments go as follows. Suppose the pure strategy bid and ask profiles of the traders lead to the market clearing price \(p=E[v]\), an informed \(L\)-trader does not sell but will buy \(M\) units of the assets when he obtains information that \(E[v|\zeta] > p\), and an informed \(H\)-trader will buy one unit when he sees \(E[v|\zeta] = p\), and \(M\) units if \(E[v|\zeta] > p\). For low information costs, if there are traders submitting prices around \(E[v]\), then information acquisition is a best response for some traders. In such a case, conditional on observing trade uninformed sellers know that they receive too little for selling the asset. This speculative threat and the resulting defensive trading behavior of some uninformed agents cause the allocative inefficiency.

Risk aversion
The following arguments show that risk neutrality is not a crucial assumption for the qualitative implications of the paper. Suppose there is a riskless asset \(S\), and a risky asset \(R\), and the agents have concave utility functions \(u\) and endowments \(\omega=(\omega_S,\omega_R)\) of the assets. In general, if \(\omega_{is}>\omega_{js}\) and \(\omega_{ir}<\omega_{jr}\), then agent \(i\) and \(j\) can realize gains from trade by reallocation of risks because agent \(i\) has a higher marginal valuation of the risky asset than agent \(j\). To

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16 Note, if the signal is noisy, this only changes the expected speculative profit and the critical value of the information cost for the different types of equilibria to arise, but not the qualitative implications.
simplify the analysis this paper assumes that agent $i$ has a valuation of $v+\Delta$ for the first unit of the risky asset that he buys.17

The strategic incentive of information acquisition under risk neutrality also arises under risk aversion because a risk averse agent also has to evaluate the gains from trade (or hedging of risks), the potential speculative loss of being uninformed, as well as the potential speculative gains from being informed. In particular, if the agents are risk averse, the information cost is low, and the signal perfectly reveals the true value of the asset, then is no trade at all because perfect information prevents hedging of risks. So Proposition 1(c) ceases to exists.

Many heterogenous agents
Suppose the traders have valuations $u_{LL}=v-\Delta$, $u_{L}=v$, and $u_{H}=v+\Delta$. If agent $LL$ and agent $H$ trade with each other, then the total trading gain is $2\Delta$ and none of the qualitative implications changes. For example, Proposition 1(b) would state that if $\frac{1}{2} \Delta < c < \frac{1}{4} (v_H-v_L)-\frac{1}{2} \Delta$, then no pure strategy BNE with trade between agent $LL$ and $H$ exists. A previous version of the paper analyzes this setting which is in some sense more symmetric in terms of preferences. A further variation is that a $H$-trader has the valuation of $v+\Delta$ for all units, and a $LL$-trader has a valuation of $v-\Delta$ for all units. Proposition 1(b) holds under the (same) condition $\frac{1}{2} M\Delta < c < M(\frac{1}{4} (v_H-v_L)-\frac{1}{2} \Delta)$. Similarly, if the information cost is not too high, no efficient equilibrium allocation exists in a large market and any of the three types of traders randomizes over information acquisition.

Costless Information
If $c=0$, then it is easy to see that there exists an efficient BNE with the following properties. All traders acquire information and all $L$-trader sells one unit and all $H$-traders buy one unit each. (i) For $M=1$, the equilibrium price is $p\in[v,v+\Delta]$. (ii) For $M>1$, the equilibrium price is $p=v$. There is a discontinuity at $c=0$ in terms of allocative efficiency.

Observability of information acquisitions
The next proposition shows that if information acquisitions are observable prior to the trading stage, then endogenous information acquisition has no adverse allocative consequences.

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17 For example, the liquidity traders in Mendelson and Tunca (2004) have a similar utility function.
Proposition 4

Suppose $N=1$, information acquisition is observable, and $k \in [0, \Delta]$. Any efficient $k$-sharing outcome is attainable as a perfect BNE irrespective of information cost.

The intuition is as follows. Since information acquisition is observable, a trader can also condition his offer strategy on the fact whether the other party is better informed or not. Suppose ex ante the traders “agree” to trade at $p=E[v]$, but the buyer acquires information. In the auction stage the seller knows that the buyer is informed. So the seller does not submit the price $s=E[v]$ anymore, but demands a high price. Since the buyer anticipates the lemons problem he himself creates by acquiring information, his best response is not to acquire information. No trader has an incentive to acquire more information than the counterpart.18 However, if information acquisition is not observable, the traders cannot target their offer strategies appropriately and are concerned about the endogenous lemons problem. Private information acquisition is unlikely to be publicly observable in a large market.

Other trading mechanisms

An interesting and challenging extension is to analyze an optimal multi-stage (direct) mechanism where a trader first announces his preference, then whether he has acquired information, and finally his information about the asset value. Or the traders may write (complex) state contingent contracts. However, the simple linear utility function of the traders should be regarded as a shortcut for the marginal utility of risk adverse traders possessing different endowments of the risky asset. If a trader has to make additional post transaction payments after the realization of the cash flow, then this type of arrangements undermines the idea of risk sharing in financial markets.

7. Conclusion

This paper analyses information acquisition in double auction markets populated by high and low valuation traders of an asset and shows that for any finite information cost, an efficient equilibrium allocation fails to exist if the number of traders and the units a trader is allowed to trade are sufficiently large or the asset is sufficiently risky. There is a large set of parameter values where any equilibrium with positive volume of trade is in mixed strategies and ex ante identically informed, rational traders evolve endogenously to noise traders, speculators, and

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18 It is easy to see that, e.g. the strategies $t_B^1=(0,b)$ with $b=E[v]$ if $n_B=0$, and $b=v_H$ if $n_B=1$; and $t_C^1=(0,s)$ with $s=E[v]$ if $n_B=0$, and $s=v_H$ if $n_B=1$ constitute a perfect BNE with $EU^B=\Delta$ and $EU^S=0$. 
defensive traders. Because of defensive trading the allocation is inefficient, i.e. not all gains from trade are realized. Because of endogenous noise trading the price is not fully revealing. The fraction of noise type traders is endogenous and depends on the information cost, the total gains from trade, and the riskiness of the asset.

This paper provides a strategic foundation for the Grossman and Stiglitz (1980) impossibility result of informationally efficient (double auction) markets as well as a strategic foundation for the common assumption of exogenous noise trading in the market microstructure literature. A question is whether the behavior one observes in large double auction markets might be interpreted as the realization of a mixed strategy equilibrium? Furthermore, this paper highlights a potential benefit of trading in non-anonymous over-the-counter markets. This paper shows that in decentralized trading, if the information cost is either low or high, then there exist equilibria where trade occurs with probability one. These trading equilibria cease to exist in a large market.

In contrast to stock trading in centralized markets, many other financial assets, such as mortgage-back securities, collateral debt obligations, credit default swaps, syndicated loans, corporate and municipal bonds are traded in a bilateral fashion. The emergence of so-called dark pools, a kind of upstairs markets organized by investment banks and where only a exclusive group of traders has access suggests that even for (liquidity motivated) block trading of stocks decentralized markets may dominate anonymous centralized markets.

A further evidence that suggests that both information aggregation and allocations in large centralized markets may not be efficient and thus trading may be subject to endogenous adverse selection problems is the existence of designated market makers. These agents are supposed to provide liquidity and explicitly forbidden to speculate. Christie and Schultz (1994) document that e.g. there were 59 and 46 market makers for the trading of such “liquid” stocks as Apple and Microsoft, respectively on the NASDAQ in April 1994.

Madhavan and Panchapagesan (2000) analyze the role of market makers for price discovery in the double auction type overnight market on the NYSE, and state that “there is strong evidence that the NYSE's designated dealer (specialist) sets a more efficient price than the price that would prevail in a pure call market using only public orders.” (p.656)19 Therefore, it is interesting to explore the implication for this model of the exogenous presence

19 Madhavan and Panchapagesan (2000, p.657) state “The process by which investors’ latent demands are translated into realized prices and volumes is a highly complex process that we are only now starting to understand. Our results add to a growing body of evidence that highlights the crucial roles of information and market structure in determining price efficiency, but there are still many important questions to be answered before we fully understand the inner workings of the black box of trading mechanisms.”
of a fourth category of players, designated market makers who attempt to earn profits from exclusively facilitating the encounter of buyers and sellers, as well as to identify the set of conditions under which such players would endogenously emerge in equilibrium. In other words, can market makers facilitate allocative and informational efficiency in double auction markets with endogenous information?

The present paper highlights some potential inefficiencies in a static large double auction market. An important function of a centralized market is the transmission of information through prices and sequential trading. This can mitigate the duplication of costly information acquisitions. Another type of costs of decentralized trading are search costs, i.e. the costs of finding a trader with the opposite trading need. (See Duffie et al. (2005).) A dark pool may represent one such solution. Another interesting extension is to allow the traders to endogenously choose whether to trade in a centralized or decentralized. Further research may provide additional insights on the working of different trading institutions as well as the competition between trading platforms when all traders are strategic and can acquire information before they trade.

Appendix

Proof of Proposition 1(c)

Statement: (i) If \( c = \frac{1}{2} \Delta \), then \( t_B = t_S = (1, v_L + \Delta, v_H) \) is the only BNE where trade occurs with probability 1 (called full trade BNE). (ii) If \( c < \frac{1}{2} \Delta \), there is a continuum of full trade BNE. (iii) In any full trade BNE both traders acquire information and the price is fully revealing.

Proof: Consider the strategy pair \( t_B = t_S = (1, v_L + \Delta - r, v_H + z) \) with \( r, z \in [0, \Delta] \). In this case

\[
EUB^B = \frac{1}{2} [v_L + \Delta - (v_L + \Delta - r)] + \frac{1}{2} [v_H + \Delta - (v_H + z)] - c = \frac{1}{2} \Delta + \frac{1}{2} (z - r) - c.
\]

If the buyer chooses \((0, b)\) with \( b = v_H + z \) then \( EU^B = \frac{1}{2} [v_L + \Delta - \frac{1}{2} (v_L + \Delta - r + v_H + z)] + \frac{1}{2} [v_H + \Delta - (v_H + z)] = \frac{1}{4} \Delta - \frac{1}{4} (v_H - v_L) - \frac{1}{4} (3z - r) \). For \( r = \Delta \) and \( z = 0 \), the buyers’ payoff is maximal and yet \( EU^B = -\frac{1}{4} (v_H - v_L) + \Delta < 0 \). If the buyer chooses \((0, b)\) with \( b = v_L + \Delta - r \) then \( EU^B = \frac{1}{2} r \). \(^{21}\)

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\(^{20}\) For example, if some traders have a continuous inflow of trading needs (such as the execution of incoming orders of costumers), they may be the ones who acquire information and determine prices. The small traders wait, observe the price and trade without costly information acquisition.

\(^{21}\) It is easy to see that the strategies \((0, b)\) where \( b < s_L, b \in (s_L, s_H) \) or \( b > s_H \) are weakly dominated.
If the seller chooses \((0, s)\) with \(s=v_L+\Delta-r\) then \(EU^\delta=\frac{1}{4}\Delta+\frac{1}{4}(z-r)-\frac{1}{4}(v_H-v_L)\). For \(r=0\) and \(z=\Delta\), the seller’s payoff is maximal and yet \(EU^\delta=\frac{1}{4}\Delta-\frac{1}{4}(v_H-v_L)<0\). If the seller chooses \((0, s)\) with \(s=v_H+z\) then \(EU^\delta=\frac{1}{2}z\).

(i) For \(c=\frac{1}{2}\Delta\), if \(r>0\), the buyer deviates to \((0, b)\) with \(b=v_L+\Delta-r\) and gets \(EU^\beta=\frac{1}{2}r\) which is larger than \(EU^\beta=0\) from choosing \((1, v_L+\Delta-r, v_H+z)\). If \(z>0\), the seller deviates to \((0, s)\) with \(s=v_H+z\) and gets \(EU^\delta=\frac{1}{4}z>0\). So \(t_B^*=t_S^*=(1, v_L+\Delta, v_H)\) is the only full trade BNE.

(ii) For \(c<\frac{1}{2}\Delta\), \(t_B^*-t_S^*=(1, v_L+\Delta-r, v_H+z)\) are best responses if the following holds: \(EU^\beta=\frac{1}{4}\Delta-\frac{1}{4}(z-r)-c>\frac{1}{2}r\) and \(EU^\delta=\frac{1}{4}\Delta+\frac{1}{4}(z-r)-c>\frac{1}{4}z\). Define \(\varepsilon=\frac{1}{2}\Delta-c\). Then for any \(r,z\in[0,\varepsilon]\), \((t_B^*, t_S^*)\) constitutes a BNE.

(iii) If \(c\leq\frac{1}{2}\Delta\), no full trade equilibrium exists in which (a) no trader acquires information or (ii) only one trader acquires information. The assumption \(\Delta<\frac{1}{4}(v_H-v_L)\) and \(c\leq\frac{1}{2}\Delta\) imply that \(c<\frac{1}{4}(v_H-v_L)-\Delta\). So no trading equilibrium without information acquisition exists. (b) Suppose that only the buyer acquires information and he chooses \((b_L, b_H)\) with \(b_L=v_L+\Delta-r\) and \(b_H=v_H+z\). If there is to be full trade the uninformed seller must choose \((0, s)\) with \(s=b_L\). For any \(r,z\in[0,\Delta]\), \(EU^\delta<0\). Analogously for \(n_B=0\) and \(n_S=1\). So no full trade occurs if only one trader acquires information. QED

**Proof of Proposition 2**

The set of mixed strategies may be very large. The proof contains two parts and proceeds as follows. Part A derives a mixed strategy BNE under the assumption that the traders can only choose three offer prices. Part B shows that the equilibrium properties in part A also hold without this restriction.

**Part A**

**Assumption A**

The traders can only choose offer prices from the set \(b, s\in\{l, m, h\}\) where \(l=v_L+\frac{1}{2}\Delta\), \(m=E[v]+\frac{1}{2}\Delta\), and \(h=v_H+\frac{1}{2}\Delta\). (These trading strategies lead to an equal-split of surplus.)

**Remark A1**

An informed buyer does not choose \(b>v_L+\Delta\) at \(v_L\), while an informed seller does not choose \(s<v_H\) at \(v_H\). An uninformed buyer does not bid more than his expected valuation, i.e.
Given Assumption A and Remark A1, one can focus on the following pure strategies that are not weakly dominated. For the buyer, these are (0, l), (0, m), (1, l, m), (1, l, h), and (1, l, l). For the seller, these are (0, m), (0, h), (1, l, h), (1, m, h), and (1, h, h). In a mixed strategies, the buyer and seller choose probability

\[ V_B^1 \text{ on } (0, l), \quad V_S^1 \text{ on } (0, h), \]
\[ V_B^2 \text{ on } (0, m), \quad V_S^2 \text{ on } (0, m), \]
\[ V_B^3 \text{ on } (1, l, m), \quad V_S^3 \text{ on } (1, m, h), \]
\[ V_B^4 \text{ on } (1, l, h), \quad V_S^4 \text{ on } (1, l, h), \]
\[ V_B^5 \text{ on } (1, l, l), \quad V_S^5 \text{ on } (1, h, h), \]

respectively. The expected payoffs of the buyer are given as follows.

\[ EU^{\beta}(0, l) = \frac{1}{2} \sigma_{S_4} + \frac{1}{2} \Delta \]
\[ EU^{\beta}(0, m) = \frac{1}{2} \sigma_{S_4} + \frac{1}{2} \Delta - \frac{1}{2} (v_H - v_L) + \frac{1}{2} \sigma_{S_2} \Delta + \frac{1}{2} \Delta (v_H - v_L) + \frac{1}{2} \Delta - \frac{1}{2} (v_H - v_L) + \frac{1}{2} \Delta \]
\[ EU^{\beta}(1, l, m) = \frac{1}{2} \sigma_{S_4} \Delta + \frac{1}{2} \sigma_{S_2} \Delta + \frac{1}{2} \Delta + \frac{1}{2} \sigma_{S_1} \Delta \]
\[ EU^{\beta}(1, l, h) = \frac{1}{2} \sigma_{S_4} \Delta + \frac{1}{2} \sigma_{S_2} \Delta + \frac{1}{2} \Delta + \frac{1}{2} \sigma_{S_1} \Delta + \frac{1}{2} \sigma_{S_3} \Delta + \frac{1}{2} \Delta - \frac{1}{2} (v_H - v_L) - c \]
\[ EU^{\beta}(1, l, l) = \frac{1}{2} \sigma_{S_4} \Delta - c \]

The expected payoffs of the seller are given as follows.

\[ EU^{\delta}(0, h) = \frac{1}{2} \sigma_{B_4} \Delta \]
\[ EU^{\delta}(0, m) = \frac{1}{2} \sigma_{B_4} \Delta - \frac{1}{2} (v_H - v_L) + \frac{1}{2} \sigma_{B_3} \Delta - \frac{1}{2} (v_H - v_L) + \frac{1}{2} \sigma_{B_2} \Delta \]
\[ EU^{\delta}(1, m, h) = \frac{1}{2} \sigma_{B_4} \Delta + \Delta + \frac{1}{2} \sigma_{B_2} \Delta + \frac{1}{2} (v_H - v_L) - c \]
\[ EU^{\delta}(1, l, h) = \frac{1}{2} \sigma_{B_4} \Delta + \frac{1}{2} \sigma_{B_3} \Delta + \frac{1}{2} \sigma_{B_2} \Delta + \frac{1}{2} (v_H - v_L) - c \]
\[ EU^{\delta}(1, h, h) = \frac{1}{2} \sigma_{B_4} \Delta - c \]

Since the pure strategy (1, l, l) is strictly dominated by the pure strategy (0, l), the buyer chooses \( \sigma_{B_5} = 0 \). Since (1, h, h) is strictly dominated by (0, h), the seller chooses \( \sigma_{S_5} = 0 \).

Step 2

(a) This step analyses the best responses of the buyer.

(i) The strategy (1, l, m) weakly dominates (1, l, h) if
\[
\begin{align*}
\Leftrightarrow \quad & \frac{1}{2} \sigma_{S2}(\frac{1}{2} \Delta + \frac{1}{2} (v_H - v_L)) - c \geq \frac{1}{2} (\sigma_{S3} + \sigma_{S4}) \frac{1}{2} \Delta + \frac{1}{2} \sigma_{S1} \frac{1}{2} \Delta + \frac{1}{2} \sigma_{S2}(\frac{1}{2} \Delta + \frac{1}{2} (v_H - v_L)) - c \\
\Leftrightarrow \quad & \frac{1}{2} \sigma_{S2}(v_H - v_L) \geq \frac{1}{4} (\sigma_{S1} + \sigma_{S3} + \sigma_{S4}) \Delta \\
\Leftrightarrow \quad & \sigma_{S2}(v_H - v_L) \geq 2(1 - \sigma_{S2}) \Delta \\
\Leftrightarrow \quad & \sigma_{S2} \geq \frac{2\Delta}{(2\Delta + v_H - v_L)} \equiv I_{(l,h)}^{(l,m)}
\end{align*}
\]

(ii) The strategy \((1,l,m)\) weakly dominates the strategy \((0,l)\) if
\[
\sigma_{S2} \geq \frac{4c}{(\Delta + v_H - v_L)} \equiv I_{l}^{(l,m)}
\]

(iii) The strategy \((1,l,m)\) weakly dominates the strategy \((0,m)\) if
\[
\sigma_{S2} \geq \frac{4c - \frac{1}{2} \sigma_{S4}(v_H - v_L)}{(v_H - v_L - \Delta)} - \sigma_{S3} \equiv I_{m}^{(l,m)}
\]

(iv) The strategy \((0,m)\) weakly dominates the strategy \((0,l)\) if
\[
\sigma_{S2} \geq \frac{(2\sigma_{S3} + \sigma_{S4})(v_H - v_L)}{4\Delta} - \frac{1}{2} \sigma_{S3} \equiv I_{l}^{m}
\]

(v) The strategy \((1,l,h)\) weakly dominates the strategy \((0,l)\) if
\[
\sigma_{S2} \geq \frac{2(4c - \Delta)}{v_H - v_L} \equiv I_{l}^{(l,h)}
\]

(vi) The strategy \((1,l,h)\) weakly dominates the strategy \((0,m)\) if
\[
\sigma_{S2} \geq \frac{8c + 2\Delta(-\sigma_{S1} + \sigma_{S3} - \sigma_{S4} - (\sigma_{S3} + \sigma_{S4})(v_H - v_L))}{(v_H - v_L - 2\Delta)} \equiv I_{l}^{(l,m)}
\]

(b) Analogously for the seller. E.g., \(\sigma_{S2} \geq I_{l}^{(l,m)}\), then \(EU^{S}(l,m,h) \geq EU^{S}(l,l,h)\).

Step 3
Claim: There exists no mixed strategy equilibrium in which the informed buyer and informed seller choose the “honest” strategy \((1,l,h)\), with positive probability.

Proof: For the buyer, \((1,l,h)\) and \((1,l,m)\) are the two potential strategies with information acquisition for being a candidate in a mixed strategy equilibrium. The buyer does not choose \((1,l,h)\) with positive probability if it is strictly dominated by \((1,l,m)\). Suppose that the strategy \((1,l,h)\) weakly dominates \((1,l,m)\), i.e. \(\sigma_{S2} \leq I_{l}^{(l,m)}\). It is easy to see that \(I_{l}^{(l,m)} < I_{l}^{(l,h)}\). Consequently, if the strategy \((1,l,h)\) weakly dominates \((1,l,m)\), then \((1,l,h)\) is strictly
dominated by the strategy \((0,l)\) because in this case \(\sigma_{S2} < I_{i}^{(l,h)}\). Therefore, if the seller randomizes such that the buyer is indifferent between \((1,l,h)\) and \((1,l,m)\); or \((1,l,h)\) dominates \((1,l,m)\), then the buyer chooses \(\sigma_{B1}=1\), i.e. he does not acquire information. Analogously for the seller, if the strategy \((1,l,h)\) weakly dominates \((1,m,h)\), then \((1,l,h)\) is strictly dominated by the strategy \((0,h)\).

Consequently, in a mixed strategy equilibrium (where information must be acquired with positive probability), the strategy \((1,l,h)\) must be a strictly dominated strategy and one must have \(\sigma_{B4} = \sigma_{S4} = 0\).  

Step 4

Claim: In a mixed strategy equilibrium the traders get zero expected payoff.

Proof: For \(\sigma_{B4} = \sigma_{S4} = 0\), \(EU^{B}(0,l) = EU^{S}(0,h) = 0\) since the buyer does not bid more and the seller does not demand less than the price \(m = E[v] + \frac{1}{2} \Delta\). In other words, if the buyer is indifferent between \((1,l,m)\) and \((0,l)\) or indifferent between \((0,m)\) and \((0,l)\), then his expected payoff is zero. In order to find a mixed strategy equilibrium in which the traders get positive expected payoffs, the following is required: For the buyer, he should be indifferent between \((1,l,m)\) and \((0,m)\); and \((0,m)\) should strictly dominate \((0,l)\), i.e. the buyer chooses \(\sigma_{B1} = 0\). The buyer is indifferent between \((1,l,m)\) and \((0,m)\) if \(\sigma_{S2} = I_{m}^{(l,m)}\). For \(\sigma_{S4} = 0\),

\[
\sigma_{S2} = \frac{4c - \frac{1}{2} \sigma_{S4}(v_{H} - v_{L})}{(v_{H} - v_{L} - \Delta)} - \sigma_{S3}
\]

\(\Rightarrow\) \(\sigma_{S2} = \frac{4c}{(v_{H} - v_{L} - \Delta)} - \sigma_{S3}\).

Since \(c < \frac{1}{4} (v_{H} - v_{L} - \Delta)\), this implies that \(\frac{c}{v_{H} - v_{L} - \Delta} < 1\). Therefore \(\sigma_{S2} + \sigma_{S3} < 1\), which means that there is some probability “left”, i.e. \(\sigma_{B1}\) must be larger than zero. In order to make the buyer indifferent between \((1,l,m)\) and \((0,m)\), the seller must choose \((0,h)\) with positive probability.

---

22 If the buyer is indifferent between \((1,l,m)\) and \((1,l,h)\), then \((0,l)\) strictly dominates both \((1,l,m)\) and \((1,l,h)\), since \(\sigma_{S2} = I_{l}^{(l,m)}\) and \(c > 0.5 \Delta\) imply \(\sigma_{S2} < I_{l}^{(l,m)}\).

23 Suppose the buyer only randomizes over \((0,l)\) and \((0,m)\). If the seller also does not acquire information, then he chooses \(s = m\) with probability 1. Given the seller’s response, the buyer’s best response is \((1,l,m)\). So there exists no mixed strategy equilibrium in which information is acquired with zero probability.

24 For the seller, he should be indifferent between \((1,m,h)\) and \((0,m)\); and \((0,m)\) should strictly dominate \((0,h)\), i.e. he chooses \(\sigma_{S1} = 0\).
On the other hand, if the seller chooses $(0, h)$ with positive probability he must be indifferent between $(0, m)$ and $(0, h)$. Since $EU^b(0, h) = 0$, $EU^s(0, m)$ must be zero, too. Otherwise, the seller is not indifferent. Consequently, the expected payoff of the seller must be zero in a mixed strategy equilibrium.\(^{25}\)

**Step 5**

**Claim:** In the only (non-degenerated) mixed strategy equilibrium the buyer randomizes over $(0, l)$, $(0, m)$ and $(1, l, m)$ according to $\sigma_B$ and the seller randomizes over $(0, h)$, $(0, m)$ and $(1, m, h)$ according to $\sigma_S$ where

\[
\sigma_B = \sigma_S = \left(1 - \frac{4c}{v_H - v_L - \Delta}, \frac{4c}{v_H - v_L + \Delta}, \frac{8c\Delta}{(v_H - v_L)^2 - \Delta^2}\right).
\]

**Proof:** In order to make the buyer indifferent between $(1, l, m)$ and $(0, l)$, the seller chooses $\sigma_{S2}$ such that $\sigma_{S2} = I_{i}^{(l, m)}$; and to make the buyer indifferent between $(1, l, m)$ and $(0, m)$, the seller chooses $\sigma_{S2}$ and $\sigma_{S3}$ such that $\sigma_{S3} = I_{i}^{(l, m)}$. So $I_{i}^{(l, m)} = I_{i}^{(l, m)}$ implies

\[
\frac{4c}{v_H - v_L + \Delta} = \frac{4c}{v_H - v_L - \Delta} - \sigma_{S3}
\]

\[
\Leftrightarrow \sigma_{S3} = \frac{8c\Delta}{(v_H - v_L + \Delta)(v_H - v_L - \Delta)} = \frac{8c\Delta}{(v_H - v_L)^2 - \Delta^2}.
\]

In addition, the seller chooses

\[
\sigma_{S1} = 1 - \sigma_{S2} - \sigma_{S3} = 1 - \frac{4c}{v_H - v_L - \Delta}.
\]

**Step 6**

**Claim:** The outcome in a mixed strategy BNE has the following properties. (i) The probability of trade is $\frac{16c^2}{(v_H - v_L)^2 - 4\Delta^2}$. (ii) The only trading price is $p = E[v] + \frac{1}{2} \Delta$ and not fully revealing.

**Proof:** The buyer randomizes over $(0, l)$, $(0, m)$ and $(1, l, m)$. The seller randomizes over $(0, h)$, $(0, m)$ and $(1, m, h)$.

(i) Trade occurs in the following events: (a) both the buyer and the seller choose $(0, m)$; (b) the buyer chooses $(0, m)$ and the seller chooses $(1, m, h)$ and the true state is $v_L$; and (c) the

---

\(^{25}\) Analogously for the buyer, he must choose $(0, l)$ with positive probability in order to make the seller indifferent between $(1, m, h)$ and $(0, m)$, i.e. his expected payoff is zero in a mixed strategy equilibrium.

\(^{26}\) Alternatively, the buyer should be indifferent between $(0, m)$ and $(0, l)$ and this yields the same condition.
buyer chooses \((1,l,m)\), the seller chooses \((0,m)\) and the true state is \(v_H\). The probability of trade is given as follows:

\[
\text{prob}(\text{trade}) = \sigma_{B_2}\sigma_{S_2} + \frac{1}{2}\sigma_{B_2}\sigma_{S_3} + \frac{1}{2}\sigma_{B_3}\sigma_{S_2} = \sigma_{B_2}\sigma_{S_2} + \sigma_{B_2}\sigma_{S_3}
\]

\[
\Leftrightarrow \quad \text{prob}(\text{trade}) = \frac{16c^2}{(v_H - v_L + 2\Delta)^2} + \frac{64c^2\Delta}{(v_H - v_L + 2\Delta)^2(v_H - v_L - 2\Delta)}
\]

\[
\Leftrightarrow \quad \text{prob}(\text{trade}) = \frac{16c^2}{(v_H - v_L)^2 - 4\Delta^2}.
\]

(ii) The buyer bids at most \(E[v] + \frac{1}{2}\Delta\) and the seller demands at least \(E[v] + \frac{1}{2}\Delta\). Therefore, no trade occurs at the prices \(v_L\) and \(v_H\). Trade only occurs if at least one trader is uninformed. If the uninformed trader observes trade, he cannot distinguish whether he makes a fair deal and realizes \(\frac{1}{2}\Delta\), or suffers a speculative loss. Although the uninformed trader updates his belief, he does not know the true state when observing \(p = E[v] + \frac{1}{2}\Delta\).

**Remark A2**

\(\sigma_{l_3}\) is the equilibrium probability of information acquisition. It decreases in \(v_H - v_L\) and increases in the information cost \(c\) which seems unintuitive. As the potential speculative profit increases and the cost of becoming informed decreases, one might expect that a trader has a higher incentive to acquire information. But in order to make the other trader indifferent between his pure strategies, equilibrium randomization requires it.

**Remark A3**

If \(c = \frac{1}{4}(v_H - v_L - \Delta)\), then in the non-degenerated mixed strategy BNE the buyer randomizes over \((0,m)\) and \((1,l,m)\) according to \(\sigma_B\) and the seller randomizes over \((0,m)\) and \((1,m,h)\) according to \(\sigma_S\) where

\[
\sigma_B = \sigma_S = \left( \frac{v_H - v_L - \Delta}{v_H - v_L + \Delta}, \frac{2\Delta}{v_H - v_L + \Delta} \right),
\]

and trade occurs with probability \(\sigma_{l_1}\), and both traders have zero expected payoff.\(^{27}\)

**Remark A4**

(a) For \(c = \frac{1}{4}(v_H - v_L - \Delta)\), as \(v_H - v_L \to \infty\), then \(c \to \infty\) and the probability that both traders choose \((0, E[v])\) converges to one and yet \(EU^B = EU^S = 0\) (prior to the randomization outcome).

\(^{27}\) In the unique pure strategy trading BNE both traders choose \((0,m)\) and \(EU^B = EU^S = 0.5\Delta\).
(b) For $c < \frac{1}{4}(v_H - v_L - \Delta)$, as $(v_H - v_L) \to \infty$, then the probability that the buyer chooses $(0, v_L)$ and the seller chooses $(0, v_H)$ converges to one.

**Part B**

Now it is assumed that the traders can choose any real number as an offer price.

**Remark B1**

From part A, a set of mixed strategy BNE from a three points distribution is the following. The buyer randomizes over $(0, l)$, $(0, m)$ and $(1, l, m)$ where $b \leq v_L + \frac{1}{2} \Delta$. The seller randomizes over $(0, h)$, $(0, m)$ and $(1, m, h)$ where $h \geq v_H + \frac{1}{2} \Delta$.

The considerations in Remark A1 imply that in search for a candidate for an equilibrium randomization with potentially positive payoffs, it suffices to focus on bid and ask prices within the following three subintervals: $I_L = [v_L, v_L + \Delta]$, $I_M = [E[v], E[v] + \Delta]$, and $I_H = [v_H, v_H + \Delta]$. See Figure 1. In particular, one can focus on the sets $\{t_B\}$ and $\{t_S\}$ of pure strategies with the following properties. For the buyer and seller these are:

(i) $T^B_L = \{(0, b): b \in I_L\}$, \hspace{1cm} $T^S_H = \{(0, s): s \in I_H\}$,

(ii) $T^B_M = \{(0, b): b \in I_M\}$, \hspace{1cm} $T^S_M = \{(0, s): s \in I_M\}$,

(iii) $T^B_{LM} = \{(1, b_L, b_H): b_L \in I_L, b_H \in I_M\}$, \hspace{1cm} $T^S_{MH} = \{(1, s_L, s_H): s_L \in I_M, s_H \in I_H\}$,

(iv) $T^B_{LH} = \{(1, b_L, b_H): b_L \in I_L, b_H \in I_H\}$, \hspace{1cm} $T^S_{LH} = \{(1, s_L, s_H): s_L \in I_L, s_H \in I_H\}$,

(v) $T^B_{HH} = \{(1, b_L, b_H): b_L, b_H \in I_L\}$, \hspace{1cm} $T^S_{HH} = \{(1, s_L, s_H): s_L, s_H \in I_H\}$.

**Figure 1**

```
I_L       I_M       I_H

v_L  v_L+\Delta  E[v]  E[v]+\Delta  v_H  v_H+\Delta
```

Denote $f$ (and $g$) as the density over the set $\{t_B\}$ and $\{t_S\}$ of pure strategies.

**Claim 1:** The equilibrium payoff is zero.

**Proof:** Observation
\[
\int_{t_B \in T_B^S} f(t_B) dt_B = \text{prob}(t_B \in T_B^S) = \text{prob}((0,b) \text{ with } v_L - A \leq b \leq v_L + A)
\]
which is “equivalent” to \(\sigma_B1\). Analogously for the other cases.

Suppose the seller actually chooses (0, s) with \(s=v_H\) (a \(s \in I_H\)) as the randomization outcome, then trade only occurs if the buyer actually chooses a strategy \(t_B \in T_{LH}^B\) with \(b_H \geq s\) and the true state is \(v=v_H\). The seller’s expected payoff is

\[
EU^S(0,s) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot \int_{t_B \in T_B^S \cap \{b_H \geq s\}} f(t_B) \cdot \left(\frac{1}{2}(b_H - s) + \Delta\right) dt_B
\]

which is the analogous expression to \(EU^S(0,v_H) = \frac{1}{2} \sigma_B + \frac{1}{2} \Delta\).

From Step 3 in Part A, a similar argument shows that in no mixed strategy equilibrium does the buyer choose a positive density over \(T_{LH}^B = \{(1,b_L,b_H): b_L \in I_L, b_H \in I_H\}\). If he is indifferent between \((1,b_L',b_H')\) and \((1,b_L'',b_H'')\) where \(b_L',b_L'' \in I_L, b_H' \in I_M\) and \(b_H'' \in I_H\), then these strategies are strictly dominated by \((0,b)\) with \(b \in I_L\). Therefore \(EU^S(0,s) = 0\) when \(s \in I_H\).

From Step 4, in order to make the buyer indifferent between choosing strategies without information acquisition (e.g. \(T_L^B = \{(0,b): b \in I_L\}\)) and strategies with information acquisition (\(T_{LH}^B = \{(1,b_L,b_H): b_L \in I_L, b_H \in I_H\}\)), the seller must choose a positive density on the strategies from the set \(T_H^S = \{(0,s): s \in I_H\}\). Consequently, the equilibrium payoff of the seller is zero.

**Claim 2:** If trade occurs, then the price is \(p \in (E[v], E[v]+\Delta]\) and not fully revealing.

**Proof:** The previous analysis shows that the buyer chooses a density \(f\) and the seller a density \(g\) that only have positive mass on the following pure strategies:

For the buyer, \(t_B \in \{(0,b): b \in I_L, I_M\} \cap \{(1,b_L,b_H): b_L \in I_L\text{ and } b_H \in I_M\}\).

For the seller \(t_S \in \{(0,s): s \in I_M, I_H\} \cap \{(1,s_L,s_H): s_L \in I_M\text{ and } s_H \in I_H\}\).

Therefore, trade only occurs if the buyer actually chooses \(b \in I_M\) and the seller actually chooses \(s \in I_M\) and \(b \geq s\). In any case \(p \in I_M\). QED

**Proof Proposition 3**

An equivalent statement of Proposition 3 is the following. Suppose \(M \geq N > 1\) and \(c < \frac{1}{2} (2N-1)(v_H - v_L)\). (i) If \(c < \frac{1}{2} NA\), there exists no pure strategy BNE with positive volume of trade. In any mixed strategy BNE the allocation is inefficient and the price is not fully
revealing. (ii) If \( c=\frac{1}{4}N\Delta \), pure strategy equilibria with positive volume of trade exist. In any such equilibrium the price is fully revealing but the allocation is inefficient.

**Remark**
From Lemma, since \( c<\frac{1}{4} (2N-1)(v_H-v_L) \), if there is positive volume of trade in equilibrium, some traders acquire information.

**Case (i):** \( c<\frac{1}{4} \Delta \): For the moment suppose that \( M=1 \). A pair of informed \( L \)- and \( H \)-trader can jointly cover its information cost by trading one unit at \( p=v+\frac{1}{4} \Delta \). Suppose the price is fully revealing and all agents trade one unit in both states. Otherwise some traders have a profitable deviation. To save on notation, if a trader does not want to buy (sell), then \((b_i,u_i)\), \((s_i,u_i)\) are omitted.

Define \( k=\frac{1}{4} \Delta \). Consider the following profile of pure strategies where \( z \) \( H \)-traders and \( r \) \( L \)-traders acquire information and all agents trade: \( n_l=1, (b_i,u_i^b)=((v_L+k, v_H+k),(1,1)); n_f=0, (b_i,u_i^f)=(v_H+k,1), \) where \( i=1,\ldots,z \) and \( l=z+1,\ldots,N \), and \( n_l=1, (s_i,u_i^s)=((v_L+k, v_H+k),(1,1)); n_n=0, (s_N,u_n^s)=(v_L,1) \) where \( j=N+1,\ldots,r \) and \( n=N+1+r+1,\ldots,2N \). In other words, uninformed \( L \)-traders submit low ask price to sell and uninformed \( H \)-traders submit high bid price to buy (to “insure” they can buy and sell one unit).

(i) Suppose \( z=r=1 \). Trader 1 and trader \( N+1 \) are informed. The market clearing price is \( p=v_L+k \) in state \( v_L \), and \( p=v_H+k \) in state \( v_H \) and each agent trades one unit. The following arguments show that these strategies do not constitute a BNE. One profitable deviation of the informed \( L \)-trader is to choose the following actions. In state \( v_H \), he places a bid price of \( v_H+k+\epsilon \) where \( \epsilon>0 \) to buy \((N-1)\) units. Now there are \((N-1)\) sell orders at the ask price \( v_L+k \); and \( N \) buy orders at the bid price \( v_H+k \) as well as \((N-1)\) units of the informed \( L \)-trader’s buy order at the bid price \( v_H+k+\epsilon \). Given this aggregate demand and supply schedule, the market clearing price is \( p=\frac{1}{2} (v_L+k+v_H+k) = E[v]+k \) because the lowest bid price is \( b'=v_H+k \) and the highest ask price is \( s'=v_L+k \) with \( b'\geq s' \). Since the \( L \)-traders submits the highest bid price he buys \((N-1)\) units. This response yields a strictly higher payoff to the informed \( L \)-trader.\(^{28}\)

\(^{28}\) Note, given any pure strategy profile leading to an allocation where all traders trade, the informed \( L \)-trader submits a bid price larger than the largest bid price of the other potential buyers. In the low state, the informed \( L \)-trader chooses to sell \((N-1)\) units at the ask price \( v_L-\epsilon \) and the resulting market clearing price is \( E[v]+k \).
(ii) So for any given number \((z,r)\)-informed traders, if an informed trader can move the price (with positive probability), then the informed trader speculates. This cannot be an equilibrium outcome.

(iii) Suppose that the number \((z,r)\)-informed traders is such that an informed trader cannot influence the price. If the price is fully revealing, then e.g. an informed \(H\)-trader deviates to \(n=0, b, u^b=1\) where \(b\) is larger than the highest bid price. He realizes his trading needs without paying the information cost.

This type of arguments hold for all \(k\in[0,\Delta]\). Consequently, there exists no pure strategy equilibrium with positive volume of trade. Only mixed strategy trading equilibria exist and the price is \(p\in[E[v],E[v]+\Delta]\) and not fully revealing.

Now suppose that \(M\geq N>1\). Then the only candidate (fully revealing) price in a pure strategy equilibrium is \(p=v\). If \(p>v\), then an informed \(L\)-trader has a profitable deviation by overbidding the other traders and buy \(N\) units instead of one unit. Therefore, some uninformed traders do not trade. This means they have not play a best response. But at the price \(p=v\), the informed \(L\)-traders cannot cover the information cost and the best response is not to acquire information. If only \(H\)-traders are informed, then they can move prices and speculate. Therefore, there is no pure strategy equilibrium with trade.

Case (ii): \(\frac{1}{2}\Delta < c < \frac{1}{2} NA\). The argument is similar. For \(M=1\), no pair of informed \(H\)- and \(L\)-trader can jointly cover their information cost by trading one unit. For an informed \(H\)-trader to cover his cost, the price must be \(p<v\). In this case the \(L\)-trader does not trade. For an informed \(L\)-trader to cover his cost, the price must be \(p>v\). In this case the informed \(H\)-trader has negative payoff. If only one side of the market acquires information, the other side of the market behave defensively. For \(M>1\), see Case (i).

Case (iii): \(c > \frac{1}{2} NA\). No pair of informed traders can cover his information cost at all. If only one side of the market acquires information, the other side of the market behave defensively.

Case (iv): \(c = \frac{1}{2} NA\). The following is a pure strategy BNE with positive volume of trade. One \(L\)-trader acquires information and all other traders do not. All uninformed \(L\)-traders demands \(s=v_H\) for selling one unit, all \(H\)-traders bid \(b=v_L+\Delta\) for buying one unit. The informed \(L\)-trader chooses to sell \(N\) units at the ask price \(s=v_L+\Delta\) in state \(v_L\) and \(s=v_H+\Delta\) in state \(v_H\). Trade occurs in state \(v_L\). All traders have zero expected payoff. **QED**
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