Elasticity of Demand and Total Revenue

• Consider a firm facing a downward sloping demand curve. The total revenue the firm receives is the price of the good multiplied by the quantity sold. That is:

\[ \text{Total revenue} = \text{Price} \times \text{Quantity Sold} \]

Suppose the firm considers increasing the price its good. There are two effects:
1. A price effect. After a price increase, each unit sold sells for a higher price, which tends to raise revenue.
2. A quantity effect. After a price increase, fewer units are sold, which tends to lower revenue.

• The price elasticity of demand tells us what happens to total revenue when prices: its size determines which effect – the price effect or the quantity effect – is stronger.

A Tollbooth Example

• Suppose the current toll on a bridge is $1.00, but that the highway department must raise extra money for road repairs. Will raising the toll to $1.10 increase or decrease total revenue?

• Suppose 1,000 cars cross the bridge each day at the current price. So total revenue is:

\[ \text{Total revenue} = 1.00 \times 1000 = $1000.00 \]

• Let’s look at three cases. The Price Elasticity of Demand is

1. inelastic, \( E_p = 0.5 \).
2. elastic, \( E_p = 2.0 \).
3. unit-elastic, \( E_p = 1.0 \).

Case I: Inelastic Demand

• Suppose the price elasticity of demand is 0.5; What effect will the 10 percent increase in the toll have on total revenue?

• We need to solve for quantity sold at $1.10

\[ E_p = \frac{\text{Price elasticity of demand}}{\text{Percentage change in quantity demanded}} \]
\[ = \frac{\text{Percentage change in price}}{Q_{1.10} - Q_{1.00}} \frac{Q_{1.00} - Q_{1.10}}{Q_{1.00}} \]

• We know \( E_p = 0.5 \) and \( Q_{1.00} = 1000 \). So

\[ -0.5 = \frac{Q_{1.10} - 1000}{950} \]

• So \( Q_{1.10} = 950 \) and total revenue at the higher toll is

\[ \text{Total revenue} = 1.10 \times 950 = $1045.00 \]

Case II: Elastic Demand

• Suppose the price elasticity of demand is 2.0; What effect will the 10 percent increase in the toll have on total revenue?

\[ E_p = \frac{\text{Percentage change in quantity demanded}}{\text{Percentage change in price}} \]
\[ = \frac{Q_{1.10} - Q_{1.00}}{Q_{1.00}} \frac{Q_{1.00} - Q_{1.10}}{1000} \]

\[ -2.0 = \frac{Q_{1.10} - 1000}{0.10} \]

• In this case \( Q_{1.10} = 800 \) and total revenue at the higher toll would be

\[ \text{Total revenue} = 1.10 \times 800 = $880.00 \]
Case III: Unit-Elastic Demand

- Suppose the price elasticity of demand is 1.0; What effect will the 10 percent increase in the toll have on total revenue?
- Enough of the math ... Let’s think for a minute. A price elasticity of demand of 1 means that a one-percent increase in the price leads to an one-percent decrease in quantity demanded.
- So if the price increase by 10 percent, quantity sold will decrease by 10 percent.
- So quantity sold at $1.10 will be 900 and total revenue will be
  \[ \text{Total revenue} = 1.10 \times 900 = 990 \approx 1000 \]

More generally ...

- If demand for a good is \textit{elastic} (the price elasticity of demand is greater than 1), an increase in price reduces total revenue. In this case, the price effect is weaker than the quantity effect.
- If demand for a good is \textit{inelastic} (the price elasticity of demand is less than 1), an increase in price increases total revenue. In this case, the price effect is stronger than the quantity effect.
- If the demand for the good is \textit{unit-elastic} (the price elasticity of demand is 1), an increase in price does not change total revenue. In this case the two effects off-set each other.

Conversely for a decrease in price...

- When demand is \textit{elastic}, the quantity effect dominates the price effect; so a decrease in the price increases total revenue.
- When demand is \textit{inelastic}, the price effect dominates the quantity effect; so a decrease in the price reduces total revenue.
- When demand is \textit{unit-elastic}, the effects exactly balance; so a decrease in the price has no effect on total revenue.