

Opacity and the Optimality of Debt for Liquidity Provision⁺

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Abstract

Asymmetric information can reduce trade (possibly to zero) due to adverse selection. Symmetric information can facilitate trade. We show that trade between agents is maximized, and welfare highest, when neither party to a transaction has any information about the future payoffs of the securities used to trade. Trade is best implemented by debt which preserves ignorance because it provides the smallest incentive for private information production, which creates adverse selection. Debt's value is also least sensitive to public signals. In this economy policies that increase transparency would reduce welfare. Finally, even if there is adverse selection in the market, debt maximizes the amount of trade. For the economy as a whole there is a systemic risk of using debt to provide liquidity: an aggregate shock, if bad enough, can be made worse by triggering private information production, causing adverse selection when debt becomes information-sensitive.

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1. Introduction

“Liquidity” refers to the ability to trade a given amount quickly without the transaction moving prices, and without an uninformed party losing money to a privately informed party. Akerlof (1970) shows that private information complicates the process of trading, reducing liquidity. Such asymmetric information means adverse selection, which reduces trade, possibly such that the market disappears. Symmetric information facilitates trade. One form of symmetric information is symmetric ignorance. In a trading context, we show that welfare is highest under symmetric ignorance. Debt optimally facilitates trade because debt provides the smallest incentive for private information production, which creates adverse selection. Even if there is adverse selection in the market, debt maximizes the amount of trade at the lowest cost to the uninformed. Finally, debt’s value is least sensitive to public signals. But, for the economy as a whole there is a systemic risk: an aggregate shock, if bad enough, can be made worse by triggering private information production, causing adverse selection when debt becomes information-sensitive.

Trade is needed when some agents’ desired dates for consumption does not match the dates when their goods are available, causing some agents to want to purchase goods from other agents by issuing securities backed by future project payoffs. To be more specific, consider an exchange economy with two dates and two agents. Agent A prefers to consume at the first date, but he has no endowment at that date. Instead, he owns a technology that pays-off a random amount at date 2. Agent B has a fixed amount of a perishable endowment at date 1 and is indifferent about when to consume. Suppose the agents learn about the realization of the date 2 payoff uncertainty at date 1. Bad news about the payoff of the project reduces what agent A can buy from agent B, but the effect of good news is bounded by the amount of goods that agent B can sell. From an ex ante point of view, convex combinations of the state contingent allocations are dominated by what can be achieved under ignorance (the expected value of the payoff can be traded). So, the agents are better off if they know nothing. Thus, our initial welfare result is that symmetric ignorance dominates symmetrically informed agents.

This first result, that ignorance is optimal, is reminiscent of Hirschleifer (1971) who shows that information can destroy the ability to co-insure when agents are risk averse. The setting here is completely different however; agents are risk neutral, but they need to trade. The problem with information is that the effect is asymmetric. Here information reduces the amount that can be traded when it is bad news, but has no compensating outcome when the news is good news.

To implement this “ignorance is bliss” allocation, agents must design a security to facilitate trade. In this exchange economy there are three reasons why society wants to avoid information production: (i) Information reduces welfare; (ii) Information is costly and has no social value; (iii) Private information production may create adverse selection. Yet an individual agent may have a strategic incentive to acquire information so as to exploit other agents. We define a “least information-sensitive security,” to be one which minimizes the incentive to produce private information. We impose no restrictions on the set of securities the agents can design, except a resource constraint that the issuer cannot repay more than the outcome of the project. We show that debt is an optimal security for the provision of liquidity because it minimizes the incentive for private information production. Intuitively, if there is bad news that may be learned privately and the payoff of the project is below the face value of the debt, then the buyer of that security receives the maximal feasible amount because debt is senior. Thus the value of information for the buyer is minimized. And the fixed face value of debt bounds the value of good news. While reminiscent of the result of Gorton and Pennacchi (1990) (also see Holmström (2008)), here debt is shown to be the optimal trading security from first principles.

But, even if there is adverse selection, i.e., one party to the transaction is privately informed, trade is still maximized with debt. Suppose, for example, that agent A, the owner of the project, is informed about the payoff of the project and agent B is uninformed. If agent B would like to maximize the chance of transacting a given amount, then it is best for agent B to offer to trade with a debt contract. With a debt contract with a face value of d and a price of d , agent A will sell that claim for sure. Agent B overpays in bad states. This adverse selection will occur with any security issued. But, with debt the expected overpayment is minimized because for debt the payout is the entire project value only for realizations below d .

These arguments show that there are two reasons why debt is the optimal security for liquidity provision in a two date economy. In markets where agents want to prevent asymmetric information from arising, debt is the optimal trading security because it is least information-sensitive. In other words, if debt trading triggers private information acquisition, then so does any other security. Secondly, even if there is adverse selection, debt is also optimal because it maximizes the amount of trade and minimizes the loss of uninformed agents.

“Trading” involves repeated transactions, so an analogous question is whether debt is also optimal in a three date economy of the following type: Agent A issues a security (a claim on the project payoff at the final date) to agent B at the first date, the primary market. Agent B then trades with agent C at the second date, the secondary market, and finally agent C (and possibly agent B) redeems the security with agent A at the final date. In this context we first consider the effect of an interim public signal about the distribution of the project. The information arrives just prior to the date 2 trading, and there is no possibility of private information production. We show that debt issued at the first date is still a contract that maximizes total welfare because its value does not change a lot (it is least information-sensitive) when public news arrives, so it can implement a larger trade when the public signal reveals bad news. In other words, debt enables more intertemporal transfers (it has a higher “carrying capacity”) than other securities.

The interim public signal changes the market price of the security as well as its information-sensitivity. If agents can acquire information and the information-sensitivity increases upon the realization of the interim public signal, then agents might redesign the debt to maintain its information-insensitivity for subsequent trade. This might involve issuing a new bond, taking the original bond as collateral, which “writes-down” i.e., has a lower face value than the original bond, with the holder of the underlying bond collateral retaining an equity part (“tranche”) and selling a senior claim on the collateral (i.e., securitization). A less drastic strategy that might work would be to sell a *pari passu* fraction of the bond.

We analyze these issues in Section 2 (welfare and information), Section 3 (the two date model), and Section 4 (the three date model), where we look at individual optimization problems to determine efficient allocations and optimal security design. In Section 5 we analyze a strategic security issuance, information acquisition, and trading game and show that in equilibrium debt is issued in the primary market. If the interim news is bad and causes the information-sensitivity of debt to rise, then in equilibrium agents either trade a fraction of the original bond to avoid triggering information production or there is information production and adverse selection. Both cases correspond to “systemic risk” because the outcome is worse than that caused only by the fundamentals. The “fundamentals” corresponds to the bad news. Instead of trading at the new expected value of the debt, agents trade much less than they could or even not at all. In this sense there is a collapse of trade.¹

¹ Arguably, this is the type of collapse which occurred in U.S. financial markets starting in August 2007; information-insensitive debt used as collateral in the sale and repurchase –“repo”–market became information-sensitive when house prices did not rise. See Gorton (2009) and Gorton and Metrick (2009).

Debt is optimal for the economy, which needs a certain amount of leverage to implement efficient trade. But, a systemic event can occur because the debt is not riskless (as in Gorton and Pennacchi (1990)). The systemic event corresponds to the information-insensitive debt becoming information-sensitive, giving rise to concerns of adverse selection, and reducing the amount of trade below what could be implemented if the agents just traded at the lower expected value of the debt. This is a different crisis mechanism than that of, for example, Kiyotaki and Moore (1997) where the collateral value is subject to a feedback effect from the initial shock causing its value to decline further. The cause of the systemic event here is also distinct from coordination failure models of bank runs based on self-fulfilling expectations, as in Diamond and Dybvig (1983).

The idea that ignorance dominates transparency in liquidity provision has many implications. The economy as a whole must create the amount of debt needed to implement the optimal amount of trade. In order to create debt is also to create the complement of debt, the equity residual. This claim has the opposite property of debt, namely it is very information-sensitive. Indeed, we show it has the maximal information-sensitivity. Someone must hold this “toxic” claim. But, trading the equity residual may trigger information acquisition which would affect the provision of liquidity. There is no traded equity residual in securitization, however, which explains why securitized bonds are the basis for the repo market. Also, our model has a role for rating agencies, which can facilitate liquidity provision by announcing information partitions (ratings) at the interim date which provide just enough information to prevent agent B from having an incentive to produce private information. We explain why rating agencies announce coarse partitions. These, and other issues, are discussed in Section 6.

Section 7 concludes.

The two issues that we focus on, liquidity and the optimality of debt, have not been previously formally linked. Diamond and Dybvig (1983) and Gorton and Pennacchi (1990) study liquidity provision but assume the existence of debt, while Townsend (1979) and Hart and Moore (1995, 1998) study the optimality of debt, but do not study liquidity. Diamond and Dybvig (1983) associate “liquidity” with intertemporal consumption smoothing and argue that a banking system with demand deposits provides this type of liquidity. But, as Jacklin (1987) argued, there is no explanation for the optimality of demand deposits in their setting, and demand deposits only arise because other markets and securities are arbitrarily ruled out. Gorton and Pennacchi argue that debt is an optimal trading security because it minimizes trading losses to informed traders when used by uninformed traders. Hence debt provides liquidity in that sense. Gorton and Pennacchi, however, focus on explaining the existence of banks, institutions that attract informed traders as equity holders, so that the uninformed traders can use the banks’ demand deposits to trade (minimizing their losses). In Gorton and Pennacchi the debt is riskless, and it is not formally shown that debt is an optimal contract.²

There are also a few papers that raise the issue of whether more information is better in the context of trading or banking. These include, for example, Andolfatto (2009), Kaplan (2006), and Pagano and Volpin (2009). Andolfatto (2009) is most closely related. He also considers an economy where agents

² Demarzo and Duffie (1999) is also related; they study the optimal security to be issued by a privately-informed issuer, and provide conditions under which that security is debt. Also, there is a large literature on debt in firms’ capital structures has developed based on agency issues in corporate finance. Townsend (1979) and Hart and Moore (1995, 1998) argue that debt is optimal for monitoring or controlling borrowing firms whose returns are not observable or verifiable, respectively. Also, see, e.g., Myers and Majluf (1984), Aghion and Bolton (1992), and Bolton and Scharfstein (1990). The settings studied by these authors are not that of trading or liquidity provision, that is, the focus is the primary market not the secondary market. Also, in most cases, these papers assume ex ante asymmetric information while we ask how security design can prevent asymmetric information from arising in the first place.

need to trade, and shows that when there is news about the value of the “money” used to trade, some agents cannot achieve their desired consumption levels. Agents would prefer that the news be suppressed. Kaplan (2006) studies a Diamond and Dybvig-type model and in which the bank acquires information before depositors do. He derives conditions under which the optimal deposit contract is non-contingent. Pagano and Volpin (2009) study the incentives a security issuer has to release information about a security, which may enhance primary market issuance profits, but harm secondary market trading. These authors assume debt contracts.

2. Welfare, Trade and Information

In the baseline setting, we consider an exchange economy with two risk neutral agents, A and B, with utility functions:

$$U_A = \alpha C_{A1} + C_{A2}$$

$$U_B = C_{B1} + C_{B2},$$

where $\alpha > 1$ is a constant and where C_{ht} denotes consumption of agent h at date t . The endowment of agent h at time t is described by the vector $\omega_h = (\omega_{h1}, \omega_{h2})$ and is given as follows: $\omega_A = (0, X)$, $\omega_B = (w, 0)$ where w is a constant and X is a random variable. The final realization of X is verifiable. Endowments are nonstorable. So, agent A has no endowment of goods at $t=1$ but receives x units of goods at $t=2$, where x is a verifiable realization of the random variable X (a “Lucas tree”). Agent B possesses w units of goods at $t=1$. The agents start with identical information about the random variable X . Assume that X is a continuous random variable with positive support on $[x_L, x_H]$ and density $f(x)$. The information about the endowments and the Lucas tree is common knowledge.

Since agents have different marginal valuations of consumption at different dates, gains from trade can be realized by a reallocation of goods. An allocation of goods can be described by a 2×2 consumption matrix denoted by $\{c\}$, where a row specifies the consumption of agent h at date t , i.e., $c_h = (c_{h1}, c_{h2})$ for $h=A, B$. An allocation $\{c^*\}$ is efficient if there exists no other allocation $\{c\}$ that increases the expected utility of one agent without reducing the expected utility of the other agent.

The set of all (feasible) allocations is given by:

$$c = \begin{bmatrix} c_{A1} & c_{A2} \\ c_{B1} & c_{B2} \end{bmatrix}$$

where (i) $c_{ht} \geq 0$ for all h and t , and (ii) $\sum_h c_{h1} \leq w$, (iii) $\sum_h c_{h2} \leq x$.

Some examples of allocations are as follows:

- (i) If $c_{B1} = w$ and $c_{A2} = x$, then the agents consume their endowments.
- (ii) If $c_{A1} = w$ and $c_{B2} = x$, then the agents consume the maximal available amount of goods at their preferred date. This yields the highest sum of payoffs (total benefit (TB)), $TB = \alpha w + x$, and satisfies individual rationality of A, if $\alpha \geq \frac{x}{w}$, i.e. $U_A(w, 0) = \alpha w \geq U_A(0, x) = x$.
- (iii) If $\alpha < \frac{x}{w}$, then the highest TB is given by:

$$c_{TB}^{\max} = \begin{bmatrix} w & x - \alpha w \\ 0 & \alpha w \end{bmatrix}.$$

We consider efficient allocations under two polar information settings. The first is Perfect Information (PI), the case where all agents know the realization x as of $t=1$. And, the second setting, Ignorance (IG), is the case where the agents learn the realization, x , only at the final date, $t=2$.

First consider the case of Perfect Information (PI), that is agents know that $X=x$ (i.e., it not a random variable). The outside option of each agent is to consume his own endowment. The individual rationality (IR) constraints are given as follows:

$$\text{Agent A: } \alpha c_{A1} + c_{A2} \geq x \quad (\text{IR}_A)$$

$$\text{Agent B: } c_{B1} + c_{B2} \geq w \quad (\text{IR}_B)$$

There are two sub-cases: $x \geq w$ and $x < w$. What is the highest payoff that agent A can obtain while agent B is willing to accept the (proposed) allocation, i.e. $U_B(c) \geq w$? This is equivalent to:

Max $\alpha c_{A1} + c_{A2}$ subject to:

$$(i) c_{ht} \geq 0, \text{ for all } h \text{ and } t, (ii) \sum_h c_{h1} \leq w, (iii) \sum_h c_{h2} \leq x, \text{ and } \text{IR}_B.$$

The solution is given by:

$$(a) x < w$$

$$(b) x \geq w$$

$$c_A^{\max} = \begin{bmatrix} x & 0 \\ w - x & x \end{bmatrix}$$

$$c_A^{\max} = \begin{bmatrix} w & x - w \\ 0 & w \end{bmatrix}.$$

This is the allocation that maximizes the utility of agent A subject to agent B obtaining his outside option. In Appendix A we characterize the full set of efficient allocations. The interpretation of these allocations is as follows. When $x \geq w$, agent A is allocated w (implicitly from agent B) and retains the residual $x-w$. But, when $x < w$, and this is known, agent A's wealth is only x , so he is allocated only that amount (from agent B). Agent B then consumes $w-x$ at $t=1$. Ex ante, agent A's expected utility is given by:

$$EU_A^{PI} = \underbrace{\alpha \int_{x_L}^w x \cdot f(x) dx + \alpha \int_w^{x_H} w f(x) dx}_{\alpha E[c_{A1}]} + \underbrace{\int_{x_L}^{x_H} \max[x - w, 0] f(x) dx}_{E[c_{A2}]}.$$

Now we turn to the other extreme: Ignorance (IG). Suppose the realization of x is not known, i.e., there is no information acquisition. X is a random variable to be drawn from a distribution F . As shown in Appendix A, the following allocations maximize the expected utility of agent A while agent B receives w :

$$(a) E[x] < w$$

$$(b) E[x] \geq w$$

$$c_A^{\max} = \begin{bmatrix} E[x] & 0 \\ w - E[x] & x \end{bmatrix}$$

$$c_A^{\max} = \begin{bmatrix} w & \max(x - s(x), 0) \\ 0 & s(x) \end{bmatrix},$$

where $s(x)$ is a function specifying what agent B gets conditional on the outcome x and where $E[s(x)]=w$. Suppose $E[x] \geq w$, ex ante, agent A's expected utility is given by:

$$EU_A^{IG} = \alpha \int_{x_L}^w w \cdot f(x) dx + \alpha \int_w^{x_H} w f(x) dx + \int_{x_L}^{x_H} \max[x - s(x), 0] f(x) dx.$$

Since $E[s(x)]=w$, comparing perfect information to ignorance shows immediately that higher expected utility is achieved under ignorance if $w < x_H$. Note that when w is larger than x , then the second and third terms in the two expected utility expressions are identical. Therefore, if $w \in (x_L, x_H)$, then $EU_A^{IG}(\max) > EU_A^{PI}(\max)$. The intuition is that when $x < w$, agent A is worse off but when $x > w$, agent A cannot consume more than w ; thus no convex combination of the two sub-cases of Perfect Information is better than Ignorance, where agent A can consume w at $t=1$. The case $E[x] < w$ is analogous. This is the intuition for the following proposition.

Proposition 1 (Perfect Information and Ignorance: Welfare Comparison): For any $F(x)$, if $x_L < w$ and $\alpha w < x_H$, ex ante the efficient allocation under Ignorance yields a strictly higher welfare than the efficient allocations under Perfect Information.

Proof: See Appendix A. //

The condition $\alpha w < x_H$ implies that $EU_B^{IG}(\max) > EU_B^{PI}(\max)$. The next lemma clearly follows.

Lemma 1.1: For any $\{\alpha, w\}$, if the distribution $F(x)$ is sufficiently risky, then ignorance yields strictly higher welfare than perfect and symmetric information.

As discussed in the Introduction, this result is clearly different than Hirschleifer's result that information can make risk averse agents worse off in a setting where agents want to co-insure, for example each other's health. Here information can destroy liquidity, the ability to transact the optimal amount between the risk neutral agents A and B.

In the economy here there is a limited amount of available goods to trade at each date, namely w . The amount w has already been determined. For example, w can be the amount of corn that was harvested. The issue agents confront is how to move, so to speak, the amount w from its owner, agent B, to the agent with preferred consumption at that date, agent A. Information can reduce the value of what agent A has to offer agent B in trade; this occurs if the information about the payoff is bad news. If the information is good news, it does not necessarily help agent A, as agent B has a limited amount of endowment to sell to agent A. There is an asymmetry: bad news reduces what agent A can buy from agent B, but the effect of good news is bounded by what agent B has to sell. Interestingly, this feature makes the utility function of risk neutral agent A concave since it has slope $\alpha > 1$ for $x \leq w$ and slope 1 for $x > w$. The agents are better off if they know nothing and (implicitly) trade at the expected value. Note, if $w < x_L$ or $\alpha w > x_H$, then ignorance and perfect information yield the same welfare.

3. Optimal Security Design: The One-Period Case

In order to implement the efficient allocation discussed above, agents need to trade. Agent A has to issue a security (i.e., a contractual claim on the outcome of X) to agent B in exchange for agent B's consumption goods, w . Implementing the welfare maximizing allocation requires that ignorance be preserved. In other words, agents must not have an incentive to produce (private) information about

the realization of X . Their incentives to produce information are related to how the value of the security might change as a function of different realizations of x . So the security design problem involves creating a security which is not (or is least) sensitive to information about x ; it minimizes the incentive to produce information. In this section, we define “information-sensitivity,” derive a security with the minimal information-sensitivity, and show that such a security is optimal for agent A to issue, i.e. maximizes his expected utility subject to the participation and information acquisition constraints of agent B.

Let S denote the set of all possible securities (contracts), i.e., functions, $s(x)$, which satisfy the resource feasibility (or limited liability) constraint, $s(x) \leq x$. Any mapping $s: X \rightarrow \mathbb{R}$ with $s(x) \leq x$ is an element of S . Some examples are:

- (i) Equity: $s(x) = \beta x$ where $\beta \in (0, 1]$ is the share on the x ;
- (ii) Debt: $s(x) = \min[x, D]$ where D is the face value of the debt;
- (iii) Step function contract: y_1 if $x \in [x_L, x_1]$, y_i if $x \in [x_{i-1}, x_i]$ where $y_i \leq x_i$.
- (iv) State contingent securities: $s(x_i) = y_i$ where $y_i \leq x_i$;
- (v) Stochastic contracts: $s(x_i) = y_i$ where $y_i: x_i \rightarrow [x_L, x_i]$ with distribution F_i .

Assumption I: $w < E[X]$.

Assumption I makes the analysis interesting in this section. Otherwise, the efficient allocation requires agent A to sell his project, with $t=2$ payoff (the tree) to agent B and it may seem that there is no reason to discuss optimal security design subsequently. However, we show below that this assumption is not crucial for the optimality result.

A. Debt as a Least Information-sensitive Security

We start with analyzing a decision problem of the following type: Suppose an agent has wealth w and can buy a security $(p, s(x))$ with $p = E[s(x)]$ where p denotes the price and $s(x)$ specifies the payoff as a function of x . Ex ante the agent only knows the distribution F of x . If the agent is informed, then he knows the true realization of x . We ask which security $s(x)$, with $p = E[s(x)]$, gives rise to the lowest value of information or in other words, which security is least information-sensitive.

Definition (The value of information): Suppose the decision of the agent is whether to buy a particular security or not. The *value of information of a buy (B) transaction* is defined as $\pi_B = EU(PI) - EU(IG)$, where $EU(PI)$ is the expected utility based on the optimal transaction decision in each state under perfect information about x (PI), and $EU(IG)$ denotes the expected utility of a buy transaction based on the initial information, ignorance of the true state (IG). An analogous definition applies to value of information of a sell transaction, π_S , where the agent must decide to either sell or not sell the asset. Formally, the value of information is given by:

$$\pi_B = \int_{x_L}^{x_H} \max[p - s(x), 0] \cdot f(x) dx \quad \text{and}$$

$$\pi_S = \int_{x_L}^{x_H} \max[s(x) - p, 0] \cdot f(x) dx.$$

Under ignorance (a risk neutral) agent is willing to buy the security since $E[s(x)] = p$. The value of information to a potential buyer (agent B) concerns the region where $s(x) < p$. That is the area where agent B is overpaying. If agent B knew that $s(x) < p$, then he would not trade and instead he would

consume the unspent amount p . Define $Q_{<} = \{x | s(x) < p\}$ to be the set of such states and define $Q_{>} = \{x | s(x) \geq p\}$. Thus, $Q_{<} + Q_{>} = Q = [x_L, x_H]$. So, the value of information for agent B (the potential buyer) is $\pi_B = EU_B(PI) - EU_B(IG)$:

$$\Leftrightarrow \pi_B = \left(\int_{Q_{<}} p \cdot f(x) dx + \int_{Q_{>}} s(x) \cdot f(x) dx \right) - \int_{Q_{>}} s(x) \cdot f(x) dx$$

$$\Leftrightarrow \pi_B = \left(\int_{Q_{<}} p \cdot f(x) dx + \int_{Q_{>}} s(x) \cdot f(x) dx \right) - \left(\int_{Q_{<}} s(x) \cdot f(x) dx + \int_{Q_{>}} s(x) \cdot f(x) dx \right)$$

$$\Leftrightarrow \pi_B = \int_{Q_{<}} (p - s(x)) \cdot f(x) dx.$$

This is the value of avoiding overpayment. For the seller, information is valuable when $s(x) > p$, that is, when the security is worth a lot, the seller would be undervaluing it were he to sell for the price p . Again, define $Q_{>} = \{x : s(x) \geq p\}$ and $Q_{<} = \{x : s(x) < p\}$. So, the value of information for the seller is $\pi_s = EU_s(PI) - EU_s(IG)$

$$\Leftrightarrow \pi_s = \left(\int_{Q_{<}} p \cdot f(x) dx + \int_{Q_{>}} s(x) \cdot f(x) dx \right) - p$$

$$\Leftrightarrow \pi_s = \int_{Q_{>}} (s(x) - p) \cdot f(x) dx.$$

This is the value of avoiding selling the security for too little.

Example: The agent's utility function is $U=c_1+c_2$ and wealth is 1.5. Suppose there are two outcomes, $s(x=L)=1$ or $s(x=H)=2$, both states are equally likely, and the price of buying this asset is 1.5. Suppose the decision of the agent is whether to buy the asset or not. If the agent buys the asset without information acquisition, or simply consumes his endowment, then $EU(IG)=1.5$. If the agent acquires information, then in state 1 he does not buy the asset and consumes his endowment at $t=1$; in state 2 he buys the asset and consumes $s(x=H)=2$ at $t=2$. Thus, $EU(PI)=0.5 \cdot 1.5 + 0.5 \cdot 2 = 1.75$ and $\pi_B=0.25$.³

Definition (Information-sensitivity of a security):⁴ Suppose an agent can buy either of two securities that have the same expected value and the same price. Security i is said to be less *information-sensitive* than security j for agent h if the value of information for buying security i is lower than the value of information for buying security j , i.e. $\pi_h^i < \pi_h^j$. (The analogous definition applies to a sell transaction.)

A *standard debt contract* is given by:

$$\begin{aligned} s^D(x) &= D & \text{if } x > D \\ s^D(x) &= x & \text{if } x \leq D. \end{aligned}$$

That is, a standard debt contract is a security that pays $s^D(x)=x$ up to a specified amount, the face value of the debt, D . Note that in the range $x \leq D$ the payoff function has slope 1 due to limited liability. If

³ The value of information depends on the point of view, buyer or seller. In the example, the value of information for seller is also 0.25. This is not a coincidence. We show below that for any distribution $F(x)$, if $p=E[s(x)]$, then the value of information is the same for the buyer and seller.

⁴ Demarzo and Duffie (1999) also define information-sensitivity in a similar spirit.

$x > D$, then the investor receives D . In order to implement trade we will be interested in the following (standard) debt contract, which has price w and face value D , where D solves the following equation:

$$w = \int_{x_L}^D xf(x)dx + \int_D^{x_H} Df(x)dx.$$

The price of debt equals its expected value. By design this debt contract can potentially implement the transaction needed to achieve the efficient allocation. That is, the amount w can be traded. Given $f(x)$ and w , D is determined.

Now we derive a contract with the minimal information-sensitivity subject to the constraint that any contract should have the same expected payoff and the prices of all contracts are $p = E[s(x)]$. (Note, if traded, the contract should implement efficient consumption, i.e. $p = E[s(x)] = w$.) In other words, we are minimizing π_B, π_S , as well as $\pi_B + \pi_S$ over a set $\{s\}$ of functions where $\{s: s(x) \leq x \text{ and } E[s(x)] = p\}$, and we require the solution to hold for any distribution F for x . In each of these cases, this is a non-trivial mathematical problem in functional space, but the solution and the proof turn out to be surprisingly simple. Let $s(x)$ be the security issued by agent A to agent B. Then:

Theorem 2: Assume for all s that $E[s^D(x)] = E[s(x)] = p$ and that $s(x) \leq x$. Debt is a least information-sensitive security.

Proof: The proof follows from three lemmas. For ease of exposition we assume that $\alpha = 1$ (the case of $\alpha > 1$ is considered in Appendix A).

Lemma 2.1: Debt is a least information-sensitive security for agent B.

Proof: Consider two securities, $s^D(x)$ and $s(x)$, where $s^D(x)$ is debt, i.e., $s^D(x) = D$, for $x \geq D$ and $s^D(x) = x$, for $x < D$. Recall that we have assumed that for all s that $E[s^D(x)] = E[s(x)] = p$, and that there is the resource constraint $s(x) \leq x$. The lemma says that debt is a contract that minimizes:

$$\pi_B = \int_{x_L}^{x_H} \max[p - s(x), 0] \cdot f(x) dx .$$

Equivalently, for all s ,

$$\int_{Q^D} (p - s^D(x)) \cdot f(x) dx \leq \int_{Q^S} (p - s(x)) \cdot f(x) dx \quad (*)$$

where $Q^D = \{x: s^D(x) \leq p\}$ and $Q^S = \{x: s(x) \leq p\}$. Note, $s^D(x) = x \geq s(x)$ for all $x \leq p$ implies

(i) $Q^D = [x_L, p] \subseteq Q^S$, (ii) $p - s^D(x) \leq p - s(x)$ for all $x \in Q^D$, and (iii) $\text{prob}(Q^D) \leq \text{prob}(Q^S)$ for all s . //

Lemma 2.2: For $\alpha = 1$, the value of information to agent A is equal to the value of information to agent B.

Proof: $E[s(x)] = p$ can equivalently be written as:

$$E[s(x) - p] = 0$$

$$\Leftrightarrow \int_{Q^<} (s(x) - p) \cdot f(x) dx + \int_{Q^>} (s(x) - p) \cdot f(x) dx = 0$$

$$\Leftrightarrow \int_{Q^>} (s(x) - p) \cdot f(x) dx = - \int_{Q^<} (s(x) - p) \cdot f(x) dx$$

$$\Leftrightarrow \int_{Q^>} (s(x) - p) \cdot f(x) dx = \int_{Q^<} (p - s(x)) \cdot f(x) dx //$$

Lemma 2.3: Debt is a least information-sensitive security for agent A.

Proof: The lemma says that no other security has a lower value of information for agent A than debt, i.e.:

$$\int_{Q^D} (s^D(x) - p) \cdot f(x) dx \leq \int_{Q^S} (s(x) - p) \cdot f(x) dx \quad (**)$$

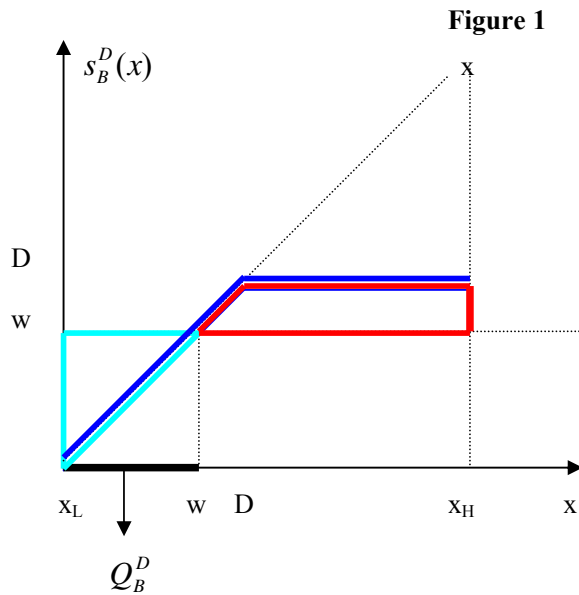
This follows from Lemmas 2.1 and 2.2. Note, Lemma 2.2 shows that for all securities s ,

$$\int_{Q^<} (p - s(x)) \cdot f(x) dx = \int_{Q^>} (s(x) - p) \cdot f(x) dx.$$

Substituting this relationship into the LHS as well as the RHS of (*) yields (**). //

End of the proof of Theorem 2.

The intuition for Theorem 2 is as follows. Agent B (the buyer) must decide to buy the bond ($p=w$, D) or not. If he knows the true value of the payoff, x , then he does not buy the bond in states where $x < w$ because his payoff is $s(x)=x < w$. Since the debt contract has slope one in this region of states, i.e. the buyer receives the maximum amount of repayment that is possible, there exists no other contract that has a smaller set of states that are information-sensitive. Figure 1 depicts the debt contract (the dark blue line). The set of states where information has value to the buyer is denoted by Q_B^D . It is easy to see that $Q_B^D \subseteq Q_B^S$ for all $s \in S$.



The value of information to the buyer is that he avoids overpaying $p=w$, when the realization of x is less than that, $x < p$. The seller of the security, agent A, benefits from being informed to the extent that he avoids paying back too much when the realization of x is larger than w . In the states where $x < w$,

the buyer must be compensated for his low payoff (the light blue triangle, evaluated with the density $f(x)$), with larger payoffs than w in states $x > w$ (the red area). The expected payoffs in these high states are exactly the states where information has value to the seller. In Appendix A, we address the case where $\alpha > 1$ and derive under the assumption that price equals expected payoff that the light blue and red areas are given by:

$$\pi_A^D = \int_{aw}^D (x - \alpha w) f(x) dx + \int_D^{x_H} (D - \alpha w) f(x) dx$$

$$\pi_B^D = \int_{x_L}^w (w - x) f(x) dx.$$

These areas are the same when $\alpha = 1$, and for $\alpha > 1$, $\pi_B > \pi_A$. The implication is that we can focus on agent B, as his value of information is always larger. (This will also ease notation later.) The following results are self-evident.

Theorem 3 (Characterization): The set $\{s\}$ of securities with $p = E[s(x)] = w$ that has the minimal information-sensitivity is given by $\{s: s(x) = x \text{ for } x \leq w \text{ and } s(x) \geq w \text{ for } x > w\}$.

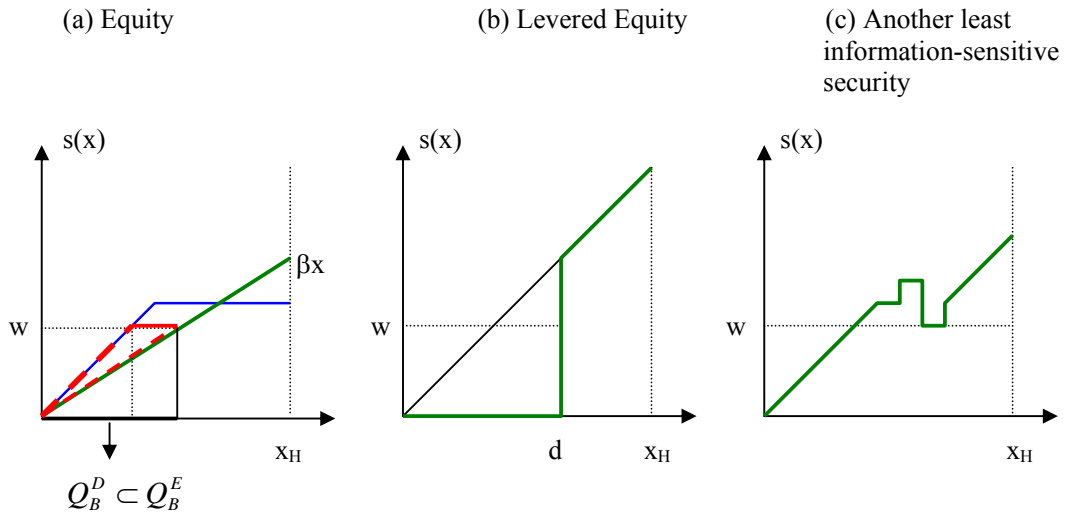
Corollary 3.1 (Equity): Equity is strictly more information-sensitive than debt.

Corollary 3.2 (Toxic Asset or Equity Residual): The security with the maximal information-sensitivity is given by: $\bar{s}(x) = 0$ for $x \in [x_L, d]$ and $\bar{s}(x) = x$ for $x \in [d, x_H]$ where d solves

$$\int_d^{x_H} x f(x) dx = w.$$

Figure 2 compares the payoff on debt to three other securities. Figure 2 shows the payoff to an equity contract, in panel (a), the payoff on levered equity in panel (b); and another least information-sensitive, debt-like, contract is shown in panel (c). In panel (a) the red triangle is the area where equity is more information-sensitive than debt. In panel (b) the information-sensitive area is the rectangle spanned by w and d and evaluated with the density is the value of information. In panel (c) the payoff is non-monotonic, compared to the flat payoff on standard debt.

Figure 2



Theorem 2 is one of the main results of the paper and states, that in markets where agents want to prevent asymmetric information from arising, debt is the optimal trading security. In other words, if the trading of debt triggers information acquisition, then so does the trade of any other security with the same expected value (or price).

Now suppose agents can privately produce information about the true value of X at the cost γ in terms of utility, what is the maximal amount of debt that can be issued (and thus the amount of goods that agent A can consume at $t=1$) without triggering information acquisition? Information acquisition is not worthwhile for agent B if $\pi_B \leq \gamma$. (Recall that $\pi_A \leq \pi_B$.)

Corollary 3.3 (Maximum Debt Issuance): The maximum amount of debt that agent A can issue at $t=1$ without triggering information acquisition by agent B is given by $\min[w, E[x], p]$ where p solves the following equation:

$$\int_{x_L}^p (p-x)f(x)dx = \gamma$$

We can think of securitization in this context. The maximum possible debt that can be issued can be increased if it is backed by a portfolio of projects. Suppose a claim is written on a portfolio of N projects, X_1, \dots, X_N with distributions F_1, \dots, F_N . Define $Y=X_1+\dots+X_N$ as the random variable with distribution F_Y . In general it is very complicated to calculate the distributions of a sum of random variables. As an example, suppose $N=2$, and X_1 and X_2 are independently and uniformly distributed on $[0,1]$. Then Y is not uniformly distributed; it has density:

$$f_Y(y) = \begin{cases} y & \text{for } 0 \leq y \leq 1 \\ 2-y & \text{for } 1 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

The information-sensitivity of a single debt contract with price $p=E[s^D(x)] < E[x]$ is $\int_0^p (p-x)f(x)dx$.

If all the X 's are independently and identically distributed, trading N individual bonds gives rise to the total value of information, $N\pi \equiv \Pi_N^{\Sigma}$. Denote $\Pi^{DP} = \int_0^{p_{DP}} (p_{DP}-y) \cdot f_{Y_N}(y)dy$ as the information-

sensitivity of a single bond backed by a portfolio of N projects where $y = \sum_{n=1}^N X_n$. Analogously, denote

$\Pi^{PD} = \int_0^{p_{PD}} (p_{PD}-s(y)) \cdot f_{Y_N}(y)dy$ is the information-sensitivity of a portfolio of bonds where

$s(y) = \sum_{n=1}^N \min[x_i, D]$. To facilitate comparison, we set $p_{DP} = p_{PD} = Np$. Also, define

$\bar{\Pi}_N^{DP} = \frac{1}{N} \int_0^{N \cdot p} (Np-y) \cdot f_{Y_N}(y)dy$, i.e. the information-sensitivity per unit of project debt.

Lemma 3.1 (Portfolio Information-sensitivity): Suppose $Y=X_1+\dots+X_N$, where X_i is independently and uniformly distributed on $[0,1]$ for $i=1,\dots, N$. Then:

(i) $\Pi_N^{DP} < \Pi_N^{PD} < \Pi_N^{\Sigma}$ for $N \geq 2$.

(ii) $\overline{\Pi}_{N+1}^{DP} < \overline{\Pi}_N^{DP}$ for all N .

Proof: See Appendix A. //

Part (i) of the Lemma shows that the information-sensitivity of a single bond, backed by the sum of the cash flows of the N projects, is lower than the information-sensitivity of a portfolio of bonds, where each backed by a single project, which in turn is lower than the sum of information-sensitivities of the individual bonds. This provides an explanation for securitization. Suppose the random variable Y is a pool of mortgages or automobile loans that are sold to a Special Purpose Vehicle (SPV). What type of claims backed by the pool of the cash flows should the SPV issue? If diversification is the main purpose, a natural candidate would be equity. But if these securities are used as collateral, say in the repo market, where information-insensitive collateral assets are important, then the optimal security is debt. Part (ii) shows that more debt can be issued if the debt portfolio is larger. See Gorton and Souleles (2006) and Gorton and Pennacchi (1993).

B. The Optimality of Debt under Adverse Selection

Theorem 2 states that debt is an optimal security in a trading environment where agents want to prevent adverse selection from arising. However, Corollary 3.3 states that the amount that can be issued without triggering information decreases in the information cost γ . Suppose that only agent B can acquire information. For γ small, agent A can issue very little. What if $\gamma=0$ so that adverse selection cannot be avoided? The next Proposition shows that debt is also optimal under adverse selection, i.e. debt maximizes the expected payoff of agent A when facing an informed agent B.

Proposition 4: Suppose agent B is informed and agent A is uninformed. The optimal contract that agent A offers agent B is a debt contract with price d and face value d , where d maximizes $(1 - F(d))d$.

Proof: The proof is in two steps. (1) We first show that debt maximizes the probability that agent A obtains any (desired) amount d of goods from agent B as well as avoids any repayment larger than d . Then, (2), we derive the optimal face value d .

Step 1: Suppose agent A offers $s^D(x) = \min[x, d]$ for the price d . Since agent B knows the true value of x , he buys $s^D(x)$ only if $x \geq d$. The set of states with no trade is $Q^D = \{x | x < d\}$. The probability of trade is $1 - F(d)$. If trade occurs agent A repays d to agent B at $t=2$. Now, note:

(i) Since $s^D(x) = x$ for $x \leq d$, there exists no other contract s where the sets of states with no trade is smaller than Q^D , i.e. $Q^D \subseteq Q^S = \{x | s(x) < d\}$ and $1 - F(d) \geq \text{prob}(\text{trade under contract } s)$ for all $s \in S$.

(ii) Consider a contract s where $s(x)=x$ for $x \leq d$ and $s(x) > d$ for some x . If trade occurs in these states, then agent A repays $s(x) > d$ to agent B.

Step 2: The expected utility of agent A is $EU_A = \alpha(1 - F(d))d + E[X] - (1 - F(d))d$. Agent A chooses d to maximize EU_A and thus $(\alpha - 1)(1 - F(d))d$, i.e. $(1 - F(d))d$. //

Analogously, we can show that if an uninformed buyer faces an informed, or a potentially informed, seller of a security, debt is also the optimal security. This case will be discussed in Section 5. To highlight the intuition, suppose the seller is informed and the uninformed buyer wants to buy an asset with expected payoff $E[s(x)] = w$ with the highest possible probability. Then a debt contract with face value D such that $E[s^D(x)] = w$ and price $p = D$ is optimal. Since $s(x) \leq D = p$ for all x , trade occurs with probability one. Since $s(x) = x$ for $s < D$, the expected overpayment is minimized. If the buyer were to buy equity with $\beta E[x] = w$ and wants trade to occur with probability one, then he must pay $p = \beta x_H > D$ and the expected overpayment is larger.

C. The Optimality of Debt in the One Period Case

As mentioned, we assume that only agent B can acquire information about the true value of x at the cost γ (though agent A can be privately informed without changing the results). We give a motivation for this assumption in the next section. This section shows that debt is an optimal security for any γ , i.e. debt maximizes the expected utility of agent A subject to the participation and information acquisition constraints of agent B.

Proposition 5: For any $\{F, \alpha, \gamma\}$, debt is optimal, i.e., it maximizes the expected utility of agent A.

Proof: In step 1 we derive optimal contracts without triggering information acquisition and in step 2 we derive optimal contracts with information acquisition. We show debt maximizes the payoff of agent A in both cases.

Step 1: The following two strategies are candidates that maximize agent A's payoff without triggering information acquisition by agent B:

Strategy I (Down-Sizing of Debt): Theorem 2 shows that if agent A wants to issue a security with $p = E[s(x)]$ avoid information acquisition by agent B, then issuing debt is optimal. From Corollary 3.3, let a contract called Contract (I) be given by:

price p' such that $\int_{x_L}^{p'} (p' - x)f(x)dx = \gamma$, and a face value D such that

$$\int_{x_L}^D xf(x)dx + \int_D^{x_H} Df(x)dx = p',$$

i.e., the expected payoff is: $E[s^{D'}(x)] = p'$.

Strategy II (Debt with Surplus Sharing): Another strategy is to offer debt with price $p = E[s(x)] + (\pi - \gamma)$ so as to "bribe" agent B not to acquire information by giving him some of the trading gains. Proposition 2 shows that debt minimizes π and thus the price agent A has to pay to agent B without triggering information. In this case agent A chooses debt with face value D to maximize $EU = \alpha E[s^D(x)] + E[x] - E[s^D(x)] - (\pi^D - \gamma)$ where π^D increases in $E[s^D(x)]$. Formally, Contract (II) is debt with face value D^S (and price p^S) that maximizes:

$$\alpha \left(\underbrace{\int_{x_L}^D xf(x)dx + \int_D^{x_H} Df(x)dx}_{\equiv q^s} \right) + E[X] - q - \underbrace{\left(\int_{x_L}^{x_H} \max[p^1-x, 0] \cdot f(x)dx - \gamma \right)}_{\equiv p^s}$$

Note the definitions of q^s and p^s for future reference. These are used in the proof of Corollary 5.1 below.

Step 2: Proposition 4 shows that debt is optimal if $\gamma=0$. If γ is sufficiently low and α not too large, then avoiding information acquisition may not be optimal. The following strategy maximizes the payoff of agent A with information acquisition of agent B.

Strategy III (Debt with Information Acquisition): Suppose agent A wants to consume the amount p^1 . Then debt is optimal since it maximizes the probability of obtaining p^1 . Facing an informed agent B the set of states with no trade is minimized since $s^D(x)=x$ for $x \leq p^1$. Given the “desired” p^1 , the optimal modified debt contract has the following form: price $p=p^1$ and a face value $p^1 + \tilde{\gamma}$ such that $\pi(p^1 + \tilde{\gamma}) = \gamma$, i.e. when agent B acquires information he just covers his information cost. Note, without information acquisition agent B would buy the contract $s^D(x)$ for the price p^1 since $E[s^D(x)] < p^1$. This can be seen as follows: Suppose $E[s^D(x)] = p^1$. Since $p^1 > p^*$ (Corollary 3.3), this implies that $\pi^D > \gamma$. Reducing the face value to $p^1 + \tilde{\gamma}$ such that $\pi(p^1 + \tilde{\gamma}) = \gamma$, implies that $E[s^D(x)] < p^1$. Formally, Contract (III) is debt with price p^1 , face value $p^1 + \tilde{\gamma}$, and an expected payoff $E[s^D(x)] < p^1$ where:

p^1 maximizes $\alpha(1 - F(p^1))p^1 + E[x] - R$ where R (the expected repayment) is given by:

$$R = \int_{p^1}^{p^1 + \tilde{\gamma}} x f(x) \Big|_{x \geq p^1} dx - \int_{p^1 + \tilde{\gamma}}^{x_H} (p^1 + \tilde{\gamma}) \cdot f(x) \Big|_{x \geq p^1} dx \text{ and}$$

$$\tilde{\gamma} \text{ solves } \pi(p^1, \tilde{\gamma}) = \gamma: \int_{p^1}^{p^1 + \tilde{\gamma}} (x - p^1) f(x) dx + \int_{p^1 + \tilde{\gamma}}^{x_H} (p^1 + \tilde{\gamma} - p^1) f(x) dx = \gamma.$$

Note that the set of debt-like contracts maximizing $\alpha(1 - F(p^1))p^1 + E[x] - R$ is given by $\{s: s(x)=x, \text{ for } x \leq p^1, E[s(x)] \text{ such that } \pi=\gamma\}$ and $p=p^1$. If $\gamma=0$, the optimal contract is unique and given by Proposition 4.

To summarize, these strategies give rise to the following payoff of agent A: $EU_A(I) = \alpha p^1 + E[x] - p^1$, $EU_A(II) = \alpha q^s + E[x] - p^s$, and $EU_A(III) = \alpha(1 - F(p^1))p^1 + E[x] - R$. Agent A chooses the strategy with the highest expected utility. In any case, debt is issued. //

Corollary 5.1: For any $\{F\}$, there exists α' and $\gamma' < \gamma^*$ with the following properties:
(i) If $\gamma \geq \gamma'$, it is optimal to issue debt that does not trigger information acquisition by agent B for any α .
(ii) If $\alpha \geq \alpha'$, it is optimal to issue debt that does not trigger information acquisition by agent B for any γ .
(iii) If $\alpha < \alpha'$ and $\gamma \geq \gamma'$, it is optimal to issue debt that does not trigger information acquisition by agent B. If $\gamma < \gamma'$, then it is optimal to issue debt with information acquisition by agent B.

Proof: See Appendix A.

Corollary 5.1 shows that Assumption I, that $w < E[x]$, is not crucial. If $w \geq E[x]$ and γ is high, then selling the whole project or issuing a “degenerate” debt contract with face value D where $D = x_H$, is optimal. If $\gamma < \gamma^*$, selling the whole project is not optimal, but issuing debt with face value $D < x_H$ is optimal.

4. Optimal Security Design: The Two-Period Case

We have shown that in the one period model debt is an optimal security for agent A to issue because debt is least information-sensitive. In this section we extend the model and analyze the issuance of a security in the primary market and the trading of that security (or a contract taking the original security as the collateral for a new security) in the secondary market. For this purpose, we analyze an economy with three dates ($t=1, 2, 3$) and three agents $\{A, B, C\}$ with utility functions:

$$U_A = \alpha C_{A1} + C_{A2} + C_{A3}$$

$$U_B = C_{B1} + C_{B2} + C_{B3}$$

$$U_C = C_{C1} + C_{C2} + \alpha C_{C3},$$

where, as before $\alpha > 1$ is a constant and where C_{ht} denotes the consumption of agent h at date t . The endowment of agent h at time t is given as follows: $\omega_A = (0, 0, X)$, $\omega_B = (w, 0, 0)$, $\omega_C = (0, w, 0)$. We assume that agent C is not present at $t=1$ but enters the economy at $t=2$, i.e. agents A and B cannot sign any contract with agent C at $t=1$. Also, we assume that at $t=2$ agent C can only trade with agent B.⁵

Given the assumed form of the utility functions and the endowments, it is socially efficient for agent A to consume at $t=1$, for agent B to consume at $t=2$, and for agent C to consume at $t=3$. In order to implement the efficient allocation, agents need to trade, starting with agent A issuing a security $s(x)$ (i.e., a claim backed by X) to agent B at $t=1$. Then agent B uses that security, or a new security $\hat{s}(y)$ which he creates, taking the original security, $y=s(x)$, as collateral, to trade with agent C at $t=2$. Finally, agent C (and possibly also agent B) redeems the security $s(x)$ by presenting it to agent A at $t=3$. Alternatively, agent C redeems the security $\hat{s}(y)$ by presenting it to agent B at $t=3$ for payment and agent B redeems the original security by presenting it to agent A.

The set of contracts: At $t=1$, the set of contracts agent A can issue to agent B is $s \in S = \{s: s(x) \leq x\}$. At $t=2$ the set of contracts agent B can trade with agent C is given by $\hat{S} = \{\hat{s} : \hat{s}(y) \leq y\}$ where $y=s(x)$ denotes the payoff of the security that agent B has bought from agent A.

Two examples of feasible contracts, which will play roles later, are:

(i) A “vertical strip,” i.e., $\hat{s}(x) = \kappa s(x)$ where $\kappa \in [0, 1]$ is a pro rata share of the original contract (i.e. agent B sells the fraction κ of his security to agent C.)

(ii) A tranche or “horizontal slice,” as follows. Suppose $s(x) = \min[x, D]$ was issued originally at $t=1$. Then, at $t=2$, agent B could create a new security using $s(x)$ as the collateral. In particular, agent B could design a new bond $\hat{s}(x) = \min[s(x), \hat{D}]$, with $\hat{D} \leq D$. This is a debt contract that writes-down the original face value D of the original debt contract to the new face value \hat{D} .

⁵ These assumptions simplify the analysis. Alternatively, we could assume $U_A = \alpha C_{A1} + \frac{1}{\alpha} C_{A2} + C_{A3}$ and $U_C = C_{C1} + \frac{1}{\alpha} C_{C2} + C_{C3}$. At $t=2$ agent C only has an incentive to trade with agent B.

The original bond is used as collateral for the new contract. In particular, the new debt contract is a senior tranche of the collateral, and agent B will hold the equity residual, with payoff $\max[s(x) - \hat{D}, 0]$. The new bond is a “horizontal slice” (“tranche”) of the collateral based on seniority.

Note that a vertical strip is an equity claim on the underlying asset $s(x)$. A horizontal slice is a debt claim on $s(x)$. Corollary 3.1 states that debt is strictly less information-sensitive than equity. These two cases, the vertical strip and the horizontal tranche will reappear later, where we will discuss their different incentive effects.

Information: We assume that at $t=1$ the agents’ prior on X is given by the (mixture) distribution F_m , where the density of F_m is given by $f_m(x) = \sum_{k=1}^K \lambda_k f_k(x)$, where $\lambda_k \geq 0$, $\sum_{k=1}^K \lambda_k = 1$, and distribution F_k has positive support on $[x_L^k, x_H^k]$. We assume that $x_H^k < x_L^{k+1}$ for all k . In other words, we assume that the support of each F_k is a partition of the support of the mixture distribution $[x_L, x_H]$, where $x_L = x_L^1$ and $x_H = x_H^K$. At $t=2$, the agents receive a public signal about the “true” distribution of X (i.e. public news about which distribution x will be drawn from).⁶ This assumption is made for tractability.

Sequence of Moves and Events: The timing of events at $t=1$ is as follows:

- t=1.0: The realization of X (namely, x) occurs but is not publicly known.
- t=1.1: Agent A makes a take-it-or-leave-it contract offer of $s(x)$ to agent B.
- t=1.2: Agents B chooses whether to produce information about the *true* x at the cost γ (in terms of utility), or not.
- t=1.3: Agent B accepts the contract or not.

If there is no trade between agents A and B, the game ends. If agent B trades with agent A, then at $t=2$ agent B has the claim $s(x)$ available to use to trade with agent C. The timing of events at $t=2$ is as follows:

- t=2.0: The public signal F_k is observed.
- t=2.1: Agent C makes a take-it-or-leave-it contract offer $\hat{s}(y)$ to agent B.
- t=2.2: Agents B chooses whether to produce information about the *true* x at the cost γ (in terms of utility), or not.
- t=2.3: Agent B accepts the offer or not.
- t=3: Agent B redeems $s(x)$ with agent A, and agent C redeems $\hat{s}(y)$ with agent B.

To fix ideas the reader may want to think of agent B as an intermediary, standing between agent A who originates a loan/project (and who can be privately informed about it) and agent C, who is a final investor. Agent B is a sophisticated agent in the sense that he can produce information, while the other

⁶ Formally, a signal structure I is a mapping from x to subsets $I(x)$ of x , denoted by $\{I_1, \dots, I_K\}$. I is a partition if (i) for $I(x) \cap I(x') \neq \emptyset$, then $I(x) = I(x')$ for all $x, x' \in X$. In other words, $I_j \cap I_k = \emptyset$. (ii) $I_1 \cup \dots \cup I_K = X$. We also assume that $x \in I(x)$ and $I(x)$ is convex. This assumption is a special (and stronger) case of First Order Stochastic Dominance (FOSD). Here, the distribution with support on I_N first-order stochastically dominates all other distributions by construction, while F_1 with support on I_1 is first-order stochastically dominated by all other distributions. See Milgrom (1981). We discuss other signal structures in Appendix B.

two agents cannot produce information.⁷ In addition, we may want to interpret the endowment and preference structure as follows: At $t=1$ bank A has a shortage of cash while bank B has excess cash to lend out against collateral $s(x)$. At $t=2$ bank B has a shortage of cash while bank C has excess cash and wants to store it for usage at $t=3$. For this purpose bank C wants to buy bank B's security.

A. The Optimality of Debt for Trading Capacity

In the previous section we showed that debt is an optimal security in the one period setting. Corollary 5.1 shows that for γ sufficiently high, any contract with $E[s(x)]=w$ is optimal. In this section we derive a *further* benefit of debt in a setting where there is no potential for adverse selection (no private information can be produced), but where there is an interim public signal. We show that debt maximizes the amount of intertemporal trade that can be implemented. The amount of intertemporal trade that can be implemented we call a security's "trading capacity."

Here is a summary of the argument. Suppose no agent can acquire information about the true value of the underlying asset X at any date, i.e., γ is prohibitively high. Since there are no adverse selection concerns, issuing any security s with price $p=w$ and payoff $E_m[s(x)]=w$ is optimal for agent A. At $t=2$, it is efficient for agent B to consume by selling the security $s(x)$ to agent C in exchange for goods. But at $t=2$, there is a public signal about the distribution of X . If the signal reveals that distribution k is the true distribution, then the market value of the security is $E_k[s(x)]$. This means that the resale price of security s fluctuates. In this section we ask: what date 1 security, $s(x)$, maximizes the expected trading capacity and thus the expected consumption of agent B at $t=2$ and the expected consumption of agent C at $t=3$ (since agent A is indifferent between issuing all securities with $p=w$ and $E[s(x)]=w$)? We solve for a contract that maximizes the expected payoff of agent C subject to $E_m[s(x)]=w$ and the participation constraint of agent B.

Proposition 6 (Debt maximizes trading capacity): Suppose agents cannot acquire private information. Then issuing debt at date 1 maximizes the payoff of agent A as well as the amount of trade between agents B and C, the trading capacity. Debt dominates any contract $s \in S$ in terms of total payoff.

Proof: At $t=1$, any contract s with $E_m[s(x)]=w$ maximizes agent A's consumption at $t=1$ (his preferred date) subject to the participation constraint of agent B. Now we show that debt maximizes (expected) trade between agents B and C at $t=2$. Suppose at $t=1$ debt and a security s has been issued, where $E_m[s^D(x)] = E_m[s(x)] = w$.

Since debt $s^D(x)$ is non-decreasing, the assumption of partial information implies that $E_k[s^D(x)] \leq E_{k+1}[s^D(x)]$ for all k . Define $\{F^<\} = \{F_1, \dots, F_k\}$ as the set of distributions where $E_k[s^D(x)] < E_m[s^D(x)] = w$. Analogously, define $\{F^>\} = \{F_{k+1}, \dots, F_N\}$ to be the set of distributions where $E_k[s^D(x)] \geq E_m[s^D(x)]$. At $t=2$, if $E_k[s^D(x)] \geq w$, agent C trades w for $\alpha E_k[s^D(x)]$ where $\alpha = \frac{w}{E_k[s^D(x)]}$. At $t=3$, agent C has an expected consumption of $\alpha E_k[s^D(x)] = w$. If $E_k[s^D(x)] < w$, then agent C buys the whole debt and consumes the rest of his endowment

⁵ This assumption keeps the equilibrium analysis tractable. Since only agent B (the responder at both dates) can acquire information, no signaling issue arises. For an analysis of a bargaining game with two sided information acquisition see Dang (2008).

$w - E_k[s^D(x)]$ at $t=2$, and he consumes the expected amount of $E_k[s^D(x)]$ at $t=3$. $EU_C[c_C^D] \geq EU_C[c_C^S]$ since:

$$\begin{aligned} & \sum_{k=1}^{k'} \lambda_k \left(\underbrace{w - E_k[s^D(x)]}_{E[c_{C2}]} + \alpha \underbrace{E_k[s^D(x)]}_{E[c_{C3}]} \right) + \sum_{k=k'+1}^K \lambda_k \left(\underbrace{0}_{E[c_{C2}]} + \alpha \underbrace{w}_{E[c_{C3}]} \right) \\ & \geq \sum_{k=1}^{k'} \lambda_k (w - E_k[s(x)] + \alpha E_k[s(x)]) + \sum_{k=k'+1}^K \lambda_k (\max[w - E_k[s(x)], 0] + \alpha \min[w, E_k[s(x)]]). \end{aligned}$$

Note, $s^D(x) = x \geq s(x)$ for all $x \leq D$, implies that $E_k[s^D(x)] \geq E_k[s(x)]$ for all $k \leq k'$. For $k > k'$, depending on $s(x)$, $E_k[s^D(x)]$ can be smaller, equal, or larger than $E_k[s(x)]$. Note, $EU_B[c_B^D] = EU_B[c_B^S] = w$ for all s . //

Proposition 6 is similar in spirit to Proposition 1. We assume that agent C has an endowment of $w_C = w$ at $t=2$. If $w < w_C < x_H$, then debt (weakly) dominates any other security. If $w_C > x_H$, then any security gives rise to the same expected trading capacity. In this case from the ex ante ($t=1$) point of view the expected amount agent C can consume at $t=3$ is w , i.e. the expected payoff of the security (that agent A has issued at $t=1$).

If the public signal is perfect, then Proposition 6 can be interpreted as the “exact” counterpart of Theorem 2 on information-sensitivity and endogenous adverse selection. In Theorem 2, information-sensitivity is the area between price p and the repayment $s(x)$ evaluated with the density. The expected trading capacity or market price of the security is the area below $s(x)$ evaluated with the density. Since debt has $s^D(x) = x$ for $x \leq p$, information-sensitivity is minimized and the trading capacity in bad states is maximized. Intuitively, Proposition 6 shows that even without any adverse selection concern, securities with a high variance of resale prices are less attractive as a collateral asset than assets with low price fluctuations. Price fluctuation (payoff variance), however, is not the same as information-sensitivity as we have defined it.

B. Information Acquisition, Information-Sensitivity, and Trading at the Interim Date

In this sub-section we assume that agent B can acquire information at $t=1$ or $t=2$, at the cost γ , about the true final payoff of the underlying asset X . We know that issuing debt at $t=1$ maximizes the utility of agent A. (Agent B is indifferent about when he consumes.) Now we analyze whether a $t=1$ debt contract also maximizes the expected utility of agent C; that is, we ask what security agent C eventually wants to use to trade with agent B, after observing the public signal about the distribution, so as to maximize his expected consumption at $t=3$. (Recall that agent C makes an offer to agent B, so agent C can be viewed as designing or asking for a specific security.)

Whether the trading of the security $s(x)$ at $t=2$ triggers information acquisition or not depends on the date 2 information-sensitivity, π , of that asset relative to the information cost γ . Since prices $p_k = E_k[s(x)]$ fluctuate with the public signal k , π also changes with that information since:

$$\pi(k) = \int_{x_L}^{x_H} \max[p_k - s(x), 0] \cdot f_k(x) dx \quad \text{where} \quad p_k = \int_{x_L}^{x_H} s(x) \cdot f_k(x) dx.$$

In Appendix C, we show that the information-sensitivity, π , of a security is a complicated object. Even with the assumption of partitional information (or First Order Stochastic Dominance) and prices are monotonic in k , π is typically non-monotonic in k .

Proposition 7: Suppose $\gamma \geq \pi^D(m)$ and debt with price w has been issued at $t=1$. If, (a), $E_k[s^D(x)] \geq E_m[s^D(x)]$ (i.e., good news), or (b), $E_k[s^D(x)] < E_m[s^D(x)]$ and $\pi^D(k) \leq \gamma$, (i.e., bad news but not so bad as to trigger information production), then trading at $t=2$ results in an efficient consumption allocation between agents B and C.

Proof: (a) In this case, there was good news. There are two sub-cases: (i) If $\pi^D(k) \leq \gamma$, i.e., information production is not profitable, then agent C will offer to buy the fraction $\kappa = \frac{w}{E_k[s^D(x)]}$ of agent B's debt for the price w (vertical strip). This is because the good news has caused the bond to rise in value so that it is worth more than w . Agent C offers to buy a strip that is just worth w . Agent B sells without information acquisition since $\kappa \pi^D(k) < \pi^D(k) \leq \gamma$. (ii) If $\pi^D(k) > \gamma$, then information production is profitable. As a result, agent C will offer to buy a new debt contract with face value $\hat{D} < D$ and price w , taking the original debt contract as the underlying collateral (horizontal slice). Agent C wants to design the new bond so as not to trigger information production by agent B. Agent B sells the horizontal slice without information acquisition since $\pi^{\hat{D}}(k) < \pi^D(m) \leq \gamma$. Note, $p_k^{\hat{D}} = p_m^D = w$ and $F_k \succ_{SD} F_m$ (i.e., stochastically dominates) imply that $\pi^{\hat{D}}(k) < \pi^D(m)$. In both these subcases, Agent B consumes w at $t=2$ and has an expected consumption of $E_k[s^D(x)] - w$ at $t=3$; and at $t=2$, agent C has no consumption and an expected consumption of w at $t=3$.

(b) In this case there is bad news, but not so bad that information production is triggered. Agent C buys the whole debt of agent B for the price $E_k[s^D(x)]$. Agent B sells without information acquisition since $\pi^D(k) \leq \gamma$. Agent B consumes the amount $E_k[s^D(x)]$ of goods at $t=2$; agent C consumes $w - E_k[s^D(x)]$ at $t=2$ and $E_k[s^D(x)]$ at $t=3$. //

Proposition 7 (a), part (i) of the proof, shows how a pro rata reduction (a vertical strip) in the amount of debt traded can keep agent B from having an incentive to produce information.⁸ The cost of information production is a fixed amount, so reducing the amount of debt traded can eliminate the incentive to produce information. A horizontal slice would also work. Part (ii) of the proof is different in that trading a vertical strip may not suffice to prevent information acquisition. The agent B needs to issue a new debt contract, where agent B retains a junior equity tranche relative to what agent C is willing to accept — the senior tranche. Horizontal slicing is more powerful because it introduces seniority: agent B retains the equity piece of the newly issued (redesigned) bond. Any information produced would be wasted because it mostly concerns the residual which agent B has to keep in any case.

We now turn to the case where the interim public signal is bad news, such that, if agent C trades the efficient amount of debt, agent B will produce private information. The next proposition is similar to Proposition 5 where the buyer of the security (agent B) can acquire information. Here the seller of the security (agent B) can acquire information.

⁸ Keep in mind that π is not monotonic in k , so that it could happen that there is “good news,” i.e., $E_k[s(x)] > E_m[s(x)]$, and still it can be profitable for agent B to produce information.

Proposition 8: Suppose $\gamma \geq \pi^D(m)$, (i.e., it does not pay to produce information initially) and debt with price w has been issued at $t=1$. At $t=2$ $E_k[s^D(x)] < E_m[s^D(x)]$ and $\pi^D(k) > \gamma$ (information production will be triggered ceteris paribus). Then as a best response, agent C either chooses:

- (i) Strategy I (Maximum debt write-down): A debt contract with face value $\hat{D} < D$ and price $\hat{p}_k^D = E_k[s^{\hat{D}}(x)]$ that agent B accepts without information acquisition;
 - (ii) Strategy II (Surplus Sharing): A debt contract with face value d where $\hat{D} < d \leq D$ and the price $p_k^d > E_k[s^d(x)]$ that agent B accepts without information acquisition;
 - (iii) Strategy III (Adverse Selection): A debt contract with face value d' where $\hat{D} < d' \leq D$ and the price $p_k^{d'} = E_k[s^{d'}(x)]$, where agent B acquires information;
- where \hat{D}, d, d' and $\hat{p}_k^D, p_k^d, p_k^{d'}$ are defined below.

Proof: See Appendix A.

Proposition 8 considers the best response of agent C when the public signal at the interim date has made the value of information high, $\pi^D(k) > \gamma$. Similar to Proposition 5, agent C compares three strategies. All the strategies involve redesign, i.e., a new bond being issued taking the bond that agent B holds, having received it from agent A, as collateral. In Strategy I agent C computes the maximal amount of debt that can be traded without triggering information production. Strategy II also avoids information production. In this case, agent C offers to pay more than the expected value of the bond, but the bond has a higher face value than in Strategy I. Strategy II may dominate Strategy I because it achieves a larger amount that is traded, and hence is more efficient. Finally, Strategy III is the case where agent C chooses a higher face value of the debt than Strategy I such that agent B does produce information, but the price offered maximizes agent C's expected utility, which depends on the chance of a trade occurring given that agent B has private information.

Propositions 7 and 8 will be of use when we determine the equilibrium of the trading game, to which we now turn.

5. Equilibrium Debt Issuance and the Possible Collapse of Debt Trading

In this section we analyze Perfect Bayesian Nash equilibrium in the three-agent and three-date economy with interim public news arrival about the distribution of x and the possibility of information acquisition by agent B. The bargaining and trading game are as specified in Section 4. We focus on the case where $\gamma \geq \pi^D(m)$. The other case is completely analogous.

Proposition 9 (Debt Equilibrium): Consider the economy $\{\alpha, \gamma, w, \{F_i\}_{i=1}^N\}$ and suppose $\gamma \geq \pi^D(m)$. Then there exists an equilibrium with the following properties:

At $t=1$, agent A consumes w by issuing debt to agent B. No information is acquired.

At $t=2$, if $E_k[s^D(x)] \geq E_m[s^D(x)]$, there is efficient trade. If $E_k[s^D(x)] < E_m[s^D(x)]$, then depending on the revealed distribution F_k , the following cases can arise:

- (i) There is efficient trade between agents B and C. No information is acquired.
- (ii) There is inefficient trade between agents B and C. No information is acquired.
- (iii) Agent B produces information and trade occurs with probability less than one.

At $t=3$, agents who own a claim on X consume the goods delivered by the claim.

Proof: At $t=1$, issuing debt maximizes the payoff of agent A and thus is a best response. At $t=2$, the following cases arise:

Case (i): There is efficient trade at $t=2$.

(a) Suppose $E_k[s^D(x)] \geq E_m[s^D(x)]$. Proposition 7(a) shows there is always efficient trade.

(b) Suppose $E_k[s^D(x)] < E_m[s^D(x)]$. There is efficient trade if

(bi) $\gamma \geq \pi^D(k)$, see Proposition 7(b); or

(bii) $EU_C(d = D) \geq \max[EU_C(\hat{D}), EU_C(d')]$, i.e. agent C chooses Strategy (II) to trade the whole debt. See Proposition 8(ii); or

(biii) $EU_C(d < D) \geq \max[EU_C(\hat{D}), EU_C(d')]$, i.e. agent C chooses Strategy (II) and spends all his w to buy a new debt contract with $d < D$. See Proposition 8(ii).

Case (ii): There is inefficient trade and no information acquisition at $t=2$, if:

(a) $EU_C(d < D) \geq \max[EU_C(\hat{D}), EU_C(d')]$, i.e. agent C chooses Strategy (II) to trade a new debt contract with face value $d < D$ and agent C has positive consumption at $t=2$. See Proposition 8(ii).

(b) $EU_C(\hat{D}) \geq \max[EU_C(d), EU_C(d')]$, i.e. agent chooses Strategy (I) to trade a new debt contract with face value $\hat{D} < D$ and agent C has positive consumption at $t=2$. See Proposition 8(i).

Case (iii): There is adverse selection at $t=2$, if $EU_C(d') \geq \max[EU_C(\hat{D}), EU_C(d)]$, i.e. agent C chooses Strategy (III) and agent B acquires information. See Proposition 8(iii). //

Under the monotonicity conditions given in Appendix B, Proposition 9 has the interpretation that at $t=2$ if there is good news, $E_k[s^D(x)] \geq E_m[s^D(x)]$, then there is efficient debt trading.⁹ With bad news, case (ii) occurs, that is, there is insufficient debt trading but no information acquisition by agent B. There is a collapse of debt trading in the sense that agents B and C trade less than the (new) market value of agent B's debt. In the numerical example below, if there is bad news agents B and C trade a senior tranche of 10% of the market value of agent B's debt, i.e. there is a 90% write-down of the original debt contract. Finally, for very bad news, agent B produces information and because of

⁹ In Proposition 9, it is not necessarily the case that decreasing k corresponds to worse outcomes; the value of information is not monotonic in k . First order stochastic dominance (FOSD) or a partitional information structure implies that the price of a security is weakly monotonic in F_k . But, that is not necessarily the case with regard to the value of information. The intuition is the following: Bad news (a distribution with more mass in the left tail) reduces the price of the security, and thus the "area" between price and $s(x)$. But on the other hand that smaller area is evaluated with more probability mass. The overall effect is ambiguous. Similarly, good news increases the price but there is less probability mass on the left tail. Our results do not require that the value of information be monotonic. Intuitively, one wants to think of lower k as corresponding to a worse state of the economy, but this interpretation requires monotonicity, which is more structure on the distributions than provided by FOSD or a partitional information structure. In Appendix B we discuss this non-monotonicity and provide conditions under which the value of information is monotonic in F_k .

adverse selection there is may be no trade at all. Both cases correspond to “systemic risk” because the outcome is worse than that caused only by the fundamentals. The “fundamentals” corresponds to the bad shock k . Instead of trading at the new expected value of the debt, agents trade much less than they could or even not at all. In this sense there is a collapse of trade.

Corollary 9.1: For any $\{F\}$ with $\pi^D(m) < \gamma < \pi^D(k)$ and $E_k[s^D(x)] < E_m[s^D(x)]$ for some k , there exist a $\alpha' > 1$ such that if $\alpha \leq \alpha'$, and the signal reveals that k is the true distribution, then there is maximum write-down of debt at $t=2$ in equilibrium.

Proof: See Appendix A.

Proposition 9 and Corollary 9.1 are perhaps best understood with an example. Suppose $F_1 \sim u[0, 0.8]$, $F_2 \sim u[0.8, 1.2]$, $F_3 \sim u[1.2, 2]$ and $\lambda_1 = \lambda_2 = \varepsilon$, and $\lambda_3 = 1 - 2\varepsilon$. Then: $f_m = 5\varepsilon/4$ for $x \in [0, 0.8]$, $f_m = 5\varepsilon/2$ for $x \in [0.8, 1.2]$, $f_m = 5(1-\varepsilon)/4$ for $x \in [1.2, 2]$ and $f_m = 0$ else. That is, these are the prior densities (for the mixture distribution) over the different intervals corresponding to the k -distributions. Suppose $\varepsilon \approx 0$, $w = 1$, $\gamma = 0.001$, and $\alpha = 1.2$. The subsequent numbers are exact up to the fourth decimal.

If debt with face value $D=1$ and price $p_m^D = 1$ is issued, then $\pi^D(m) = 0$.¹⁰ In this example, equilibrium outcomes are as follows.

(i) If F_1 is the true distribution, then $p_1^D = 0.4$, $\pi^D(1) = 0.1$. (a) If agent C proposes to buy the whole debt for the price $p_1^D = 0.4$, agent B acquires information and sells when $x \leq p_1^D$ and $EU_C = w + \text{prob}(x < p_1^D) \cdot (\alpha E[x | x < p_1^D] - p_1^D) = 1 + 0.5(1.2 \cdot 0.2 - 0.4) = 0.92$. (b)

If agent C proposes to buy the whole debt for the price $p_1^{ss} = p_1^D + \pi^D(1) - \gamma = 0.499$, then agent B sells his debt with probability 1 and without information acquisition and $EU_C = 1 + (1.2 \cdot 0.4 - 0.499) = 0.981$. Thus there is no efficient trade, i.e., although agent C has enough endowment he chooses not to buy the whole debt of agent B. (c) If agent C proposes to write-down to $\hat{p}_1^D = 0.04$ and $\hat{D}_1 \approx 0.0411$, agent B sells without information acquisition and $EU_C = (1 - \hat{p}_1^D) + \alpha \hat{p}_1^D = 1.008$. Thus buying a new debt contract, i.e.

$\hat{\kappa} = \frac{\hat{p}_k^D}{p_k^D} = \frac{0.04}{0.4} = 0.1$ or 10% percent of agent B's expected cash flow as a senior tranche, dominates the other two strategies. There is no adverse selection but insufficient trade.

(ii) If F_2 is the true distribution, then $p_2^D = 0.95$, $\pi^D(2) = 0.0281$. If agent C chooses to buy the whole debt for the price $p_2^D = 0.95$, agent B acquires information and $EU_C = 1 + \frac{1.5}{4}(1.2 \cdot 0.875 - 0.95) = 1.00375$. If agent C chooses to buy the whole debt for the price $p_2^{ss} = p_2^D + \pi^D(2) - \gamma = 0.9771$, then $EU_C = 1 + 1.2 \cdot 0.95 - 0.9771 = 1.1629$. If agent C chooses maximum down writing, and proposes to buy debt with $\hat{p}_2^D = 0.8283$ and

¹⁰ If equity with $\beta \approx \frac{5}{8}$ and $p_m^E = 1$ is issued, then $\pi^E(m) \approx 0.0625$ and this triggers information acquisition.

$\hat{D}_2 = 0.8294$, then $EU_C = (1 - \hat{p}_2^D) + \alpha \hat{p}_2^D = 1.1657$. Thus agent C also proposes maximum write-down.

In this example, if there is good news (i.e., $F=F_3$), there is efficient trade between agents B and C at $t=2$. If there is bad news (i.e., $F=F_2$), then the market price of debt drops from 1 to 0.95 and agent C buys a senior tranche of 87.2% of agent B's debt. This can be interpreted as a haircut of 13%. If there is very bad news (i.e., $F=F_1$), then the market price of debt is 0.4 and agent C buys a senior tranche of 10% of agent B's debt, i.e. a new debt contract with market price of 0.04 (backed by the original debt), which corresponds to a haircut of 90%.

Proposition 10: The debt equilibrium is a second best outcome, i.e., it is constrained efficient.

Proof: Agent A is indifferent between issuing any contract $s(x)$ with $E_m[s(x)]=w$ at $t=1$. Thus a debt contract maximizes his payoff. Propositions 7 and 8 state that debt is optimal for agents B and C. We (only) have to prove that for $E_k[s^D(x)] < E_m[s^D(x)]$ and $\pi^D(k) > \gamma$, from the point of view of agents B and C, issuing debt at $t=1$ also weakly dominates any other contract s .

Suppose contract s has been issued at $t=1$. Note, Proposition 6 states that $E_k[s^D(x)] \geq E_k[s(x)]$.

(a) Suppose $\pi^S(k) \leq \gamma$, i.e., trading the whole contract s does not trigger information acquisition. Here is a “replication strategy” given debt has been issued: Agent C can propose to buy a new debt contract with face value $\hat{D} < D$ (taking the original debt contract as the underlying collateral) and where the price equals the market value of contract s , i.e. $p^{\hat{D}} = E_k[s^{\hat{D}}(x)] = E_k[s(x)]$. In this case $\pi^{\hat{D}}(k) < \pi^S(k) \leq \gamma$. (Proposition 2 states that debt is a least information-sensitive security given two securities with the same expected payoff.) Debt is at least as good as any contract s .

(b) Suppose $\pi^S(k) > \gamma$, i.e., trading the whole contract s triggers information acquisition. Since $E_k[s^D(x)] \geq E_k[s(x)]$, with a date 1 debt contract agent C can replicate any new contract s' that takes contract s as the underlying collateral. Thus $p^{\hat{D}} = E_k[s^{\hat{D}}(x)] = E_k[s'(x)]$ and $\pi^{\hat{D}}(k) \leq \pi^{S'}(k)$. More precisely, for $x \leq D$, by redesigning the original debt contract, $s^D(x)=x$, agent C can replicate the payoff of any contract $s(x)$ or any redesign contract $s'(y)$ with $y=s(x)$. For $x > D$, contract $s(x)$ may generate a higher repayment in states where $s(x) > s^D(x) = D$. But in these states, the privately informed agent B only sells if agent C offers at least $p=s(x) > D$. But under the debt contract $s^D(x)$, if agent C proposes the price $p=D$, agent B sells in all states and agent C's expected consumption at $t=3$ is $E_k[s^D(x)]$. Under the contract s , agent C does not always sell, if $s(x) > D$ for some x . Thus issuing debt at $t=1$ is optimal. //

Proposition 10 states that issuing a debt contract at date 1 is as good as any contract if $E_k[s^D(x)] \geq E_m[s^D(x)]$, and allows agents B and C to replicate the expected payoff of any contract at date 2 when $E_k[s^D(x)] < E_m[s^D(x)]$. In other words, debt issuance at $t=1$ maximizes the flexibility to redesign contracts at $t=2$. A graphical illustration is given in Figure C1 (b) in Appendix C, that compares debt and equity.

6. Discussion

In this section we briefly discuss modeling assumptions and then some issues raised by the analysis above.

A. Modeling Assumptions

Three assumptions are crucial for the analysis. First, the cost of producing information is a fixed amount, γ . This is the source of the suddenness of the financial crisis when information-insensitive debt becomes information-sensitive. It is this threshold which is crossed. Second, the general welfare result that ignorance is Pareto-optimal depends on the endowments, w , being predetermined. We suggested that the endowments be thought of as the harvest; this is the maximum amount that is available. More generally, the economy has a certain amount of consumption goods already produced and available. Although more can be produced, that is all that is available at time t . Finally, the optimality of debt depends on limited liability, which is the source of the 45 degree line determining the payoff on debt when $x < D$. Without limited liability there would be information-sensitivity about the other assets that the debtor might have to back the claim.¹¹

B. Financial Crisis

We have shown that for the economy as a whole debt allows trade to occur efficiently. Debt implements trade because it is information-insensitive; it prevents adverse selection because it minimizes the incentive to produce private information. A richer society is one in which the endowments, w , are larger. In such a society more debt would be required to implement trade. For society, this leverage is optimal. Wealth results in leverage, but, the debt is not riskless. And, because it is not riskless, a type of regime switch can occur where the debt becomes information-sensitive. This occurs because of the fixed cost of producing information, the γ threshold. We have shown that this can result in a crisis. The crisis in our model is not the arrival of bad news per se. The “crisis” refers to the outcome which is worse than the fundamentals. The “fundamentals” corresponds to the bad shock k , which then causes agent C to ask for a write down of debt or even “worse” agent B to produce information and there is adverse selection. In this sense there can be a collapse of trade. The model and the analysis above provide a way to understand the size of the crisis and what is meant by the description of it as systemic. As mentioned in the Introduction, this crisis is very different from coordination failure models.

C. Equity and Leverage

Creation of debt for the provision of liquidity means that the economy must have leverage, and some agent must hold the levered equity residual. Above, this was held by agent A. There are several issues to be discussed with regard to levered equity. The issues raised here are discussed further in Dang, Gorton, and Holmström (2009).

The first issue arises if there is a cost to agent A associated with this leverage, say a bankruptcy cost (in utility terms) if $x < D$ at $t=3$. In that case, agent A may choose to issue less debt ex ante for fear of the bankruptcy cost and the economy is worse off (compared to the case of no bankruptcy cost). A more realistic cost would be one associated with the possibility that agent A had to sell the residual, possibly resulting in information externalities.

¹¹ Easterbrook and Fischel (1985) describe limited liability as the “fundamental principle of corporate law” (p.89).

Secondly, suppose information has a social value. If the model is extended to allow for a second project, requiring investment at the interim date, then there would be a capital market at the interim date. Agent A would need to issue a security to raise capital. But, then at date 2 there is a potential negative informational externality, as such trade, or lack of trade, may reveal information. Again, the economy is potentially worse off due to the informational externality.

D. Transparency versus Opacity

Symmetric information avoids adverse selection. But, our initial welfare result is stronger than this because it says that symmetric ignorance dominates symmetrically informed agents. This is because the economy has a limited amount of available goods to trade, namely w . The amount w has already been determined. Consequently the effect of (full or partial) information is asymmetric. Bad news reduces what agent A can buy from agent B, but the effect of good news is bounded by what agent B has to sell. The agents are better off if they know nothing and (implicitly) trade at the expected value of the claim.

In this economy government policies that increase transparency would reduce welfare. This would seem to be counter to the intuition built from the idea of efficient markets. But, that theory does not say what the optimal amount of information that should be available is, but only that security prices reflect whatever information is available.

E. Rating Agencies

Rating agencies are a puzzle. Why do they exist? The standard version of “efficient markets” in equities has agents becoming privately informed and trading on their information. Prices are informative and there is no need for rating agencies. Why are debt markets different? Also, why do rating agencies only produce coarse signals, when as the critics have pointed out, risk is multi-dimensional? Our model can address these questions.

One of the possible equilibrium outcomes is the possibility that agent B produces information and trade is reduced. A rating agency can minimize this welfare-reducing outcome, possibly by enough to justify the fee of γ charged by the rating agency for information production. In this subsection we sketch how this would work, but for brevity we do not present formal results.

The rating agency is a firm which commits to announce ratings just after the realization of the interim aggregate signal. For each possible distribution k that could be realized, the rating agency commits at date 1 to a set of partitions $\{I(k)\}$ of the support of distribution F_k . These are the ratings. Upon the realization of distribution k , the agency truthfully announces the rating (partition that contains x).

How could this help? Imagine that the distribution that is realized is one for which agent B would choose to produce information. If the agency has chosen its partitions correctly, then conditional on the announcement of the partition/rating, the value of information to agent B can decline sufficiently so that he does not find it optimal to produce information; welfare is improved. This is the mechanism by which the ratings can help.

The rating agency’s optimization problem, however, is very complicated. On the one hand, partitions cannot be too fine because information destroys trade. On the other hand, partitions cannot be too coarse or else agent B will still have an incentive to produce information when we would prefer that he not produce information.

F. Complex Securities

In the financial crisis, many securitization types of securitized assets were used as collateral for repo. These bonds are complicated. The internal workings of the cash flows from the underlying portfolios of loans are allocated in complicated ways, and the underlying loans themselves are complicated. See Gorton (2008). These securitization bonds were also used as the assets in other structures, such as collateralized debt obligations and structured investment vehicles.

Why such were complicated structures used? Our model sheds light on this issue. Clearly, if complexity raises the cost of producing information, raises γ , this can be welfare improving. Suppose that agent A could choose a level of complexity for the security designed at $t=1$. This corresponds to choosing some γ less than a given maximum. For large w , agent A would always choose to issue the most complex security, the one with the maximum γ because this maximizes the amount of debt that will be accepted by agent B without triggering information production.

G. Lender-of-Last-Resort

What exactly is the role of the lender-of-last resort? In our set-up this is clear. The lender-of-last-resort's role is to exchange information-insensitive debt for information-sensitive debt, possibly at a subsidized price to prevent information production, or, to make the private debt, which has become information-sensitive, information-insensitive. This prevents the crisis from being worse than the shock k . A lender-of-last-resort can prevent the deleterious effects of the switch to adverse selection. If the lender-last-resort were to purchase agent B's bond by issuing a riskless bond to agent B in exchange, such that there was no incentive to produce private information, adverse selection could be avoided. Or, if the central bank simply guaranteed the bond at a value such that agent B did not produce information then the same goal would be accomplished. In any case, the central bank would have to have some ability to tax at the final date as the proceeds from agent A's project might not cover the central bank's debt or guarantee. But, as presently constituted the model has no agents to tax at the final date.

7. Conclusion

Even before deposit insurance, checks changed hands without due diligence about the banks backing them. Billions of dollars are traded in sale and repurchase (repo) markets overnight, very quickly, every day, without extensive due diligence (i.e., information production) on the bonds used as collateral. Much corporate debt is purchased and traded based only on ratings. Trade is facilitated by a lack of information, in fact, by ignorance. We showed that opacity Pareto-dominates (even partial) transparency when agents trade.

Debt is the optimal contract for providing liquidity. It is optimal in three senses. First, with respect to public signals, it retains the most value and so produces the most intertemporal carrying capacity. Second, when costly private information can be produced, causing adverse selection, debt minimizes the incentive to produce private information and so reduces the adverse selection. Finally, when there is adverse selection, debt is optimal in maximizing the amount of consumption that can be achieved via trade. In the first two cases, debt is optimal because it least information-sensitive. In the third case, debt is optimal because it maximizes the amount traded. But, while debt is optimal there can be a collapse of trade when the public signal causes information-insensitive debt to become information-sensitive. In that case, less is traded than would be traded were no information privately produced.

We propose a measure of information-sensitivity which is a kind of measure of tail risk. For a buyer, it is defined as the expected overpayment in "bad" states, i.e. the expected sum of overpayment in all

states where $s(x) < p$. Analogously, for a seller information-sensitivity is the expected loss due to charging too little in “good states, i.e. the expected total loss in all states where $s(x) > p$. By taking this ex ante interpretation of potential ex post realized losses, we can use this definition as measure of liquidity. With respect to this measure debt is the optimal security for liquidity provision.

Systemic crises concern debt. The crisis that can occur with debt is due to the fact that the debt is not riskless. A bad enough shock can cause information insensitive debt to become information sensitive, make the production of private information profitable, and trigger adverse selection. Instead of trading at the new and lower expected value of the debt given the shock, agents trade much less than they could or even not at all. There is a collapse of trade. The onset of adverse selection is the crisis.

Appendix A: Proofs

Proposition 1 (Perfect Information and Ignorance: Welfare Comparison): Assuming that $x_L < w \leq x_H$, the ex ante efficient allocation under Ignorance yields a strictly higher welfare than the efficient allocations under Perfect Information.

Proof: The proposition is proven following a lemma.

Lemma A1 (Characterization of the Full Set of Efficient Allocations): Suppose the agents have symmetric and perfect information. The set of efficient allocations satisfying individual rationality is given as follows:

$$\begin{pmatrix} c_{A_t} \\ c_{B_t} \end{pmatrix} = \beta_{A_t} \cdot \begin{pmatrix} c_{A1}^{\max} \\ c_{A2}^{\max} \end{pmatrix} + \beta_{B_t} \cdot \begin{pmatrix} c_{B1}^{\max} \\ c_{B2}^{\max} \end{pmatrix}$$

where $\beta_{h_t} \geq 0, \sum_h \beta_{h_t} = 1$ and $t=1,2$. The notation c_{hk}^{\max} is a vector; it refers to the efficient allocation when agent h obtains the maximum consumption, indicated by column k of the 3×3 consumption matrix. For example, c_{A1}^{\max} refers to the first column of the consumption matrix, c_A^{\max} when A obtains the maximal consumption under the efficient allocation. The full set of efficient allocations will be determined as the description of the linear combinations of two cases, where each of the two agents obtains the maximum surplus.

For $x \geq w$ and $\alpha \geq \frac{x}{w}$ then:

$$c_A^{\max} = \begin{bmatrix} w & x-w \\ 0 & w \end{bmatrix} \quad c_B^{\max} = \begin{bmatrix} \frac{x}{\alpha} & 0 \\ w - \frac{x}{\alpha} & x \end{bmatrix}$$

For $x \geq w$ and $\alpha < \frac{x}{w}$, then:

$$c_A^{\max} = \begin{bmatrix} w & x-w \\ 0 & w \end{bmatrix} \quad c_B^{\max} = \begin{bmatrix} w & x - \alpha w \\ 0 & \alpha w \end{bmatrix}$$

For $x < w$ then:

$$c_A^{\max} = \begin{bmatrix} x & 0 \\ w-x & x \end{bmatrix} \quad c_B^{\max} = \begin{bmatrix} \frac{x}{\alpha} & 0 \\ w - \frac{x}{\alpha} & x \end{bmatrix}$$

Proof of Lemma A1:

Case 1.1 (Perfect Information): $w \leq x$

The outside option of each agent is to consume his endowment. The IR constraints are given as follows:

$$\text{Agent A:} \quad \alpha c_{A1} + c_{A2} \geq x \quad (\text{IR}_A)$$

$$\text{Agent B:} \quad c_{B1} + c_{B2} \geq w \quad (\text{IR}_B)$$

Agent A: Max $\alpha c_{A1} + c_{A2}$ subject to constraints (i) $c_{ht} \geq 0$ for all h and t; (ii) $\sum_h c_{h1} \leq w$; and (iii) $\sum_h c_{h2} \leq x$, and IR_B . The solution is given by:

$$c_A^{\max}(x \geq w) = \begin{bmatrix} w & x - w \\ 0 & w \end{bmatrix}.$$

Agent B: Maximizing U_B subject to constraints (i)-(iii), and IR_A , yields:

$$c_B^{\max}(x \geq w, \alpha \geq \frac{x}{w}) = \begin{bmatrix} \frac{x}{\alpha} & 0 \\ w - \frac{x}{\alpha} & x \end{bmatrix} \quad c_B^{\max}(x \geq w, \alpha < \frac{x}{w}) = \begin{bmatrix} w & x - \alpha w \\ 0 & \alpha w \end{bmatrix}.$$

It is easy to see that $c_{A1} = \frac{x}{\alpha}$ is the “cheapest” way to satisfy IR_A . This implies that $c_{B1} = w - \frac{x}{\alpha}$. The set of all efficient allocations satisfying individual rationality constraints are given by the convex combinations of the allocation matrices.

Case 1.2 (Perfect Information): $w > x$

Note, for $x < w$, the cheapest way to satisfy IR_B is to set $c_{B2} = w$ and $c_{B1} + x = w$. Therefore,

$$c_A^{\max}(x < w) = \begin{bmatrix} x & 0 \\ w - x & x \end{bmatrix}.$$

To maximize the utility of agent B, note that the cheapest way to satisfy IR_A is to set $c_{A1} = \frac{x}{\alpha}$. Therefore,

$$c_B^{\max}(x < w) = \begin{bmatrix} \frac{x}{\alpha} & 0 \\ w - \frac{x}{\alpha} & x \end{bmatrix}.$$

These matrices show the maximum an agent can consume under all parameter constellations. A convex combination yields the set of all efficient allocations satisfying individual rationality under perfect information.

Case 2: Ignorance

The outside options of agents A and B, respectively, are:

$$EU_A = E[X] \quad EU_B = w$$

Under ignorance, an allocation at $t=1$ cannot be contingent on the realization of X. For the allocation at $t=2$, we have to specify what agents can consume, i.e. describing potential contracts $s(x)$ that satisfy individual rationality.

Agent A

$$c_A^{\max}(x_L < w < E[X]) = \begin{bmatrix} w & \max(0, x - s_B(x)) \\ 0 & s_B(x) \end{bmatrix}$$

where $E[s_B(x)] = w$.

Agent B

$$c_B^{\max}\left(x_L < w < E[X], \alpha \geq \frac{E[X]}{w}\right) = \begin{bmatrix} \frac{E[X]}{\alpha} & 0 \\ w - \frac{E[X]}{\alpha} & x \end{bmatrix}$$

$$\rightarrow EU_B^{IG} = w - \frac{E[x]}{\alpha} + E[x]$$

$$c_B^{\max}\left(x_L < w < E[X], \alpha < \frac{E[X]}{w}\right) = \begin{bmatrix} w & s_A(x) \\ 0 & \max(0, x - s_A(x)) \end{bmatrix}$$

where $E[s_A(x)] = E[X] - \alpha w$.

$$\rightarrow EU_B^{IG} = \alpha w \quad .//$$

Proof of Proposition 1 (Comparison of Ignorance to Perfect Information)

Comparison for agent A: See analysis in text.

Comparison for agent B:

(i) $\alpha w \geq x_H$. Then

$$c_B^{\max}(PI, x < w) = \begin{bmatrix} \frac{x}{\alpha} & 0 \\ w - \frac{x}{\alpha} & x \end{bmatrix} \quad c_B^{\max}(PI, x \geq w, \alpha \geq \frac{x}{w}) = \begin{bmatrix} \frac{x}{\alpha} & 0 \\ w - \frac{x}{\alpha} & x \end{bmatrix}$$

$$EU_B^{PI} = \int_{x_L}^w (w - \frac{x}{\alpha} + x) \cdot f(x) dx + \int_w^{x_H} (w - \frac{x}{\alpha} + x) \cdot f(x) dx = w - \frac{E[x]}{\alpha} + E[x]$$

$$EU_B^{IG} = w - \frac{E[x]}{\alpha} + E[x]$$

(ii) $\alpha w < x_H$. Then

$$c_B^{\max}(PI, x < w \text{ or } x \geq w, \alpha \geq \frac{x}{w}) = \begin{bmatrix} \frac{x}{\alpha} & 0 \\ w - \frac{x}{\alpha} & x \end{bmatrix} \quad c_B^{\max}(PI, x \geq w, \alpha < \frac{x}{w}) = \begin{bmatrix} w & x - \alpha w \\ 0 & \alpha w \end{bmatrix}$$

$$EU_B^{PI} = \int_{x_L}^{\alpha w} (w - \frac{x}{\alpha} + x) \cdot f(x) dx + \int_{\alpha w}^{x_H} \alpha w \cdot f(x) dx$$

Case (a) : $\alpha w \geq E[x]$

$$EU_B^{IG} = w - \frac{E[x]}{\alpha} + E[x] = \int_{x_L}^{\alpha w} (w - \frac{x}{\alpha} + x) \cdot f(x) dx + \int_{\alpha w}^{x_H} (w - \frac{x}{\alpha} + x) \cdot f(x) dx$$

$$EU_B^{IG} > EU_B^{PI}$$

$$\Leftrightarrow \int_{x_L}^{\alpha w} (w - \frac{x}{\alpha} + x) \cdot f(x) dx + \int_{\alpha w}^{x_H} (w - \frac{x}{\alpha} + x) \cdot f(x) dx > \int_{x_L}^{\alpha w} (w - \frac{x}{\alpha} + x) \cdot f(x) dx + \int_{\alpha w}^{x_H} \alpha w \cdot f(x) dx$$

$$\Leftrightarrow \int_{\alpha w}^{x_H} (w - \frac{x}{\alpha} + x) \cdot f(x) dx > \int_{\alpha w}^{x_H} \alpha w \cdot f(x) dx$$

$$\Leftrightarrow \int_{\alpha w}^{x_H} (x - \frac{x}{\alpha}) \cdot f(x) dx > (\alpha - 1) \int_{\alpha w}^{x_H} w \cdot f(x) dx$$

$$\Leftrightarrow \frac{\alpha-1}{\alpha} \int_{\alpha w}^{x_H} x \cdot f(x) dx > (\alpha - 1) \int_{\alpha w}^{x_H} w \cdot f(x) dx$$

$$\Leftrightarrow \int_{\alpha w}^{x_H} \frac{x}{\alpha} \cdot f(x) dx > \int_{\alpha w}^{x_H} w \cdot f(x) dx$$

Case (b): $\alpha w < E[x]$

$$EU_B^{IG} = \alpha w = \int_{x_L}^{\alpha w} \alpha w \cdot f(x) dx + \int_{\alpha w}^{x_H} \alpha w \cdot f(x) dx$$

$$EU_B^{IG} > EU_B^{PI}$$

$$\Leftrightarrow \int_{x_L}^{\alpha w} \alpha w \cdot f(x) dx + \int_{\alpha w}^{x_H} \alpha w \cdot f(x) dx > \int_{x_L}^{\alpha w} (w - \frac{x}{\alpha} + x) \cdot f(x) dx + \int_{\alpha w}^{x_H} \alpha w \cdot f(x) dx$$

$$\Leftrightarrow \int_{x_L}^{\alpha w} \alpha w \cdot f(x) dx > \int_{x_L}^{\alpha w} (w - \frac{x}{\alpha} + x) \cdot f(x) dx$$

$$\Leftrightarrow (\alpha - 1) \int_{x_L}^{\alpha w} w \cdot f(x) dx > \frac{\alpha-1}{\alpha} \int_{x_L}^{\alpha w} x \cdot f(x) dx$$

$$\Leftrightarrow \int_{x_L}^{\alpha w} w \cdot f(x) dx > \int_{x_L}^{\alpha w} \frac{x}{\alpha} \cdot f(x) dx \quad //$$

Proof of Proposition 2 when $a > 1$

Lemma 2.1 shows that debt is a least information-sensitive security for the buyer. Now, we will show that the value of information for agent A (the seller of the bond) is not larger than the value of information for agent B (the buyer of the security) given the debt contract, i.e. $\pi_B^D = \pi_A^D$ for $\alpha=1$ and $\pi_B^D > \pi_A^D$ for $\alpha > 1$.

Step 0

Suppose agent A sells and agent B buys the bond with face value D, expected payoff and price both equal w at t=0. Trade without information acquisition gives rise to:

$$EU_A(IG) = \alpha w + \int_D^{x_H} (x - D) f(x) dx$$

$$EU_B(IG) = w$$

Step 1

Given agent A can sell the contract (w,D), what is the payoff to A from acquiring information and trading optimally in each state? There are two cases.

Case 1: Suppose $\alpha w \leq D$.

If agent A sees $x < \alpha w$, he accepts the contract since $\alpha w + \min[x - D, 0] = \alpha w > x$. If agent A sees $x = \alpha w$, he is indifferent. If agent A sees $x > \alpha w$, agent A does not trade since $\alpha w + \min[x - D, 0] = \alpha w + x - D < x$. In other words, agent A does not sell the security (w, D) if he knows that he has to pay back more than αw while consuming w only yields αw . The expected utility (prior to learning x) is:

$$EU_A(I) = \int_{x_L}^{\alpha w} \alpha w f(x) dx + \int_{\alpha w}^{x_H} x f(x) dx$$

$$\Leftrightarrow EU_A(I) = \int_{x_L}^{x_H} \alpha w f(x) dx + \int_{\alpha w}^{x_H} (x - \alpha w) f(x) dx$$

$$\Leftrightarrow EU_A(I) = \alpha w + \int_{\alpha w}^{x_H} (x - \alpha w) f(x) dx$$

The value of information for agent A is

$$\pi_A = \alpha w + \int_{\alpha w}^{x_H} (x - \alpha w) f(x) dx - \left(\alpha w + \int_D^{x_H} (x - D) f(x) dx \right)$$

$$\Leftrightarrow \pi_A = \int_{\alpha w}^{x_H} (x - \alpha w) f(x) dx - \int_D^{x_H} (x - D) f(x) dx$$

$$\Leftrightarrow \pi_A = \int_{\alpha w}^D (x - \alpha w) f(x) dx + \int_D^{x_H} (x - \alpha w) f(x) dx - \int_D^{x_H} (x - D) f(x) dx$$

$$\Leftrightarrow \pi_A = \int_{\alpha w}^D (x - \alpha w) f(x) dx + \int_D^{x_H} (D - \alpha w) f(x) dx$$

Case 2: $\alpha w > D$.

In this case, information has no value at all to the seller. Suppose $x \geq \alpha w > D$. If agent A trades, he is exchanging D for w . If he trades, then his utility is $\alpha w + (x - D)$. If he does not trade, his utility is x . Note, $\alpha w + x - D > x$ since $\alpha w - D > 0$. Consequently, he always trades, i.e. better information has no value to the seller at all. For Step 3 below, it is sufficient to analyze Case 1 ($\alpha w \leq D$).

Step 2

Given that agent B can buy the contract (w, D) , what is the payoff to agent B if he acquires information and trades optimally in each trade? If agent B sees $x < w$ he does not trade. If agent B sees $x = w$ he is indifferent. If agent B sees $x > w$, he trades. The expected payoff (prior to learning x) is:

$$EU_B(I) = \int_{x_L}^w w f(x) dx + \int_w^D x f(x) dx + \int_D^{x_H} D f(x) dx.$$

$$\pi_B = \int_{x_L}^w w f(x) dx + \int_w^D x f(x) dx + \int_D^{x_H} D f(x) dx - w$$

$$\Leftrightarrow \pi_B = \int_w^D x f(x) dx + \int_D^{x_H} D f(x) dx - \int_w^{x_H} w f(x) dx$$

$$\Leftrightarrow \pi_B = \int_w^D (x - w) f(x) dx + \int_D^{x_H} (D - w) f(x) dx$$

Note, $w = \int_{x_L}^D x f(x) dx + \int_D^{x_H} D f(x) dx.$

and substituting in $\pi_B = \int_{x_L}^w w f(x) dx + \int_w^D x f(x) dx + \int_D^{x_H} D f(x) dx - w$ yields

$$\Leftrightarrow \pi_B = \int_{x_L}^w wf(x)dx + \int_w^D xf(x)dx + w - \int_{x_L}^D xf(x)dx - w$$

$$\Leftrightarrow \pi_B = \int_{x_L}^w (w-x)f(x)dx$$

Step 3

This step shows that $\pi_B \geq \pi_A$.

$$\int_w^D (x-w)f(x)dx + \int_D^{x_H} (D-w)f(x)dx \geq \int_{aw}^D (x-\alpha w)f(x)dx + \int_D^{x_H} (D-\alpha w)f(x)dx$$

$$\Leftrightarrow \int_w^D (x-w)f(x)dx - \int_D^{x_H} wf(x)dx \geq \int_{aw}^D (x-\alpha w)f(x)dx - \int_D^{x_H} \alpha wf(x)dx$$

$$\Leftrightarrow \int_D^{x_H} (\alpha w - w)f(x)dx \geq \int_{aw}^D (x-\alpha w)f(x)dx - \int_w^D (x-w)f(x)dx$$

$$\Leftrightarrow \int_D^{x_H} \underbrace{(\alpha w - w)}_{\geq 0} f(x)dx \geq \int_w^D \underbrace{(w - \alpha w)}_{\leq 0} f(x)dx - \int_w^D \underbrace{(x-w)}_{\geq 0} f(x)dx$$

For $\alpha=1$, $\pi_B = \pi_A$ and for $\alpha>1$, $\pi_B > \pi_A$. QED

Proof of Lemma 3.1 (Portfolio Information-sensitivity)

We prove part (ii) first. Note that $p < E[x]=0.5$ and for X uniformly distributed on $[0,1]$, $f(x)=1$.

For $N=2$, we have $\bar{\Pi}_2^{DP} < \bar{\Pi}_1^{DP} = \Pi_1^\Sigma = \pi$ since

$$\Leftrightarrow \frac{1}{2} \int_0^{2p} (2p-y) \cdot f_y(y)dy < \int_0^p (p-x)f(x)dx$$

$$\Leftrightarrow \int_0^{2p} (2p-y) \cdot ydy < 2 \int_0^p (p-x)dx$$

$$\Leftrightarrow p < \frac{3}{4}$$

For $N>2$, we have:

$$\bar{\Pi}_N^{DP} = \frac{1}{N} \int_0^{Np} (Np-y) \cdot f_{Y_N}(y)dy \quad \text{where} \quad f_{Y_N}(y) = \frac{1}{(N-1)!} \sum_{k=0}^N (-1)^k \binom{N}{k} [(y-k)_+]^{N-1}$$

Numerically we show that $\bar{\Pi}_{N+1}^{DP} < \bar{\Pi}_N^{DP}$ for all N .

(i) Part (ii) shows that $\bar{\Pi}_N^{DP} < \Pi_N^\Sigma$. And $\Pi_N^{DP} = \int_0^{Np} (Np-y) \cdot f_{Y_N}(y)dy$

$$< \Pi_N^{PD} = \int_0^{Np} (Np-s(y)) \cdot f_{Y_N}(y)dy \quad \text{since} \quad y = \sum_{n=1}^N x_n \geq s(y) = \sum_{n=1}^N \min[x_n, D] \quad //$$

Proof of Corollary 5.1: (i) If $\gamma \geq \gamma^* \equiv \pi^D$ where π^D is the information sensitivity of debt with price $p=E[s^D(x)]=w$, then it is clearly optimal to choose Strategy (I) with $p=E[s^D(x)]=w$ for any α .

(ii) Strategy II dominates Strategy III, if

$$\alpha q^s + E[x] - p^s \geq \alpha(1 - F(p^I))p^I + E[x] - R$$

$$\Leftrightarrow \alpha q^s - (q^s + \pi^s - \gamma) \geq \alpha(1 - F(p^I))p^I - R$$

For γ close to zero ($\gamma \approx 0$), if $EU_A(II) \geq EU_A(III)$

$$\Leftrightarrow \alpha q^s - q^s - \pi^s \geq \alpha(1 - F(p^I))p^I - (1 - F(p^I))p^I$$

$$\Leftrightarrow (\alpha - 1)q^s - \pi^s \geq (\alpha - 1)(1 - F(p^I))p^I$$

$$\Leftrightarrow \alpha \geq 1 + \frac{\pi^s}{q^s - (1 - F(p^I))p^I} \equiv \alpha'$$

Note, $EU_A(II)$ is increasing in γ and $EU_A(III)$ is decreasing in γ because R is increasing in γ . So if $\alpha \geq \alpha'$, then agent C chooses either Strategy I (debt with $p = E[s^D(x)]$) or Strategy II (debt with $p > E[s^D(x)]$), i.e. issuing debt without triggering information acquisition. (If $\gamma \approx 0$, then $E[s^D(x)] = w$ and $p = E[s^D(x)] + \pi^D$.)

(iii) If $\alpha < \alpha'$, there exists γ' , such that if $\gamma < \gamma'$, Strategy (III) dominates Strategy (II). We compare $EU_A(III)$ with $EU_A(I)$. Note $\alpha(1 - F(d))d - R > \alpha p^I - p^I$, if $\gamma \approx 0$. (In this case, $p^I \approx 0$.) So if γ is sufficiently small, agent A chooses Strategy (III), i.e. issuing debt that triggers information acquisition by agent B. //

Proof of Proposition 8: (i) Corollary 5.1 states that the maximal amount that agent C can buy at $t=2$ where $p_k = E_k[s(x)]$ and that does not trigger information acquisition is given by \hat{p}_k^D which solves:

$$\int_{x_L^k}^{\hat{p}_k^D} (\hat{p}_k^D - s^D(x)) f_k(x) dx = \gamma,$$

and the associated face value \hat{D} solves:

$$\hat{p}_k^D = \int_{x_L}^{\hat{D}} x f_k(x) dx + \int_{\hat{D}}^{x_H} \hat{D} f_k(x) dx.$$

This strategy implies that agent C pays $\hat{p}_k^D = E_k[s^{\hat{D}}(x)]$ at $t=2$ and has an expected consumption of $E_k[s^{\hat{D}}(x)]$ at $t=3$. Thus $EU_C(\hat{D}) = (w - E_k[s^{\hat{D}}(x)]) + \alpha E_k[s^{\hat{D}}(x)]$.

(ii) Agent C chooses a surplus sharing offer, i.e. an offer that gives agent B some of the trading surplus by proposing a price $p^d > E_k[s^d(x)]$. This is another strategy that can avoid information acquisition by agent B. Suppose agent C offers the price $p^d = E_k[s^d(x)] + \varepsilon$ to buy a new debt contract (taking the original bond as the underlying collateral) with face value $\hat{D} < d \leq D$ such that $\pi^d(k) = \gamma$, where $\pi^d(k)$ is the value of information associated with that contract. If agent B accepts the offer, then $EU_B = E_k[s^D(x)] + \varepsilon$. If agent B acquires information and trades optimally, then

$EU_B = E_k[s^D(x)] + \text{prob}(\text{trade})\varepsilon + \pi^d(k) - \gamma$ where $\text{prob}(\text{trade}) = \text{prob}(Q)$
with $Q = \{x : s^D(x) \leq p^d\}$. Agent B does not acquire information if $\varepsilon \geq \text{prob}(\text{trade})\varepsilon + \pi^d(k) - \gamma$.
Define $\varepsilon^* = \frac{1}{1 - \text{prob}(\text{trade})}(\pi^d(k) - \gamma)$.¹²

This strategy yields $EU_C(d) = w - E_k[s^d(x)] - \pi^d + \alpha E_k[s^d(x)]$. The optimal surplus sharing offer has a face value d and price p_k^d that maximizes:

$$(\alpha - 1) \int_{x_L}^{x_H} \min[x, d] f_k(x) dx - \int_{x_L}^{x_H} \max[x - p_k^d, 0] \cdot f_k(x) dx$$

and $\pi^d(k) = \gamma : \int_{x_L}^{x_H} \max[x - p_k^d, 0] \cdot f_k(x) dx = \gamma$.

(iii) Agent C proposes to buy a new debt contract with face value d' where $\hat{D} < d' \leq D$, the price is $p_k^{d'}$, and $E_k[s^{d'}(x)]$. Agent B acquires information and $EU_B = E_k[s^D(x)] + \pi^{d'}(k) - \gamma$ and $EU_C(d') = w + \text{prob}(\text{trade}) \cdot (\alpha \cdot E[x | x < p_k^{d'}] - p_k^{d'})$ where $\text{prob}(\text{trade}) = \text{prob}(Q)$ with $Q = \{x : x \leq p_k^{d'}\}$. The optimal debt contract with information acquisition has a price $p_k^{d'}$ that maximizes: $\text{prob}(\text{trade}) \cdot (\alpha \cdot E[x | x < p_k^{d'}] - p_k^{d'})$ and the associated face value d' solves

$$p_k^{d'} = \int_{x_L}^{d'} x f_k(x) dx + \int_{d'}^{x_H} d' f_k(x) dx.$$

In the cases (ii) and (iii) debt gives rise to the smallest π , i.e. for any contract s with $E[s(x)] = E[s^D(x)]$, $\pi^D \leq \pi^s$. Since debt has the lowest information-sensitivity and thus minimizes the expected overpayment, debt weakly dominates any contract $s \in S$. Consequently, these are the three potential best responses. Agent C compares $EU_C(\hat{D})$, $EU_C(d)$, and $EU_C(d')$ and chooses the strategy with the highest expected utility. //

Proof of Corollary 9.1: (i) Efficient trade between agents B and C arises if agent C chooses Strategy (II), described in either case (bii) or (biii) in Proposition 8.

(bii) $EU_C(d = D) = w - p_k^d + \alpha p_k^d = w - E_k[s^D(x)] - \pi^D + \alpha E_k[s^D(x)]$

(biii) $EU_C(d < D) = w - p_k^d + \alpha p_k^d = w - E_k[s^d(x)] - \pi^d + \alpha E_k[s^d(x)] = \alpha E_k[s^d(x)]$

Consider Strategy (I), i.e., the maximum write-down of debt, which yields:

$$EU_C(\hat{D}) = w - \hat{p}_k^D + \alpha \hat{p}_k^D = w - E_k[s^{\hat{D}}(x)] + \alpha E_k[s^{\hat{D}}(x)].$$

Note, $E_k[s^{\hat{D}}(x)] < E_k[s^d(x)] < E_k[s^D(x)] < w$.

The maximum debt write-down dominates strategy (bii) if:

¹² Note, if $p^d = d$, then agent B always sells without information acquisition.

$$\begin{aligned}
& -E_k[s^{\hat{D}}(x)] + \alpha E_k[s^{\hat{D}}(x)] > -E_k[s^D(x)] - \pi^D + \alpha E_k[s^D(x)] \\
\Leftrightarrow & E_k[s^D(x)] - E_k[s^{\hat{D}}(x)] + \pi^D > \alpha E_k[s^D(x)] - \alpha E_k[s^{\hat{D}}(x)] \\
\Leftrightarrow & \frac{E_k[s^D(x)] - E_k[s^{\hat{D}}(x)] + \pi^D}{E_k[s^D(x)] - E_k[s^{\hat{D}}(x)]} > \alpha \\
\Leftrightarrow & 1 + \frac{\pi^D}{E_k[s^D(x)] - E_k[s^{\hat{D}}(x)]} > \alpha.
\end{aligned}$$

The maximum debt write-down dominates strategy (biii) if:

$$\begin{aligned}
& w - E_k[s^{\hat{D}}(x)] + \alpha E_k[s^{\hat{D}}(x)] > \alpha E_k[s^d(x)] \\
\Leftrightarrow & w - E_k[s^{\hat{D}}(x)] > \alpha E_k[s^d(x)] - \alpha E_k[s^{\hat{D}}(x)] \\
\Leftrightarrow & \frac{w - E_k[s^{\hat{D}}(x)]}{E_k[s^d(x)] - E_k[s^{\hat{D}}(x)]} > \alpha,
\end{aligned}$$

Since $w > E_k[s^d(x)]$, the LHS is larger than 1. //

Appendix B: Optimality of Debt When Information Production Results in Partial Information

In the main text we consider the case where agents either obtain perfect information or are ignorant. Now we suppose that the information that agents learn does not reveal the true realization of X . Instead, the agents receive a signal that is informative, but it provides less than perfect information. We discuss two types of signals: (i) a mean preserving spread; that is, an agent receives a noisy signal of the type $\phi = x + \varepsilon$, where ε is a random with $E[\varepsilon] = 0$; and (ii) an agent learns information about which distribution is relevant, where x initially can be drawn from more than one distribution. Debt remains a least information-sensitive security.

Proposition B1: If the agent receives a noisy signal, then Propositions 2 and 3 hold.

Proof: Upon observing ϕ , the expected payoff of the security is $E[s(x) | \phi]$. The buyer does not buy the security $s(x)$, if he observes $E[s(x) | \phi] < w$. Since $E[x | \phi] = x$, the same arguments as given in the proof of Proposition 2 show that debt gives rise to the smallest set of states where information has value to the buyer and for any of these states $p - E[s(x) | \phi] \leq p - E[s(x)]$. Consequently, Propositions 2 and 3 hold under this information structure. //

Proposition B2: Suppose the signal induces a posterior distribution where the support of each posterior distribution is a partition of the state space X . (See Section 4 for details.) Consider the feasible set of securities $S = \{s: s(x) \leq x, p = E[s(x)] = w\}$. Then debt is the least information-sensitive security in the set S , i.e. $\pi_A^D \leq \pi_A^S$ and $\pi_B^D \leq \pi_B^S$ for all $s \in S$.

Proof: (i) Debt is a least information-sensitive security for agent B; $\pi_B^D \leq \pi_B^S$. For a security s , define $\{F^<\}$ as the set of distributions where $E_k[s(x)] < E_m[s^D(x)] = w$. Analogously, define $\{F^>\}$ to be the set of distributions where $E_k[s(x)] \geq E_m[s(x)]$. Given a debt contract, agent B does not buy the bond

for the price w if he observes $k \in \{F\}^<$. The value of information is $\pi_B^D = \sum_{k \in \{F^{D<}\}} (w - E_k[s^D(x)]) \cdot \lambda_k$.

The value of information of another security $s(x)$ is $\pi_B^S = \sum_{k \in \{F^{S<}\}} (w - E_k[s(x)]) \cdot \lambda_k$. Proposition 6

shows that $\{F^{D<}\} \subseteq \{F^{S<}\}$ and $E_k[s^D(x)] \leq E_k[s(x)]$ for all $k \in \{F^{D<}\}$. Thus $\pi_B^D \leq \pi_B^S$ for all s .

(ii) Debt is a least information-sensitive security for agent A; i.e.

$$\pi_A^D = \sum_{k \in \{F^{D>}\}} \lambda_k (E_k[s^D(x)] - w) \leq \sum_{k \in \{F^{S>}\}} \lambda_k (E_k[s(x)] - w) = \pi_A^S$$

For all s (including debt), we have

$$\sum_{k \in \{F^{<}\}} \lambda_k E_k[s(x)] + \sum_{k \in \{F^{>}\}} \lambda_k E_k[s(x)] = E_m[s(x)]$$

$$\Leftrightarrow \sum_{k \in \{F^{<}\}} \lambda_k (E_k[s(x)] - w) + \sum_{k \in \{F^{>}\}} \lambda_k (E_k[s(x)] - w) = E_m[s(x)] - w$$

$$\Leftrightarrow \sum_{k \in \{F^{>}\}} \lambda_k (E_k[s(x)] - w) = E_m[s(x)] - w + \sum_{k \in \{F^{<}\}} \lambda_k (w - E_k[s(x)])$$

Now compare the information-sensitivity of debt with a security s ,

$$\pi_A^D = \sum_{k \in \{F^{D>}\}} \lambda_k (E_k[s^D(x)] - w) = E_m[s^D(x)] - w + \sum_{k \in \{F^{D<}\}} \lambda_k (E_k[s^D(x)] - w)$$

$$\pi_A^S = \sum_{k \in \{F^{S>}\}} \lambda_k (E_k[s(x)] - w) = E_m[s(x)] - w + \sum_{k \in \{F^{S<}\}} \lambda_k (E_k[s(x)] - w)$$

Case (i) shows that $\sum_{k \in \{F^{D<}\}} \lambda_k (w - E_k[s^D(x)]) \leq \sum_{k \in \{F^{S<}\}} \lambda_k (w - E_k[s(x)])$. Thus $\pi_A^D \leq \pi_A^S$.

Note that there exist posterior distributions where debt is not least information-sensitive. Although the information economics literature typically assumes that the information an agent can acquire is given by an exogenous set of posterior distributions, but in reality the decision about how much information to learn is a creative process and thus information structures are likely to be endogenous. We argue that “generically” debt gives rise to the lowest value of information.

Appendix C: The Non-Monotonicity of the Value of Information

The value of information is in general non-monotonic in the distribution k . In this appendix we discuss this non-monotonicity and provide conditions under which the value of information is monotonic in F_k . Basically, the issue concerns the tail of the distributions, F_k . Stochastic dominance and even partitional information structures do not put enough structure on the (left) tail, but this is the relevant part of the distribution with regard to the information-sensitivity of debt.

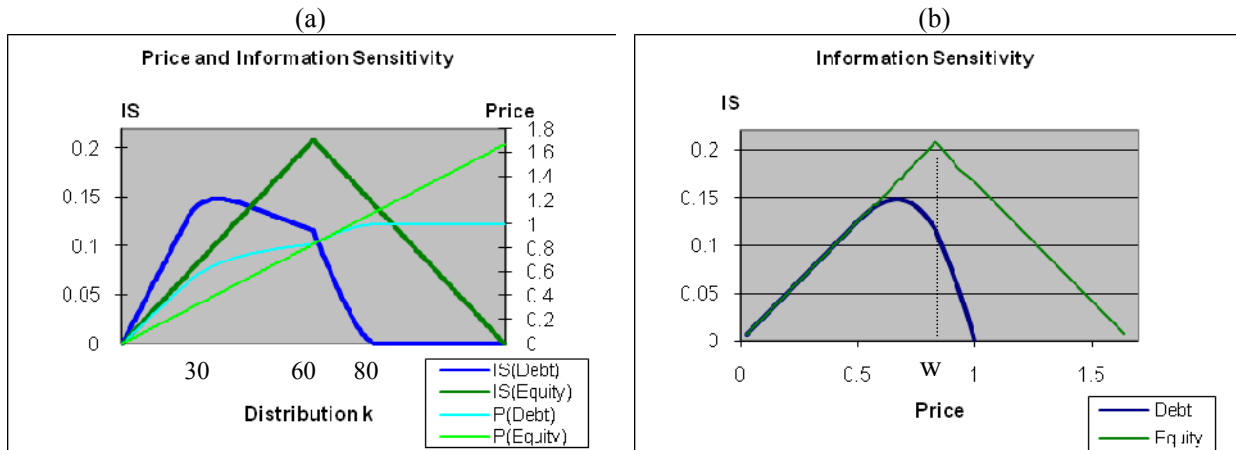
Lemma 2.2 shows that the value of information for the buyer and the seller of a security is the same for $\alpha=1$. Thus the value of information of a security s at $t=2$ is given by:

$$\pi(k) = \int_{x_L}^{x_H} \max[p_k - s(x), 0] \cdot f_k(x) dx \text{ where } p_k = \int_{x_L}^{x_H} s(x) \cdot f_k(x) dx.$$

The intuition is the following: Bad news (distribution with more mass on the left tail) reduces the price of the security, and thus the “area” between price and $s(x)$. But on the other hand that smaller area is evaluated with more probability mass. The overall effect is ambiguous. If we add an additional posterior distribution to the numerical example in section 5, such that $F_1 \sim u[0, 0.2]$, $F_2 \sim u[0.2, 0.8]$, $F_3 \sim u[0.8, 1.2]$, $F_4 \sim u[1.2, 2]$, then prices are increasing in k but, $\pi(4) < \pi(1) < \pi(3) < \pi(2)$.

The following example (with non-partitional information but satisfying FOSD), which includes both debt and equity, illustrates that $\pi(k)$ is a complicated object. Suppose $F_1 \sim u[0, 0.05]$, $F_2 \sim u[0, 0.1]$, ..., $F_{59} \sim u[0, 2.95]$, $F_{60} \sim u[0, 3]$, $F_{61} \sim u[0.05, 3]$, ..., $F_{119} \sim u[2.95, 3]$, and $\lambda_i = \frac{1}{119}$, $w = \frac{5}{6}$. Then: $F_m \sim u[0, 3]$, $f_m = 1/3$ for $x \in [0, 3]$ and $f_m = 0$ else. At $t=1$, if debt with face value $D=1$ is issued then $p_m^D = \frac{5}{6}$, $\pi^D(m) \approx 0.116$ and if equity ($\beta = \frac{5}{9}$) with price $p_m^E = \frac{5}{6}$ is issued, then $\pi^E(m) \approx 0.2083$. At $t=2$: if $F_k = F_{30} \sim u[0, 1.5]$, then $p^D(k=30) = \frac{2}{3}$, $\pi^D(k=30) \approx 0.1482$ and $p^E(k=30) = \frac{5}{12}$, $\pi^E(k=30) \approx 0.1042$. Note that for F_{60} , for example, the value of information for equity (0.1042) is lower than the value of information for debt (0.1482) but so does the price of equity (and the amount that can be potentially traded). Furthermore, in each case the value of information is non-monotonic in k . Figure B1 (a) plots price and information-sensitivity as a function of the posterior distribution k . Figure B1 (b) plots the information-sensitivity in the (p, π) space.

Figure C1



As Figure B1 (a) illustrates, depending on the distribution, the information-sensitivity of debt can be larger or smaller than the information-sensitivity of equity. Stochastic dominance does not imply an ordering for the date 2 information-sensitivity of a security as well as the information-sensitivity across securities.

What is the intuition for the non-monotonicity of the information-sensitivity of a security in the distribution k ? Consider a debt contract. If the distribution k is such that $p_k = E_k[s^D(x)] = D$ or is close to D , then $\pi^D(k) = 0$ or is close to zero. There is very little probability mass or none at all in

the left tail, i.e. $f_k(x) \approx 0$ for $x \in [0, D]$. Thus $\pi(k) = \int_{x_L}^{p_k} (p_k - x) f_k(x) dx = 0$ or is close to zero. On the other hand, if the distribution k is such that $p_k = E_k[s^D(x)]$ is close to x_L , then there is a lot of probability mass around x_L , thus $\pi^D(k)$ is also close to zero.

Furthermore, information-sensitivity is non-monotonic in prices since the price function is weakly increasing in k . See Figure B1(b) which also shows that $\pi^D(p) \leq \pi^E(p)$ for any given price. Note for $k < m$, $p_k^D = p_m^E$ only if $k < m$. In this example, there exist prices such that $\pi^D(p) = \pi^E(p)$ since for $k < 20$, the posterior distribution k only has positive support on $[0, D]$ with $D=1$ and in this range debt has a slope of one and is “equity”.

Figure B1 (a) also shows that for a debt contract with face value D , if distribution k has support such that $x_L^k \geq D$ (i.e. $k \geq 80$ where $F_{80} \sim u[1, 3]$), then $\pi^D(k) = 0$. Note, $x_L^k \geq D$ implies that debt is riskless, i.e. $s^D(x) = D$ and $p_k^D = D$. This observation is one of the results in Gorton and Pennacchi (1990). In contrast to debt, the information-sensitivity of equity is $\pi^E(k) > 0$ for any distribution k where $x_L^k \neq x_H^k$. Note, for $x_L^k \neq x_H^k$, $p_k^E = \int_{x_L^k}^{x_H^k} \beta x \cdot f_k(x) dx = \beta E_k[x]$. There exists a set of x such that $s^E(x) < p_k^E$, since $\beta x < p_k^E = \beta E_k[x]$ and $x < E_k[x]$ for $x \in [x_L, E[x]]$. Thus $\pi_k^E > 0$.

Our main results do not require that the value of information be monotonic. Intuitively, one wants to think of lower k as corresponding to a worse outcome for the economy. This requires monotonicity, that is, more structure on the distributions than provided by stochastic dominance. Two conditions guarantee that this is the case. For future reference we label these as the *monotonicity conditions*.

Consider

$$\pi^D(m) - \pi^D(k) = \int_{x_L}^{p_m} (p_m - x) \cdot f_m(x) dx - \int_{x_L}^{p_k} (p_k - x) \cdot f_k(x) dx.$$

Suppose at $t=1$ debt with face value D has been issued for the price $p_m = E_m[s^D(x)]$. $F_k \succ F_m$ implies $p_k \geq p_m$, and $\pi^D(k) \leq \pi^D(m)$ if:

$$\begin{aligned} & \int_{x_L}^{p_m} (p_m - x) \cdot f_m(x) dx - \int_{x_L}^{p_m} (p_m - x) \cdot f_k(x) dx - \int_{x_L}^{p_k} (p_k - p_m) \cdot f_k(x) dx - \int_{p_m}^{p_k} (p_k - x) \cdot f_k(x) dx \\ \Leftrightarrow & \int_{x_L}^{p_m} (p_m - x) \cdot (f_m(x) - f_k(x)) dx - \int_{x_L}^{p_m} (p_k - p_m) \cdot f_k(x) dx - \int_{p_m}^{p_k} (p_k - x) \cdot f_k(x) dx \geq 0. (+) \end{aligned}$$

In the other case, $F_m \succ F_k$ implies $p_m \geq p_k$ and $\pi^D(k) \geq \pi^D(m)$ if:

$$\begin{aligned} & \int_{x_L}^{p_k} (p_m - p_k) \cdot f_m(x) dx + \int_{x_L}^{p_k} (p_k - x) \cdot f_m(x) dx + \int_{p_k}^{p_m} (p_m - x) \cdot f_m(x) dx - \int_{x_m}^{p_k} (p_k - x) \cdot f_k(x) dx \\ \Leftrightarrow & \int_{x_L}^{p_k} (p_m - p_k) \cdot f_m(x) dx + \int_{p_k}^{p_m} (p_m - x) \cdot f_m(x) dx - \int_{x_m}^{p_k} (p_k - x) \cdot (f_m(x) - f_k(x)) dx \leq 0. (++) \end{aligned}$$

The conditions (+) and (++) are the monotonicity conditions.

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