Collateral Crises*

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Abstract

How can a small shock sometimes cause a large crisis when it does not at other times? Financial fragility builds up over time because it is not optimal to always produce costly information about counterparties. Short-term, collateralized, debt (e.g., demand deposits, money market instruments) -private money- is efficient if agents are willing to lend without producing costly information about the value of the collateral backing the debt. But, when the economy relies on this informationally-insensitive debt, information is not renewed over time, generating a credit boom during which firms with low quality collateral start borrowing. During the credit boom output and consumption go up, but there is increased fragility. A small shock can trigger a large change in the information environment; agents suddenly produce information about all collateral and find that much of the collateral is low quality, leading to a decline in output and consumption. A social planner would produce more information than private agents, but would not always want to eliminate fragility.

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1 Introduction

Financial crises are hard to explain without resorting to large shocks. But, the recent crisis, for example, was not the result of a large shock. The Financial Crisis Inquiry Commission (FCIC) Report (2011) noted that with respect to subprime mortgages: “Overall, for 2005 to 2007 vintage tranches of mortgage-backed securities originally rated triple-A, despite the mass downgrades, only about 10% of Alt-A and 4% of subprime securities had been ‘materially impaired’—meaning that losses were imminent or had already been suffered—by the end of 2009” (p. 228-29). Park (2011) calculates the realized principal losses on the $1.9 trillion of AAA/Aaa-rated subprime bonds issued between 2004 and 2007 to be 17 basis points as of February 2011.\(^1\) The subprime shock was not large. But, the crisis was large: the FCIC report goes on to quote Ben Bernanke’s testimony that of “13 of the most important financial institutions in the United States, 12 were at risk of failure within a period of a week or two” (p. 354). A small shock led to a systemic crisis. The challenge is to explain how a small shock can sometimes have a very large, sudden, effect, while at other times the effect of the same sized shock is small or nonexistent.

One link between small shocks and large crises is leverage. Financial crises are typically preceded by credit booms, and credit growth is the best predictor of the likelihood of a financial crisis.\(^2\) Furthermore, often house prices rise concurrently with the credit boom. Clearly, economy-wide high leverage is related to system-wide fragility, but this begs the question of why there was a credit boom to start with and why is it related house prices?

In this paper we develop a theory of financial crises, based on the dynamics of the production and evolution of information in short-term debt markets, which is private money (e.g., demand deposits and money market instruments). We explain how credit booms arise, leading to financial fragility where a small shock can have large consequences. We build on the micro foundations provided by Gorton and Pennacchi (1990) and Dang, Gorton, and Holmström (2011) who argue that short-term debt,

\(^1\)Park (2011) examined the trustee reports from February 2011 for 88.6% of the notional amount of AAA subprime bonds issued between 2004 and 2007.

\(^2\)See, for example, Claessens, Kose, and Terrones (2011), Schularick and Taylor (2009), Reinhart and Rogoff (2009), Borio and Drehmann (2009), Mendoza and Terrones (2008) and Collyns and Senhadji (2002). Jorda, Schularick, and Taylor (2011) (p. 1) study 14 developed countries over 140 years (1870-2008): “Our overall result is that credit growth emerges as the best single predictor of financial instability.”
in the form of bank liabilities or money market instruments, is designed to provide transactions services by allowing trade between agents without fear of adverse selection. This is accomplished by designing debt to be “information-insensitive,” that is, such that it is not profitable for any agent to produce private information about the assets backing the debt, the collateral. But, in a financial crisis there is a sudden loss of confidence in short-term debt in response to a shock; it becomes information-sensitive, and agents produce information, and determine whether the backing collateral is good or not.

In this paper we build on these micro foundations to investigate the role of such information-insensitive debt in the macro economy. We do not explicitly model the trading motive for short-term information-insensitive debt. Nor do we explicitly include financial intermediaries. We assume that households have a demand for such debt and we assume that the short-term debt is issued directly by firms to households to obtain funds and finance efficient projects. The debt that firms issue is backed by collateral. In reality, this collateral may be a portfolio of loans or a portfolio of bonds, or specific bonds, as in private bank notes in the Free Banking Era prior to the U.S. Civil War or modern day sale and repurchase agreements (repo), short-term debt in which the lender receives bonds as collateral.

Information production about the backing collateral is costly to produce, and agents do not find it optimal to produce information at every date. The key dynamic in the model concerns how the perceived quality of collateral evolves if (costly) information is not produced. Collateral is subject to idiosyncratic shocks so that over time, without information production, the perceived value of all collateral tends to be the same because of mean reversion towards a “perceived average quality,” such that some collateral is known to be bad, but it is not known which specific collateral is bad. Agents endogenously select what to use as collateral. Desirable characteristics of collateral include a high perceived quality and a high cost of information production. In other words, optimal collateral would resemble a complicated, structured, claim on housing or land, e.g., a mortgage-backed security.

When information is not produced and the perceived quality of collateral is high enough, firms with good collateral can borrow, but in addition some firms with bad collateral can borrow. In fact, consumption is highest if there is never information production, because then all firms can borrow, regardless of their true collateral quality. The credit boom increases consumption because more and more firms receive
financing and produce output. In our setting opacity can dominate transparency and the economy can enjoy a blissful ignorance. If there has been information-insensitive lending for a long time, that is, information has not been produced for a long time, there is a significant depreciation of information in the economy - all is grey, there is no black and white - and only a small fraction of collateral with known quality.

In this setting we introduce aggregate shocks that may decrease the perceived value of collateral in the economy. It is not the leverage per se that allows a small negative shock to have a large effect. The problem is that after a credit boom, in which more and more firms borrow with debt backed by collateral of unknown type (but with high perceived quality), a negative aggregate shock affects more collateral than the same aggregate shock would affect when the credit boom was shorter or if the value of collateral was known. Hence, the size of the downturn depends on how long debt has been information-insensitive in the past.

A negative aggregate shock reduces the perceived quality of all collateral. This may or may not trigger information production. If, given the shock, households have an incentive to learn the true quality of the collateral, firms may prefer to cut back on the amount borrowed to avoid costly information production, a credit constraint. Alternatively, information may be produced, in which case only firms with good collateral can borrow. In either case, output declines because the short-term debt is not as effective as before the shock in providing funds to firms.

In our theory, there is nothing irrational about the credit boom. It is not optimal to produce information every period, and the credit boom increases output and consumption. There is a problem, however, because private agents, using short-term debt, do not care about the future, which is increasingly fragile. A social planner arrives at a different solution because his cost of producing information is effectively lower. For the planner, acquiring information today has benefits tomorrow, which are not taken into account by private agents. When choosing an optimal policy to manage the fragile economy, the planner weights the costs and benefits of fragility. Fragility is an inherent outcome of using the short-term collateralized debt, and so the planner chooses an optimal level of fragility. This is often discussed in terms of whether the planner should “take the punch bowl away” at the (credit boom) party. The optimal policy may be interpreted as reducing the amount of punch in the bowl, but not taking it away.

We are certainly not the first to explain crises based on a fragility mechanism. Allen
and Gale (2004) define fragility as the degree to which “...small shocks have disproportionately large effects.” Some of the literature focuses on the magnification of shocks, that is, small shocks always generate large effects, while other literature focuses on the sudden collapse of the economy in which small shocks sometimes generate large effects, and sometimes do not. Our work tackles both aspects of fragility.

Among papers that highlight the magnification type of fragility, Kiyotaki and Moore (1997) show that leverage can have a large amplification effect. In their setting, the amount that can be borrowed is also limited by the available collateral. A fundamental shock causes asset cash-flows to fall, which causes the borrower’s equity to fall more, due to the high leverage. Then, going forward, the borrower’s investment capacity falls because the collateral is worth less. This in turn leads more of the asset to be owned by a second-best user. But, the value of the asset to the second-best user is less than that of the constrained borrower, causing asset values to fall, reinforcing the spiral. This mechanism relies on the feedback effects over time to collateral value, while our mechanism is about a sudden informational regime switch. In our setting, there is a sudden change in the information environment; agents produce information and some collateral turns out to be worthless, or firms cut back on their borrowing to prevent information production.

Krishnamurthy (2009) outlines two amplification mechanisms which can explain how a small shock could have a large effect. The first mechanism is a feedback effect to agents’ balance sheets from fire sales prices: a small negative shock causes agents to sell assets, which depresses asset prices, requiring further asset sales, and so on. The initial asset sales may be required due to leverage, capital requirements or margin requirements. This is similar to Kiyotaki and Moore, as the asset values decline due to the feedback effect. The second mechanism concerns Knightian uncertainty which can increase when unexpected events occur, especially in the context of financial innovation that is perhaps not very well understood. Agents may respond by withdrawing from these investments and seek safe assets. In our setting, leverage and complexity in the economy are endogenous (complexity corresponds to a higher cost of producing information). Agents withdraw in a way when there is a negative shock, but this is because of the endogenous credit constraint or because information is produced and firms with bad collateral cannot borrow. In our setting, complexity corresponds to the choice of collateral, where desirable collateral has a high cost of information production. When a shock causes information to be produced, it is the
realization that much collateral is bad that reduce output.

Papers that focus on the sudden collapse type of fragility are based on multiplicity of equilibria. Diamond and Dybvig (1983) show that banks are vulnerable to random external events (sunspots) when beliefs about the solvency of banks are self-fulfilling. Lagunoff and Schreft (1999) show that a wave of pessimism about future conditions may trigger a collapse in the presence of interconnected banks. A critique of sunspots is their lack of predictive power and the irrelevance of payoff relevant fundamentals in selecting among equilibria. In this respect Ordonez (2011) use a global game refinement to show that small changes in fundamentals may suddenly move the financial system towards regions where reputation concerns and market discipline collapse, leading to excessive risk-taking; Allen and Gale (2004) show that the only equilibria that are robust to the introduction of small liquidity shocks are those with non-trivial sunspot activity. Our work departs from this literature because fragility evolves endogenously over time and it is not based on equilibria multiplicity but by switches between uniquely determined information regimes.

Our paper is also related to the literature on leverage cycles developed by Geanakoplos (1997, 2010) and Geanakoplos and Zame (2010). Their work relies on low volatility and innovation for the buildup of leverage, and a jump in uncertainty for the sudden decline. In our paper, crises are generated by a negative aggregate shock in the expected value of collateral, which generates a jump in uncertainty. Furthermore, we explicitly derive real effects and welfare implications from endogenous changes in information regimes.

Finally, there are a number of papers in which agents choose not to produce information ex ante and then may regret this ex post. Examples are the work of Hanson and Sunderam (2010) and Pagano and Volpin (2010), who study ex ante incentives to become informed or provide information versus ex post needs for information when a bad state of the world is realized. Once the bad state of the world occurs it is too late to become informed and there can be a market shut-down. In the next Section we present the model and study debt decisions by a single firm. In

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3See also Andolfatto (2010) and Andolfatto, Berensten, and Waller (2011)
Section 3 we study the aggregate and dynamic implications of information sensitive-
ness. In Section 3.4 we illustrate the results with a numerical example. We consider
policy implications in Section 4 and the choice of collateral in Section 2.3. In Section
5 we present some brief empirical evidence. In Section 6, we conclude.

2 A Single Period Model

2.1 Setting

There are two types of agents in the economy, each with mass 1 – firms and house-
holds – and two types of goods – *numeraire* and */land*. Agents are risk neutral and
derive utility from consuming numeraire at the end of the period. While numeraire
is productive and reproducible – it can be used to produce more numeraire – land is
not. Since numeraire is also used as */capital* we denote it by *K*.

Only firms have access to an inelastic fixed supply of non-transferrable managerial
skills, which we denote by */L*`. These skills can be combined with numeraire in a
stochastic Leontief technology to produce more numeraire, */K*`

\[
K' = \begin{cases} 
A \min\{K, \, L^*\} & \text{with prob. } q \\
0 & \text{with prob. } (1 - q) 
\end{cases}
\]

We assume production is efficient, \(qA > 1\). Then, the optimal scale of numeraire in
production is simply by \(K^* = L^*\).

Households and firms not only differ on their managerial skills, but also on their
initial endowment. On the one hand, households born with an endowment of nu-
meraire \(\bar{K} > K^*\), enough to sustain optimal production in the economy. On the other
hand, firms born with land (one unit of land per firm), but no numeraire.\(^4\)

Even when non-productive, land has a potential value. If land is */good*, it delivers
\(C\) units of */K* at the end of the period. If land is */bad*, it does not deliver anything.
Observing the quality of land costs \(\gamma\) units of numeraire. We assume a fraction \(\hat{p}\) of

\(^4\)This is just a normalization. We can alternatively assume firms also have an endowment of num-
meraire \(\bar{K}_{firms}\) where \(\bar{K}_{firms} < K^* < \bar{K} + \bar{K}_{firms}\).
land is good. At the beginning of the period, different units of land $i$ can potentially have different perception about being good. We denote these priors $p_i$ and assume them commonly known by all agents. To fix ideas it is useful to think of an example. Assume oil is the intrinsic value of land. Land is good if it has oil underneath, with a market value $C$ in terms of numeraire. Land is bad if it does not have any oil underneath. Oil is non-observable at first sight, but there is a common perception about the probability each unit of land has oil underneath, which is possible to confirm by drilling the land at a cost $\gamma$.

In this simple setting, resources are in the wrong hands. Households only have numeraire while firms only have managerial skills, but production requires both inputs in the same hands. Since production is efficient, if output were verifiable it would be possible for households to lend the optimal amount of numeraire $K^*$ to firms using state contingent claims. In contrast, if output were non-verifiable, firms would never repay and households would never be willing to lend.

We will focus in this later case in which firms can hide the numeraire. However we will assume firms cannot hide land, what renders land useful as collateral. Firms can promise to transfer a fraction of land to households in the event of not repaying numeraire, which relaxes the financial constraint from output non-verifiability.

The perception about the quality of collateral then becomes critical in facilitating loans. To be precise, we will assume that $C > K^*$. This implies that all land that is known to be good can sustain the optimum loan, $K^*$. Contrarily, all land that is known to be bad is not able sustain any loan. But more generally, how much a firm with a piece of land that is good with probability $p$ is able to borrow? Is information about the true value of the collateral generated or not?

### 2.2 Optimal loan for a single firm

In this section we study the optimal short-term collateralized debt for a single firm, considering the possibility that lenders may want to produce information about collateral. In this paper we study a single sided information problem, since the borrower does not having resources in terms of numeraire to learn about the collateral. In a companion paper, Gorton and Ordonez (2012) extend the model to allow both borrowers and lenders being able to individually learn the collateral value.
Since firms can compute the incentives of households to acquire information, they optimally chooses between debt that triggers information production or not. Triggering information production (information-sensitive debt) is costly because it raises the cost of borrowing to compensate the monitoring cost $\gamma$. However, not triggering information production (information-insensitive debt) may also be costly because it may imply less borrowing to discourage lenders to produce information. This trade-off determines the information-sensitiveness of the debt and, ultimately the volume of information in the economy and the information dynamics.

### 2.2.1 Information-Sensitive Debt

Lenders can learn the true value of the borrower’s land by paying an amount $\gamma$ of numeraire. When information is generated, it becomes public at the end of the period, but not immediately. This introduces incentives for households to obtain information before lending and individually take advantage of such information before it becomes common knowledge. Assume lenders are competitive.$^5$

\[ p(qR_{IS} + (1 - q)x_{IS}C - K) = \gamma. \]

where $K$ is the size of the loan, $R_{IS}$ is the face value of the debt and $x_{IS}$ the fraction of land posted by the firm as collateral.

In this setting debt is risk-free. It is clear the firm should pay the same in case of success or failure. If $R_{IS} > x_{IS}C$, the firm would always default, handing in the collateral rather than repaying the debt. Contrarily, if $R_{IS} < x_{IS}C$ the firm would always sell the collateral directly at a price $C$ and repay lenders $R_{IS}$. This condition pins down the fraction of collateral posted by a firm, as a function of $p$:

\[ R_{IS} = x_{IS}C \quad \Rightarrow \quad x_{IS} = \frac{pK + \gamma}{pC} \leq 1 \]

This implies it is feasible for firms to borrow the optimal scale $K^*$ only if $\frac{pK^* + \gamma}{pC} \leq 1$, or $p \geq \frac{\gamma}{C-K^*}$. If this condition is not fulfilled, the firm can only borrow $K = \frac{pC - \gamma}{p} < K^*$ when posting the whole unit of good land as collateral. Finally, it is not feasible to borrow at all if $pC < \gamma$.

$^5$It is simple to modify the model to sustain this assumption. For example if only a fraction of firms have skills $L^*$, there will be more lenders than borrowers.
Expected net profits (net of the land value $pC$) from information-sensitive debt, are

$$E(\pi|p, IS) = p(qAK - x_{IS}C).$$

plugging $x_{IS}$ in equilibrium,

$$E(\pi|p, IS) = pK^*(qA - 1) - \gamma.$$

Intuitively, with probability $p$ collateral is good and sustains $K^*(qA - 1)$ numeraire in expectation and with probability $(1 - p)$ collateral is bad and does not sustain any borrowing. The firm always has to compensate for the monitoring costs $\gamma$.

It is optimal for firms to borrow the optimal scale as long as $pK^*(qA - 1) \geq \gamma$, or $p \geq \frac{\gamma}{K^*(qA - 1)}$. Combining the conditions for optimality and feasibility, if $\frac{\gamma}{K^*(qA - 1)} > \frac{\gamma}{C - K^*}$ (or $qA < C/K^*$), whenever the firm wants to borrow, it is feasible to borrow the optimal scale $K^*$ if the land is found to be good. We will assume this condition holds, simply to minimize the kinks in the following profit function.

$$E(\pi|p, IS) = \begin{cases} pK^*(qA - 1) - \gamma & \text{if } p \geq \frac{\gamma}{K^*(qA - 1)} \\ 0 & \text{if } p < \frac{\gamma}{K^*(qA - 1)} \end{cases}.$$

### 2.2.2 Information-Insensitive Debt

Another possibility for firms is to borrow without triggering information acquisition. Still it should be the case that lenders break even in equilibrium

$$qR_{II} + (1 - q)p x_{II} C = K.$$

subject to debt being risk-free, $R_{II} = x_{II}pC$. Then

$$x_{II} = \frac{K}{pC} \leq 1.$$

For this contract to be information-insensitive, we have to guarantee that lenders do not have incentives to deviate, to check the value of collateral and to lend at the contract provisions only if the collateral is good. Lenders want to deviate if the expected gains from acquiring information, evaluated at $x_{II}$ and $R_{II}$, are greater than the loses.
\( \gamma \) from acquiring information. Lenders do not have incentives to deviate if
\[
p(qR_{II} + (1 - q)x_{II}C - K) < \gamma \quad \Rightarrow \quad (1 - p)(1 - q)K < \gamma,
\]

More specifically, by acquiring information the lender only lends if the collateral is good, which happens with probability \( p \). If there is default, which occurs with probability \( (1 - q) \), the lender can sell at \( x_{II}C \) a collateral that was obtained at \( px_{II}C = K \), making a net gain of \( (1 - p)x_{II}C = (1 - p)\frac{K}{p} \).

It is clear from the previous condition that the firm can discourage information acquisition by reducing borrowing. If the condition is not binding at \( K = K^* \), then there are no strong incentives for lenders to produce information. If the condition is binding, the firm will borrow as much as possible given the restrictions of not triggering information acquisition,
\[
K = \frac{\gamma}{(1 - p)(1 - q)}.
\]

Information-insensitive borrowing is characterized by the following debt size:
\[
K(p|II) = \min \left\{ K^*, \frac{\gamma}{(1 - p)(1 - q)}, pC \right\}.
\]  

(1)

Expected profits, net of the land value \( pC \), from information-insensitive debt are
\[
E(\pi|p, II) = qAK - x_{II}pC,
\]
and plugging \( x_{II} \) in equilibrium
\[
E(\pi|p, II) = K(p|II)(qA - 1).
\]  

(2)

Intuitively, in this case profits are certain and given by the additional numeraire generated by restricted borrowing. Explicitly considering the kinks, profits are:
\[
E(\pi|p, II) = \begin{cases} 
K^*(qA - 1) & \text{if } K^* \leq \frac{\gamma}{(1 - p)(1 - q)}, \\
\frac{\gamma}{(1 - p)(1 - q)}(qA - 1) & \text{if } K^* > \frac{\gamma}{(1 - p)(1 - q)}, \\
pC(qA - 1) & \text{if } pC < \frac{\gamma}{(1 - p)(1 - q)}. 
\end{cases}
\]

The first kink is generated by the point at which the constraint to avoid information
production is binding when evaluated at the optimal loan size $K^*$; this occurred when financial constraints start binding more than technological constraints. The second kink is generated by the constraint $x_{II} \leq 1$, below which the firm is able to borrow up to the expected value of the collateral $pC$ without triggering information production.

2.2.3 Borrowing Inducing Information or Not?

Figure 1 shows the ex-ante expected profits, net of the expected value of land, under these two information regimes, for each possible $p$. From comparing these profits we obtain the values of $p$ for which the firm prefers to borrow with an information-insensitive loan (II) or with an information-sensitive loan (IS).

![Figure 1: A Single Period Expected Profits](image)

The cutoffs highlighted in Figure 1 are determined in the following way:

1. The cutoff $p^H$ is the belief that generates the first kink of information-insensitive profits, below which firms have to reduce borrowing to prevent information:

$$p^H = 1 - \frac{\gamma}{K^*(1-q)}.$$  

(3)
2. The cutoff $p_{II}^L$ comes from the second kink of information-insensitive profits,\(^6\)

$$p_{II}^L = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{\gamma}{C(1 - q)}}. \quad (4)$$

3. The cutoff $p_{IS}^L$ comes from the kink of information-sensitive profits

$$p_{IS}^L = \frac{\gamma}{K^*(qA - 1)}. \quad (5)$$

4. Cutoffs $p^{Ch}$ and $p^{CI}$ are obtained from equalizing the profit functions of information-sensitive and insensitive loans and solving the quadratic equation

$$\gamma = \left[ pK^* - \frac{\gamma}{(1 - p)(1 - q)} \right] (qA - 1). \quad (6)$$

There are only three regions of financing. Information-insensitive loans are chosen for collateral with high and low values of $p$. Information-sensitive loans are chosen for collateral with intermediate values of $p$.

To understand how these regions depend on the information cost $\gamma$, the five arrows in the figure show how the different cutoffs and functions move as we reduce $\gamma$. If information is free ($\gamma = 0$), all collateral is information-sensitive (i.e., the IS region is $p \in [0, 1]$). Contrarily, as $\gamma$ increases, the two cutoffs $p^{Ch}$ and $p^{CI}$ converge and the IS region shrinks until it disappears (i.e., the II region is $p \in [0, 1]$) when $\gamma$ is large enough (specifically, when $\gamma > \frac{K^*}{C}(C - K^*)$).

Then, borrowing for each belief $p$, conditional on $\gamma$ is,

$$K(p|\gamma) = \begin{cases} 
K^* & \text{if } p^{H} < p \\
\frac{\gamma}{(1-p)(1-q)} & \text{if } p^{Ch} < p < p^{H} \\
pK^* - \frac{\gamma}{(qA-1)} & \text{if } p^{CI} < p < p^{Ch} \\
\frac{\gamma}{(1-p)(1-q)} & \text{if } p_{II}^L < p < p^{CI} \\
pC & \text{if } p = p_{II}^L \end{cases}$$

\(^6\)The positive root for the solution of $pC = \gamma/(1 - p)(1 - q)$ is irrelevant since it is greater than $p^{H}$, and then it is not binding given all firms with a collateral that is good with probability $p > p^{H}$ can borrow the optimal level of capital $K^*$ without triggering information acquisition.
2.3 The Choice of Collateral

In this extension collateral is heterogenous in two dimensions, the expected value of land \( p \) and the cost of information acquisition \( \gamma \). If firms can freely choose the difficulty to monitor collateral \( \gamma \), then this extension is helpful in thinking about which collateral is more likely to observe sustaining borrowing.

Above we derived borrowing for different \( p \) and fixed \( \gamma \). Similarly, we can derive borrowing for different \( \gamma \) and fixed \( p \). The next Proposition summarizes their properties.

**Proposition 1** Effects of \( p \) and \( \gamma \) on borrowing.

Consider collateral characterized by the pair \((p, \gamma)\). The reaction of borrowing to these variables depends on the financial constraint and information sensitiveness.

1. Fix \( \gamma \).
   
   (a) No financial constraint: Borrowing is independent of \( p \).
   
   (b) Information-sensitive regime: Borrowing is increasing in \( p \).
   
   (c) Information-insensitive regime: Borrowing is increasing in \( p \).

2. Fix \( p \).

   (a) No financial constraint: Borrowing is independent of \( \gamma \).
   
   (b) Information-sensitive regime: Borrowing is decreasing in \( \gamma \).
   
   (c) Information-insensitive regime: Borrowing is increasing in \( \gamma \) if higher than \( pC \) and independent of \( \gamma \) if \( pC \).

The proof is in Appendix A.1. Figure 2 shows these regions and the borrowing possibilities for all combinations \((p, \gamma)\).

If it were possible for borrowers to choose the difficulty for lenders to monitor a collateral with belief \( p \), then they would set \( \gamma > \gamma^H_1(p) \) for that \( p \), such that \( p > p^H(\gamma) \) and the borrowing is \( K^* \), without information acquisition.

This analysis suggests that, endogenously, an economy would be biased towards using collateral with relatively high \( p \) and relatively high \( \gamma \). Agents in an economy with increasing needs for collateral will start using first collateral that is perceived to be
of high quality, to move later towards using collateral of worse quality but making information acquisition difficult and expensive. Even when outside the scope of our paper, this framework can shed light in rationalizing security design and the complexity of modern financial instruments.

2.4 Aggregation

The expected consumption of a household that lends to a firm with land that is good with probability $p$ is $\overline{K} - K(p) + E(\text{repay}|p)$. The expected consumption of a firm that borrows using land that is good with probability $p$ is $E(K'|p) - E(\text{repay}|p)$. Aggregate consumption is the sum of the consumption of all households and firms. Since $E(K'|p) = qAK(p)$

$$W_t = \overline{K} + \int_0^1 K(p)(qA - 1)f(p)dp$$

where $f(p)$ is the distribution of beliefs about collateral types in the economy and $K(p)$ is monotonically increasing in $p$.

In the unconstrained first best (the case of verifiable output, for example) all firms borrow and operate with $K^*$, regardless of beliefs $p$ about the collateral. This implies that the unconstrained first best aggregate consumption is

$$W^* = \overline{K} + K^*(qA - 1)$$
Since collateral with relative low $p$ is not able to sustain loans of $K^*$, the deviation of consumption from the unconstrained first best critically depends on the distribution of beliefs $p$ in the economy. When this distribution is biased towards low perceptions about collateral values, financial constraints hinder the productive capacity of the economy. This distribution also introduces heterogeneity in production, purely given by heterogeneity in collateral and financial constraints, not by heterogeneity in technological possibilities.

In the next section we study how this distribution of $p$ endogenously evolves over time, and how that affects the dynamics of aggregate production and consumption.

3 Dynamics

In this section we nest the previous analysis for a single period in an overlapping generation economy. The purpose is to study the evolution of the distribution of collateral beliefs that determines the level of production in the economy at every period.

We assume each unit of land changes quality over time, mean reverting towards the average quality of collateral in the economy, and study how endogenous information acquisition shapes the distribution of beliefs over time. First, we study the case without aggregate shocks to collateral, in which the average quality of collateral in the economy does not change, and discuss the effects of endogenous information production on the dynamics of credit. Then, we introduce aggregate shocks that reduce the average quality of collateral in the economy and generate crises, and study the effects of endogenous information on the size of crises and the speed of recoveries.

3.1 Extended Setting

We assume an overlapping generation structure, with a mass 1 of risk neutral individuals who live for two periods. These individuals born as households (when "young"), with endowment of numeraire $\bar{K}$ but not managerial skills, and become firms when "old", with managerial skills $L^*$, but no numeraire to use in production.

We assume the numeraire is non-storable and land is storable until the moment its intrinsic value (either $C$ or 0) is extracted. Since land can be transferred across generations, that firms hold land is a natural implication. When young, individuals use
their endowment of non-storable numeraire to buy land, which is useful when old as collateral to borrow productive numeraire.

This is reminiscent of the role of fiat money in overlapping generations, with the critical difference that land is intrinsically value, is subject to imperfect information about its quality and is used as collateral. As in those models, we have multiple equilibrium based on multiple paths of rational expectation land prices. In Appendix A.2 we discuss this multiplicity of prices.

We impose restrictions that simplify the price of a unit of land with belief $p$, to include just the expected intrinsic value $pC$, and not its potential role as collateral. This equilibrium has the advantage of isolating the dynamics generated by information acquisition from the better understood dynamics generated by beliefs about future prices of collateral. Still, the information dynamics we focus in this equilibrium remains in other equilibria, when the price of land is increasing in $p$.

The first of these restrictions is that information can be produced only at the beginning of the period, not at the end. This assumption simplifies the price of land and also justifies that firms prefer to post land as collateral rather than sell land at the risk of information production. The second assumption is that each seller of land (each old individual at the end of the period) matches with a unique buyer who has the bargaining power (makes a take-it-or-leave-it offer). This implies that sellers will be indifferent between selling the unit of land at $pC$ or consume $pC$ in expectation.\footnote{It is simple to modify the model to sustain this assumption. For example, if a small fraction of households inherit an endowment of new land, there will be more firms selling land than households buying land. Since sellers who do not sell just deplete their unsold land, the mass of land sustaining production in the economy is invariant. In Appendix A.2 we relax this assumption.}

Under these assumptions, the single period analysis repeats over time. The only changing state variable linking periods is the distribution of beliefs about collateral. This evolving distribution may generate credit booms but also credit crises. Hence, a critical difference with models where credit booms and crises arise from bubbles in the price of each unit of collateral, in this paper the price of each unit of collateral is its fundamental value, and credit booms and crashes arise from the units of land that can be used as collateral in the economy.
3.2 No Aggregate Shocks

We impose a specific process of idiosyncratic mean reverting shocks that are useful in characterizing analytically the dynamic effects of information production on aggregate consumption. First, we assume idiosyncratic shocks are observable, but not their realization, unless information is produced. Second, we assume the probability land faces an idiosyncratic shock is independent of its type. Finally, we assume the probability land becomes good, conditional on having an idiosyncratic shock, is also independent of its type. These assumptions are just imposed to simplify the exposition. The main results of the paper are robust to different processes, as long as there is mean reversion of collateral in the economy.

Specifically, we assume that initially (at period 0) there is perfect information about which collateral is good and which is bad. In every period, with probability $\lambda$ the true quality of each unit of land remains unchanged and with probability $(1 - \lambda)$ there is an idiosyncratic shock that changes its type. In this last case, land becomes good with a probability $\hat{p}$, independent of its current type. Even when the shock is observable, the realization of the new quality is not, unless a certain amount of the numeraire good $\gamma$ is used to learn about it.

In this simple stochastic process for idiosyncratic shocks, and in the absence of aggregate shocks to $\hat{p}$, this distribution has a three-point support: 0, $\hat{p}$ and 1. The next Proposition shows the evolution of aggregate consumption depends on the borrowing of $\hat{p}$, which can be either in the information sensitive or insensitive region.

**Proposition 2** Evolution of aggregate consumption in the absence of aggregate shocks.

Assume there is perfect information about land types in the initial period. If $\hat{p}$ is in the information-sensitive region ($\hat{p} \in [p^{Cl}, p^{Ch}]$), consumption is constant over time and is lower than the unconstrained first best. If $\hat{p}$ is in the information-insensitive region, consumption grows over time if $\hat{p} > \hat{p}_h^*$ and $\hat{p} < \hat{p}_l^*$, where $\hat{p}_l^*$ and $\hat{p}_h^*$ are the solutions to the quadratic equation $\frac{\gamma}{(1-\hat{p})^2} = \hat{p}^* K^*$.

**Proof**

1. $\hat{p}$ is information-sensitive ($\hat{p} \in [p^{Cl}, p^{Ch}]$)
In this case, information about the fraction \((1 - \lambda)\) of collateral that gets an idiosyncratic shock is reacquired every period \(t\). Then \(f(1) = \lambda \hat{p}, \ f(\hat{p}) = (1 - \lambda)\) and \(f(0) = \lambda(1 - \hat{p})\). Considering \(K(0) = 0\),

\[
W_{t}^{IS} = \bar{K} + [\lambda \hat{p}K(1) + (1 - \lambda)K(\hat{p})] \ (qA - 1). \tag{7}
\]

Aggregate consumption \(W_{t}^{IS}\) does not depend on \(t\); it is constant at the level at which information is reacquired every period.

2. \(\hat{p}\) is information-insensitive \((\hat{p} > p^{Ch} \text{ or } \hat{p} < p^{Cl})\)

Information of collateral that suffers an idiosyncratic shock is not reacquired and at period \(t\), \(f(1) = \lambda^t \hat{p}, f(\hat{p}) = (1 - \lambda^t)\) and \(f(0) = \lambda^t(1 - \hat{p})\). Since \(K(0) = 0\),

\[
W_{t}^{II} = \bar{K} + [\lambda^t \hat{p}K(1) + (1 - \lambda^t)K(\hat{p})] \ (qA - 1). \tag{8}
\]

Since \(W_{0}^{II} = \bar{K} + \hat{p}K(1)(qA - 1)\) and \(\lim_{t \to \infty} W_{t}^{II} = \bar{K} + K(\hat{p})(qA - 1)\), the evolution of aggregate consumption depends on \(\hat{p}\). A credit boom ensues and aggregate consumption grows over time, whenever \(K(\hat{p}) > \hat{p}K(1)\), or

\[
\frac{\gamma}{(1 - \hat{p}^*)} > \hat{p}^* K^*. 
\]

Q.E.D.

This result is particularly important if the economy have collateral such that \(\hat{p} > p^{H}\). In this case consumption grows over time towards the unconstrained first best. When \(\hat{p}\) is high enough, the economy has in average enough collateral to sustain production at the optimal scale. Information about collateral types is then pervasive because restrain firms with bad collateral from producing. As information is lost in the economy good collateral implicitly subsidize bad collateral and with time all firms are able to produce.

### 3.3 Aggregate Shocks

Now we introduce negative aggregate shocks that transform a fraction \((1 - \eta)\) of good collateral into bad collateral. As with idiosyncratic shocks, the aggregate shock is
observable, but which good collateral changes type is not. When the shock hits, there is a downward revision of beliefs for collateral. This is, after the shock, collateral with belief \( p = 1 \), gets revised downwards to \( p' = \eta \) and collateral with belief \( p = \hat{p} \) gets revised downwards to \( p' = \eta \hat{p} \).

Based on the discussion about the endogenous choice of collateral, which justifies that collateral would be constructed to maximize borrowing and prevent information acquisition, we focus on the case where, prior to the negative aggregate shock, the average quality of collateral is good enough such that there are no financial constraints (that is, \( \hat{p} > p^H \)).

In the next Proposition we show that the longer the economy did not face a negative aggregate shock, the larger the consumption loss when such a shock does occur.

**Proposition 3** The larger the boom and the shock, the larger the crisis.

Assume \( \hat{p} > p^H \) and a negative aggregate shock \( \eta \) in period \( t \). The reduction in consumption \( \Delta(t|\eta) \equiv W_t - W_{t|\eta} \) is non-decreasing in shock size \( \eta \) and non-decreasing in the time \( t \) elapsed previously without a shock.

**Proof** Assume a negative aggregate shock of size \( \eta \). Since we assume \( \hat{p} > p^H \), the average collateral does not induce information. Aggregate consumption before the shock is given by equation (8). Aggregate consumption after the shock is:

\[
W_{t|\eta} = \bar{K} + \left[ \lambda^t \hat{p} K(\eta) + (1 - \lambda^t) K(\eta \hat{p}) \right] (qA - 1).
\]

Defining the reduction in aggregate consumption as \( \Delta(t|\eta) = W_t - W_{t|\eta} \)

\[
\Delta(t|\eta) = [\lambda^t \hat{p} [K(1) - K(\eta)] + (1 - \lambda^t) [K(\hat{p}) - K(\eta \hat{p})]] (qA - 1).
\]

That \( \Delta(t|\eta) \) is non-decreasing in \( \eta \) is straightforward. That \( \Delta(t|\eta) \) is non-decreasing in \( t \) follows from

\[
\hat{p} [K(1) - K(\eta)] \leq [K(\hat{p}) - K(\eta \hat{p})]
\]

which holds because \( K(\hat{p}) = K(1) \) (by assumption \( \hat{p} > p^H \)) and \( K(p) \) is monotonically decreasing in \( p \).

The intuition for this Proposition is the following. Pooling implies that bad collateral is confused with good collateral. This allows for a credit boom because firms with
bad collateral gets credit that they would not obtain otherwise. Firms with good collateral effectively subsidize firms with bad collateral since good collateral still gets the optimal leverage, while bad collateral is able to leverage more.

However, pooling also implies that good collateral is confused with bad collateral. This puts good collateral in a weaker position in the event of negative aggregate shocks. Without pooling, a negative shock reduces the belief a collateral is good from \( p = 1 \) to \( p' = \eta \). With pooling, a negative shock reduces the belief a collateral is good from \( p = \hat{p} \) to \( p' = \eta \hat{p} \). Good collateral gets the same credit regardless of having beliefs \( p = 1 \) or \( p = \hat{p} \). However credit may be very different under \( p = \eta \) and \( p = \eta \hat{p} \). Furthermore, after a negative shock to collateral, either a high amount of the numeraire is used to produce information, or borrowing is excessively restricted to avoid such information production.

If we define “fragility” as the probability aggregate consumption declines more than a certain value, then the next corollary immediately follows from Proposition 3.

**Corollary 1** Given a structure of negative aggregate shocks, the fragility of an economy increases with the number of periods the debt in the economy has been informationally-insensitive, hence increases with the fraction of collateral that is of unknown quality.

In the next Proposition we show that information acquisition speeds up recoveries.

**Proposition 4** Information and recoveries.

Assume \( \hat{p} > p^H \) and a negative aggregate shock \( \eta \) in period \( t \). The recovery is faster when information is generated after the shock when \( \eta \hat{p} < \eta p \equiv \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\gamma}{K(1-q)}} \), where \( p^{Ch} < \frac{1}{2} < p^H \). This is, \( W_{t+1}^{IS} > W_{t+1}^{II} \) for all \( \eta \hat{p} < \eta p \) and \( W_{t+1}^{IS} \leq W_{t+1}^{II} \) otherwise.

**Proof** If the negative shock happens in period \( t \), the belief distribution is \( f(\eta) = \lambda^t \hat{p} \), \( f(\eta \hat{p}) = (1 - \lambda^t) \) and \( f(0) = \lambda^t (1 - \hat{p}) \).

In period \( t + 1 \), if information is acquired (IS case), after idiosyncratic shocks realize, the belief distribution is \( f_{IS}(1) = \lambda \eta \hat{p} (1 - \lambda^t) \), \( f_{IS}(\eta) = \lambda^{t+1} \hat{p} \), \( f_{IS}(\hat{p}) = (1 - \lambda) \), \( f_{IS}(0) = \lambda (1 - \lambda^t \hat{p} - \eta \hat{p}) \). Hence, aggregate consumption at \( t + 1 \) in the IS scenario is

\[
W_{t+1}^{IS} = K + \left[ \lambda \eta \hat{p} (1 - \lambda^t) K^* + \lambda^{t+1} \hat{p} K(\eta) + (1 - \lambda) K(\hat{p}) \right] (qA - 1)
\]
In period \( t + 1 \), if information is not acquired (II case), after idiosyncratic shocks realize, the belief distribution is \( f_{II}(\eta) = \lambda^{t+1}\hat{p} \), \( f_{II}(\hat{p}) = (1 - \lambda) \), \( f_{II}(\eta\hat{p}) = \lambda(1 - \lambda^t) \), \( f_{II}(0) = \lambda^{t+1}(1 - \hat{p}) \). Hence, aggregate consumption at \( t + 1 \) in the II scenario is,

\[
W_{t+1}^{II} = K + [\lambda^{t+1}\hat{p}K(\eta) + \lambda(1 - \lambda^t)K(\eta\hat{p}) + (1 - \lambda)K(\hat{p})](qA - 1). \tag{10}
\]

Taking the difference between aggregate consumption at \( t+1 \) between the two regimes

\[
W_{t+1}^{IS} - W_{t+1}^{II} = \lambda(1 - \lambda^t)(qA - 1)[\eta\hat{p}K^* - K(\eta\hat{p})]. \tag{11}
\]

This expression is non-negative for all \( \eta\hat{p}K^* \geq K(\eta\hat{p}) \), or alternatively, for all \( \eta\hat{p} < \overline{\eta\hat{p}} = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\gamma}{K^*(1-q)}} \). From equation (6), \( p^{Ch} < \overline{\eta\hat{p}} < p^{II} \). Q.E.D.

The intuition for this Proposition is the following. When information is acquired after a negative shock, not only a lot of resources are spent in acquiring information but also a fraction \( \eta\hat{p} \) of collateral can sustain the maximum borrowing \( K^* \). When information is not acquired after a negative shock, collateral that remains with belief \( \eta\hat{p} \) will restrict credit in the following periods, until beliefs move back to \( \hat{p} \). This is equivalent to restricting credit proportional to monitoring costs in subsequent periods. Not producing information causes a kind of debt overhang going forward. The Proposition generates the following Corollary.

**Corollary 2** There exists a range of negative aggregate shocks \( (\eta \) such that \( \eta\hat{p} \in [p^{Ch}, \overline{\eta\hat{p}}] \) in which agents do not acquire information, but recovery would be faster if they did.

The next Proposition describes the evolution of the standard deviation of beliefs in the economy during a credit boom.

**Proposition 5** During a credit boom, the standard deviation of beliefs declines.

**Proof** Assume at period 0 the belief distribution is \( f(0) = 1 - \hat{p} \) and \( f(1) = \hat{p} \). The original variance of beliefs is

\[
Var_0(p) = \hat{p}^2(1 - \hat{p}) + (1 - \hat{p})^2\hat{p} = \hat{p}(1 - \hat{p}).
\]
At period $t$, during a credit boom, the belief distribution is $f(0) = \lambda^t(1 - \hat{p})$, $f(\hat{p}) = 1 - \lambda^t$ and $f(1) = \lambda^t\hat{p}$. Then, at period $t$ the variance of beliefs is
\[
Var_t(p|\Pi) = \lambda^t[\hat{p}^2(1 - \hat{p}) + (1 - \hat{p})^2\hat{p}] = \lambda^t\hat{p}(1 - \hat{p}),
\]
decreasing in the length of the boom $t$.

Finally, the next Proposition describes the evolution of the standard deviation of beliefs in the economy during a crisis.

**Proposition 6** The longer the boom, the larger the increase in dispersion after a crisis.

For a negative aggregate shock $\eta$ that triggers information about collateral with belief $\eta\hat{p}$, the increase of the standard deviation of beliefs is increasing in the length of the credit boom $t$.

**Proof** Assume a shock $\eta$ at period $t$ that triggers information acquisition about collateral with belief $\eta\hat{p}$. If the shock is "small" ($\eta > p^{Ch}$), there is no information acquisition about collateral known to be good before the shock. If the shock is "large" ($\eta < p^{Ch}$), there is information acquisition about collateral known to be good before the shock. Now we study these two cases when the shock arises after a credit boom of length $t$.

1. $\eta > p^{Ch}$. The distribution of beliefs in case information is generated is given by
\[
f(0) = \lambda^t(1 - \hat{p}) + (1 - \lambda^t)(1 - \eta\hat{p}), \quad f(\eta) = \lambda^t\hat{p} \quad \text{and} \quad f(1) = (1 - \lambda^t)\eta\hat{p}.
\]
Then, at period $t$ the variance of beliefs with information production is
\[
Var_t(p|IS) = \lambda^t\hat{p}(1 - \hat{p})\eta^2 + (1 - \lambda^t)\eta\hat{p}(1 - \eta\hat{p}),
\]
increasing in the length of the boom $t$.

2. $\eta < p^{Ch}$, The distribution of beliefs in case information is produced is given by
\[
f(0) = \lambda^t(1 - \hat{p}) + (1 - \lambda^t)(1 - \eta\hat{p})(1 - \eta\hat{p}), \quad \text{and} \quad f(1) = (1 - \lambda^t(1 - \hat{p}))\eta\hat{p}.
\]
Then, at period $t$ the variance of beliefs with information production is
\[
Var_t(p|IS) = \lambda^t\hat{p}(1 - \hat{p})\eta^2\hat{p} + (1 - \lambda^t(1 - \hat{p}))\eta\hat{p}(1 - \eta\hat{p}),
\]
increasing in the length of the boom $t$. 

Q.E.D.
Then
\[ Var_t(p|IS) - Var_t(p|II) = (1 - \lambda^I(1 - \hat{p}))\eta\hat{p}(1 - \eta\hat{p}) - \lambda^I\hat{p}(1 - \hat{p})(1 - \eta^2\hat{p}), \]
also increasing in the length of the boom \( t \).

The change in the variance of beliefs also depends on the size of the shock. For very large shocks \( (\eta \to 0) \) the variance can decline. This decline is lower the larger is \( t \).

Q.E.D.

### 3.4 Numerical Illustration

We illustrate our dynamic results with a numerical example. We assume idiosyncratic shocks happen with probability \( (1 - \lambda) = 0.1 \), in which case the collateral becomes good with probability \( \hat{p} = 0.92 \). Other parameters are \( q = 0.6 \), \( A = 3 \) (investment is efficient and generates a return of 80\%), \( \bar{K} = 10 \), \( L^* = K^* = 7 \) (the endowment is large enough to allow for optimal investment), \( C = 15 \) and \( \gamma = 0.35 \).

Given these parameters we can obtain the relevant cutoffs for our analysis. Specifically, \( p^H = 0.88 \), \( p^L = 0.06 \) and the region of beliefs \( p \in [0.22, 0.84] \) is information sensitive. Figure 3 plots the ex-ante expected profits with information sensitive and insensitive debt, and the respective cutoffs.

Using these cutoffs in each period, we simulate the model for 100 periods. At period 0 there is perfect information about the true quality of all collateral in the economy. Over time, idiosyncratic shocks make information to vanish unless it is replenished. The dynamics of consumption arises from the dynamics of belief distribution.

We introduce a negative aggregate shock that transforms a fraction \( (1 - \eta) \) of good collateral into bad collateral in periods 5 and 50. We also introduce a positive aggregate shock that transforms a fraction 0.25 of bad collateral into good collateral in period 30. We compute the dynamic reaction of consumption in the economy for different sizes of negative aggregate shocks, \( \eta = 0.97 \), \( \eta = 0.91 \) and \( \eta = 0.90 \). We will see that small differences in the size of a negative shock may have large dynamic consequences.

Figure 4 shows the evolution of the average quality of collateral for the three negative and the positive aggregate shocks we assume. Aggregate shocks have a temporary
effect on the quality of collateral because mean reversion make average quality converge back to $\hat{p} = 0.92$. We choose the size of the negative aggregate shocks to guarantee that $\eta\hat{p}$ is above $p^H$ when $\eta = 0.97$, is between $p^{Ch}$ and $p^H$ when $\eta = 0.91$ and is less than $p^{Ch}$ when $\eta = 0.90$.

Figure 5 shows the evolution of aggregate consumption for the three negative aggregate shocks. A couple of features are worth noting. First, if $\eta = 0.97$, the aggregate shock is small enough such that it does not constraint borrowing and does not modify
the evolution of consumption. Second, the positive shock does not affect the evolution of consumption either. Since \( \hat{p} > p^H \) a further improvement in average beliefs does not further relax financial constraints.

As proved in Proposition 3, if \( \eta = 0.91 \) or \( \eta = 0.90 \), the reduction in consumption from the shock in period 50, when the credit boom is mature and information is scarce, is larger than the reduction in consumption when the shock happens in period 5. Consumption suffers not only from dropping from a higher position but also by dropping to an even lower level. The reason is that the shock reduces financing for a larger fraction of collateral when information has vanished over time.

As proved in Proposition 4, a shock \( \eta = 0.91 \) does not trigger information production, but a shock \( \eta = 0.90 \) does. Even when these two shocks generate consumption crashes of similar magnitude, recovery is faster when the shock is slightly larger and information is replenished.

Finally, Figure 6 shows the evolution of the dispersion of beliefs about the collateral, a measure of available information in the economy. As proved in Proposition 5, a credit boom is correlated with a reduction in the dispersion of beliefs. As proved in Proposition 6, given that after many periods without a shock most collateral looks the same, the information acquisition triggered by a shock \( \eta = 0.90 \) generates a larger increase in dispersion in period 50 than in period 5.
3.5 Discussion

Here we briefly discuss some issues that may have occurred to the reader. We have motivated the model’s structure based on appealing to the micro foundations of Dang, Gorton, and Holmström (2011), where the best transaction medium is short-term debt. In our model, as it stands, the land could simply be sold by the old generation (the borrowers) to the young generation (the lenders). This is because we did not include a need for the young to have a transactions medium to use to shop during their first period, and before the output is realized. If there was such a market, the young would need to use the collateralized claims on the firm as “money.” That is the idea of short-term debt as money. For simplicity we did not include such a market.

In the model the firms are also uninformed about their own collateral quality. Like the households they do not produce information every period because it is costly. We view this as realistic. There may be other reasons to think that firms could differ in ways which are unobservable to the households, so that there are firm types. This is a well-studied setting and we do not include it here. The main reason for this omission is that we have abstracted from the financial intermediaries, which would be screening firms and issuing liabilities to the households for use as money. This is a subject for future research.

What about other reasons for producing information? We have eliminated all other possible model embellishments and complications in order to focus attention on the
endogenous dynamics of information production in the economy with regard to short-term debt. Clearly, however, there are other reasons why information should be produced. For example, firms might want to produce information in order to learn their best investment opportunities. The interaction of such information production with the possible production of information about the firm’s collateral potentially raises interesting issues. For example, producing information about firms not only induces more efficient investment but also leads to less borrowing in expectation. This is also a subject of future research.

Finally, it is worth noting the differences between our model and a recent literature in which credit constraints or other frictions generate “over borrowing.” In some of these settings private agents do not internalize the effects of their own leverage in depressing collateral prices in case of shocks that trigger fire sales. Since a shock is an exogenous unlucky event, the policy implications are clear: there should be less borrowing. Examples of this literature would include Lorenzoni (2008), Mendoza (2010) and Bianchi (2011). In contrast to these settings, there is nothing necessarily bad about leverage in our model, compared to these models. First, leverage always relaxes endogenous credit constraints. Second, fire sales are not an issue. In our setting the efficient outcome may be fragility.

4 Policy Implications

In this section we discuss optimal information production when a planner cares about the discounted consumption of all generations and faces the same information restrictions and costs that households and firms. Welfare is measured by

\[ U_t = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} W_\tau. \]

First, we study the economy without aggregate shocks, and show that a planner would like to produce information for a wider range of collateral \( p \) than short-lived agents. Then, we study the economy with negative aggregate shocks, and show that a planner is more likely to trigger information acquisition than decentralized agents. However, when expected shocks are not very large or likely, it may be optimal for the planner to avoid information production, riding the credit boom even when facing
the possibility of collapses.

4.1 Ex-Ante Policies in the Absence of Aggregate Shocks

The next Proposition shows that, when \( \beta > 0 \), the planner wants to acquire information for a wider range of beliefs \( p \).

**Proposition 7** The planner’s optimal range of information-sensitive beliefs is wider than the the decentralized range of information-sensitive beliefs.

**Proof** Denote the expected discounted consumption sustained by a unit of collateral with belief \( p \) if producing information (IS) as

\[
V^{IS}(p) = pK^*(qA - 1) - \gamma + \beta[\lambda(pV(1) + (1 - p)V(0)) + (1 - \lambda)V(\hat{p})] + pC
\]

and expected discounted consumption if not producing information (II) as

\[
V^{II}(p) = K(p)(qA - 1) + \beta[\lambda V(p) + (1 - \lambda)V(\hat{p})] + pC
\]

We can solve for

\[
V^{IS}(p) = \frac{pK^*(qA - 1)}{1 - \beta \lambda} - \gamma + Z(p, \hat{p})
\]

and

\[
V^{II}(p) = \frac{K(p)(qA - 1)}{1 - \beta \lambda} + Z(p, \hat{p}),
\]

where

\[
Z(p, \hat{p}) = \beta \left[ \frac{\lambda(1 - \lambda)}{1 - \beta \lambda} + (1 - \lambda) \right] V(\hat{p}) + pC.
\]

The planner decides to acquire information if \( V^{IS}(p) > V^{II}(p) \), or

\[
\gamma(1 - \beta \lambda) < [pK^* - K(p)](qA - 1),
\]

while, as shown in equation (6), individuals decide to acquire information when

\[
\gamma < [pK^* - K(p)](qA - 1),
\]

which effectively means the decision rule for the planner is the same that the decision rule for decentralized agents, with \( \beta > 0 \) for the planner and \( \beta = 0 \) for agents. Q.E.D.
The cost of information is effectively lower for the planner, since acquiring information has the additional gain of enjoying more borrowing in the future if the collateral is found to be good. The difference between the planner and the agents widens with the government discounting ($\beta$) and with the probability that the collateral remains unchanged ($\lambda$).

The planner can align incentives easily by subsidizing information production by an fraction $\beta \lambda$ from lump sum taxes on individuals, such that, after the subsidy, the cost of information production agents face is effectively $\gamma(1 - \beta \lambda)$.

4.2 Ex-Ante Policies in the Presence of Aggregate Shocks

In this section we assume that the planner assigns a probability $\mu$ a negative shock occurs next period. The next two propositions summarize how the incentives to acquire information change with the probability and the size of aggregate shocks.

**Proposition 8** Incentives to acquire information in the presence of aggregate shocks increases with the probability of the shock $\mu$ if $p[K^* - K(\eta)] \leq [K(p) - K(\eta p)]$, and decreases otherwise.

**Proof** Without loss of generality we assume the negative shock can happen only once. Expected discounted consumption sustained by a unit of collateral with belief $p$ if producing information (IS) is

$$V^{IS}(p) = pK^*(qA - 1) + \beta[(1 - \mu)[\lambda (pV(1) + (1 - p)V(0)) + (1 - \lambda)V(\hat{p})]$$

$$+ \mu[\lambda (pV(\eta) + (1 - p)V(0)) + (1 - \lambda)V(\eta \hat{p})] + pC,$$

and if not producing information (II) is

$$V^{II}(p) = K(p)(qA - 1) + (1 - \mu)[\lambda V(\hat{p}) + (1 - \lambda)V(\eta \hat{p})] + pC.$$ 

We can solve for

$$V^{IS}(p) = \frac{pK^*(qA - 1)}{1 - \beta \lambda} - \frac{\beta \lambda \mu}{1 - \beta \lambda} p[K^* - K(\eta)](qA - 1) + Z(p, \hat{p})$$

and

$$V^{II}(p) = \frac{K(p)(qA - 1)}{1 - \beta \lambda} - \gamma - \frac{\beta \lambda \mu}{1 - \beta \lambda} [K(p) - K(\eta p)](qA - 1) + Z(p, \hat{p}).$$
Naturally, the expectation of aggregate shocks reduce expected consumption in both situations. The effect on information production depends on which one drops more. The Proposition arises straightforwardly from comparing $V_{IS}(p)$ and $V_{II}(p)$. Q.E.D.

To build intuition, assume $\eta$ is such that $K(\eta p) < K(p)$ and $K(\eta) = K^*$, for example if the shock is small and $p = p^H$. In this case, the aggregate shock, regardless of its probability, does not affect the expected discounted consumption of acquiring information, but reduces the expected discounted consumption of not acquiring information. In this case, producing information relaxes the borrowing constraint in case of a future negative shock, and when that shock is more likely, there are more incentives to acquire information.

**Proposition 9** Incentives to acquire information in the presence of aggregate shocks increases with the size of the shock (decreases with $\eta$) if $\frac{\partial K(\eta p)}{\partial \eta} \leq p \frac{\partial K(\eta)}{\partial \eta}$, and decreases otherwise.

**Proof** Define $DV(p) = V_{IS}(p) - V_{II}(p)$, which measures the incentives to acquire information. Taking derivatives with respect to $\eta$, incentives to acquire information increase with the size of the shock (decrease with $\eta$) if

$$\frac{\partial DV(\eta|p)}{\partial \eta} = \frac{\beta \lambda \mu}{1 - \beta \lambda} \left[ \frac{\partial K(\eta p)}{\partial \eta} - p \frac{\partial K(\eta)}{\partial \eta} \right] \leq 0.$$

Q.E.D.

The effect is clearly non-monotonic in the size of the shock. For example, at the extreme of very large shocks ($\eta = 0$), in which all collateral becomes bad, the incentives to produce information in fact decline, since the condition in that case becomes

$$\gamma \frac{1 - \beta \lambda}{1 - \beta \lambda \mu} < pK^* - K(p),$$

increasing the effective cost of acquiring information. In this extreme case, the planner still wants to acquire more information than decentralized agents, but less than in the absence of an aggregate shock (since $(1 - \beta \lambda) \leq \frac{1 - \beta \lambda}{1 - \beta \lambda \mu} \leq 1$).

The previous two propositions show there are levels of $p$ for which, even in the presence of a potential future negative shock the planner prefers not producing information, maintaining a high level of current output rather than avoiding a potential reduction in future output. This result is summarized in the following Corollary.
Corollary 3 The possibility of a negative aggregate shock does not necessarily justify acquiring information, and reducing current output to insure against potential future crises.

This corollary suggests that there are conditions under which it is efficient to accept potential reductions in future consumption in order to obtain guaranteed increases in current consumption. This result is consistent with the findings of ?) who show that "high growth paths are associated with the undertaking of systemic risk and with the occurrence of occasional crises."

4.3 Ex-Post Policies

Now we study ex-post policies, conditional on a realized aggregate shock. Naturally these policies affect the results in the previous section, since if they are effective in helping the economy recover, they render ex-ante information acquisition to relax borrowing constraints less important in the presence of aggregate shocks.

We consider policies that are intended to boost the expected quality of collateral after a negative aggregate shock. The effectiveness of such a policy depends on how fast the government is able to react to the negative shock, for example guaranteeing the quality of the collateral. This policy manifests itself as a positive aggregate shock in which a fraction $\alpha$ of bad collateral becomes good one period after the negative aggregate shock, for example collateral guarantees by the government.

The next Proposition shows that, if there is a positive aggregate shock after a negative aggregate shock that takes the average collateral $\eta \hat{p}$ to a new higher level above $p^H$, the recovery from the negative shock is faster if at the same time the government prevents information production as a response to the negative shock.

**Proposition 10** Ex-post policies are more effective if information acquisition is avoided.

Assume a negative aggregate shock $\eta$ that induces information acquisition (this is $\eta \hat{p} \in [p^{Cl}, p^{Ch}]$), immediately followed by a positive **policy** of size $\alpha$ that makes firms able to borrow $K^*$ (this is $p' = \eta \hat{p} + \alpha (1 - \eta \hat{p}) > p^H$). This policy is more effective in speeding up recovery if information were not acquired. More specifically $\Delta^{II} > \Delta^{IS}$ (where $\Delta^{II} \equiv W_{t+1 | \alpha}^{II} - W_{t+1}^{II}$ and $\Delta^{IS} \equiv W_{t+1 | \alpha}^{IS} - W_{t+1}^{IS}$).
The proof is in Appendix A.3. The intuition relies on the speed of information recovery. Assume all collateral has the same belief and an aggregate negative shock induces information that sorts out the quality of collateral. In this case, a successful policy that improves average quality does not have a big impact. It does not increase borrowing for the good collateral and only helps marginally the bad collateral. Contrarily, if the aggregate negative shock does not induce information, a successful policy that improves average quality increase the borrowing both of the good and the bad collateral.

Figure 7 introduces a policy that boosts the average quality of collateral in the numerical illustration of the previous section. Specifically it assumes a policy $\alpha = 0.25$ in period 51, right after a negative shock. As can be seen, this policy is more effective in speeding up recovery when the negative shock did not induce information.

This implies that, if the planner has access to a policy to deal with a crises, such as guaranteeing collateral use, that policy is more effective if the original shock does not induce information acquisition in the economy. How can the government prevent information acquisition after a crisis? Possibly introducing a lending facility, financed through household taxation, that covers the difference between the optimal borrowing and the level of borrowing that in equilibrium would induce information.
5 Some Empirical Evidence

In this section we briefly and simply examine the central prediction of the model, using U.S. historical data. The model has a number of predictions. We focus on the prediction from Proposition 5, that during a credit boom the standard deviation of beliefs declines. If information about collateral decays, because no information is produced, then the standard deviation of beliefs is shrinking and lending is increasing, leading to higher output. The basic empirical strategy is to examine the correlation between the growth in credit creation, and the change in the standard deviation of beliefs from the trough of a business cycle to the next business cycle peak. In order to increase the sample size, as many business cycles as possible should be included.

The model has a number of other predictions as well. However, some are hard to operationalize since information production is not observable. Other predictions concern what happens after a shock. But, the model does not include bank clearinghouses, for example, so the post-shock predictions may be problematic. Thus, we focus on the main prediction of the model, the link between the change in beliefs and growth of credit and output.

There are a number of complications in implementing a test. We need to measure credit creation and beliefs. With regard to credit creation, there are no consistent time series that span a long period of U.S. history for credit creation, so we are forced to examine sub-periods and use less precise measures. We will look at banks’ total assets for most of the period, but to include the pre-Civil War period we will also look at industrial output. In the model, credit creation and output grow one-for-one. The output data, however, are annual. The bank total assets data are five or six times a year from 1863 to 1923 and four times a year thereafter.

As for beliefs, we need a proxy for the distribution of perceived collateral quality. In the model, perceived collateral quality is linked to output. For simplicity the model is one in which firms have a constant expected marginal product of capital, but in terms of the empirical work, we want to imagine that firms have concave production technologies. In this case, expected returns can vary depending on the perceived quality of collateral. Also, realistically there are likely more than three types of collateral. We can proxy for beliefs with the standard deviation of the cross section of stock returns. The idea is that at each date we calculate the stock return over a given period (annual, monthly) and then for that date we calculate the standard deviation of this cross sec-
tion of stock returns. We then have a time series of the cross section of stock returns. Over time, as information is decaying, the standard deviation of the cross section of stock returns should be shrinking.

The focus of our empirical analysis is on the period leading up to a crisis, the credit boom prior to a negative shock. So, we examine business cycles. In the period prior to the U.S. Civil War, Davis’s annual data results in a different business cycle chronology than that of the National Bureau of Economic Research (NBER); see Davis (2006). The NBER dating has not been revised since first announced (see also discussion in Davis (2004)). There are some NBER cycles that Davis does not include. Some of Davis’ cycles do not display much of a downturn.\footnote{With regard to the cycles with peaks at 1811, 1822, and 1836, Davis (2004, p. 1203) states that these cycles had losses “that do not exceed the minimum postbellum loss.”} Where the NBER and Davis agree on the cycle existence, there is also agreement on the date of the peaks. There is most disagreement about the trough dates. We date the start of the credit boom as the trough, so this is potentially problematic. We focus on Davis’s chronology, as it is the most current. This gives us a pre-Civil War sample of nineteen cycles, omitting the wartime cycles, which are somewhat special as the shock was arguably the start of the war rather than anything else.\footnote{Davis (2004, p. 1203) says of the wartime cycles: “Two Civil War cycles (1861 and 1865 troughs) are omitted. Although their inclusion would not meaningfully affect calculations.”}

The period from 1790-1915 includes the National Banking Era, 1863-1914, which was followed by the Federal Reserve System, starting in 1914. It also includes the Civil War period, and the period prior to the Civil War where banks were overseen by the states and issued their own private money. The year 1837, following President Andrew Jackson’s veto of the re-charter of the Second Bank of the United States, marks the beginning of the Free Banking Era, during which some states allowed free entry into banking. Also, as explained below, because of data limitations with stock price data, we look at the following five periods: (1) 1823-1914, using annual data on output; (2) 1837-1914, using annual data on output; (3) 1863-1914, the National Banking Era, using National Banks’ total assets; (4) the Federal Reserve period, 1914-2010, using banks’ total assets; and (5) the whole period from 1863-2010. The bank total assets data are four, five, or six times a year.\footnote{The Davis data is an index of real industrial production. We do not deflate the nominal asset values for lack of data, which is only annual. But, since we are calculating the change in total assets over short periods this should not be a problem.} \footnote{Until the 1920s the bank regulators examined the banks at random times, usually five times a year.}
We examine the pre-Fed period using three measures of credit growth. First, we will use the annual real index of American industrial production, 1790-1915, produced by Davis (2004). Davis (2004) uses annual physical-volume data on 43 manufacturing and mining industries to construct an annual index. We use the industrial production index through the year 1914, after which the Federal Reserve System is in existence. This series has the advantage that it extends back to 1790, but has the disadvantages that it is annual and it is a measure of output, rather than credit. However, in the model credit growth translates into output.\footnote{\textsuperscript{12}}

The second measure of credit growth is based on banks’ total assets.\footnote{\textsuperscript{13}} Data on National Banks’ total assets from October 1863 until 1976 are from the Reports of the Comptroller of the Currency. From 1976 to 2011 the total assets data are from the Comptroller of the Currency Reports of Income and Condition (the “Call Reports”), which covers all federally insured banks. The set of federally insured banks is larger than the set of National Banks (which excludes state banks), so the two series are not consistent. This requires us to determine when to splice them together. We chose 1976, which means that we lose one business cycle, January 1980 peak to July 1980 trough; July 1981 was the next peak. That is, we picked a very short cycle to omit.

The third measure of credit growth is simply the number of years or months from trough to peak. We use this as a supplement to Davis’ measure.

As discussed above, we will proxy for agents’ beliefs about collateral quality using the standard deviation of the cross section of stock returns. The standard deviation of the cross section of stock returns has been previously used as a measure of “uncertainty” by Lougani, Rush, and Tave (1990), Brainard and Cutler (1993), and Bloom, Floetotto, and Jaimovich (2009). Here, however, we have a precise meaning for this “uncertainty.” The idea is that the standard deviation of the cross section of stock returns should decline during the credit boom, as more and more firms are borrowing based on collateral with a perceived value of $\hat{p}$. That is, the firms are increasingly viewed as being of the same quality. The work cited above takes this measure of uncertainty as exogenous, whereas we are trying to determine how it arises endogenously.

For the period 1815-1925, we use New York Stock Exchange stock price data, col-
lected by Goetzmann, Ibbotson, and Peng (2001). Because Davis’ data are annual, we convert the monthly standard deviations to annual by simple averaging. The year 1837, following President Andrew Jackson’s veto of the re-charter of the Second Bank of the United States, marks the beginning of the Free Banking Era, during which some states allowed free entry into banking. We will look at two periods, 1823-1914 and 1839-1914. For the period 1926-2011, we use data from the Center for Research in Security Prices.

Figures 8 and 9 show two suggestive examples of the trough to peak episodes, the first for an early boom and the second for the most recent trough to peak episode. In the vertical axis we plot the level of total bank assets and the level of the standard deviation of the cross section of stock returns when it has been H-P filtered. In the horizontal axis we plot the number of observations between the trough date and the peak date.

Figure 8: Example: Trough: May 1885 to Peak: March 1887

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14 When monthly values are missing, the annual average is the average over the remaining months. The entire year 1867 is missing; its annual value was interpolated.

15 Also, the early part of the stock series has very few companies.

16 The first relevant trough is 1812 and the next peak is 1823. That period, from 1815, is used to calculate the first cumulative change, so the dating is given as 1823.
5.1 Post Civil War Measuring Credit with Industrial Production

We start by examining the means of the variables over the trough to the peak of the business cycles during the period. The period 1823-1914 has 13 cycles, and the period 1837-1914 has eleven cycles. The measure of the credit boom, the cumulative change in the Davis Index, should be positive on average, whereas the cumulative change in beliefs should be negative.

Table 1 shows the cumulative change in beliefs averaged over the 13 cycles during 1823-1914 is positive not negative. Also shown is the average number of years from trough to peak. Table 1 also shows that the cumulative change in the standard deviation of cross section of stock returns (called “Beliefs” below) is negatively correlated with the credit boom, measured either with the cumulative change in the Davis Index or simply with number of years from trough to peak. As the boom grows, the standard deviation of the cross section of stock returns should fall, as more firms are perceived to be of quality $\hat{\rho}$.

We now turn to looking at the National Banking Era, 1863-1914, and the Federal Reserve Era, 1914-2010.
5.2 Post Civil War Measuring Credit with Bank Total Assets

During the National Banking Era, 1863-1914, there were 12 cycles. During the Federal Reserve period, 1914-2010, there were 17 cycles. As above, we first examine the means of the variables from trough to peak, averaged over the cycles. The data are finer now, being four, five, or six times a year. In this section we present two measures of the standard deviation of the cross section of stock returns, one is the raw measure and the other is a HP filtered version of the series, using a smoothing parameter of 1400.

Table 2 shows that, from trough to peak, the change in Beliefs are negative on average, but they are not during the National Banking Era based on unfiltered data. The table also shows the correlations in all periods are as predicted, regardless of how Beliefs are measured.

If we think of one trough to peak episode as an observation and ask if the above correlations are statistically significant, then we find that they are not. This is perhaps not too surprising as the samples are very small. Instead of treating the cumulative trough to peak variables as observations we could look at the change in each variable, total assets, beliefs or H-P filtered beliefs, during the trough to peak, in a panel. In that case we are analyzing 29 cycles with 359 observations, 330 if we look at one lag. We examined the panel regression of the change in the total assets, each observation period (four, five or six times a year, depending on the period, on the change in one of the measures of beliefs. In both cases the results are a negative coefficient on beliefs (contemporaneous or lagged), but not statistically significant.

**Summary** It is not possible to observe information production, so the amount of information in the economy at any date is hard to measure. We have suggested that
the cross section of the standard deviation of stock returns can be used as a proxy for agents’ beliefs about the distribution of the quality of collateral in the economy. The distribution depends on information production. If, as we have argued, the belief distribution is shrinking as more time passes since information was produced, then a credit boom ensues, until a shock at the peak. We view the above evidence as preliminary and suggestive. It is suggestive because it demonstrates that a model which is driven by unobservable beliefs can be tested if a proxy for these beliefs can be found. The evidence suggests that the cross section of volatility is related to the unobservable choice of whether to produce information in the economy. The endogeneity of the amount of information in the economy appears to be linked to the growth of credit and output. There is clearly more research to be done.

6 Conclusions

What determines the amount of credit (leverage) in an economy? What is the role of information in determining that credit? We argued that leverage and information are linked, and this link is the basis for financial fragility, which is defined as the susceptibility of the economy to small shocks having large effects.

What determines the information in an economy? It is not optimal for lenders to produce information every period about the borrowers because it is costly. In that case, the information about the collateral degrades over time. Instead of knowing which borrowers have good collateral and which bad, all collateral starts to look alike. These
dynamics of information result in a credit boom in which firms with bad collateral start to borrow. During the credit boom, output and consumption rise, but the economy becomes increasingly fragile. The economy becomes more susceptible to small shocks. If information is produced after such a shock, firms with bad collateral cannot access credit. Alternatively, if information is not produced, firms are endogenously credit constrained to avoid information production.

Why did complex securities play a leading role in the recent financial crisis? Agents choose (and construct) collateral that has a high perceived quality when information is not produced and collateral that has a high cost of producing information. For example, to maximize borrowing firms will tend to use complex securities linked to land, such as mortgage-backed securities. As we showed this increases fragility over time.

We focus on exogenous shocks to the expected value of collateral to trigger crises. However it is straightforward to show real exogenous shocks to productivity trigger crises in the same way than shocks to collateral. Furthermore, if we assume additional borrowing is used to finance projects with decreasing marginal productivity, then crises may be generated endogenously as the boom grows, without the need for any exogenous shock. Under this assumption, a larger credit boom truly increases the fragility of the economy.

We cannot measure the amount of information in the economy, or whether information has been produced. But, our empirical work shows that the standard deviation of the cross section of stock returns seems to be a reasonable proxy for the time-varying distribution of perceived collateral value in the model. We presented evidence for the predicted link between the beliefs and credit booms, looking at almost two hundred years of U.S. business cycles. The evidence, while preliminary, suggests that it is possible to test models driven by unobservable beliefs. This is a subject for further research.
References


A Appendix

A.1 Proof of Proposition 1

Point 1 is a direct consequence of $K(p | \gamma)$ being monotonically increasing in $p$ for $p < p^H$ and independent of $p$ for $p > p^H$.

To prove point 2 we derive the function $\tilde{K}(\gamma | p)$, which is the inverse of the $K(p | \gamma)$, and analyze its properties. Consider first the extreme in which information acquisition is not possible (or $\gamma = \infty$). In this case the limit to financial constraints is the point at which $K^* = p C$; lenders will not acquire information but will not lend more than the expected value of collateral, $p C$. Then, the function $\tilde{K}(\gamma | p)$ has two parts. One for $p \geq \frac{K^*}{C}$ and the other for $p < \frac{K^*}{C}$.

1. $p \geq \frac{K^*}{C}$:

\[
\tilde{K}(\gamma | p) = \begin{cases} 
\frac{p C}{(1-p)(1-q)} & \text{if } \gamma \leq \gamma^H_1 \\
\frac{p K^*}{(1-p)(1-q)} - \frac{\gamma}{(q A - 1)} & \text{if } \gamma^H_1 < \gamma < \gamma^L
\end{cases}
\]

where $\gamma^H_1$ comes from equation 3. Then

\[
\gamma^H_1 = K^*(1 - p)(1 - q)
\]

and $\gamma^L$ comes from equation 6. Then

\[
\gamma^L = p K^* \frac{(1 - p)(1 - q)(q A - 1)}{(1 - p)(1 - q) + (q A - 1)}
\]

2. $p < \frac{K^*}{C}$:

\[
\tilde{K}(\gamma | p) = \begin{cases} 
\frac{p C}{(1-p)(1-q)} & \text{if } \gamma \leq \gamma^H_2 \\
\frac{p K^*}{(1-p)(1-q)} - \frac{\gamma}{(q A - 1)} & \text{if } \gamma^H_2 < \gamma < \gamma^L
\end{cases}
\]

where $\gamma^H_2$ in this region comes from equation 4. Then

\[
\gamma^H_2 = p(1 - p)(1 - q) C
\]

and $\gamma^L$ is the same as above.

It is clear from the function $\tilde{K}(\gamma | p)$ that, for a given $p$, borrowing is independent of $\gamma$ in the first region, it is increasing in the second region (information-insensitive regime) and it is decreasing in the last region (information-sensitive regime).

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A.2 Land Prices that Include the Value of Land as Collateral

In the main text the price of land just reflects its outside option, or fundamental value, since we assumed buyers have all the negotiation power and make take-it or leave-it offers. In this extension we generalize the results assuming Nash bargaining between buyers and sellers, where the sellers’ negotiation power $\theta \in [0, 1]$ determines how much they can extract from the surplus of buyers (in the main text we assumed $\theta = 0$). To simplify the exposition in the main text we also assumed no discounting (i.e., $\beta = 1$). In this extension we assume a generic discount factor $\beta \in [0, 1]$.

First we assume the case without aggregate shocks and then we discuss how the introduction of aggregate shocks just enter into prices as an expectation. We denote the price of a unit of land with perceptions $p$ as $Q(p)$.

The surplus of land for the seller is just its outside option

$$J_S(p) = pC.$$

The surplus of land for the buyer is the expected profit from a firm plus the expected price of the land. If $p$ is such that debt is information-sensitive, the surplus is

$$J_B(p|IS) = E(\pi|p, IS) + \lambda[pQ(1) + (1 - p)Q(0)] + (1 - \lambda)Q(\hat{p}),$$

where $E(\pi|p, IS) = [pK(1) + (1 - p)K(0)](qA - 1) - \gamma$.

If $p$ is such that debt is information-insensitive, the surplus is

$$J_B(p|II) = E(\pi|p, II) + \lambda Q(p) + (1 - \lambda)Q(\hat{p}),$$

where $E(\pi|p, II) = K(p)(qA - 1)$.

Then

$$Q(p) = \beta[\theta J_B(p) + (1 - \theta)J_S(p)]$$

since $Q(p) = J_S(p) + \theta(J_B(p) - J_S(p))$.

1. Borrowing as a function of land price

Firms can compute the possible borrowing with both information-sensitive and insensitive debt and determine which one is higher. In the main text we impose the price of land as the sellers’ outside option and we determine the optimal borrowing as a function of that price. Now the price of land also depends on the optimal borrowing, and then they should be determined simultaneously.

$^{17}$In the main text we did not need an explicit name since the price of land $p$ was just $pC$. 
In the case of information-sensitive debt, \( R_{IS}(1) = x_{IS}(1)Q(1) \) and \( R_{IS}(0) = x_{IS}(0)Q(0) \) because debt is risk-free. Lenders break even when,

\[
p[x(1)Q(1) - K(1)] + (1 - p)[x(0)Q(0) - K(0)] = \gamma
\]

where \( x(1)Q(1) \geq K(1) \) and \( x(0)Q(0) \geq K(0) \).

In the case of information sensitive debt, \( R_{II}(p) = x_{II}(p)Q(p) \) because debt is risk-free. Lenders break even when,

\[
x(p)Q(p) = K(p)
\]

with the constraint that

\[
p[x(p)(qQ(p) + (1 - q)Q(1)) - K(p)] \leq \gamma
\]

or, which is the same as

\[
K(p) \leq \frac{\gamma}{(p\frac{Q(1)}{Q(p)} - p)(1 - q)}.
\]

In the main text, where \( \theta = 0, Q(1) = C, Q(p) = pC \) and then \( K(p) \leq \frac{\gamma}{(1-p)(1-q)} \).

2. Solving Borrowing and Land Price Simultaneously

We now show how to solve simultaneously for optimal borrowing and the land price.

1. When \( \gamma > 0 \), firms with collateral \( p = 0 \) and \( p = 1 \) prefer to borrow without producing information.

This is clear because knowing the type of the collateral (which is the case with \( p = 0 \) and \( p = 1 \)), it does not make sense for the borrower to pay \( \gamma \).

2. \( K(1) = K^* \)

Since \( K(1) \) is not financially constrained in the information-insensitive case, then \( K(1) = K^* \).

3. Determination of \( K(\hat{p}), Q(\hat{p}) \) and \( Q(1) \).

   (a) \( \hat{p} \) is information-insensitive and \( K^* \leq \frac{\gamma}{(\hat{p}\frac{Q(1)}{Q(\hat{p})} - \hat{p})(1 - q)} \): This implies \( K(\hat{p}) = K^* \) and

   \[
   Q(\hat{p}) = \frac{\beta(1 - \theta)\hat{p}C + \beta\theta K^*(qA - 1)}{1 - \beta\theta}.
   \]

   (b) \( \hat{p} \) is information-insensitive and \( K^* > \frac{\gamma}{(\hat{p}\frac{Q(1)}{Q(\hat{p})} - \hat{p})(1 - q)} \): Since \( Q(\hat{p}) \) and \( Q(1) \) just depend on \( K(\hat{p}) \), it is obtained from equation 16.
(c) \( \hat{p} \) is information-sensitive: When information reveals the collateral is bad, and assuming the firm maximizes borrowing \( x_{1s}(0) = 1 \). The following two equations jointly determine \( K(0) \) and \( K(\hat{p}) \):

\[
K(\hat{p}) = \hat{p}K^* + (1 - \hat{p})K(0) - \frac{\gamma}{(qA - 1)},
\]

(17)

\[
K(0) = Q(0) = \frac{\beta \theta [K(0)(qA - 1) + (1 - \lambda)Q(\hat{p})]}{1 - \beta \theta \lambda},
\]

(18)

where \( Q(\hat{p}) \) just depends on \( K(\hat{p}) \).

In these three cases \( K(\hat{p}) \) is solvable, and the prices

\[
Q(\hat{p}) = \frac{\beta (1 - \theta)\hat{p}C + \beta \theta K(\hat{p})(qA - 1)}{1 - \beta \theta},
\]

and

\[
Q(1) = \frac{\beta (1 - \theta)C + \beta \theta [K^*(qA - 1) + (1 - \lambda)Q(\hat{p})]}{1 - \beta \theta \lambda},
\]

are well-defined. Similarly, expected profits for \( \hat{p} \) in both the cases of information-sensitive and insensitive can be computed such that firms choose the highest possible amount of borrowing.

4. Determination of \( K(0) \) and \( Q(0) \).

These are determined by

\[
K(0) = Q(0) = \frac{\beta \theta [K(0)(qA - 1) + (1 - \lambda)Q(\hat{p})]}{1 - \beta \theta \lambda}.
\]

5. Determination of \( K(p) \) and \( Q(p) \).

There are two cases, from which the firm chooses the highest possible borrowing:

(a) \( p \) is information-insensitive:

\[
K(p) = \frac{\gamma}{(pQ(1) - p)(1 - q)}.
\]

(b) \( p \) is information-sensitive:

\[
K(p) = pK^* + (1 - p)K(0) - \frac{\gamma}{(qA - 1)}.
\]
where in both cases $Q(p)$ only depends on $K(p)$,

$$Q(p) = \frac{\beta(1 - \theta)pC + \beta\theta[K(p)(qA - 1) + (1 - \lambda)Q(\hat{p})]}{1 - \beta\theta\lambda}.$$ 

The determination of which regions are information-sensitive and insensitive is similar to the case in the main text. Expected profits with information-sensitive debt is linear while expected profits with information-insensitive debt depend on the shape of the land prices.

### 3. Potential Multiplicity

In the previous steps we show how to solve the optimal borrowing when land prices are endogenous. However these steps do not guarantee uniqueness of the solution (for example under information-insensitiveness, equation (16) does not imply uniqueness). The intuition is the following: If there is no confidence that in the future low quality collateral can be used to sustain borrowing, this will reduce the price of the collateral, reinforcing the fact that it will not be able to sustain such a borrowing. This “complementarity” between the price of collateral and borrowing capabilities is what creates potential multiplicity.

An interesting example is the extreme opposite to the one assumed in the main text, this is $\theta = 1$. In this extreme case, the potential multiplicity takes a very clear form. Assume an equilibrium where all collateral sustain borrowing of $K^*$ without producing information, regardless of the perception $p$ that land is good. If this is the case, the price for all collateral is independent of $p$,

$$Q(p) = \frac{\beta K^*(qA - 1)}{1 - \beta}.$$ 

Given these prices, borrowing without information acquisition is not binding because $Q(1) = Q(\hat{p})$ and then $K^* < \frac{\gamma}{(p-p)(1-q)} = \infty$. As conjectured, all collateral can borrow $K^*$ regardless of $p$. In general, a larger $\theta$ allows for the existence of an equilibrium that sustains a lot of credit without information acquisition, but fragile to beliefs about whether land with low $p$ can sustain high credit.

### A.3 Proof Proposition 10

As in Proposition 4, if the negative shock happens in period $t$, the distribution in period $t$ is: $f(\eta) = \lambda^t\hat{p}$, $f(\eta\hat{p}) = (1 - \lambda^t)$ and $f(0) = \lambda^t(1 - \hat{p})$.

1. Without information, in period $t + 1$ the distribution of beliefs is $f_{II}(\eta) = \lambda^{t+1}\hat{p}$, $f_{II}(\eta\hat{p}) = \lambda(1 - \lambda^t)$, $f_{II}(\hat{p}) = (1 - \lambda)$ and $f_{II}(0) = \lambda^{t+1}(1 - \hat{p})$. 

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A policy $\alpha$ introduced at $t+1$ change beliefs from $\eta$ to $\alpha+\eta(1-\alpha)$, from $\hat{p}$ to $\alpha+\hat{p}(1-\alpha)$, from $\eta\hat{p}$ to $\alpha+\eta\hat{p}(1-\alpha)$ and from 0 to $\alpha$.\footnote{The same results hold if the policy is introduced in subsequent periods.} The distribution of beliefs then becomes: $f_{II}(\alpha + \eta(1-\alpha)) = \lambda^{t+1}\hat{p}$, $f_{II}(\alpha + \eta\hat{p}(1-\alpha)) = \lambda(1 - \lambda^t)$, $f_{II}(\alpha + \hat{p}(1-\alpha)) = (1 - \lambda)$ and $f_{II}(\alpha) = \lambda^{t+1}(1 - \hat{p})$.

Since we assume $\hat{p} > p^H$ and $\eta > p^H$, the positive shock does not affect borrowing for those beliefs. Since we assume $\alpha + \eta\hat{p}(1-\alpha) > p^H$, borrowing increases from $K(\eta\hat{p})$ to $K^\star$. Similarly, borrowing of known bad collateral increases from 0 to $K(\alpha)$.

Only individual beliefs change, not their distribution. Then, using equation (10), we can compare the aggregate consumption with and without policy,

$$
\Delta_{II}^{II} \equiv W_{t+1|\alpha}^{II} - W_{t+1}^{II} = \lambda(qA - 1)[(1 - \lambda^t)(K^\star - K(\eta\hat{p})) + \lambda^t(1 - \hat{p})K(\alpha)]. \tag{19}
$$

2. With information production, in period $t + 1$ the distribution of beliefs is $f_{IS}(\eta) = \lambda\eta\hat{p}(1 - \lambda^t)$, $f_{IS}(\eta) = \lambda^{t+1}\hat{p}$, $f_{IS}(\hat{p}) = (1 - \lambda)$, $f_{IS}(0) = \lambda(1 - \lambda^t\hat{p}) - \eta\hat{p}(1 - \lambda^t)$.

After the policy, beliefs change from $\eta$ to $\alpha + \eta(1-\alpha)$, from $\hat{p}$ to $\alpha + \hat{p}(1-\alpha)$, and from 0 to $\alpha$. Also, beliefs 1 remain 1. Since we assume $\hat{p} > p^H$ and $\eta > p^H$, the positive shock does not affect the borrowing for those beliefs. Borrowing for bad collateral increase from 0 to $K(\alpha)$. Again, we can compare the aggregate consumption with and without policy,

$$
\Delta_{II}^{IS} \equiv W_{t+1|\alpha}^{IS} - W_{t+1}^{IS} = \lambda(qA - 1)[(1 - \lambda^t\hat{p}) - \eta\hat{p}(1 - \lambda^t)]K(\alpha). \tag{20}
$$

Taking the difference between equations (19) and (20),

$$
\Delta_{II}^{II} - \Delta_{II}^{IS} = \lambda \left[ (1 - \lambda^t)[K^\star - K(\eta\hat{p})] + \left[ \lambda^t(1 - \hat{p}) - (1 - \lambda^t\hat{p}) + \eta\hat{p}(1 - \lambda^t) \right] K(\alpha) \right]
= \lambda(1 - \lambda^t) [K^\star - K(\eta\hat{p}) - (1 - \eta\hat{p})K(\alpha)].
$$

In the range of interest, where $\eta\hat{p} < p^{Ch}$ and there are incentives for information production, avoiding information production would imply $K(\eta\hat{p}) \leq \eta\hat{p}K^\star - \frac{\gamma}{(qA - 1)}$. Using this upper bound to evaluate the expression above, we obtain that the increase in borrowing at $t + 1$ induced by the policy is larger when no information is acquired than when information is acquired.

$$
\Delta_{II}^{II} - \Delta_{II}^{IS} \geq \lambda(1 - \lambda^t) \left[ K^\star - \eta\hat{p}K^\star + \frac{\gamma}{(qA - 1)} - (1 - \eta\hat{p})K(\alpha) \right]
\geq \lambda(1 - \lambda^t)(1 - \eta\hat{p}) [K^\star - K(\alpha)] + \frac{\gamma}{(qA - 1)(1 - \eta\hat{p})} > 0.
$$