

# The Evolution of Latent Health over the Life Course

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## **Abstract**

We propose a new method to estimate rich dynamic models of health that exploits longitudinal observations of multiple health measures. Our method adapts and combines two techniques first developed in different contexts. In a first step, we use factor analytic methods to estimate a series of age-specific static measurement models that determine how underlying latent health is related to observed discrete and continuous measures. This step also recovers the unconditional nonparametric distribution of latent health at each age. During the second step, we estimate a stochastic dynamic model of latent health using the method of simulated moments. Specifically, we simulate the dynamic health process and use the previously estimated measurement model to derive an implied set of moments that can be compared with moments in the data. We demonstrate the method by estimating health processes using data from the Health and Retirement Study. Our findings demonstrate

how important it is to use multiple health measures to estimate dynamic models of health. Our estimates which are based on multiple health measures display significantly less persistence in health than do the standard estimates obtained using single measures of health.

## 1 Introduction

All decisions that elderly individuals make depend on their health and how their health changes with age. As physical health deteriorates, the capacity to work declines, leading some to retire from the work-force.<sup>1</sup> Health also affects how much the elderly consume and save, both because health enters directly into preferences and because the elderly save for future, uncertain medical expenditures. How agents behave as they approach old age depends crucially on their current health, on what they expect about their future health, and on how uncertain they are about their future health. Thus, to understand the economic choices the elderly make one needs to understand how health evolves with age. For this reason, a number of researchers in economics have in recent years modeled and estimated how health evolves with age. In particular, Contoyannis, Jones and Rice (2004), Halliday (2008, 2010), and Heiss, Börsch-Supan, Hurd, and Wise (2009) use panel data to estimate latent health model, typically relying on a single health measure such as the

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<sup>1</sup>There is a large literature on the role of health in retirement. In a recent contribution, Bound, Stinebrickner, and Waidmann (2010) find that the health plays a central role in retirement behavior. They report that at age 62, those in poor health exit the labor force at 5 times the rate of those in good health.

self-rated health status (SRHS) or a disability indicator (DI). It is crucial for these estimates that the measure used contains relatively little noise, that it is unbiased, and that it captures most aspects of health. Assuming that the SHRS and the DI satisfy these criteria, these studies suggest that health is very persistent. This is because DI and SRHS are themselves very persistent. It is unclear whether health measured more broadly has the same dynamic characteristics. To answer this question, one requires a broad and diverse set of health measures. At the same time, there is a need for maintaining a simple structure of the dynamic model of health in order to use the estimated health models in the analysis of economic behaviors. Our contribution in this paper is to develop and estimate dynamic models of health that are sufficiently tractable to be useful in standard structural models of economic behavior and are at the same time based on multiple health measures that provide a broad, encompassing description of physical health.<sup>2</sup>

Increasingly individual micro-data sets, often with a panel dimension, collect additional measures of health beyond the SRHS. For example, the Health and Retirement Study (HRS) contains additional self-reported health measures such as an Index of the Ability to perform Daily Activities (IADL) and an index of the ability to perform task that require general physical strength (the "large muscle index"). Further, the HRS collects multiple clinical measures such as a grip strength measure, a timed walk measure, and

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<sup>2</sup>We do not consider mental health in this paper. However, it is straightforward (even if numerically expensive) to extent the analysis to include mental health measures.

a measure of lung functioning. All these measures contain information that can be exploited to describe health more broadly than if one simply relies on the SRHS. When we estimate our dynamic model, we use information from five different measures of health available in the HRS panel, including three clinical measures. As such, our estimates are based on a much richer empirical basis than existing approaches.

We conceive of health as a single index that evolves stochastically over time. This health index maps into measures of health that are observed in the data. The challenge is to estimate both how health and health measures are jointly distributed and how the health index itself evolves stochastically with age. We split this problem into two parts: we first use a factor analytic approach to estimate how health and health measures are jointly distributed within age. Next, we estimate the stochastic process of health itself.

Our approach has a number of methodological advantages. We are robust to misspecifying the dynamic process of health when we estimate the joint distribution of health and health measures. This is because we separately estimate this joint distribution and the process of health. We allow the relation between health and health measures to change with age. For example, the same level of physical health might be reported as "poor health" at age 50 and "excellent health" at age 85. We anchor how health is distributed at different ages, we use the clinical measures. A third advantage of our approach is that we exploit continuous measures of health. This allows us to identify a non-parametric distribution of latent health. In contrast with previous work,

we do not impose normality but instead allow the distribution of the health to exhibit skew and thick tails. We find these features of non-normality to be important in the cross-section.

We consider three alternative dynamic models of health. The simplest model treats the health index as a first-order autoregressive process.<sup>3</sup> The second specification augments the basic model with an endogenous mortality equation. That is, individual survival is allowed to depend on health and we estimate this dependence in the model. The third model allows for measurement specific random effects. This model is motivated by the observation that the auto-correlations of measurements across age are much larger than those observed within age across measurements. To account for this fact, we allow for individual random effects specific to each measurement.

The basic specification displays a surprising degree of persistence in health. The autoregressive parameter is close to one<sup>4</sup> and the variance in the health innovations is very low. Only among older men (age 75+) is there evidence for regression to the mean. Once we allow for endogenous mortality, we find more rapid declines in health among the oldest population which is par-

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<sup>3</sup>We also considered a model that allows for an asymmetry in the regression to the mean in the dynamic process. That is, we allowed the autoregressive component to be of different strength depending on whether the individual is healthier or sicker than the average individual in the population. This model was motivated by the intuitive notion that recovery from sickness differs qualitatively from health differences between individuals that are relatively healthy. When we estimated that model, we did not find strong evidence for asymmetries in the regression to the mean and therefore are not reporting estimates from that model here.

<sup>4</sup>Only among older men (age 75+) is there evidence for regression to the mean in the base model.

tially offset by dynamic selection from the bottom of the health distribution. Allowing for endogenous mortality however has little effect on how much persistence we find in health. We still observe regression to the mean only among the oldest mean and we find little variance in health innovations. However, once we allow for measurement specific random effects, we find evidence for regression to the mean for both genders and among most age-groups. We also find that the variance of health innovations increases by up to an order of magnitude for all ages and both genders. Models with measurement specific random effects fit the data substantially better. Our findings support the need for multiple measurements of health in estimating the dynamics of health in order to distinguish persistence in individual health measurements from persistence in health overall.

While we find much less persistence in the random effects model, this finding should not be overstated. Regardless of the model we estimate, we find large health differences in the population at age 50. And, there is a distinct tendency for these differences to persist over the remainder of the life-cycle.

Our work is related to multiple literatures which we can not adequately review in the space available here. One important literature has made use of the multiple health measures in the HRS to address concerns that justification bias in self-rated health might give rise to the large observed relations between self-rated health and retirement behavior. Here, the work by John Bound and co-authors (in particular, Bound (1991) and Bound, Stinebrick-

ner, and Waidmann (2010)) deserves particular mention.<sup>5</sup> These studies rely on exogeneity assumptions related to objective health measures to implement either structural or instrumental variable methods to estimate effects of health on retirement that are free of justification bias. The dynamics of health or the relation between latent health and the observed health measures itself have typically not been the focus of these studies. More closely related to our work is the recent work on the dynamics of health exemplified by Contoyannis, Jones, and Rice (2004), Halliday (2008, 2010), and Heiss, Börsch-Supan, Hurd, and Wise (2009). These studies attempt to estimate latent variable models of health similar to our basic specification.<sup>6</sup> The fundamental difference between these studies and ours is that the former rely on single indicators of health such as a self-rated health status measure (Contoyannis et al. (2004), Halliday (2010) and Heiss, et al. (2009)) or an inability to work measure (Heiss et al (2009)), sometimes combined with a mortality equation. They are therefore based on substantially less information than our approach, are sensitive to the choice of measure used to represent health, and do not fit the empirical joint distribution of the multiple health measures within and across age. By construction, they cannot distinguish between persistence of latent health and persistence of the unique health measures they use. Some of the conclusions on persistence from these papers have to be

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<sup>5</sup>See also Blau and Gilleskie (2001)

<sup>6</sup>Related literature (Adda, Banks, and von Gaudecker (2009), Adams, Hurd, McFadden, Merrill, and Ribeiro (2003), Borsch-Supan, Heiss, and Hurd (2003), Contoyannis, Jones, and Rice (2004), Halliday (2008)) studies how health and income evolve jointly using dynamic panel data methods.

modified once one exploits a broader set of health variables.

Overall, our paper hopes to contribute to what is, in the words of Halliday (2010), still "very much a fledgling field" by expanding the empirical basis for dynamic latent health models to include multiple health measures. We propose an estimation method and provide estimates of dynamic models of health that successfully describe how health measures at different ages are jointly distributed. At the same time, these dynamic models still maintain a tractable, single index specification for health.

We begin our exploration in Section 2 by describing the data. In Section 3, we present the static factor model that relates health measures to the underlying health index. We show estimates of this model are in Section 4. Section 5 presents the dynamic specifications and Section 6 discusses the method we use to estimate the dynamic models. In Section 7, we show estimates of the dynamic model and conclude in Section 8.

## **2 Measures of Health in the Health and Retirement Study (HRS)**

The Health and Retirement Study (HRS) is uniquely suited for studying the dynamics of health among the elderly population. It contains detailed health measures for a panel of individuals that have been followed for up to 16 years. In this section, we describe the sample and health measures we use in our study. In particular, we motivate why present reduced form results that

motivate why we rely on multiple health measures to study the dynamics of health. These results also motivate how we specify the dynamics of health in our preferred model.

We argue that the traditional focus on SRHS misses a lot of the useful information on health contained in the HRS. This argument is supported by two observations: there is a lot of systematic variation in the additional health measures that can not be explained by SRHS and this variation correlates with important outcomes such as subsequent mortality, labor force participation and weekly earnings. We then use Principle Component Analysis (PCA) to show that much of the information in the available health measures can be captured using a single index model of health augmented by a set of factors that are specific to the various health measures. In the cross-section, these measurement specific components are akin to uncorrelated measurement error. However, dynamically the measurement specific components correlate across age. Overall, our descriptive analysis of these measures therefore leads us to formulate single index factor models of health in the cross-section that, in our preferred specification, contain individual measurement specific random effects.

## **2.1 The Data**

The HRS is a large representative longitudinal survey with both clinical and subjective measures of health. The clinical measures are continuous, which will allow us to estimate non-parametric distributions of health. The

subjective measures are categorical variables. Every two years since 1992, the HRS has surveyed nearly 20,000 respondents representing the US population aged 50 and older. In order to maintain a representative sample of the population aged 50+, new birth cohorts are enrolled every 6 years. We base our study on the 9 surveys conducted between 1992 and 2008.<sup>7</sup>

Our data is based on the RAND HRS data files, the HRS Tracker files and the physical measure files for the 2004-2008 waves. The RAND files (version H) are a user friendly version of the HRS made available by the RAND corporation. We obtain from it the subjective health variables, age, and height. In addition, we merge in 3 objective health measures (peak expiratory air flow, hand grip strength and timed walking speed data) from the 2004-2008 physical measure files. The HRS Tracker files (version 2.0, January 2008) provide vital statistics based on the National Death Index.<sup>8</sup>

Table 1 documents summary statistics for our sample. We use 158,595 observation years obtained from a total of 29,723 individuals. About 5,769 individuals report data for all 9 surveys. The average age in sample is 67 and ca. 44% of respondents are male. The data from the National Death Index (NDI) covers the period from 1992 to 2004 and applies to 25,803 individuals. About 26% of these individuals have died by 2004. Their average age of

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<sup>7</sup>We use data from annual surveys between 1992 and 1996 and the biannual surveys between 1996 and 2008.

<sup>8</sup>The mortality data in the tracker files is based on finder files submitted to NCHS in 1995, 2000, 2002, and 2004. Based on the information in the tracker files, we can determine the vital status and the year and month of death up to 2004.

death was just over 78 years old. A final control used in this analysis is a measure of individual height. We have demeaned this variable by gender. The standard deviation in individual height is just over 7 centimeters.

The HRS collects a multitude of subjective health measures and we chose two from amongst these: an index of large muscle strength (LMI) and the SRHS. The SRHS are reported on a 5 point scale from poor to excellent. The LMI is derived from variables indicating difficulties in four tasks: sitting for two hours; getting up from a chair; stooping, kneeling or crouching; and pushing or pulling a large object.<sup>9</sup> In addition to the subjective health measures, we use 3 clinical health measures collected since 2004. Hand grip strength measures general muscle strength and of the presence of arthritis and other conditions in the hand. Hand grip strength has been shown to be related to general physical and medical status and predicts mortality. The grip strength variable is roughly log normally distributed and we therefore use the log of hand grip strength. The measure of lung function, peak expiratory air flow, is a measure of obstructive lung disease. Declines in peak expiratory air flow have been shown to be related to mortality, as well as cognitive and physical decline. Finally, the timed walk, which has only be collected from individuals aged 65+, has been shown to be a highly reliable measure of functional capacity and it predicts major health outcomes.

As documented in Table 1, we find that respondents on average report

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<sup>9</sup>The SRHS is available for all waves in the study, whereas the LMI is not available for waves 1 and for a subset of the sample in wave 2.

difficulties in 1.3 (out of four) tasks that make up the LMI and they report themselves to be of good health. The LMI and SRHS indicate that the population is slightly less healthy after 2004, but this is likely due to the fact the population is somewhat older during these later years. As is also evident from Table 1, we have many fewer observations for the objective health measures. This is because these measures have only been collected since 2004 and they have only been administered to (random) subsets of the sample. For example, the expiratory air flow and log grip strength measures have been collected from about 3,000 respondents in 2004 and 7,000 each in 2006 and 2008. And, unfortunately, only a few individuals were administered the objective measurements in two adjacent years - which makes them less useful to estimate dynamic models. Finally, the timed walk is only available for individuals aged 65 and older.<sup>10</sup> For these reasons, the subjective measures will be more influential in our estimation results.

## 2.2 The Case for Going Beyond SRHS

Just because measures are available does not mean that they need be used. It is plausible that a summary measure such as the SRHS captures most of the relevant information on health. In that case, using additional health variables make the analysis more complex without adding much additional information. We therefore will now show that SRHS does not fully capture

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<sup>10</sup>We use the timed walk to estimate the static measurement model, but we do not (currently) use it to estimate the dynamic model.

the systematic variation in the health measures included in our analysis and that these additional measures provide additional information for explaining outcomes such as mortality, work participation, and earnings.

Table 2 shows two correlation matrices for males in our sample (results for females are similar and available upon request). Panel A shows the correlations for all 5 health measures in the data after residualizing on gender, height of the individual, and a full set of age indicators. All these correlations are highly statistically significant. The largest is the correlation between the two subjective measures - the SHRS and the LMI. Generally, the correlations between clinical measures and between subjective measures are higher than those between clinical and subjective measures.

Panel B then shows that the additional health measures are correlated with each other even after residualizing on the SRHS. While the correlations in measures do decline, they all remain positive and highly significant. Clearly, these measures are systematically related even after removing the component explained by the SHRS and they contain additional information about the health of individual.

In Table 3 we relate these health measures to meaningful outcomes such as mortality, whether an individual works, and weekly wages among those working. In cols 1 and 2, we estimate a hazard model relating mortality to SRHS and LMI and test whether the LMI is significant.<sup>11</sup> We then con-

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<sup>11</sup>We can not relate mortality to the physical measures, since these are only collected in the HRS subsequent to 2004 and the mortality data is only available up to 2004.

sider regression models of work participation and weekly earnings. We report results with and without the clinical measures, since the latter are only available for a small subset of the sample.

The SRHS measure clearly contains a lot of information for both the mortality and the work decision. For instance, mortality risk conditional on reporting "very good" or "excellent" health is only about 1/5th of the mortality risk for those reporting "poor" health. Similarly, work participation rates and wages are substantially higher among those reporting better health. The magnitude of the relation between SRHS and these outcomes is large enough so that a significant proportion of the data is explained by SRHS.

The other health measures however also explain relevant proportions of the data. For instance, the partial R-squared associated with the additional health measures is about one-half to two-thirds as large as the R-squared associated with the SRHS. And, these additional measures are highly statistically significant explanatory variables for mortality, work, and to a lesser degree wages. Clearly, the full vector of health measures explains much larger fractions of the variation in outcomes than does the SRHS measure on its own.

### **2.3 Health as a Scalar?**

Our approach summarizes how health measures are correlated in the cross-section using a single health factor. Using the correlations of health measures of individuals of the same age, we estimate a factor model. This produces

estimates of how the scalar "health" is distributed and how the health measures are distributed conditional on the health factor. Together these two components explain the joint distribution of health measures within age.

One might question whether a single index model can capture the correlation-structure across health measures observed in the cross-section. To answer this question we perform a principle component analysis (PCA) on the correlation matrix. We perform this analysis separately by 5-year age group and gender and present the results for the males 65 to 69 years old in Table 4.<sup>12</sup>

In PCA, the eigenvalues give the proportion of the total variation in the health measures that is explained by the associated orthogonal factors (principal components).<sup>13</sup> The principal components are ordered by the sizes of their eigenvalues - that is by the amount of variation explained by each. From Table 4, we see that the first principal component explains 38% of the total variation in the data. Examining the remaining eigenvalues, we find that these accounts for a much smaller proportion of the variation and that they account for roughly equivalent amounts (between 11 and 19%). This pattern is consistent with a data-structure in which a single factor (health) explains the cross-correlations across the measures and the remaining variation is captured by orthogonal, measurement specific factors.

Concerning the factor loadings, we find that all health measures load on

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<sup>12</sup>The results from this age group are similar to those obtained for females and for those from other age groups and are available from us upon request.

<sup>13</sup>Our results are based on the correlation matrix and are thus scale invariant and assign equal importance to the variation in each measure.

the first principal component with roughly equal coefficients. We interpret this as evidence that there is indeed a systematic scalar source of variation that drives a lot of the variation in health measures and can reasonably be interpreted as "health". There is an interesting pattern among the loadings on the second principal component. The loadings on the second component for the clinical measures have the opposite sign of the loadings for the subjective measures. This reflects the relatively high correlation within clinical and within self-reported health measures that we already commented upon when examining Table 2: some respondents tend to self-report worse health than seems indicated by their clinical measures. This results in relatively high agreement between the two self-reported health measures as well as between the three clinical measures, while there is less agreement across self-reported and clinical measures. A more complete analysis would require a second factor to subjective attitudes towards health that are not confirmed in the physical measures. We find it difficult to interpret the remaining principal components.

Overall, the patterns in the eigenvalues and eigenvectors from PCA suggest that a model with a single health factor can account for much of how health measures across individuals covary. Given our preference for parsimony, we refrain from using higher dimensional factor models and instead concentrate our efforts on a single factor model.

## 2.4 Correlations in Health Measures across Age

The same health measures from different ages are highly correlated. Table 5 shows the pairwise correlations between our health measures within age as well as the correlations of the health measures with those obtained during the previous survey. The SHRS measure has an autocorrelation of 0.66 - indicating a significant amount of persistence. The autocorrelations for LMI is 0.62 and those for the physical measures are similarly high. At the same time, and as noted above, the correlations within age across measures are much smaller - the largest being 0.44 for the SRHS and LMI measures. The correlations across age and measures are still smaller. These features of the correlations suggest a significant systematic component for each measure that persists over time and is specific to each measures. Our preferred dynamic specification will therefore include measurement specific random effects.

In the next two Sections, we describe the static model and its estimation before we turn to the dynamic model.

## 3 The Static Measurement Model

For each 5-year age-group and gender we wish to represent the joint distribution of subjective and clinical health measures using a single factor model. We refer to this scalar factor as health and denote it by  $h_a$ , where  $a$  denotes age. The factor model specifies how health is distributed and how the health measures relate to individual height and age, to the factor  $h_a$ , and to mea-

surement specific stochastic error. The parameters to be estimated for each 5 year age-group are the parameters entering the distribution of  $h_a$  as well as the parameters of each measurement equation.<sup>14</sup>

To develop this structure, we need additional notation. Let  $Y_a$  stand for a  $m_1$ -vector of categorical health indicators  $y_{j,a}$  and  $X_a$  for a  $m_2$ -vector of continuous health measures  $x_{j,a}$ . The total number of categorical and continuous measurements is  $m = m_1 + m_2$ . Each categorical variable  $y_{j,a} \in Y_a$  is assumed to reflect an underlying latent index  $\tilde{y}_{j,a}$ . The categorical variables are ordered and have  $K_j$  segments defined by cutoffs  $c_{j,g}^k$ . Thus, each  $y_{j,a} \in Y_a$  is linked to its latent counterpart  $\tilde{y}_{j,a} \in \tilde{Y}_a$  :

$$y_{j,a} = \sum_{k=1}^{K_j} 1(\tilde{y}_{j,a} \geq c_{j,g}^{k-1}) \quad (1)$$

where  $1(\cdot)$  is an indicator function taking the value 1 if the condition in parentheses is true and 0 otherwise. Collect the latent indices  $\tilde{y}_{j,a}$  and the continuous measurement variables  $x_{j,a}$  in a vector  $Z_a$ .

We then relate the measurements  $Z_a$  to the health factor  $h_a$  as well as  $r$  additional controls  $Q_a$ , and a vector of measurement specific errors  $\epsilon_a$ . The controls  $Q_a$  include age (within age group) and height. Height proxies for the direct effect of physical capacity that is unrelated to health on the clinical

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<sup>14</sup>The youngest age group covers the ages 50–54. The oldest age-group covers a longer interval and includes all respondents aged 85 to 100. We drop respondents older than 100 years.

measures such as log grip strength, expiratory airflow and walking speed.

$$Z_a = \begin{pmatrix} X_a \\ \tilde{Y}_a \end{pmatrix} = \alpha_g + \Lambda'_g h_a + \Theta'_g Q_a + \epsilon_a \quad (2)$$

Equations (1) and (2) define the measurement model. Here,  $\alpha_g$  denotes a m-vector of intercepts,  $\Lambda_g$  denotes a m-vector of factor loadings,  $\Theta_g$  denotes an m-by-r matrix of regression coefficients and  $\epsilon_a$  a m-vector of independently distributed measurement errors.<sup>15</sup> The parameters to be estimated include the factor loadings, the intercepts for the continuous measure, the cut-offs for the categorical measures, the regression coefficients, the variances of  $\epsilon_a$  for the continuous measures, as well as the parameters that govern the distribution of the health factor  $h_a$ .<sup>16</sup> All of the parameters in this measurement model are subscripted by g because we estimate the measurement model separately for each gender and 5-year age group. Thus, we rely on only a cross-section of health measures for age group g to identify the "static" parameters  $(\alpha_g, \Lambda_g, Q_g, Var_g(\epsilon_a))$  and the distribution  $F_g(h_a)$ .

Consider how this measurement model is identified.<sup>17</sup> We need to identify the parameters  $(\alpha_g, \Lambda_g, Q_g)$ , the vector of the variances of  $\epsilon_a$ , and the distribution  $F_g(\cdot)$  of the latent health variable  $h_a$ . Standard factor analytic results imply that, assuming that we have 3 or more measurement variables,

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<sup>15</sup>The assumption of independence is stronger than the uncorrelatedness assumption typical in factor analysis. It is required to obtain identification of the distribution of  $h_a$  without parameteric assumptions.

<sup>16</sup>For the ordered categorical measures, the usual normalizations imply that the intercept is 0 and the variance of the measurement error is 1.

<sup>17</sup>We provide a more complete identification argument in Appendix 1.

we can identify these parameters up to 2 normalizations, one for the scale and one for the location of the distribution of the latent health variable. We normalize the intercept and the factor loading of the log grip strength variable.

Because we normalize the intercept and slope of the log grip strength variable implies that health is measured in units of log grip strength. That is, a unit increase in health implies an increase in expected log grip strength by one unit. Further, the average of health conditional on age equals the average log grip strength for that age. To the extent that the log grip strength is an objective, interpretable measure of health, we can use it to compare the level of health across ages. However, grip strength itself is difficult to interpret in terms of outcomes that individuals care about. Therefore, we will use the estimates of the mortality model obtained in section 6 to renormalize the health factor and describe the distribution of health in units of predicted mortality.

A second set of normalizations accounts for the fact that the subjective measures are ordinal, categorical variables without scale and location. We normalize the intercepts and the error variances of the measurement equations dealing with categorical variables to 0 and 1 respectively.

As we show in the appendix, our approach has the major advantage that we can non-parametrically identify how health is distributed. This result relies on having more than one continuous measures of health such as the log grip strength or the expiratory air flow measure. In practice we need to

impose some parametric functional form. We assume that the latent health variable is distributed as a mixture of normal random variables with different means and variances:

$$F_g(h_a) = p_g * N(\mu_{1,g} + \beta_{1,g}a, \sigma_{1,g}^2) + (1 - p_g) N(\mu_{1,g} + \beta_{1,g}a, \sigma_{2,g}^2) \quad (3)$$

The means of these normal distributions depend linearly on age. The distribution of health conditional on age therefore depends on 7 parameters. These are the mixture probability  $p_g$  as well as the 3 parameters that govern each of the mixture distributions. Mixtures of normal distribution are very flexible and can accommodate skewness, thick tails and bimodalities.

## 4 Estimating the Static Measurement Model

To estimate the static model, we pool all observations in the HRS from the same age group and gender. Furthermore, we combine ages into 8 age bins, ranging from 50-54 to 85+. For each of the age groups, we estimate the specification described in Section 3. We estimate this model using Mplus and have searched intensively over the parameter space to find global maxima. Mplus provides us with asymptotic standard errors which we report in this Section. When we combine the static and the dynamic models, we will bootstrap the estimation procedure to obtain consistent standard errors of all the model parameters.

Depending on the age-group, the measurement model contains either 25 or 29 parameters for a total of 440 parameters. These are too many to discuss in this paper and we therefore present estimates for males and females from 3 age-groups (50-54, 65-69, 80-84) only. The estimates results from the other age-groups are comparable and available from us upon request. Table 6 reports point estimates for the parameters in the measurement model (2).

Table 6

Across all age groups, the estimated parameters are qualitatively similar even if they sometimes differ in magnitude. For instance, the factor loading for SRHS is typically about one-third larger than that for LMI and about 4-7 times the size of the (normalized) loading on the log grip strength variable. The loadings on the air flow and the walking speed measures are by contrast 1-2 times the size of the loading on the log grip strength variable. Similarly, the error variances are quite similar across age-groups. Generally, the loadings and the error variances are precisely estimated.

Consider next the regression coefficients on age for the subjective health measures. These age coefficients are based only on how health responses vary with age within age-group net of changes in the actual health factor. The health factor itself also depends on age and we present estimates of these associations below. The changes in the health factor with age (within age-group)<sup>18</sup> are driven by the average decline in clinical measures. The

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<sup>18</sup>The changes in the health factor across age-groups (not across age within age-group)

regression coefficients on age for the subjective health measures represent changes in subjective health reported by agents that deviates from changes in the clinical measures. Positive coefficients on age such as those observed in the data signify that individuals report better health as they age relative to the physical capabilities as reflected in the clinical measures. This pattern is consistent with subjective health reports that are relative to others of the same age.

The regression coefficients on height are positive for the physical measures, but less so for the subjective measures. For the LMI, we find either a zero coefficient or a negative relation with height. For SRHS, we find a positive impact of height. However, comparing the regression coefficient on height with the factor loading for SRHS, we find that the relation between height and SHRS is much weaker than that between the health factor and SRHS. For the clinical measures, we find that height has a strong impact, especially on the grip strength variable, suggesting that taller individuals are generally stronger. By controlling for height in the measurement model we remove any long-run correlations between height and health. It is not clear that this is the appropriate specification, since height itself may be correlated with health. Nevertheless, to the extent that height is fixed (we measure height as the maximum height reported in the panel and it is thus fixed across age), we believe that the dynamic specifications estimated below 

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reflect the average decline in the normalized health measure, that is the log grip strength measure.

will be unaffected.

In Table 7, we show how the measurement model fits the data using moments obtained from 50-54 year old females. Clearly, the model does a fits the joint distribution of the large muscle index and the self-reported health variable well. Because we only have the log grip strength variable and the expiratory air flow variable for the years 2004 and 2008, and only for a subset of observations during those years, the number of observations for which these variables are available is much smaller ( $N=842$  vs.  $N=11,254$ ). This implies that the parameter estimates of the static model will primarily be driven by the subjective health measures. Nevertheless, we also capture most of the joint variation in the clinical measures. We interpret these results as evidence that the static measurement model in fact captures the variation in health measures within an age-group very well.

Table 7

In Figure 1 shows how average observed and fitted health measures as well as the health factor vary with age.

Figure 1

The model fits the declines in the averages of the various health measures well. The decline in the health factor by construction mirrors the decline in the log grip strength variable closely.

Consider now how the health factor is distributed. For this, recall that the measurement model is normalized against the log grip strength measure. Consequently, a unit change in the health factor corresponds to a unit change in the expected log grip strength.<sup>19</sup> Thus, the distribution in the health factor is meaningful to the extent that one can interpret the distribution of expected grip strength. We acknowledge that grip strength itself is not a particularly interesting outcome. We will therefore revisit the distribution of health after we have estimated the mortality model and will then express health in terms of predicted mortality rates.

Nevertheless, it is of interest to examine the distribution of health even when normalized against the log grip strength variable because it informs us on asymmetries in the distribution of health and whether it is necessary to account for these asymmetries. The distribution of health is governed by 7 parameters: the variance as well as the intercept and age-coefficient for the mean of the two mixture distributions in addition to the weight placed on each of the component distributions. Table 8 presents these estimates for the same 3 age-groups and both genders. The parameters themselves are difficult to interpret, so we also display the means, standard deviation, and skew. We plot the estimated densities in Figure 2.

Table 8

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<sup>19</sup>The grip strength variable itself has been standardized to have a mean of 0 and a standard deviation of 1 across the entire population.

## Figure 2

Mean health according to this health factor declines with age and is lower for females than for males. This gender difference reflects the disadvantage in physical strength that females have relative to males. We believe that these gender differences should not be interpreted as reflecting real health differences - they underscore the need to separately estimate the health models for males and females.

Health is not normally distributed. Rather, health is characterized by left skew. This skew is stronger for the younger population, but we find significant deviations from normality even at older ages. Further, we find health among males is skewed more heavily to the left. Our finding that the distribution of health is non-normal stands in contrast to the normality assumptions standard in the literature on the dynamics of health (eg. Halliday (2010) and Heiss et al. (2009)).

In summary, we find that our health model fits the data in the age-conditional cross-sections well and we find support that the underlying distribution of health is not normally distributed. In addition, the measurement model (2) delivers an estimate of the conditional distribution of measures based on health that we require to estimate the dynamic model.

## 5 The Dynamics of Health

We estimate three dynamic models of health. All three impose that the health scalar  $h_a$  follows a first order autoregressive process with drift:

$$h_{a+1} = \mu_g + \rho_g h_a + v_a \quad (4)$$

The parameters that govern this annual process are the constant  $\mu_g$ , the auto-regressive parameter  $\rho_g$  and the variance of the innovation  $\sigma_{v,g}^2$ . The innovation  $v_a$  captures idiosyncratic shocks to individuals' health and is assumed to be normally distributed. The parameters are indexed by  $g$ , which denotes the 5-year age group of the individual in year  $a$ . This allows the dynamic process governing health to change with age.<sup>20</sup>

To start the random process (4) we require a distribution  $F(h_0)$  that describes how health is distributed in the initial period. For this initial distribution, we use our estimate of how health is distributed among 50-54 year old males and females respectively. This distribution is described in Table 7 and Figure 2 above.

Our basic dynamic specification consists of this initial distribution together with the dynamic equation (4). By combining this basic specification with the measurement model (2) we obtain an implied joint distribution of health measures. This joint distribution depends on the dynamic parameters

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<sup>20</sup>In principle, we could allow the parameters to vary for each age of the individuals, but limited sample size forces us to group individuals into 5-year age groups.

$(\mu_g, \rho_g, \sigma_{v,g}^2)$  and on the parameters of the measurement model.

Our second model augments the basic specification with a mortality equation:

$$\Pr(s_a = 1) = \Phi(\alpha_{0,g} + \alpha_{1,g}h_a) \quad (5)$$

where  $s_a$  is an indicator for whether an individual survives to age  $a + 1$  conditional on being alive at age  $a$ , and  $\Phi(\cdot)$  denotes the cumulative distribution function of the standard normal. Again, we allow the parameters to vary by age-group. Estimating the dynamic model now means estimating  $(\mu_g, \rho_g, \sigma_{v,g}^2)$  as well as  $(\alpha_{0,g}, \alpha_{1,g})$  for all age-groups.

Our third model is motivated by the correlation patterns documented in Table 5. The autocorrelations within health measures across age are much higher than the correlations observed between health measures within or across age. We assume that the measurement error  $\varepsilon_a^m$  pertaining to the measure  $m$  at age  $a$  is composed of an age-constant random effect  $\chi^m$  and an age specific measurement error  $\omega_a^m$ .<sup>21</sup>

$$\varepsilon_a^m = \chi^m + \omega_a^m \quad (6)$$

We assume that  $\chi^m$  is normally distributed with variance  $\sigma_{\chi,m}^2$  and is uncorrelated with (i) the health factor  $h_a$ , (ii) all other random effects  $\chi^{k \neq m}$  and (iii) all age specific measurement errors  $\omega_a^k$ . Furthermore, we assume that the measurement error  $\omega_a^m$  is normally distributed with variance  $\sigma_{\omega,m,g}^2$ .

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<sup>21</sup>These heterogeneous  $\varepsilon_{a,i}^m$  are collected in the the random vector  $\epsilon_a$  in (2).

This variance is allowed to vary across age groups  $g$ . Estimating the model with random effects thus requires estimating an additional 4 age-invariant parameters  $\sigma_{\chi,m}^2$  that govern the variance of the random effects.<sup>22</sup>

We will now describe the general method we use to estimate our three dynamic models.

## 6 Estimating the Dynamic Model

In this section we describe how we estimate the parameters of the dynamic process. Our general approach combines a separately estimated comprehensive measurement model with the method of simulated moments to estimate the dynamic process of health.

### 6.1 A Simulated Method of Moments Approach to Estimating the Dynamic Latent Health Process

We propose a simulation based algorithm that minimizes the distance between moments obtained from simulated health measurement data and moments of the empirical distribution of measures. Let  $\tilde{Z}$  denote a simulated panel data-set containing measures of health for individuals at different ages. Based on  $\tilde{Z}$  we can compute simulated moments  $\tilde{M}(\theta)$  that depend on the parameter vector  $\theta$  governing the dynamic model of health.

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<sup>22</sup>The variances of the age-specific measurement error terms do not need to be estimated as they are determined by the variances of the random effects and the error terms estimated for static model (2) above.

We construct the same moments from the observed data and denote these  $M$ . Our estimator then chooses  $\widehat{\theta}$  to minimize the distance  $D(M - \widetilde{M}(\theta)) = (M - \widetilde{M}(\theta))' W (M - \widetilde{M}(\theta))$  where  $W$  is an appropriately chosen weighting matrix.

The contents of  $\theta$  depend on the model that is estimated. For the most basic model, the parameter vector  $\theta = \{\mu_g, \rho_g, \sigma_{v,g}^2\}_{g=0}^G$  comprises 24 parameters. When we correct for mortality,  $\theta$  contains also the mortality parameters for each age-group:  $\theta = \{\mu_g, \rho_g, \sigma_{v,g}^2, \alpha_{0,g}, \alpha_{1,g}\}_{g=0}^G$ . In that case  $\theta$  has dimension 40. Finally, the random effects model depends on a parameter vector  $\theta$  that consists of 44 parameters:  $\left\{ \{\mu_g, \rho_g, \sigma_{v,g}^2, \alpha_{0,g}, \alpha_{1,g}\}_{g=0}^G, \{\sigma_{\chi,m}^2\}_{m=1}^M \right\}$ .

At the core of the estimator is an algorithm that lets us construct  $\widetilde{M}(\theta)$ . Before we explain this algorithm, note that we have separately estimated how the latent health variable maps into the health measures  $Z_a$  (as shown in Section 3 and 4). We can thus treat this mapping as known. Furthermore, note that the static measurement model estimated at age 50 provides an estimate of the initial distribution  $F(h_0)$  of health for the starting age.

The algorithm to construct the simulated moments of measurements  $\widetilde{M}(\theta_1)$  implied by any parameter realization  $\theta_1$  consists of the following steps:

Step 1 Generate draws of initial health  $\widetilde{h}_0$  for a large simulated sample of individuals by drawing from the estimated distribution  $F_0(\cdot)$  of the latent health variable at the initial age  $a = 0$ .

Step 2 If the dynamic model contains random effects, use  $\theta_1$  to generate  $\{\chi_m\}_{m=1}^M$

for each individual in the simulated sample.

Step 3 Use the dynamic model with the parameters  $\theta_1$  to draw  $\tilde{h}_{a+1}|\tilde{h}_a$  for each individual. This generates a simulated panel of health histories.

Step 4 Use the implied survival probability  $\Phi\left(\alpha_{0,g} + \alpha_{1,g}\tilde{h}_a\right)$  to simulate the mortality process and generate a sample of survivors.

Step 5 For each age in the panel, use the estimates from the static measurement model together with the sample of random effects  $\chi_m$  to draw  $\tilde{z}_a|\tilde{h}_a, \chi_m$  for the sample of survivors.

This leaves us with a panel of measurements  $\{\tilde{z}_a\}_{a=0}^A$  from which we can generate  $\tilde{M}(\theta_1)$  and for which we can generate the distance  $D(M - \tilde{M}(\theta_1))$ . It should be clear that this algorithm can be implemented for very complex dynamic processes. The main constraints that prevent us from estimating richer dynamic models than our current specifications are computational.

## 6.2 Implementing the Simulated Moments Algorithm

In order to implement the above algorithm we must choose the appropriate set of moments  $M(\theta)$ . By construction, the measurement model matches the cross-sectional distribution of the health measures in a given age group. The moments that are available to estimate the dynamic models are therefore moments from the joint distribution of health measures across time. The

HRS imposes a further restriction in that observations are spaced two years apart. We therefore match the following moments for each age-group:

1. For each continuous objective measure we match
  - (a) average change in the measure from age  $a$  to  $a + 2$ . (2 moments)
  - (b) variances in the objective measures in  $a + 2$  (2 moments).
  - (c) The covariances of the measures (both within and across measures) between  $a$  and  $a+2$  (3 moments).
2. For each categorical subjective measure, we match the entire intertemporal transition matrix. Both the large muscle index and the self-rated health index variable have 5 support points and there are therefore 25 transition probabilities. The transition probabilities within each row of the transition matrix must sum to 1 implying 5 restrictions on the transition matrix. This means that each of the ordinal categorical variables contributes 20 moments and we have an additional 20 moments by going across categories (60 moments).
3. We also use the expected value of each continuous variable in  $a + 2$  conditional on the support of each categorical variable in  $a$ . (20 moments)
4. To identify the survival process, we also match
  - (a) mean of the objective measures in  $a$ , conditional on dying before period  $a + 2$  (2 moments)

- (b) marginal distribution for each of the subjective measures, also conditional on dying before period  $a + 2$  (8 moments)
- (c) unconditional mortality rate (1 moment)

For each age group, we thus have a total of 98 moments to estimate  $\theta$ . The estimated parameters are asymptotically consistent regardless of the weighting matrix chosen. However, in finite sample, the choice of the weighting matrix will result in different estimates. We use a grid search to find good starting values and thus need to compare the minimized criterion function across different starting values to ensure that we find the global minimum. This precludes using the conventional two-step Hansen optimal weighting scheme, because in this scheme the minimized criterion depends on the weight matrix, which is itself a function of the initial parameters. Instead, we use the inverse of the variance matrix of the observed moments. This weighting scheme places more weight on those moments that are more precisely estimated.

We bootstrap the standard errors at the individual level using 100 replications. We estimate both the measurement model and the dynamic model within each bootstrap replication and thus obtain standard errors that account for the estimation error in the measurement model.

## 7 Estimates of the Dynamic Model

### 7.1 The Fit of the Model

In Tables 9, 10, and 11, we present the parameters of the three dynamic models of health. We will discuss what these parameters mean in more detail below. At this point, we simply consider how well these models fit the dynamic aspects of the data.

We compare the observed and the fitted correlation matrices of the 4 health measures used to estimate the dynamic models: the log grip strength, the expiratory air flow, the SRHS and the LMI. In Table 12, we show for women between 65 and 69 how the observed and the predicted correlations compare.<sup>23</sup>

As discussed in Section 2, health measures correlate highly across age within but not across measures. These high correlations within but not between measures motivated the random effects model. Unsurprisingly, the basic specification and the mortality corrected model do not capture these high autocorrelations specific to individual measures. Instead, we find that these two basic models obtain very similar correlations across age regardless whether consider the correlation within the same measure or across different measures. For example, the basic and the mortality model predict that the autocorrelation of the expiratory air flow measure is about 0.17. These models find a similar correlation of the expiratory air flow measure with the log

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<sup>23</sup>We get a similar fit for the other groups.

grip strength measure across age and a larger one for the correlation with the two subjective measures. These findings are clearly counterfactual. By contrast, the random effects model fits the data quite well. It allows both for the high autocorrelations within health measures as well as for the lower correlations across health measures and time. The overall dynamic fit of the model is thus substantially improved by including measurement specific random effects in the error component of the models.

## **7.2 The Units of Health: More on the Distribution of Health**

Because of an arbitrary normalization, health is measured in units of log grip strength. Any linear transformation of the health factor is equally capable of capturing the correlation in health measures. And, furthermore, once we describe the health dynamics with a factor  $h$  that follows some dynamic model, we are free to take an arbitrary monotone transformation  $g(h)$  to assign meaning to the latent variable health. For example, rather than measuring health in units of log grip strength, we could normalize health we might use an estimate of the mortality model (eq. 5) to measure health in predicted mortality rates. Transformation the health factor  $h$  into mortality units is useful because mortality units are a more familiar outcome and because it is easier to attach value to mortality risk.

The first panel of Table 13 reproduces the health distribution based on

the log grip strength normalization and previously shown in Table 7 and Figure 2. The second and third panel display the distribution of health in mortality units. The second panel is based on expected mortality itself and is measured in annual percent mortality risk, whereas the third panel shows the distribution of mortality risk relative to the age and gender group specific average mortality risk. In Figure 3, we show how health as measured in predicted mortality is distributed.

Both average mortality and also the variation in predicted mortality risk in the population rises sharply with age. The average mortality risk for females aged 80-84 is for instance about 5 times the risk for women aged 50-54, while the standard deviation in mortality risk for the older group exceeds that of younger women by a factor of 6. At the same time, the health distribution displays right skew for all ages, so that there is a sizeable minority that has dramatically higher risk in all age groups. The skew is somewhat smaller for the older group, but nevertheless the increased dispersion in mortality risks means that there are large differences in predicted mortality in this population at all ages. For example, the 90-10 ratio in predicted mortality is between 1.75 and 2 for all age and gender groups.

As Table 13 and Figure 3 reveal, the health distribution is skewed leading the average predicted mortality rate to exceed that at the median by only between 5 and 10 percent depending on the age and gender group. We judge this degree of skew to be relatively small compared to the overall variation in health observed in this data. There is substantial dispersion in health as

measured by expected mortality rates within age and gender group.

### 7.3 The Persistence and Volatility of Health

How persistent are health differences between individuals across age? Can the sick and sickly expect to recover over time or are their health difficulties likely to remain with them? We answer these questions based on the 3 specifications described above, and our answers differ depending on the specification. In particular, we find that the two models that do not account for the measurement specific random effects predict more persistence and significantly less volatility in health than does our preferred specification that accounts for the persistence that is specific to each measurement. Our results suggest that dynamic models that do not allow for measurement specific persistent differences will attribute much of the persistence in health measures observed across individuals to general health, thus overestimating how stable health differences are in the population.

Consider first the estimates of the basic specification (Table 9) as well as the mortality corrected specification (Table 10).<sup>24</sup> For both models, the parameter estimates display a considerable degree of persistence. The AR-1 parameter is typically close to one, suggesting that health closely resembles a

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<sup>24</sup>In each table, the last column summarizes the average decline in health implied by the parameters. This decline is calculated by taking  $E[h_{a+1}] - E[h_a] = E[\mu + \rho h_a + \varepsilon_{a+1}] - E[h_a] = \mu + (\rho - 1) E[h_a]$  and depends on the two parameters  $(\mu, \rho)$  as well as the observed mean health for each age:  $E[h_a]$ . For the mortality model, this decline is calculated without conditioning on survival - it shows what would happen to average health in the population if individuals were not subject to mortality.

random walk. Only for older men do we find AR-1 coefficients that are below 1, but only for the oldest group can we reject a random walk. Overall, these estimates suggest that individuals do not recover from health difficulties at the frequency we are considering here. Instead of observing regression to the mean in health, existing health differences in the population seem to persist as individuals age. The volatility estimated for these specifications is likewise small. At the annual level, the standard deviation in the innovations is typically below one-tenth and sometimes below one-hundredth of the overall standard deviation in health observed in the population (see Table 8). These estimates of the basic specification thus suggest that health is a very stable process and that health differences between individuals are very persistent across ages.

The estimates of the random effects model in Table 11 however lead us to revise this conclusion. The auto-regressive parameters for women are uniformly smaller than those reported in Table 10. Among older women and especially men, the point estimates of  $\rho$  suggests a strong tendency for health to regress back to the mean. Health is also much more volatile than previously thought. Almost uniformly, the estimated variances in innovations from the random effects model exceed those obtained in the basic specifications; some of them are much greater, particularly for older individuals.

Overall, there is still a substantial degree of persistence in the population, but compared to the estimates in Table 9 and 10, the more complete model allows for a significant degree of regression to the mean among men.

And, the observed volatility now implies that a non-trivial fraction of the population experiences large declines in health relative to the remainder of the population. This is particularly true among the elderly.

## 8 Conclusion

In this paper, we have proposed a new simulation based method of estimating how health evolves as individuals age. Our approach splits the problem into two parts. The first is a static measurement stage that recovers for each age how latent health (conditional on controls) is distributed. The static stage also delivers age-specific estimates of how latent health maps into observed health measurements. The second stage of the approach focuses on the dynamics. Using the estimates of the measurement model, we can generate simulated joint distributions of the manifest measurement variables implied by our model of dynamic health and our estimated measurement model. The estimation proceeds by choosing parameters of the dynamic model that minimize differences between cross-age moments in the simulated and observed distributions of measures.

To demonstrate our method, we have estimated a measurement model using 5 health measure available in the HRS and, based on this measurement model, estimate three different dynamic models of health. Our estimates of the static measurement model allow us to consider non-parameteric estimates of the distribution of health. Using expected mortality to set the scale

of the health distribution, we find that health is non-normally distributed and displays significant left skew, reflecting the observation that a significant fraction in the population registers low values on multiple or all health measures. We also find that the variance in expected mortality increases significantly alongside the average mortality rate as individuals age.

Our simplest dynamic health model displays a very high degree of persistence. According to this model, health evolves as a random walk with very little volatility. When we correct for endogenous mortality selection, we find very little change in the dynamics of health. The persistence is still very large and health still follows a dynamic process that closely resembles a random walk. However, these two models fail to adequately describe the dynamics of health measures observed in the data. In particular, these two models are not capable of reproducing the high degree of autocorrelation in specific health measures observed in the data. To account for this high degree of correlation observed within measures across age, we allow for measurement specific random effects and find that introducing these random effects substantially improves model fit. When we allow for random effects specific to the measures, health displays a greater tendency to regress to the mean and is substantially more volatile across age.

## 9 Appendix: Identification of the Basic Model

In this appendix, we discuss the identification of the static measurement model as well as of the basic dynamic model.<sup>25</sup> The parameters that need to be identified are the dynamic parameters  $(\mu_a, \rho_a, \sigma_{v,a}^2)$ , the initial distribution of health  $F(h_0)$ , as well as the parameters from the measurement model  $(\alpha_a, \Lambda_a, c_{j,a}^{k-1})$ . We will show that these parameters are identified up to a normalization on the intercept and factor loading for one of the continuous measurement equations, as well as the standard normalizations on variances and intercepts of categorical measurement equations. We assume that we have access to at least 2 continuous measurement variables and 3 additional continuous or categorical measurement variables.

First, we appeal to standard factor analytic arguments and assert that with 3 continuous and categorical measurement variables, we can identify the parameter vectors  $(\alpha_a, \lambda_a)$  and the variances of  $\epsilon_a$  up to a normalization of one intercept and one factor loading. We will impose these normalizations on the same measurement equation at all ages.

We did not restrict the distribution of  $F(h_a)$  and we therefore need to show, using Kotlarski's Theorem, that  $F(h_a)$  can be non-parametrically identified using two continuous measurements only. Use the first and second continuous measurement for this identification argument:  $x_{1,a}$  and  $x_{2,a}$ . We

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<sup>25</sup>To simplify the notation, we omit the direct dependence of the measurement equations on age and on height. Extending the identification to allow for control variables is unproblematic.

have normalized the factor loading and intercept on the first and thus have

$$\begin{aligned}x_{1,a} &= h_a + \varepsilon_{1,a} \\x_{2,a} &= \alpha_{2,a} + \lambda_{2,a}h_a + \varepsilon_{2,a}\end{aligned}$$

Since  $(\alpha_2, \lambda_2)$  are identified, we can write:

$$\begin{aligned}x_{1,a} &= h_a + \varepsilon_{1,a} \\ \frac{x_{2,a} - \alpha_{2,a}}{\lambda_{2,a}} &= h_a + \frac{\varepsilon_{2,a}}{\lambda_{2,a}}\end{aligned}$$

and can treat the left hand side of both of these equations as observed. Kotlarski's Theorem implies that if  $(h_a, \varepsilon_{1,a}, \frac{\varepsilon_{2,a}}{\lambda_{2,a}})$  are jointly independent and  $E[\varepsilon_{1,a}] = E[\frac{\varepsilon_{2,a}}{\lambda_{2,a}}] = 0$ , then the marginal distribution of  $h$  can be identified from the joint distribution of  $(x_{1,a}, x_{2,a})$ . Therefore,  $F(h_a)$  and the parameters of the measurement equations are identified.

We have yet to discuss the identification of the parameters of the dynamic equation (4). For this purpose, we will restrict attention to two adjacent ages  $(a, a + 1)$ . First, note that we can identify the parameters  $\{\mu_a\}$  using the marginal distributions of health  $h_a$  directly:

$$\begin{aligned}\mu_0 &= E[h_0] \\ E[h_{a+1}] &= \mu_a + \rho_a E[h_a]\end{aligned}$$

Now, from equation (4) we get:

$$\begin{aligned}
E[h_{a+1}|Z_a] &= \mu_a + \rho_a E[h_a|Z_a] + E[\varepsilon_{a+1}|Z_a] \\
\implies \rho_a &= \frac{E[h_{a+1}|Z_a] - \mu_a}{E[h_a|Z_a]}
\end{aligned}$$

$E[h_a|Z_a]$  can be directly obtained using the parameter estimates from the measurement model. However, we do not have direct estimates of  $E[h_{a+1}|Z_a]$ .<sup>26</sup> However, we have the following:

$$\begin{aligned}
E[Z_{a+1}|Z_a] &= E[\alpha_{a+1} + \Lambda_{a+1}h_{a+1} + \varepsilon_{a+1}|Z_a] \\
&= \alpha_{a+1} + \Lambda_{a+1}E[h_{a+1}|Z_a] \\
\Leftrightarrow E[h_{a+1}|Z_a] &= (\Lambda'_{a+1}\Lambda_{a+1})^{-1}\Lambda'_{a+1}(E[Z_{a+1}|Z_a] - \alpha_{a+1})
\end{aligned}$$

where the right hand side can be obtained using the estimated factor loading matrices and the data. Therefore  $\rho_a$  is identified.

To identify  $\sigma_a^2$ , consider the following expression:

$$\begin{aligned}
Var(h_{a+1}|Z_a) &= V(\mu_a + \rho_a h_a + \varepsilon_{a+1}|Z_a) \\
&= \rho_a^2 V(h_a|Z_a) + V(\varepsilon_{a+1}|Z_a) \\
\implies \sigma_a^2 &= Var(h_{a+1}|Z_a) - \rho_a^2 V(h_a|Z_a)
\end{aligned}$$

Again, we obtain  $V(h_a|Z_a)$  directly from the measurement model and we

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<sup>26</sup>Note that  $E[h_{a+1}|Z_a] \neq E[E[h_{a+1}|Z_{a+1}]|Z_a]$ .

need to concern ourselves only with finding  $Var(h_{a+1}|Z_a)$ . For this purpose, we again use the joint distribution of the measurement equations.

$$\begin{aligned}
V(Z_{a+1}|Z_a) &= V(\alpha_{a+1} + \Lambda_{a+1}h_{a+1} + \varepsilon_{a+1}|Z_a) = \Lambda_{a+1}V(h_{a+1}|Z_a)\Lambda'_{a+1} + V(\varepsilon_{a+1}|Z_a) \\
&= \Lambda_{a+1}V(h_{a+1}|Z_a)\Lambda'_{a+1} + V(\varepsilon_{a+1}) \\
&\Leftrightarrow V(h_{a+1}|Z_a) = (\Lambda'_{a+1}\Lambda_{a+1})^{-1}\Lambda'_{a+1}(V(Z_{a+1}|Z_a) - V(\varepsilon_{a+1}))\Lambda_{a+1}(\Lambda'_{a+1}\Lambda_{a+1})^{-1}
\end{aligned}$$

where again the right-hand side is observed or estimable from the static model.

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Table 1: Summary Statistics

	<i>Full Sample: 1992-2008</i>		<i>Only 2004-2008</i>	
	<i>Mean</i>	<i>Std Dev</i>	<i>Mean</i>	<i>Std Dev</i>
Self-reported Health	3.14	1.16	3.09	1.13
Large Muscle Index	3.74	1.32	3.66	1.33
Exp. Air Flow	na	na	0.00	1.00
Log Grip Strength	na	na	0.00	1.00
Log Walking Speed	na	na	0.00	1.00
Age	66.95	10.61	68.58	10.57
Height	0.00	0.07	0.03	0.07
Fraction: Male	0.44		0.44	
Observations	158,595		53,902	

Reported are summary statistics for the full FHS sample as well as for the 2004-2008 years only. The objective health measures are available during the 2004-2008 period and for subsamples of the total sample. We have recoded the Large Muscle Index and the log speed variable so that higher values represent better health. The Large Muscle Index is code as 5 minus the number of (up to) 4 activities for which a respondent reported difficulties. The Self-reported Health Status takes integer values running from 1 to 5. The objective measures are standardized to mean zero and standard deviation 1 in the full sample. The height of individuals is standardized within gender to have a mean of zero and is measured in meters. The physical measures are available for only about 17,000 respondent years for the grip strength

Table 2: Standard Dichotomous Health Measures Capture Only Part of Information in Health Measures

Panel A: Health Measures Residualized on Gender, Height, and Age.

	<i>Self Rated Health</i>	<i>Large Muscle</i>	<i>Peak Expiratory</i>	<i>Log Grip Strength</i>	<i>Log Walking Speed</i>
Self Rated Health Status	1				
Large Muscle Index	0.44	1			
Peak Expiratory Air Flow	0.28	0.18	1		
Log Grip Strength	0.19	0.19	0.27	1	
Log Walking Speed	0.24	0.21	0.27	0.23	1

Panel B: Health Measures Residualized on SHRS, Gender, Height, and Age.

	<i>Self Rated Health</i>	<i>Large Muscle</i>	<i>Peak Expiratory</i>	<i>Log Grip Strength</i>	<i>Log Walking Speed</i>
Self Rated Health Status	na				
Large Muscle Index	na	1			
Peak Expiratory Air Flow	na	0.06	1		
Log Grip Strength	na	0.11	0.23	1	
Log Walking Speed	na	0.12	0.22	0.19	1

A, the health measures have been residualized using a regression of the various health variables on a full set of age dummies, gender, and individuals height. In Panel B, the health measures have been residualized using the same set of controls as above and also the SRHS measure. All of the reported correlations are highly statistically significant with p-values of less than 0.0001.

Table 3 Health Measures, Mortality, and Labor Market Outcomes

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
		Mortality		Work				Weekly Wages			
		Female	Male	Female		Male		Female		Male	
SRHS	Fair	0.52** [0.03]	0.48** [0.03]	0.13** [0.01]	0.13** [0.02]	0.17** [0.01]	0.13** [0.03]	36.15 [58.03]	-25 [338.86]	144.56 [122.89]	85.2 [217.20]
	Good	0.29** [0.02]	0.27** [0.02]	0.24** [0.01]	0.23** [0.02]	0.28** [0.01]	0.23** [0.02]	76.43 [56.11]	126.43 [332.64]	214.01+ [117.94]	157.54 [209.52]
	Very Good	0.19** [0.02]	0.18** [0.02]	0.29** [0.01]	0.27** [0.02]	0.33** [0.01]	0.26** [0.03]	175.20** [56.36]	187.47 [334.13]	402.52** [118.44]	235.87 [212.24]
	Excellent	0.18** [0.02]	0.15** [0.02]	0.31** [0.01]	0.29** [0.03]	0.37** [0.01]	0.32** [0.03]	231.93** [57.93]	410.79 [342.52]	461.10** [121.83]	360.61 [221.15]
LMI	1	0.98 [0.07]	0.97 [0.08]	0.09** [0.01]	0.08** [0.02]	0.13** [0.01]	0.13** [0.03]	72.83+ [42.97]	216.66 [203.02]	95.6 [138.04]	30.94 [237.23]
	2	0.85* [0.07]	0.80* [0.07]	0.13** [0.01]	0.11** [0.02]	0.19** [0.01]	0.15** [0.03]	60.09 [40.59]	21.36 [193.72]	73.26 [129.14]	-42.51 [223.53]
	3	0.83* [0.07]	0.81* [0.07]	0.15** [0.01]	0.10** [0.02]	0.22** [0.01]	0.15** [0.03]	69.49+ [39.78]	84.92 [191.28]	169.05 [125.89]	116.3 [218.68]
		0.69** [0.06]	0.74** [0.06]	0.17** [0.01]	0.14** [0.02]	0.24** [0.01]	0.20** [0.03]	124.61** [38.68]	149.6 [185.32]	325.12** [123.06]	309.44 [213.10]
	Log Grip Strength				0.03** [0.01]		0.02* [0.01]		33.1 [66.91]		89.82 [60.93]
	Exp. Air Flow				0.03** [0.01]		0.03** [0.01]		114.28+ [64.18]		87.44* [36.57]
Test Statistics (Degrees of Freedom) on Exclusion Tests (Chi-squares for Mortality, F-tests for Work and Wages)											
All Measures	810.15 (8)**	900.29 (8)**	900.29 (8,72100)**	58.26 (10,7829)**	815.81 (8,57457)**	52.28 (10,5701)**	17.68 (8,25222)**	2.32 (10,2639)**	14.76 (8,24787)**	5.25 (10,2263)**	
SRHS	545.92 (4)**	613.86 (4)**	614.40 (4,72100)**	50.50 (4,7829)**	671.92 (4,57457)**	39.30 (4,5701)**	19.57 (4,25222)**	2.53 (4,2639)**	12.29 (4,24787)**	1.73 (4,2263)**	
Other Measures (LMI,	23.56 (4)**	20.48 (4)**	196.16 (4,72100)**	17.17 (6,7829)**	243.80 (4,57457)**	18.50 (6,5701)**	4.74 (4,25222)**	1.14 (6,2639)	8.62 (4,24787)**	4.58 (6,2263)**	
Observations	53,956	42,369	72,138	7,869	57,495	5,741	25,260	2,679	24,825	2,303	

Columns 1 and 2 display hazard rates relating mortality up to 2004 to Health Measures estimated with a log-linear correction for the age of individuals. Columns 3-10 report OLS estimates. The dependent variables in column 3-6 are an indicator for working and in column 7-10 the weekly wage conditional on working for pay. Standard Errors are reported below the point estimates in Brackets. Stars denote significance levels: \*\* p<0.01, \* p<0.05, + p<0.1.

Table 4 Principal Component Analysis of Health Measures for High School Graduates Age 65-69

Component:	(1)	(2)	(3)	(4)	(5)
Eigenvalue	1.91	0.97	0.83	0.72	0.57
Proportion of Variation	0.38	0.19	0.17	0.14	0.11
Current / Preceding Eigenvalue		0.51	0.91	0.87	0.79
	Eigenvector				
Large Muscle Index	0.46	-0.59	0	0.3	0.6
Self-rated Health Status	0.51	-0.44	0.12	-0.24	-0.7
Peak Expiratory Airflow	0.47	0.35	-0.16	-0.72	0.34
Log Grip Strength	0.42	0.37	-0.63	0.49	-0.21
Log Walking Speed	0.38	0.45	0.75	0.31	0.01

Reported are the eigenvalues and the eigenvectors of the correlation matrix of the health measures for male high school graduates between age 65 and 69. Below the eigenvalues, the share in the overall variation is reported and below this the ratio of the eigenvalue of the current principal component with the preceding one. The principal component analysis is based on 894 respondents with observed values for all 5 health measures.

Table 5 The Correlations in Health Measures within and across Age (Males)

		Large Muscle Index	Self-rated Health Status	Peak Expiratory Airflow	Log Grip Strength	Log Walking Speed
Age	Large Muscle Index	1				
	Self-rated Health Status	0.44	1			
	Peak Expiratory Airflow	0.18	0.28	1		
	Log Grip Strength	0.19	0.19	0.27	1	
	Log Walking Speed	0.21	0.24	0.27	0.23	1
Age-2	Large Muscle Index	0.62	0.41	0.20	0.16	0.25
	Self-rated Health Status	0.41	0.66	0.27	0.13	0.21
	Peak Expiratory Airflow	0.18	0.30	0.62	0.18	0.45
	Log Grip Strength	0.15	0.18	-0.00	0.74	0.05
	Log Walking Speed	0.15	0.12	0.12	0.08	0.49

Reported are pairwise correlations in residualized health measures within age and between age and age-2. The measures are residualized using height and a full set of age dummies. Note that the correlations across age within clinical measures are often based on few measures due to fact that only subsamples of the HRS waves in more recent years include clinical measures. For example the correlations of the log walking speed variables with the other clinical variables are based only 19 and 23 observations respectively.

Table 6: Static Measurement Equation for Selected Groups

Age-Group	Male			Female		
	50-54	65-69	80-84	50-54	65-69	80-84
	Parameters of Measurement Equations					
	Factor Loadings					
Self-rated Health Status	7.118 (1.858)	4.422 (0.591)	3.382 (0.585)	3.884 (0.576)	4.859 (0.582)	4.161 (0.665)
Large Muscle Index	5.867 (1.342)	3.992 (0.480)	2.230 (0.315)	3.557 (0.471)	3.449 (0.357)	3.092 (0.407)
Peak Exp. Air Flow	1.749 (0.467)	2.157 (0.263)	1.141 (0.176)	0.953 (0.133)	1.003 (0.112)	0.983 (0.139)
Log Walking Speed	na	1.577 (0.216)	1.744 (0.242)	na	1.856 (0.203)	2.503 (0.393)
	Error Variances					
Log Grip Strength	0.318 (0.069)	0.329 (0.097)	0.413 (0.080)	0.336 (0.050)	0.359 (0.027)	0.451 (0.043)
Peak Exp. Air Flow	0.654 (0.054)	0.779 (0.039)	0.770 (0.045)	0.341 (0.022)	0.307 (0.012)	0.271 (0.014)
Log Walking Speed	na	0.619 (0.051)	0.548 (0.055)	na	0.639 (0.054)	0.724 (0.069)
	Regression Coefficients Height					
	1.54 (0.275)	1.474 (0.230)	0.922 (0.303)	1.345 (0.219)	0.396 (0.223)	0.307 (0.270)
Large Muscle Index	0.134 (0.249)	-0.138 (0.199)	-0.426 (0.285)	0.154 (0.215)	-0.63 (0.200)	-0.589 (0.249)
Log Grip Strength	2.176 (0.279)	2.332 (0.230)	2.279 (0.378)	1.903 (0.303)	1.557 (0.280)	2.421 (0.385)
Peak Exp. Air Flow	2.885 (0.449)	2.803 (0.378)	2.027 (0.496)	1.949 (0.292)	1.303 (0.238)	0.728 (0.292)
Log Walking Speed	na	0.644 (0.349)	1.040 (0.505)	na	0.549 (0.346)	0.502 (0.583)
	Regression Coefficients Age					
Self-rated Health Status	-0.029 (0.101)	0.161 (0.034)	0.085 (0.043)	0.033 (0.034)	0.115 (0.032)	0.13 (0.037)
Large Muscle Index	-0.010 (0.084)	0.129 (0.031)	0.054 (0.030)	0.050 (0.032)	0.099 (0.023)	0.092 (0.027)
Obs.	6,859	10,590	4,779	11,291	11,930	7,560

The Table shows the estimates of the static measurement model for selected age-groups and gender obtained using Mplus. Reported are analytic standard errors.

Table 7: Fit of Static Measurement Model for Females age 50 to 54

Panel A: Bivariate Frequency Table for Subjective Health Measures

		Large Muscle Index (LMI)				
		4	3	2	1	0
SRHS	Poor	3.17 / 2.92	1.56 / 1.54	0.84 / 0.97	0.56 / 0.52	0.45 / 0.39
	Fair	3.30 / 3.13	3.12 / 3.11	2.92 / 2.94	2.61 / 2.61	2.92 / 3.26
	Good	1.95 / 2.04	3.59 / 3.25	5.35 / 4.85	5.87 / 6.09	11.26 / 11.28
	Very Good	0.81 / 0.68	1.93 / 2.09	3.98 / 4.18	6.79 / 6.73	17.61 / 17.63
	Excellent	0.23 / 0.09	0.57 / 0.61	1.37 / 1.80	3.48 / 3.45	13.76 / 13.88

reported difficulties with the following activities: sitting for two hours, getting up from a chair, stooping or kneeling or crouching, pushing or pulling a large object. Each cell reports observed / predicted frequencies. The number of observations with non-missing values on both the SRHS and LMI among females aged 50-54 is 11,254.

Panel B: Variance Covariances for Objective Health Measures

	Log Grip	Exp. Air Flow
Log Grip	0.43 / 0.44	
Exp. Air Flow	0.13 / 0.10	0.42 / 0.44

Reported are the observed / estimated covariances of the objective measures available for this age-education group. The number of observations in non-missing values on both log grip and expiratory airflow variables among females aged 50-54 is 842.

Table 8: The Distribution of Health for Selected Age Groups

		Male			Female		
Age-Group		50-54	65-69	80-84	50-54	65-69	80-84
		Intercept and Age-regression Coefficient and Variance					
Class 1	Intercept	0.755 (0.765)	3.650 (0.552)	3.897 (1.369)	1.376 (0.567)	1.764 (0.474)	1.712 (0.828)
	Age	0.007 (0.015)	-0.040 (0.008)	-0.049 (0.017)	-0.028 (0.011)	-0.032 (0.007)	-0.032 (0.010)
	Variance	0.028 (0.014)	0.021 (0.007)	0.076 (0.038)	0.106 (0.028)	0.032 (0.007)	0.012 (0.006)
Class 2	Intercept	1.318 (0.755)	3.042 (0.896)	2.096 (1.178)	0.705 (0.419)	0.873 (0.600)	2.230 (0.856)
	Age	0.000 (0.014)	-0.037 (0.012)	-0.021 (0.014)	-0.011 (0.008)	-0.025 (0.009)	-0.043 (0.011)
	Variance	0.006 (0.003)	0.023 (0.030)	0.028 (0.011)	0.002 (0.000)	0.023 (0.019)	0.048 (0.016)
		Logit Parameter and implied Probability of Class 1					
Logit Parameter		-0.792 (0.343)	1.316 (1.157)	-0.159 (0.623)	0.663 (0.103)	1.292 (0.686)	-0.585 (0.478)
Class 1 Probability		0.31	0.79	0.46	0.66	0.78	0.36
		Implied Means, Std Dev, and Skew in the Distribution of Health Factors					
		1.256	0.882	0.150	-0.011	-0.472	-1.157
Standard Deviation		0.147	0.229	0.335	0.288	0.251	0.269
Skew		-1.121	-0.558	-0.568	-0.608	-0.337	-0.343
Observations		6,859	10,590	4,779	11,291	11,930	7,560

Reported are the estimated parameters pertaining to the health factor distribution from the static measurement model described in the text. Also reported are moments from the implied health factor distribution. In parentheses are standard errors.

Table 9: Estimates of a Simple Aging Process:  $h_{a+1}=\mu+\rho h_a+\varepsilon$  - No Mortality Correction

Females						
Age	$\mu$	$\rho$	$\sigma$	$\alpha_0$	$\alpha_1$	$E[h_{a+1}]-E[h_a]$
50-54	-0.032	0.995	0.017	No Mortality Model Estimated		-0.032
	[-0.05,-0.02]	[0.89,1.03]	[0.015,0.022]			
55-59	-0.027	0.970	0.012			
	[-0.05,-0.01]	[0.89,1.03]	[0.004,0.018]			
60-64	-0.032	1.022	0.013			
	[-0.05,-0.02]	[0.90,1.05]	[0.012,0.021]			
65-69	-0.030	1.020	0.013			
	[-0.07,-0.02]	[0.94,1.03]	[0.013,0.019]			
70-74	-0.079	0.965	0.020			
	[-0.11,-0.02]	[0.94,1.04]	[0.009,0.022]			
75-79	-0.024	1.036	0.007			
	[-0.12,-0.01]	[0.94,1.05]	[0.006,0.019]			
80-84	-0.024	1.028	0.010			
	[-0.11,-0.01]	[0.96,1.04]	[0.010,0.017]			
85-100	-0.116	0.954	0.028			
	[-0.12,-0.09]	[0.95,0.97]	[0.028,0.030]			

Age	$\mu$	$\rho$	$\sigma$	$\alpha_0$	$\alpha_1$	$E[h_{a+1}]-E[h_a]$
50-54	-0.108	1.077	0.016	No Mortality Model Estimated		-0.012
	[-0.19,0.09]	[0.87,1.09]	[0.001,0.019]			
55-59	-0.103	1.069	0.012			
	[-0.17,0.02]	[1.00,1.12]	[0.001,0.014]			
60-64	0.011	0.955	0.014			
	[-0.09,0.09]	[0.89,1.04]	[0.011,0.018]			
65-69	-0.057	1.025	0.014			
	[-0.09,0.03]	[0.93,1.04]	[0.010,0.018]			
70-74	-0.063	1.011	0.017			
	[-0.10,-0.02]	[0.94,1.08]	[0.015,0.020]			
75-79	-0.033	0.936	0.019			
	[-0.07,-0.00]	[0.86,1.02]	[0.017,0.023]			
80-84	-0.017	0.880	0.019			
	[-0.05,0.02]	[0.66,0.98]	[0.018,0.030]			
85-100	-0.107	0.799	0.023			
	[-0.15,-0.05]	[0.43,0.95]	[0.015,0.037]			

approach described in the paper. Below the parameter estimates are the 95% confidence intervals obtained by bootstrapping with 100 replications. The change in mean health in the last column is obtained using the parameter estimates presented in the same row.

Table 10: Estimates of a Mortality Corrected Aging Process:  $h_{a+1}=\mu+\rho h_a+\varepsilon$

Females							
Age	$\mu$	$\rho$	$\sigma$	$\alpha_0$	$\alpha_1$	$E[h_{a+1}]-E[h_a]$	Mortality
50-54	-0.031	0.994	0.017	8.167	1.682	-0.031	0.000
	[-0.05,-0.02]	[0.89,1.04]	[0.015,0.022]	[5.1,10.7]	[1.56,2.71]		
55-59	-0.027	0.970	0.012	10.411	2.569	-0.023	0.000
	[-0.05,-0.01]	[0.89,1.04]	[0.004,0.018]	[2.9,10.7]	[0.80,2.51]		
60-64	-0.032	1.021	0.013	7.884	1.676	-0.038	0.000
	[-0.06,-0.02]	[0.90,1.05]	[0.012,0.019]	[3.0,10.6]	[1.33,2.70]		
65-69	-0.030	1.019	0.013	9.924	1.879	-0.039	0.000
	[-0.07,-0.02]	[0.94,1.03]	[0.013,0.019]	[2.7,10.6]	[0.59,2.55]		
70-74	-0.080	0.964	0.023	7.162	1.703	-0.058	0.000
	[-0.11,-0.02]	[0.92,1.04]	[0.011,0.023]	[3.0,10.2]	[1.25,2.61]		
75-79	-0.023	1.037	0.006	7.626	2.152	-0.055	0.000
	[-0.11,-0.01]	[0.95,1.05]	[0.006,0.019]	[3.3,10.0]	[1.27,2.80]		
80-84	-0.024	1.031	0.010	3.751	1.597	-0.060	0.027
	[-0.11,-0.01]	[0.96,1.04]	[0.010,0.018]	[2.5,6.8]	[0.60,2.26]		
85-100	-0.120	0.956	0.028	2.935	1.051	-0.060	0.069
	[-0.19,-0.09]	[0.91,0.97]	[0.022,0.030]	[2.1,3.6]	[0.53,1.44]		

Males							
Age	$\mu$	$\rho$	$\sigma$	$\alpha_0$	$\alpha_1$	$E[h_{a+1}]-E[h_a]$	Mortality
50-54	-0.108	1.077	0.016	8.666	2.425	-0.012	0.000
	[-0.24,0.11]	[0.87,1.14]	[0.003,0.020]	[0.7,8.7]	[0.37,3.46]		
55-59	-0.102	1.069	0.012	0.458	1.801	-0.021	0.005
	[-0.16,-0.02]	[1.00,1.12]	[0.001,0.014]	[-0.2,1.8]	[0.81,2.62]		
60-64	0.006	0.957	0.015	1.228	1.106	-0.038	0.009
	[-0.09,0.08]	[0.89,1.04]	[0.011,0.018]	[0.7,1.9]	[0.61,1.72]		
65-69	-0.058	1.025	0.014	0.648	1.936	-0.037	0.011
	[-0.10,0.03]	[0.93,1.08]	[0.008,0.017]	[0.2,1.2]	[1.32,2.70]		
70-74	-0.065	1.011	0.018	1.087	1.417	-0.057	0.019
	[-0.11,-0.010]	[0.93,1.08]	[0.015,0.020]	[0.7,1.3]	[0.95,1.95]		
75-79	-0.048	0.963	0.019	1.048	1.693	-0.065	0.034
	[-0.08,-0.01]	[0.87,1.04]	[0.015,0.024]	[0.9,1.3]	[1.26,1.99]		
80-84	-0.024	0.894	0.020	1.263	1.648	-0.042	0.062
	[-0.07,0.01]	[0.74,1.02]	[0.015,0.026]	[1.2,1.5]	[1.27,2.25]		
85-100	-0.097	0.799	0.023	1.385	1.702	-0.082	0.104
	[-0.13,-0.05]	[0.52,0.97]	[0.005,0.033]	[1.2,2.2]	[1.48,3.61]		

Presented are parameter estimates of the dynamic model obtained using the simulation approach described in the paper. Below the parameters are 95% Confidence intervals obtained by bootstrapping with 100 replications. The change in mean health in the last column is obtained using the parameter estimates presented in the same row. This is the change in health in the population including descendants (i.e. without removing descendants from the population on which the decline in health is calculated).

Table 11: The Random Effects Model

Females							
Standard Deviations of Random Effects							
Expiratory Air Flow	Log Grip Strength	SRHS	LMI				
0.41	0.38	0.71	0.71				
[0.32,0.44][0.26,0.41][0.70,0.75][0.69,0.72]							
Autoregressive Parameters							
Age	$\mu$	$\rho$	$\sigma$	$\alpha_0$	$\alpha_1$	$E[h_{a+1}]-E[h_a]$	Mortality
50-54	-0.029	0.929	0.062	8.189	1.696	-0.028	0.000
	[-0.044,-0.017]	[0.85,0.99]	[0.060,0.063]	[3.29,10.82]	[1.47,2.77]		
55-59	-0.030	0.949	0.028	10.419	2.575	-0.022	0.000
	[-0.051,-0.011]	[0.86,1.00]	[0.021,0.027]	[2.87,10.42]	[0.85,2.52]		
60-64	-0.034	0.999	0.038	8.065	1.681	-0.034	0.000
	[-0.063,-0.021]	[0.83,1.02]	[0.036,0.041]	[3.07,10.70]	[1.48,2.74]		
65-69	-0.041	0.984	0.021	9.941	1.909	-0.034	0.000
	[-0.083,-0.035]	[0.90,1.00]	[0.021,0.026]	[2.72,10.69]	[0.65,2.56]		
70-74	-0.093	0.938	0.013	7.219	1.710	-0.055	0.000
	[-0.138,-0.048]	[0.88,0.98]	[0.003,0.020]	[2.95,10.32]	[1.27,2.71]		
75-79	-0.046	1.003	0.007	7.678	2.196	-0.049	0.000
	[-0.014,-0.091]	[0.95,1.04]	[0.006,0.022]	[3.23,10.06]	[1.40,2.85]		
80-84	-0.099	0.964	0.090	3.757	1.609	-0.058	0.027
	[-0.087,-0.113]	[0.89,0.97]	[0.090,0.090]	[3.18,6.72]	[1.23,2.33]		
85-100	-0.205	0.893	0.147	2.943	1.073	-0.058	0.072
	[-0.189,-0.372]	[0.79,0.90]	[0.145,0.146]	[2.17,3.70]	[0.57,1.55]		

Males							
Standard Deviations of Random Effects							
Expiratory Air Flow	Log Grip Strength	SRHS	LMI				
0.67	0.33	0.74	0.75				
[0.50,0.78][0.21,0.39][0.74,0.84][0.74,0.81]							
Autoregressive Parameters							
Age	$\mu$	$\rho$	$\sigma$	$\alpha_0$	$\alpha_1$	$E[h_{a+1}]-E[h_a]$	Mortality
50-54	0.008	0.989	0.059	8.846	2.431	-0.006	0.000
	[-0.071,0.265]	[0.77,1.05]	[0.016,0.029]	[0.25,8.78]	[0.35,3.54]		
55-59	-0.072	1.046	0.021	0.458	1.801	-0.019	0.005
	[-0.143,0.036]	[0.96,1.11]	[0.012,0.020]	[-0.20,1.78]	[0.83,2.72]		
60-64	0.025	0.940	0.038	1.228	1.131	-0.036	0.009
	[-0.067,0.103]	[0.87,1.02]	[0.014,0.021]	[0.64,1.90]	[0.61,1.72]		
65-69	-0.035	0.999	0.024	0.648	1.960	-0.036	0.010
	[-0.081,0.057]	[0.90,1.05]	[0.027,0.033]	[0.12,1.08]	[1.35,2.70]		
70-74	-0.024	0.957	0.009	1.083	1.444	-0.055	0.018
	[-0.069,0.040]	[0.87,1.03]	[0.006,0.018]	[0.62,1.34]	[0.97,2.02]		
75-79	-0.032	0.933	0.023	0.996	1.818	-0.062	0.033
	[-0.046,0.034]	[0.76,0.96]	[0.097,0.099]	[0.88,1.24]	[1.27,2.20]		
80-84	-0.005	0.832	0.090	1.338	1.771	-0.034	0.051
	[-0.036,0.024]	[0.73,0.93]	[0.160,0.162]	[1.18,1.51]	[1.30,2.30]		
85-100	-0.093	0.795	0.145	1.421	1.871	-0.078	0.100
	[-0.058,-0.133]	[0.50,0.89]	[0.250,0.251]	[1.35,2.08]	[1.57,3.95]		

described in the paper. Below the parameters are 95% Confidence intervals obtained by bootstrapping with 100 replications. The change in mean health in the last column is obtained using the parameter estimates presented in the same row.

**Table 12: Observed and Predicted Autocorrelation Matrices for Females age 65-69**

<b>Observed Empirical Moments</b>					
		Period t+2			
		Exp. Air Flow	Grip Strength	SRHS	LMI
Period t	Exp. Air Flow	0.75	0.20	0.18	0.10
	Log Grip Str	0.13	0.41	0.17	0.11
	SRHS	0.23	0.20	0.69	0.43
	LMI	0.11	0.19	0.43	0.65

<b>Predicted Moments</b>					
Basic Specification					
		Period t+2			
		Exp. Air Flow	Grip Strength	SRHS	LMI
Period t	Exp. Air Flow	0.17	0.16	0.30	0.23
	Log Grip Str	0.15	0.16	0.27	0.22
	SRHS	0.30	0.26	0.54	0.44
	LMI	0.24	0.22	0.45	0.38

Mortality Corrected Model					
		Period t+2			
		Exp. Air Flow	Grip Strength	SRHS	LMI
Period t	Exp. Air Flow	0.17	0.16	0.30	0.23
	Log Grip Str	0.15	0.16	0.26	0.21
	SRHS	0.30	0.26	0.54	0.44
	LMI	0.24	0.22	0.45	0.37

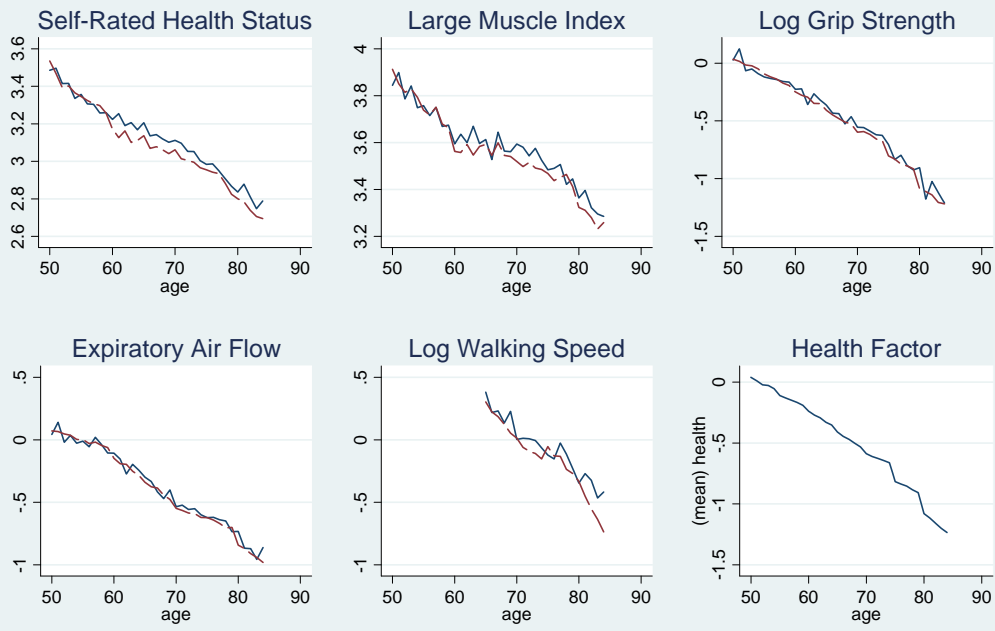
Measurement Specific Random Effect Model					
		Period t+2			
		Exp. Air Flow	Grip Strength	SRHS	LMI
Period t	Exp. Air Flow	0.61	0.15	0.29	0.23
	Log Grip Str	0.15	0.67	0.27	0.21
	SRHS	0.29	0.26	0.58	0.43
	LMI	0.23	0.21	0.44	0.53

The table shows the correlations of health measures across HRS-interviews for females aged 65-69. The top panel displays the empirically observed moments, whereas the bottom 3 panels show the

Table 13: The Distribution of Health under Different Normalizations

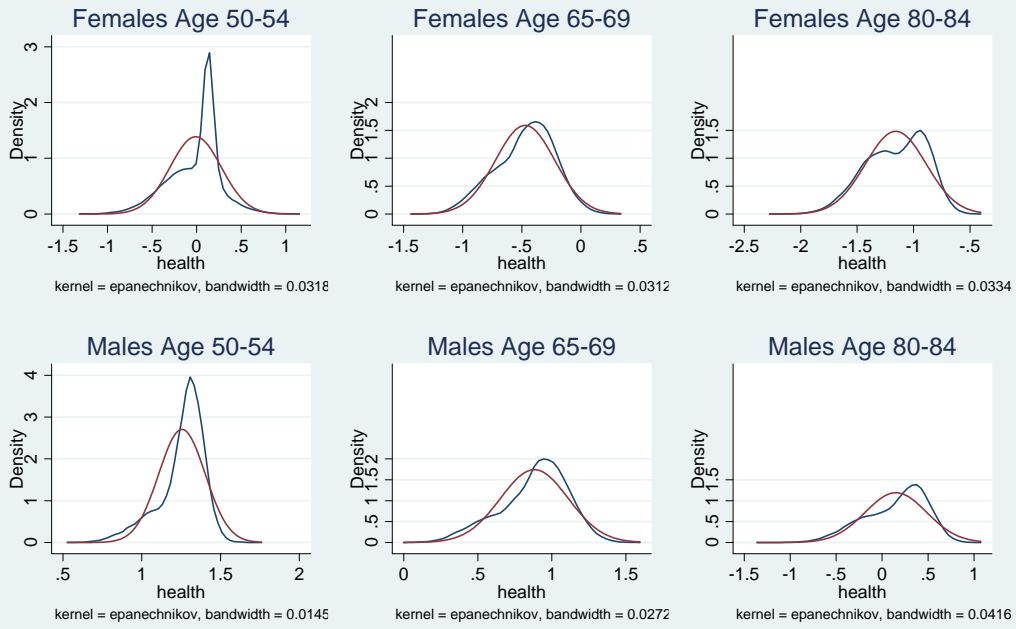
	Implied Means, Std Dev, and Skew in the					
	Male			Female		
Age-Group	50-54	65-69	80-84	50-54	65-69	80-84
	Log Grip Strength					
Mean	1.26	0.88	0.15	-0.01	-0.47	-1.16
Standard Deviation	0.15	0.23	0.34	0.29	0.25	0.27
Skew	-1.12	-0.56	-0.57	-0.61	-0.34	-0.34
	Predicted Mortality (percent)					
Mean	0.95	1.79	7.60	0.83	1.47	4.39
Standard Deviation	0.13	0.69	3.81	0.18	0.35	1.19
Skew	1.50	1.30	1.30	1.25	0.82	0.78
	Relative Mortality Risk (to Mean)					
Mean	1.00	1.00	1.00	1.00	1.00	1.00
Standard Deviation	0.14	0.36	0.25	0.22	0.24	0.27
Skew	1.50	1.30	1.30	1.25	0.82	0.78

### Figure 1: Mean Female Health Measures by Age



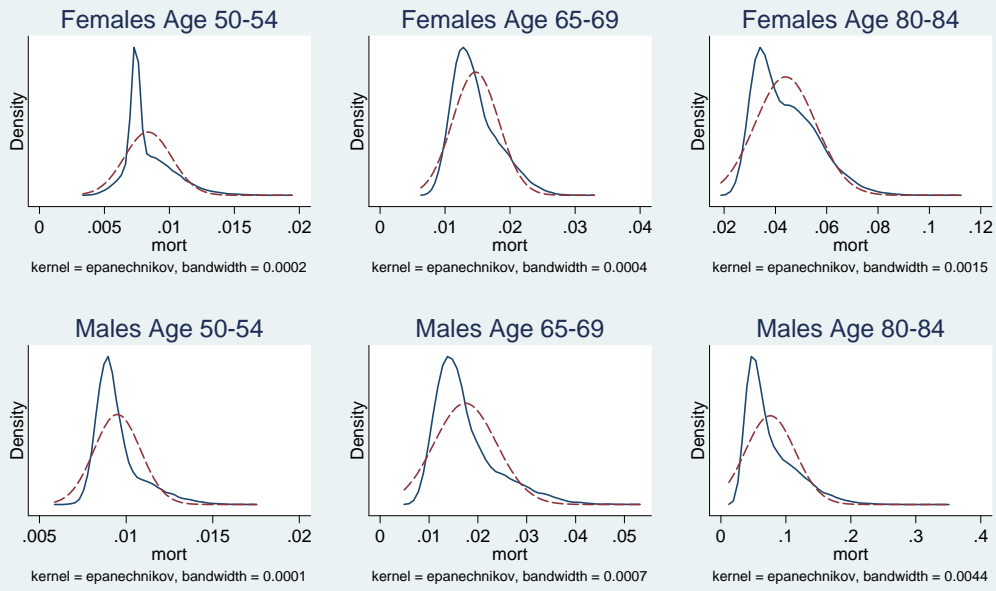
Solid lines depict the observed and broken line the simulated health measures

Figure 2: Estimated Health Factor Densities



Also shown are normal densities with same mean and variance.

Figure 3: Estimated Health Densities  
In Units of Predicted Mortality



Also shown are normal densities with same mean and variance.