Problem Set 4: Due Wednesday, February 9

1. In their paper, “The Inflation Tax in a Real Business Cycle Model,” page 738, Cooley and Hansen state that “it is not possible to invoke the second welfare theorem to compute an equilibrium for the economy studied in this paper.”

(a) Explain what it means to “invoke the second welfare theorem” to compute an equilibrium. Exactly what calculations are the authors referring to?

(b) Using equilibrium conditions for the model stated in the paper, explain why these calculations will not locate an equilibrium for the model of this paper.

2. After the first major OPEC-induced oil price increase in 1973, U.S. real GNP fell by 0.5 percent in 1973-74, and then again by 1.3 percent in 1974-75. During the same period, the price level rose by 9 percent (73-74) and 9.8 percent (74-75).

(a) Account for these events as best you can within the framework offered by the Cooley-Hansen model. What pattern of shocks would have produced the observed magnitude and sign of the GNP and price level responses? How are these shocks connected to an increase in oil prices?\(^1\)

(b) Within this framework, what would have been the appropriate response of the U.S. monetary authority to the OPEC price increase?

3. Suppose the consumption index is \( C_i = \left[ \int_0^1 C_{i,j}^{\sigma - 1} \, dj \right]^{\frac{1}{\sigma - 1}} \), where \( C_{i,j} \) is the \( i \)th individual’s consumption of good \( j \). Suppose an individual has amount \( X_i \) to spend on goods. Thus the budget constraint is \( X_i = \int_0^1 P_j C_{i,j} \, dj \). Let the price index, \( P \), be defined as: \( P \equiv \left[ \int_0^1 P_j^{1 - \sigma} \, dj \right]^{\frac{1}{1 - \sigma}} \).

(a) Find the first order condition for the problem of maximizing \( C_i \) subject to the budget constraint. Solve for \( C_{ij} = \left( \frac{P_k}{P_j} \right)^\sigma C_{ik} \) for \( j \neq k \).

(b) Use the budget constraint to find \( C_{ij} \) in terms of \( P_j \) and \( X_i \).

(c) Substitute your result in part (b) into the expression for \( C_i \) and show \( C_i = \frac{X_i}{P} \).

(d) Use your results in parts (b) and (c) to show \( C_{ij} = \left( \frac{X_i}{P} \right) \left( \frac{P_k}{P} \right)^{-\sigma} \).

(e) What is the economic interpretation of the expression you just derived?

---

\(^1\)The Cooley-Hansen model makes no reference to oil or oil prices, or indeed any international trade. The trick here is to interpret the oil price shock as one of the two shocks in the model: a technology shock or a money supply shock.
4. Consider an endowment economy with no uncertainty. Each period the economy is endowed with \( y \) units of nonstorable output. A representative household consumes two types of goods, a “cash good” \( c \) and a “credit good” \( x \). A government consumes \( g_t \) in output each period. Hence

\[
  c_t + x_t + g_t = y. \tag{1}
\]

There are two types of financial assets, fiat money and one-period bonds. Both are issued by the government and held by households.

The government finances its stream of consumption expenditures via lump-sum taxes, \( \tau_t \), borrowing and money creation. It faces a sequence of budget constraints:

\[
g_t + B_t = \tau_t + \frac{B_{t+1}}{R_t} + \frac{M_{t+1} - M_t}{p_t}
\]

where \( B_0 \) and \( M_0 \) are given. Here \( B_t \) is government indebtedness to the households, denominated in time \( t \) goods, maturing at the beginning of period \( t \), and \( M_t \) is the stock of money that the government has issued as of the beginning of period \( t \). The relative price of goods to money is \( p_t \); and \( R_t \) is the real gross rate of return on one-period bonds held from time \( t \) to time \( t+1 \).

The household evaluates streams of the cash and credit goods according to the function:

\[
\sum_{t=0}^{\infty} \beta^t u(c_t, x_t).
\]

The household’s budget constraint is then:

\[
c_t + x_t + \frac{b_{t+1}}{R_t} + \frac{m_{t+1}}{p_t} = y - \tau_t + b_t + \frac{m_t}{p_t}
\]

where \( m_{t+1} \) is nominal balances held between times \( t \) and \( t+1 \); and \( b_t \) is the real value of one-period government bonds holding that are issued at date \( t-1 \) and mature at date \( t \).

The cash good must be purchased with money, hence the representative household faces a cash-in-advance constraint:

\[
p_t c_t \leq m_t.
\]

and household cannot issue money: \( m_t \geq 0 \). Let \( m_0 \) and \( b_0 \) be given.

Please do the following:

(a) Define an equilibrium with valued fiat currency and one-period bonds.

(b) Let \( R_{mt} \equiv p_t/p_{t+1} \) denote the real gross rate of return on money held from \( t \) to \( t+1 \) (i.e. it is the inverse of the inflation rate.) Derive the household’s first order conditions and solve for \( R_t \) and the ratio of \( R_{mt} \) to \( R_t \) in terms of marginal utilities. Why are these two rates of return not equated?
(c) Let the agents utility function take the form

\[ u(c_t, x_t) = \frac{c_t^{1-\delta}}{1-\delta} + \frac{x_t^{1-\alpha}}{1-\alpha}. \]

Solve for a stationary equilibrium in which

\[
\frac{p_t}{p_{t+1}} = R_m \quad \forall t \geq 0
\]

\[ R_t = R \quad \forall t \geq 0 \]

\[ c_t = c \quad \forall t \geq 0 \]

\[ x_t = x \quad \forall t \geq 0 \]

\[ \tau_t = \tau \quad \forall t \geq 0 \]

Solve for the demand function for real money balances \( m/p = f(R_m, R, x). \)

(d) Note that the government budget constraint can now be written as:

\[ g - \tau + B(R-1)/R = f(R_m)(1 - R_m). \]

Provide an economic interpretation for the term \( f(R_m)(1 - R_m). \)