Problem Set 3: Due Wednesday, February 2
Adding One-Period Bonds to the Cash in Advance Model

Consider an endowment economy with no uncertainty. Each period the economy is endowed with \( y_t \) units of nonstorable output. A representative household consumes two types of goods, a “cash good” \( c \) and a “credit good” \( x \) subject to

\[
    c_t + x_t = y_t. \tag{1}
\]

The household evaluates streams of the two goods according to the function:

\[
    \sum_{t=0}^{\infty} \beta^t u(c_t, x_t).
\]

There are two types of financial assets, fiat money issued by the government and one-period bonds issued and held by households. Bonds that are issued at date \( t \) and mature at date \( t+1 \) sell at a price \( q_t \). Bonds are priced in equilibrium such that they are in zero net supply. The relative price of goods to money is \( p_t \).

Assume the aggregate money supply follows the law of motion

\[
    M_{t+1}^s = (1 + \pi) M_t^s \tag{2}
\]

with \( \pi M_t^s \) new cash injected into the economy each period via lump-sum transfers. Let \( m_t \) denote the fraction of the total money supply held by the representative household at time \( t \), so the household’s per-period budget constraint is:

\[
    (1 + \pi)m_{t+1} = m_t + b_t + p_t y_t - p_t c_t - p_t x_t - q_t b_{t+1} + \pi \tag{3}
\]

The cash good must be purchased with money, hence the representative household faces a cash in advance constraint:

\[
    p_t c_t \leq m_t - q_t b_{t+1}. \tag{4}
\]

Please do the following:

1. Define a stationary equilibrium with valued fiat currency and one-period bonds.

2. Show that in equilibrium

\[
    \frac{u_x(c, x)}{u_c(c, x)} = q = \frac{1}{1 + r} = \frac{\beta}{1 + \pi}
\]

where \( r \) is the one period interest rate and \( \pi \) is the inflation rate.

3. Use the approximation \( \log(1 + z) \approx z \) to verify the Fisher equation \( r \approx \rho + \pi \) where \( \rho \) is the pure rate of time preference (i.e. \( \beta = \frac{1}{1 + \rho} \)).
Assume the utility function takes the form

\[ u(c, x) = A [\delta c^{\gamma} + (1 - \delta)x^{\gamma}]^{-\frac{1}{\gamma}} \]

so \( \frac{1}{1-\gamma} \) is the elasticity of substitution between cash and credit goods.

4. Using equation (1) and your answer to question 2, derive a demand function for real money balances \( m/p \) as a function of \( y \) and \( r \). Verify that real money demand is proportional to income and decreasing in the interest rate. Is this money demand expression consistent with the quantity theory of money? If so, is velocity increasing or decreasing in \( r \)?

5. Suppose you wanted to estimate the parameters \( \delta \) and \( \gamma \). Describe an estimation strategy (in broad terms). What data would you want to collect?