

# Dynamic factor models of consumption, hours and income

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### Summary

This paper addresses two questions in the economics of intertemporal choice. First, what are the key factors that drive fluctuations in income and what are the time paths of their effects? Second, how do consumers respond to these factors? We answer these questions by estimating dynamic factor models of consumption, hours, wages, unemployment, and income that account for measurement error and the fact that variables used in the study are measured at different time intervals and/or are aggregates for the calendar year. We pay special attention to a dynamic factor representation of a joint life cycle model of consumption and labour supply, which permits us to quantify the effect of wages, unemployment, and other factors on the marginal utility of income as well as to estimate the substitution effects of wage changes on labour supply and consumption. © 2002 University of Venice

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### Introduction

This paper addresses two long-standing questions in the economics of intertemporal choice. First, what are some of the key factors that drive fluctuations in income and what are the time paths of their effects? Second, how do consumers respond to these factors?

We take two approaches to these questions. First, we estimate how a household's income process is affected by changes in wages, hours of work and unemployment. Then we use Friedman's (1957) permanent income model to interpret the consumption response to changes in the income process and the factors which affect it. There are many studies that examine the response of consumption to changes in income, as well as to specific events, such as illness and job loss, that induce income changes, but the magnitudes of the responses are hard to interpret without a model of the income process.

The permanent income hypothesis assumes that labour supply is exogenous. When labour supply is endogenous, it is desirable to decompose consumption and labour supply responses to changes in exogenous factors into substitution and income effects. In our second approach, we use a life cycle model of consumption and labour supply to estimate the response of consumption and labour supply to changes in the marginal utility of income induced by a typical wage, price or unemployment shock.<sup>†</sup> Little empirical research on these consumption and labour supply responses to such shocks has been conducted. This was true when we completed a draft of this paper more than a decade ago, (Altonji, Martins and Siow (1987), hereafter AMS), and it is still true.

Our econometric models are vector moving average representations of the consumption, hours, wages, unemployment, and

<sup>&</sup>lt;sup>†</sup> The pure intertemporal substitution responses to wages, prices, and interest rates (with the marginal of utility of income held constant) can and have been estimated without a model of wage, price and interest rate behaviour. (See for example, Heckman and MaCurdy (1980), MaCurdy (1981), Hansen and Singleton (1983) and Browning et al. (1985) and Altonji (1986)). Furthermore, following Hall (1978), many studies have tested versions of the permanent income and life cycle models by examining whether past information about wages, interest rates and other budget constraint determinants is related to changes in consumption. These studies of "excess sensitivity", surveyed in Hayashi (1987) and more recently in Deaton (1992), Browning and Lusardi (1996), do not require a detailed model of the income process either. See Deaton (1985), Mayer (1972) and Hayashi (1987) for discussions and references to the permanent income hypothesis. See Altonji (1986), Blundell (1986), Browning et al. (1985), Ghez and Becker (1975), Heckman (1974), Heckman and MaCurdy (1980), MaCurdy (1981, 1983), and more recently Blundell and MaCurdy (1999) for detailed discussions and references to the literature on life cycle models.

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income processes.<sup>†</sup> Structural models of consumer behaviour place restrictions on the autocovariances and cross covariances of these variables. We estimate a model's parameters by fitting the theoretical covariances of the model to sample covariances that are estimated from data. The data are from the Panel Study of Income Dynamics (PSID).

The vector moving average representation of the data has a number of advantages. First, it provides a convenient framework for incorporating measurement error in the empirical analysis.‡ Second, by working with a multivariate model of income (which is of independent interest), we can examine the response of consumption to identifiable sources of income variation that differ in their persistence.§ Third, within a life cycle framework, we can estimate the response of consumption and labour supply to changes in the marginal utility of income induced by a typical wage or unemployment shock, and we can identify the sources of variation in the marginal utility of income. Fourth, we can account for nonstationarity in a simple way. Fifth, in many micro panel data sets, including the PSID, the variables relevant to a study are measured at different time intervals and/or are aggregates for the calendar year. We show how quarterly dynamic factor models can be restricted with polynomial distributed lag structures to cope with this problem.

Our main empirical findings are the following.

 $\dagger$  Factor models of income include Baker (1997), Lillard and Weiss (1979), Hause (1980), Kearl (1988), MaCurdy (1982a and b), Abowd and Card (1987 and 1989) and Chowdhury and Nickell (1985).

<sup>±</sup> Duncan and Hill (1984) have provided some direct evidence on the importance of measurement error by comparing the responses of employees of a single large firm with the records of the employer. They find that measurement error accounts for 16.8 % of the variance in the earnings level. Under reasonable assumptions, these would translate into a much larger percentage of the variance in the first difference of earnings. Measurement error in nonlabour income is likely to be an even more serious problem. Mellow and Sider (1984) use matched employer/employee responses to show the existence of considerable measurement error in the survey data. Altonji (1986) provides evidence, of substantial measurement error in the first difference of the log of earnings divided by hours and in hours of work. For the same data, Altonji and Siow (1987) found that the life cycle model may be wrongly rejected if measurement error in the income variable is ignored, and found that the ordinary least squares estimate of the regression coefficient relating the change in consumption to the change in income is only one third of the estimate obtained using an instrumental variables estimator to account for measurement error. In his survey, Hayashi (1987) concludes that measurement error is a major issue in micro panel studies of consumption and liquidity constraints. For a recent survey of research on measurement error, see Bound, Brown and Mathiowetz (2000).

§ Holbrook and Stafford (1971) analysed the link between the level of consumption and various components of family income using one year of consumption data and three years of income data for a cross section of families.

- 1. There is substantial measurement error in income in all our models (45% to 75% of the variance of the change in measured income). The lower estimates are based on models that account for non-synchronization in the data. We are less successful in obtaining precise estimates of measurement error in measured wages and work hours.
- 2. Innovations in the wage, unemployment and work hours explain surprisingly little of the remaining variance in the change in family income. This lack of explanatory power is consistent with results from descriptive regressions. When we account for non-synchronization in the data, the explanatory power of the economic variables that determine income improves.
- 3. After two years, a shock to unemployment of the head of household has essentially no effect on income. In contrast, 95% of the effect of a shock to the wage remains after 2 periods. The corresponding fraction for work hours is about two-thirds, suggesting that work hours adjustments (conditional on unemployment) are more permanent than unemployment shocks.
- 4. The zero restrictions implied by the permanent income model for the covariance structure of the data are not rejected. There is little evidence that lagged factors affect the change in consumption.<sup>†</sup>
- 5. Allowing for endogenous labour supply, we provide an estimate of the total variance of the innovation in the marginal utility of income. Wage and unemployment innovations together explain over 40% of the total variance. Wage innovations are responsible for most of this variance. This is consistent with the evidence that wage innovations have a substantial variance and have a substantial and persistent effect on income.
- 6. We do not find much evidence that substitution effects are important. Our estimates suggest that the intertemporal labour supply elasticity is small. The point estimates are actually negative but are not significantly different from 0. We obtain a small negative estimate of the cross substitution effect of the wage on consumption. The point estimate suggests

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<sup>&</sup>lt;sup>†</sup> HM found that the change in consumption responds to the lagged change in income using the PSID, and this result is frequently cited as evidence against a simple rational expectations permanent income model. The bulk of the evidence from time series data is consistent with their results (See Deaton, 1985). However, our finding that this evidence for the PSID is not robust is consistent with the results of our earlier paper (Altonji and Siow, 1987). In that paper we obtain different evidence on the effect of the lagged change in the log of income on the change in the log of consumption with different samples, although the empirical magnitude of the effect was small in all cases. Zeldes (1989) findings on the relationship between change in the log of consumption and the lagged value of the log of income are also sensitive to the details of the specification and sample. See Attanasio (1999) for a recent summary of the evidence.

that consumption and leisure are complements, but the null hypothesis of intra period separability of consumption and leisure cannot be rejected.

The paper proceeds as follows. Section 1 provides an overview of the dynamic factor models studied in the paper. Section 2 and the Appendix discuss the restrictions on the dynamic factor model implied by models of consumer behaviour. Section 3 presents a methodology for taking account of time aggregation and nonsynchronization in estimating such models. Section 4 discusses estimation methods and the data. In Section 5 we discuss the estimates of the covariance stationary model, the properties of the income process, and the response of consumption to current and lagged income shocks. Sections 6 presents estimates of the life cycle model of consumption and labour supply. Section 7 concludes.

### 1. An overview

This paper estimates various dynamic factor models of annual consumption, income and hours. We use the real wage and unemployment as additional indicators of factors which drive these variables.<sup>†</sup> Throughout the paper,  $\Delta$  is the first difference operator,  $C_t$  is consumption in year t,  $Y_t$  is real family income in year t,  $W_t$  is the real wage in year t,  $Z_t$  is 2000 plus hours of unemployment in year t for the head of household, and  $N_t$  is the head's annual work hours in year t. For notational convenience, subscripts for individuals are left implicit. For X = C, W,Y,Z, and  $N, X_t^*$  is the measure of  $X_t$  at t. We estimate models using first differences, with the exception of unemployment  $Z_t$ , which we do not first difference. In analyzing the RE-life cycle model, we replace  $Y_t$  with labour earnings of the head of household  $Y_t^n$ . For convenience, we sometimes refer to the first difference of a variable as the variable itself.

We analyse consumption, hours and income using various dynamic factor models that are nested in the following general model:

A GENERAL DYNAMIC FACTOR MODEL OF CONSUMPTION, INCOME AND HOURS

General consumption model ( $\Delta C_t^*$ ):

 $\Delta C_t^* = \beta_{cw0} u_{wt} + \beta_{cw1} u_{wt-1} + \beta_{cw2} u_{wt-2} + \beta_{cz0} u_{zt} + \beta_{cz1} u_{zt-1}$ 

 $\dagger$  We also experimented with hours of illness, but it did not contribute much to explaining the variables of interest.

$$\begin{aligned} &+ \beta_{cz2} u_{zt-2} + \beta_{cn0} u_{nt} + \beta_{cn1} u_{nt-1} + \beta_{cn2} u_{nt-2} \\ &+ \beta_{cy0} u_{yt} + \beta_{cy1} u_{yt-1} + \beta_{cy2} u_{yt-2} + \beta_{cc0} u_{ct} \\ &+ \beta_{cc1} u_{ct-1} + \beta_{cc2} u_{ct-2} \end{aligned}$$
(1.1a)

### General income model

Income Equation  $(\Delta Y_t^*)$ :

$$\Delta Y_{t}^{*} = \beta_{yw0} u_{wt} + \beta_{yw1} u_{wt-1} + \beta_{yw2} u_{wt-2} + \beta_{yz0} u_{zt} + \beta_{yz1} u_{zt-1} + \beta_{yz2} u_{zt-2} + \beta_{yn0} u_{nt} + \beta_{yn1} u_{nt-1} + \beta_{yn2} u_{nt-2} + \beta_{yy0} u_{yt} + \beta_{yy1} u_{yt-1} + \beta_{yy2} u_{yt-2} + \Delta \varepsilon_{yt}$$
(1.1b)

Annual Work Hours Equation  $(\Delta N_t^*)$ :

$$\begin{split} \Delta \mathbf{N}_{t}^{*} &= \beta_{nw0} \mathbf{u}_{wt} + \beta_{nwl} \mathbf{u}_{wt-1} + \beta_{nw2} \mathbf{u}_{wt-2} + \beta_{nz0} \mathbf{u}_{zt} \\ &+ \beta_{nz1} \mathbf{u}_{zt-1} + \beta_{nz2} \mathbf{u}_{zt-2} + \beta_{nn0} \mathbf{u}_{nt} \\ &+ \beta_{nn1} \mathbf{u}_{nt-1} + \beta_{nn2} \mathbf{u}_{nt-2} + \beta_{ny0} \mathbf{u}_{yt} \\ &+ \beta_{ny1} \mathbf{u}_{yt-1} + \beta_{ny2} \mathbf{u}_{yt-2} + \Delta \varepsilon_{nt} \end{split}$$
(1.1c)

Wage Equation  $(\Delta W_t^*)$ :

$$\Delta \mathbf{W}_{t}^{*} = \beta_{ww0} \mathbf{u}_{wt} + \beta_{ww1} \mathbf{u}_{wt-1} + \beta_{ww2} \mathbf{u}_{wt-2} + \Delta \varepsilon_{wt}$$
(1.1d)

Unemployment Equation  $(Z_t^*)$ :

$$Z_{t}^{*} = \beta_{zz0}u_{zt} + \beta_{zz1}u_{zt-1} + \beta_{zz2}u_{zt-2}$$
(1.1e)

The factors  $u_{ct},\ u_{wt},\ u_{zt},\ u_{nt},$  and  $u_{yt}$  are assumed to have the following properties:

 $Var(u_{it}) = 1$  i = c,y,w,z,n(Normalization of the variances to 1)  $u(u_1, u_{i-1}) = 0$   $i = a_1 u_1 u_2 n_1 k \neq 0$ 

$$\begin{split} Cov(u_{it},u_{it-k}) &= 0 \quad i = c,y,w,z,n; k \neq 0 \\ (No \ serial \ correlation) \\ Cov \ (u_{it},u_{jt-k}) &= 0 \quad i \neq j; for \ all \ k \end{split}$$

(0 cross covariances)

The measurement error (ME) components have the properties:

$$\begin{split} &Var(\epsilon_{it})=\sigma_i^2 \quad i=y,\!w,\!n\\ &Cov(\epsilon_{it},\epsilon_{it-k})=0 \quad i=y,\!w,\!n; k\neq 0 (No \; serial \\ & correlation \; in \; ME) \end{split}$$

## $Cov(\varepsilon_{it}, \varepsilon_{jt}) = 0$ $i \neq j$ , (ME are unrelated)

### $Cov(u_{it}, \varepsilon_{it-k}) = 0$ for all i,j,k.(ME are unrelated

### to true variables)

The  $\beta_{ijk}$  are the response coefficients or "factor loading" relating the variable i to factor j lagged k periods. For example,  $\beta_{ywl}$  is the response of income to the wage factor  $u_{wt-1}$ . We restrict the analysis to second-order vector moving average (MA) models because autocovariances and cross covariances among the variables are very small after two lags. We work with a dynamic factor framework rather than a VAR regression model for  $\Delta C_t^*$ ,  $\Delta Y_t^*$ ,  $\Delta W_t^*$ ,  $Z_t^*$ ,  $\Delta N_t^*$  in part because it is very difficult to accommodate measurement error in the latter framework.

Equations (l.ld) and (l.le) specify that wages  $\Delta W^*_t$  and unemployment  $Z^*_t$  are autonomous processes that are driven only by their own factors. The zero correlation between wages and unemployment implied by this assumption is tested below. We use the same equations for wages and unemployment in all of our empirical models.

Equation (l.la) specifies that consumption  $\Delta C_t^*$  depends on the current and lagged wage, unemployment, hours of work, and income factors. We also include current and lagged values of an independent consumption factor  $u_{ct}$  that captures consumption shocks unrelated to the rest of the model as well as measurement error in consumption. Equations (l.lb) and (l.lc) specify that income  $\Delta Y_t^*$  and hours  $\Delta N_t^*$  depend on the current and lagged wage, unemployment, hours, and income factors. Specific structural models of consumption, income and hours imply additional restrictions on the factor loadings in the consumption, income and hours equations.

The general model also allows for serially uncorrelated measurement errors  $\varepsilon_{yt}$ ,  $\varepsilon_{wt}$ , and  $\varepsilon_{nt}$  in the measures  $Y_t^*$ ,  $W_t^*$  and  $N_t^*$  of the variables  $Y_t$ ,  $W_t$ , and  $N_t$ . In specific cases, we experiment with allowing  $\varepsilon_{yt}$ ,  $\varepsilon_{wt}$ , and  $\varepsilon_{nt}$  to be first-order moving average measurement errors.

The above model cannot be estimated with our data. Instead we consider versions that incorporate restrictions that are implied by various economic models of consumption and hours. For each economic model, we first test the zero restrictions implied by the theory against a model which imposes only covariance stationarity on the data. Then we estimate the economic model, test the overidentifying restrictions against larger factor models and also models which only impose covariance stationarity, and evaluate the parameter estimates. Without further ado we turn to those models, beginning with the permanent income model of consumption, and then turning to life cycle consumption and labour supply model.

### 2. Structural models of consumer behaviour

In this section, we consider the implications of models of consumer behaviour for the parameters of the dynamic factor model.

### 2.1 consumption and income when labour supply is exogenous.

Many consumption studies assume that hours worked and income are determined independently of consumption. Consequently, in some models we restrict the hours equation of the general income model in (l.lc) to be

### Exogenous Work Hours Equation ( $\Delta N_t^*$ ):

$$\Delta \mathbf{N}_{t}^{*} = \beta_{nn0} \mathbf{u}_{nt} + \beta_{nn1} \mathbf{u}_{nt-1} + \beta_{nn2} \mathbf{u}_{nt-2} + \beta_{nz0} \mathbf{u}_{zt} + \beta_{nzl} \mathbf{u}_{zt-1} + \beta_{nz2} \mathbf{u}_{zt-2} + \Delta \varepsilon_{nt}.$$
(1.1c')

We will refer to the income model consisting of equation (l.1c') for hours and equations (l.lb, l.ld, 1.1e) for income, wages and unemployment as the "income model with exogenous hours". These zero restrictions imposed by (1.1c') will be tested.

The basic idea of the rational expectations-permanent income hypothesis (RE-PIH) is that consumers base consumption today on lifetime resources and lifetime needs and form expectations rationally. This has two implications. The first is that the change in consumption only depends on innovations in income, not anticipated changes, assuming that the preferences are separable across years and consumption is nondurable. In this case, the change in consumption only depends on the contemporaneous factors affecting income:  $u_{wt}$ ,  $u_{zt}$ ,  $u_{nt}$ , and  $u_{yt}$ .<sup>†</sup> This implies the following consumption equation:

RE-PIH Consumption Equation ( $\Delta C_t^*$ ):

$$\Delta C_{t}^{*} = \beta_{cw0} u_{wt} + \beta_{cz0} u_{zt} + \beta_{cn0} u_{nt} + \beta_{cy0} u_{yt} + \beta_{cc0} u_{ct} + \beta_{cc1} u_{ct-1} + \beta_{cc2} u_{ct-2}.$$
(2.1.1)

<sup>†</sup> We follow Hall and Mishkin (1982) and others in applying these 0 restrictions on  $\beta_{cy1}$  and  $\beta_{cy2}$  using data on family income, but they only hold for nonasset income. The asset component of total income is influenced by consumption preferences, which influence savings. As a result, the income shocks and consumption preference shocks could be correlated, in which case lags of  $u_{yt}$  may be related to  $\Delta C_t^*$ . See the Appendix. In practice there is little evidence of this, perhaps because we use food data.

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The second implication is that the size of factor loadings  $\beta_{cw0}$ ,  $\beta_{cz0}$ ,  $\beta_{cn0}$ , and  $\beta_{cy0}$  are increasing functions of the magnitude and the persistence of the effects of  $u_{wt}$ ,  $u_{zt}$ ,  $u_{nt}$ , and  $u_{yt}$  on income. Under special assumptions about preferences, consumption is equal to the annuity value of wealth plus the annuity value of the discounted value of current and future income. In this case,  $\beta_{cw0}$ ,  $\beta_{cz0}$ ,  $\beta_{cn0}$ , and  $\beta_{cv0}$  must satisfy

$$\beta_{cj0} = \alpha(\beta_{yj0} + \rho\beta_{yj1} + \rho^2\beta_{yj2}) \quad \mathbf{j} = \mathbf{w}, \mathbf{z}, \mathbf{n}, \mathbf{y}$$
(2.1.2)

where  $\rho$  is the inverse of one plus the market interest rate and  $\beta_{yj0} + \rho\beta_{yj1} + \rho^2\beta_{yj2}$  is the change in permanent income induced by  $u_{jt}$ . The parameter  $\alpha$  is the marginal propensity to consume food out of permanent income and we add it to account for the fact we only have data on food expenditures. Equation (2.1.2) is valid only when income is measured in levels.

An important focus in AMS is on estimating the consumption equation with (2.1.2) imposed. This amounts to an extension of the innovative panel data studies of Hall and Mishkin (1982; hereafter HM) and Bernanke (1984), who estimate income and consumption jointly but assume that income is measured without error, to the case of a multivariate model of income. This permits one to account for measurement error and examine the response of consumption to identifiable sources of income variation that differ in their persistence. However, the analysis in AMS is unsatisfactory in a number of ways. First, the assumption of quadratic utility rules out a precautionary savings motive, which has received considerable emphasis in more recent work. Second the assumption of exogenous labour income is problematic. Third, the framework used in AMS, HM and other structural analyses of the RE-PIH is unsatisfactory because it treats the rate at which consumers discount future income as the key parameter to be estimated while at the same time maintaining that this discount rate is equal to the market rate of interest faced by the consumers. In this circumstance, any substantial difference between the discount rate estimate and consumer borrowing and lending rates should be interpreted as a rejection of the model rather than as a consistent estimate of the discount rate. Fourth, our estimates of the discount rate are often imprecise and are sensitive to the details of the specification.

For all of these reasons, we de-emphasize permanent income type models. In the Appendix we drop the assumption that the subjective discount rate may differ from the market interest rate and derive the implications for consumption, family income, wealth, and savings of a more general model that nests the permanent income model and Keynesian model. We also briefly summarize our experience with estimating the consumption equation when (2.1.2) is imposed. In this paper, we measure income primarily in logs and so we use (2.1.2) only as a guide to interpreting the relative effects of different innovations on income.

### 2.2 the life cycle model of consumption and labour supply

The permanent income model ignores endogenous labour supply. When labour supply is endogenous, it is desirable to separate out the intertemporal substitution and within period substitution effects of the wage rate on consumption and labour supply from the "income effect" of this variable. To study these effects, we study a standard life cycle model of consumption and labour supply behaviour under uncertainty. Following most of the literature, we incorporate intertemporal separability of preferences.† Using loglinear approximations to the marginal utility of income constant (Frisch) demand equations for hours and consumption and the intertemporal optimality condition for expected utility maximization, one obtains the following model for the first difference equations for hours and consumption (e.g. Altonji (1986), McCurdy (1983)).

$$\begin{aligned} \mathbf{ln\lambda}_{t} &= \mathbf{ln\lambda}_{t-1} - \mathbf{lnr}_{t-1,1} + \eta_{t} \end{aligned} \tag{2.2.1} \\ \Delta \mathbf{N}_{t}^{*} &= \mathbf{constant} + \beta_{n} \Delta \mathbf{W}_{t} + (\beta_{n} + \beta_{nc}) \Delta \mathbf{P}_{t} - (\beta_{n} + \beta_{nc}) \mathbf{r}_{t-1,1} \\ &+ (\beta_{n} + \beta_{nc}) \eta_{t} + \beta_{nc0} \mathbf{u}_{zt} + \beta_{nc1} \mathbf{u}_{zt-1} + \beta_{nc2} \mathbf{u}_{zt-2} \end{aligned}$$

$$+ \beta_{nn} \eta_{n+1} + \beta_{n+1} + \beta_{n+1}$$

$$+ p_{nn0}u_{nt} + p_{nnl}u_{nt-1} + p_{nn2}u_{nt-2} + \Delta\varepsilon_{nt}$$
(2.2.2)

$$\Delta C_t^* = constant + \beta_{cn} \Delta W_t + (\beta_c + \beta_{cn}) \Delta P_t - (\beta_c + \beta_{cn}) r_{t-1,1}$$

+ 
$$(\beta_{c} + \beta_{cn})\eta_{t} + \beta_{cc0}u_{ct} + \beta_{ccl}u_{ct-1} + \beta_{cc2}u_{ct-2}$$
 (2.2.3)

- $\lambda_t$ : marginal utility of income at date t.
- $r_t$ : nominal interest rate at date t.
- $\eta_t$ : innovation in the log of the marginal utility of income.
- $N_t^*$ : log of measured labour supply at date t.
- $C_t^*$ : log of measured consumption at date t.
- W<sub>t</sub>: log of the real wage at date t.
- $P_t$ : log of the price level at date t.
- $\varepsilon_{nt}$ : serially uncorrelated measurement error.

Equation (2.2.1) shows the evolution of the expected value of the marginal utility of income,  $\lambda_t$ . Under rational expectations,  $\eta_t$ , the innovation in the marginal utility of income, is uncorrelated with information known to the consumer in t - 1.

Equation (2.2.2) is the first difference of the Frisch labour supply function. Equation (2.2.3) is the Frisch consumption equation in

 $<sup>\</sup>dagger$  Hotz et al. (1988), Eichenbaum et al. (1988) and Blundell (1986) relax the separability assumption.

first differences. With intertemporal separability of preferences,  $\Delta N_t$  and  $\Delta C_t$  depend upon current assets and the distribution of future wages and prices only through changes in  $\ln\lambda_t$ . The variables  $u_{nt}$  and  $u_{ct}$  are taste shifters. Measurement error in consumption is incorporated in  $u_{ct}$ . In estimation, we assume that  $u_{nt}$  and  $u_{ct}$  are uncorrelated. This implies a zero correlation between the changes in labour supply and consumption preferences that are not captured by the demographics variables we control for. The variable  $u_{zt}$  is the factor driving unemployment. It may reflect changes in labour supply preferences and/or constraints on hours of work.†

The parameter  $\beta_n$  is the intertemporal labour supply elasticity. The parameters  $\beta_n + \beta_{nc}$  and  $\beta_c + \beta_{cn}$  are the intertemporal substitution effects of changes in the nominal interest rate on labour supply and consumption (respectively). Strict concavity of preferences and the assumption that consumption and leisure are normal goods imply  $\beta_n + \beta_{nc} > 0$  and  $\beta_c + \beta_{cn} < 0$ . (See Heckman (1974).) Symmetry of the constant cross-substitution effects implies that the elasticity  $\beta_{cn}$  is approximately equal to  $\beta_{nc}(N_tW_t/C_t)$ . Under the assumption of intraperiod separability,  $\beta_{cn} = \beta_{nc} = 0$ .

Shocks to budget parameters such as the wage rate and to preferences have "income" effects on consumption and labour supply through  $\ln \lambda_t$ . The value of  $\ln \lambda_t$  is determined by the parameters of the utility function, the individual's wealth level and expectations about the distribution of current and future values of wages, interest rates, prices, and the preference shifters. Since an analytical solution for  $\ln \lambda_t$  and the innovation  $\eta_t$  does not exist in the case of uncertainty and time varying preferences, we simply specify  $\eta_t$  as an unrestricted linear function of unanticipated changes in exogenous (with respect to preferences) factors affecting income and unanticipated changes in preferences. Specifically,

$$\eta_{t} = \beta_{\eta w 0} \mathbf{u}_{wt} + \beta_{\eta z 0} \mathbf{u}_{zt} + \beta_{\eta v 0} \mathbf{u}_{vt} + \mathbf{u}_{\eta t}, \qquad (2.2.5)$$

where  $u_{wt}$  is the wage innovation,  $u_{zt}$  is the unemployment innovation, and  $u_{yt}$  is the innovation in components of earnings not directly related to changes in the wage rate or work hours, and  $u_{\eta t}$  is a residual factor with variance  $\sigma_{\eta t}^2$ . In anticipation of the empirical specifications used below, in (2.2.5) we impose the assumptions that  $u_{ct}$  and  $u_{nt}$  do not affect  $\eta_t$ . This will be true only

<sup>&</sup>lt;sup>†</sup> As Ham (1986) and others have discussed, the presence of hours constraints may bias the estimates of the labour supply parameters, particularly if  $u_{zt}$  is correlated with the other factors in the model. Problems may also arise if, as in the Lucas and Rapping model of unemployment, hours of unemployment are intrinsically related to labour supply decisions and vary with the wage rate. We are ignoring these considerations. Note that we cannot reject the hypothesis that covariances between the wage change and unemployment are 0.

if consumers have perfect foresight about consumption and labour supply preferences (We were not able to estimate models which relax this assumption.). The exclusion of lagged values  $u_{wt}$ ,  $u_{zt}$  and  $u_{yt}$  from (2.2.5) is implied by the assumption of RE, which implies that  $\eta_t$  is uncorrelated with information known in  $t - 1.\ddagger$ 

Substitute (2.2.5) for  $\eta_t$ , and (l.ld) for  $\Delta W_t$ , in the first differenced consumption and labour supply equations (2.2.2) and (2.2.3). After suppressing constant terms this leads to

$$\begin{split} \Delta C_{t}^{*} &= \beta_{cn}(\beta_{ww0}u_{wt} + \beta_{ww1}u_{wt-1} + \beta_{ww2}u_{wt-2}) + (\beta_{c} + \beta_{cn}) \\ &\times (\beta_{\eta w0}u_{wt} + \beta_{\eta z0}u_{zt} + \beta_{\eta y0}u_{yt}) + (\beta_{c} + \beta_{cn}) \\ &\times (u_{\eta t} - r_{t-1,1} + \Delta P_{t}) + \beta_{cc0}u_{ct} \\ &+ \beta_{cc1}u_{ct-1} + \beta_{cc2}u_{ct-2} \\ \Delta N_{t}^{*} &= \beta_{n}(\beta_{ww0}u_{wt} + \beta_{ww1}u_{wt-1} + \beta_{ww2}u_{wt-2}) + (\beta_{n} + \beta_{nc}) \\ &\times (\beta_{\eta w0}u_{wt} + \beta_{\eta z0}u_{zt} + \beta_{\eta y0}u_{yt}) + (\beta_{n} + \beta_{nc}) \\ &\times (u_{\eta t} - r_{t-1,1} + \Delta P_{t}) + \beta_{nz0}u_{zt} + \beta_{nz1}u_{zt-1} \\ &+ \beta_{nz2}u_{zt-2} + \beta_{nn0}u_{nt} + \beta_{nn1}u_{nt-1} \\ &+ \beta_{nn2}u_{nt-2} + \Delta\varepsilon_{nt} \end{split}$$
(2.2.7)

We estimate versions of (2.2.6) and (2.2.7) along with the wage and unemployment equations (l.ld and l.le). Unfortunately, the parameters  $\beta_{\eta w0}$ ,  $\beta_{\eta z0}$ ,  $\beta_{\eta y0}$ ,  $\sigma_{\eta}^2$ ,  $\beta_c$  and the parameter  $\beta_{nc}$  are not identified unless one imposes the symmetry restriction  $\beta_{cn} = \beta_{nc}(N_t W_t)/C_t$ . We provide one set of estimates with this restriction imposed and  $N_t W_t/C_t = 4$ . We also estimate the model with intraperiod nonseparability between food consumption and labour supply imposed ( $\beta_{cn} = \beta_{nc} = 0$ .)

Finally, one may make use of the fact that a measure of the change in labour earnings  $\Delta Y_t^{n*}$  is available that is measured independently of  $\Delta N_t$  and  $\Delta W_t$  by combining (2.2.7) and (l.ld) to form the equation

$$\Delta Y_t^{n*} = (1 + \beta_n)(\beta_{ww0}u_{wt} + \beta_{ww1}u_{wt-1} + \beta_{ww2}u_{wt-2}) + (\beta_n + \beta_{nc})$$
$$\times (\beta_{\etaw0}u_{wt} + \beta_{\etaz0}u_{zt} + \beta_{\etay0}u_{yt}) + (\beta_n + \beta_{nc})$$

 $\ddagger$  As Chamberlain (1984) pointed out and Hayashi (1985) observed in a similar context, the rational expectations hypothesis does not imply that the forecast error  $\eta_t$  is uncorrelated with past information when the distribution is taken across households rather than over time for a given household. If the effect of an aggregate disturbance on the marginal utility of income is systematically related to lagged values of  $u_{wt}$ ,  $u_{zt}$  and  $u_{yt}$ , then these determinants will be correlated with  $\eta_t$  in a short panel even when the main effects of aggregate shocks are removed using time dummies. A similar problem would arise in HM's analysis or in the work with the RE-PIH model discussed above.

$$\times (\mathbf{u}_{\eta t} - \mathbf{r}_{t-1,1} + \Delta \mathbf{P}_{t}) + \beta_{nz0}\mathbf{u}_{zt} + \beta_{nz1}\mathbf{u}_{zt-1} + \beta_{nz2}\mathbf{u}_{zt-2} + \beta_{nn0}\mathbf{u}_{nt} + \beta_{nn1}\mathbf{u}_{nt-1} + \beta_{nn2}\mathbf{u}_{nt-2} + \beta_{yy0}\mathbf{u}_{yt} + \beta_{yy1}\mathbf{u}_{yt-1} + \beta_{yy2}\mathbf{u}_{yt-2} + \Delta\varepsilon_{yt}$$
(2.2.8)

The restrictions in (2.2.8) are not satisfied by the data. Consequently, we report estimates of labour earnings without restricting  $\Delta Y_t^{n*}$ , using a specification which is analogous to equation (l.lb) for family income. The equation is

$$\Delta Y_{t}^{n*} = \beta_{yw0} u_{wt} + \beta_{yw1} u_{wt-1} + \beta_{yw2} u_{wt-2} + \beta_{yz0} u_{zt} + \beta_{yz1} u_{zt-1} + \beta_{yz2} u_{zt-2} + \beta_{yn0} u_{nt} + \beta_{yn1} u_{nt-1} + \beta_{yn2} u_{nt-2} + \beta_{yy0} u_{yt} + \beta_{yy1} u_{yt-1} + \beta_{yy2} u_{yt-2} + \Delta \varepsilon_{yt}$$
(2.2.9)

The effects of the interest rate and the price change are removed using year dummies. We are ignoring cross sectional variation in the after tax interest rate.

By estimating the system consisting of (2.2.6, 2.2.7, 1.1d, 1.1e and 2.2.9), we can get estimates of  $\beta_n$ ,  $\beta_c$ ,  $\beta_{cn}$  and  $\beta_{nc}$ . We pay special attention to the responses of  $\Delta C_t^*$  and  $\Delta N_t^*$  to the shocks  $u_{wt}$ ,  $u_{zt}$ , and  $u_{yt}$  via their effects on  $\eta_t$ .<sup>†</sup> From the same intuition that leads to (2.1.2) in the permanent income case, we expect that more permanent shocks have larger effects on  $\eta_t$  than transitory ones, just as persistent shocks to income induce larger changes in permanent income than do transitory shocks.

### 3. Time aggregation and non-synchronous measurements

In many micro panel data sets, the variables relevant to a study are measured at different time intervals. For example, in the PSID, individuals are interviewed at yearly intervals. The consumption measure and the hourly wage measure refer to the time of the survey (typically in March or April) while family income and hours unemployed refer to the calendar year which precedes the survey date. This poses a problem because the inconsistency of the timing may weaken the relationship between the change in family income and the change in the wage, and distort the dynamic relationship between the two. Furthermore, the differences in the timing of the consumption, wage, income, and unemployment

 $<sup>\</sup>dagger$  To our knowledge, this paper is the first to estimate the contribution of particular factors to the variance in the innovation of the marginal utility of income. Using aggregate time series data, Attfield and Browning (1985) provide estimates of the covariance of innovation of the marginal utility of income with price changes.

variables may affect the estimates of the relative response of consumption and hours to the various factors, particularly because the marginal utility of income should not respond to lagged income innovations if expectations are rational. Since the consumption change is measured a few months after the income change and hours change measures, part of  $u_{yt}$  may be past information. Consequently, estimates of the response of consumption to the income factor  $u_{yt}$  may be understated. (Many tests of the RE-PIH model hinge on the issue of timing of information about income. See the survey by Hayashi (1987) as well as more recent surveys by Browning and Lusardi (1996) and Deaton (1992)). In addition, the use of annual values rather than the unavailable quarterly values may cause problems.

HM recognized the problem of non-synchronization and made adjustments within the annual framework of their model to deal with the problem. We treat the problems of non-synchronous timing and time aggregation by specifying quarterly dynamic factor series models for the determinants of consumption and income and then aggregating where this is appropriate. Given the inherent data limitations, we impose Almon (1962) polynomial distributed lag structures on the coefficients of the quarterly dynamic factor models.

Our model is as follows. Let

$$\begin{split} W_{t,i} - W_{t,i-1} &= \sum_{j=0}^{7} \beta_{wwj} u_{wt,i-j} \end{split}$$
(3.1)  
$$Z_{t,i} &= \sum_{j=0}^{7} \beta_{zzj} u_{zt,i-j} \\ N_{t,i} - N_{t,i-1} &= \sum_{j=0}^{7} (\beta_{nzj} u_{zt,i-j} + \beta_{nnj} u_{nt,i-j}) \\ Y_{t,i} - Y_{t,i-1} &= \sum_{j=0}^{7} (\beta_{ywj} u_{wt,i-j} + \beta_{yzj} u_{zt,i-j} + \beta_{ynj} u_{nt,i-j} + \beta_{yyj} u_{yt,i-j}) \end{split}$$

where

 $\begin{array}{l} W_{t,i} = \text{wage rate in the i'th quarter of year t} \\ Z_{t,i} = \text{hours of unemployment in the i'th quarter of year t} \\ N_{t,i} = \text{work hours in the i'th quarter of year t} \\ Y_{t,i} = \text{Income in the i'th quarter of year t} \end{array}$ 

In the model above, the data are generated at a quarterly rate (i runs from 1 to 4). For example, the difference of Z in the i'th quarter of year t from Z in the i - 1'st quarter of year t is a seventh

order moving average process (a two-year process). At the risk of some confusion, the time subscript t.i - j refers to the observation j quarters prior to the i'th quarter of year t. Thus, t-1.i and t.i-4 both refer to the i'th quarter of year t-1.

We have restricted the consumption analysis to models in which only the innovations in wages, unemployment, work hours, and other income sources matter. This will be true only if consumers are rational, and preferences are separable between consumption and leisure. In this case the quarterly consumption change is

$$C_{t.i} - C_{t.i-1} = \beta_{cw0} u_{wt.i} + \beta_{cz0} u_{zt.i} + \beta_{cn0} u_{nt.i} + \beta_{cy0} u_{yt.i},$$

where  $C_{t,i} =$ food consumption in the i'th quarter of year t.†

The life cycle labour supply-consumption model implies a set of additional restrictions. However, they are much more complicated and in practice we experienced difficulties in estimating the model with the restrictions imposed, so we omit them.

The data are only available at annual intervals, and  $Z_t^*$ ,  $N_t^*$  and  $Y_t^*$  are annual averages:

$$\begin{split} W_{t}^{*} - W_{t-1}^{*} &\equiv W_{t.1} - W_{t-1.1} + \Delta \varepsilon_{wt} \end{split} \tag{3.3}$$

$$\begin{aligned} Z_{t}^{*} &\equiv \sum_{i=1}^{4} Z_{t-1.i} \\ N_{t}^{*} - N_{t-1}^{*} &\equiv \sum_{i=1}^{4} (N_{t-1.i} - N_{t-2.i}) + \Delta \varepsilon_{nt} \\ Y_{t}^{*} - Y_{t-1}^{*} &\equiv \sum_{i=1}^{4} (Y_{t-1.i} - Y_{t-2.i}) + \Delta \varepsilon_{yt} \\ C_{t}^{*} - C_{t-1}^{*} &\equiv C_{t.1} - C_{t-1.1} + \beta_{cc0} u_{ct} + \beta_{cc1} u_{ct-1} + \beta_{cc2} u_{ct-2} \end{aligned}$$

For the PSID data,  $W_t^*$  is the reported wage rate at the time of the survey (typically March or April), which we approximate as the first quarter wage.  $Z_t^*$  is the reported total hours of unemployment in the calendar year preceding the survey date.  $N_t^*$  is total work hours on the main job in the calendar year preceding the survey.

 $\dagger$  In the quarterly case the restriction (2.1.2) for the permanent income specification of the change in consumption becomes

$$\beta_{ck0} = \alpha \sum_{j=0}^{7} \rho_{q}^{j} \beta_{ykj}; \mathbf{k} = \mathbf{w}, \mathbf{z}, \mathbf{n}, \mathbf{y}$$
(3.2)

 $\rho_q \equiv$  quarterly discount factor

 $Y_t^*$  is reported total annual income in the calendar year preceding the survey date.  $C_t^*$  is the annual rate of food consumption reported in the week of the survey, which we interpret as the rate of consumption for the first quarter.

Given the available data (i.e. data as defined in (3.3)), we cannot hope to recover all the parameters of the model in (3.1). Instead, we impose an Almon lag structure on (3.1). In particular we impose:

$$\begin{split} \beta_{ikj} &= a_{0it} + a_{lik}j + a_{2ik}j^2 \quad (3.4) \\ &ik = ww,zz,nz,nn,yw,yz,yy; j = 0, 1, 2, 3, 4, 5, 6, 7 \end{split}$$

We have reduced the number of free parameters in each of the quarterly moving averages from 8 to 3. If we have three years of consecutive data as defined in (3.3), then the model consisting of (3.1), (3.2) and (3.4) can be estimated. The differences in timing and aggregation of the different variables help identify the quarterly lag structure from annual observations. The model implies that several of the covariances at three-year lags will be nonzero, and so we add the relevant moments to the set of sample moments used in estimation.

### 4. Data and econometric methodology

The structural parameters of the models are estimated by fitting the theoretical auto-covariances and cross-covariances implied by the models to the corresponding sample moments of the variables. Chamberlain (1984) contains a comprehensive discussion of these estimators.<sup>†</sup>

The estimation procedure minimizes a quadratic form  $(S - \Sigma(\Pi))'\Omega(S - \Sigma(\Pi))$  where S is the vector of distinct sample covariance elements,  $\Sigma(\Pi)$  is the vector of predicted covariance elements, considered as a function of the vector of parameters  $\Pi$  (e.g.  $\beta_{ijk}$ 's and  $\sigma_i$ 's in (1.2)).  $\Omega$  is the identity matrix in the case of unweighted least squares estimates, and a consistent estimate of the inverse of the fourth moment matrix of the underlying data in the case of optimal minimum distance estimates (OMD). In practice, we follow a number of previous studies and use the inverse of the empirical fourth moment matrix of the underlying data, V, when computing OMD estimates. The unweighted least squares case amounts to running a nonlinear regression of the individual sample covariances in S against the elements of  $\Sigma(\Pi)$ . The optimal minimum distance estimator (OMD) is analogous to fitting this relationship by generalized least squares.

 $\dagger$  Abowd and Card (1989, Appendix A) provide a clear exposition of the issues which are relevant to the present paper.

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We use the OMD estimator rather than maximum likelihood under the assumption of normality, which was used by HM and Bernanke. We do so because our preliminary data analysis, for both levels and logs, indicated that the data are nonnormal. Specifically, we calculated the Kolmogorov-Smirnov test statistic for the null hypothesis of normality for each variable in each year (e.g.  $\Delta C_{1979}$ ). The null hypothesis was rejected in *every* case at a marginal significance level less than 0.01. We also found that the empirical fourth moment of a given variable,  $x_t$ , is 1.5 to 10 times larger than  $3^*(var(x_t))^2$  even though these quantities should be approximately equal if  $x_t$  is normally distributed.

Unfortunately, there are also drawbacks to the OMD estimator. In particular, sampling error in the fourth moments is likely to be correlated with sampling error in the second moments. If this is true, it may be preferable to use a simpler weighting scheme to estimate the models than the full GLS transformation used in the OMD case. Altonji and Segal (1996) analyse this issue and show that in many cases OMD performs worse then unweighted minimum distance. For this reason, we also estimate our models with the diagonal elements of  $\Omega$  set to the inverse of the average of the diagonal elements of V corresponding to a given type of covariance (e.g., the variance of the income change, the covariance of the wage change with the consumption change, etc.).<sup>†</sup> In this case all off diagonal elements of  $\Omega$  are set to 0. This amounts to fitting the model by a form of weighted least squares, which hereafter we will refer to as WLS. The average of the estimated fourth moments for the various years corresponding to each of the moments in equation (4.1) below is used as the weight for the particular moments.

Chamberlain shows how tests of parameter restrictions can be conducted when the OMD estimator is used. Let  $E(S) = \Sigma(\Pi)$ , where the vector  $\Pi$  has dimension K. Suppose restrictions on  $\Sigma(\Pi)$ imply E(S) = G(l) where the vector l has dimension L<K. Then if the restrictions hold,  $d_1 - d_2 \rightarrow \chi^2(K-L)$ , where

$$\begin{split} \mathbf{d}_2 &= \mathbf{m}(\mathbf{S} - \boldsymbol{\Sigma}(\boldsymbol{\Pi}))' \mathbf{A}(\mathbf{S} - \boldsymbol{\Sigma}(\boldsymbol{\Pi})), \\ \mathbf{d}_1 &= \mathbf{m}(\mathbf{S} - \mathbf{G}(l))' \mathbf{A}(\mathbf{S} - \mathbf{G}(l)), \end{split}$$

<sup>†</sup> For example, consider the variance of the wage change in each of our five sample years. For each year the fourth moments of the wage change form the basis of our estimates of the variance of the wage variance. These fourth moments are the elements of the diagonal of V corresponding to the estimated variances of the wage in each of the five years. We set the five elements of the diagonal of  $\Omega$  corresponding to the five wage variances equal to the inverse of the average of the five fourth moments of the wage change.

m is the number of observations, and A is set to  $V^{-1}$ . Newey (1985) provides a  $\chi^2$  goodness of fit test that is valid when WLS rather than OMD is used.

The least restricted model that can be estimated is the nonstationary model in which each moment in S is given its own parameter. In this case  $\Sigma(\Pi)$  and S have the same dimension. All covariance stationary models which we experimented with are overwhelmingly rejected against this model.

As a second bench mark and to provide a convenient data summary, we also use the covariance stationary model.

### COVARIANCE STATIONARY MODEL

$$Cov(I_t^*, M_{t+j}^*) = \theta_{IMj} \ I, M = \Delta C, \Delta Y, \Delta W, Z, \Delta N; j = -2, -1, 0, 1, 2 \eqno(4.1)$$

We use the covariance stationary model to judge the fit of the structural models, for two related reasons. First, tests based upon the test statistic above using the empirical fourth moment matrix to form the  $\chi^2$  statistic indicate that the data are nonstationary in the covariances. Therefore our various economic models will be rejected just due to the fact that they are stationary models. Second, there are indications that the fourth moment matrix may be too imprecisely estimated to permit reliable tests of the restricted models against the nonstationary model using the test procedure discussed above. (See AMS for details.)

Our use of stationary structural models in the face of evidence of nonstationarity raises the possibility of inconsistency in the estimates of the parameters of income and consumption equations. We checked this in several ways. First, we estimated models in which the variance of  $u_{ct}$ ,  $u_{yt}$ ,  $u_{nt}$ ,  $u_{zt}$ , and  $u_{wt}$  were permitted to depend on a common year specific scalar. This typically resulted in a significant improvement in the fit of the models (although the modified models were also overwhelmingly rejected against the unrestricted nonstationary model). However, the response coefficients of the income equations and the consumption equation did not change very much. We experimented with other ways of introducing nonstationarity into the dynamic factor models, with little change in the estimates of the income and consumption equations despite improvements in the fit of the model.

Second, we introduced dummy variables for the moments for which there was a departure from stationarity at the .05 significance level. This is analogous to excluding the problematic moments from the analysis. We identified these moments using a step-wise regression procedure to estimate the stationary model. The procedure resulted in the introduction of dummy variables for

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about 6% of the moments. The fit of the dynamic factor models generally improved to the point where they cannot be rejected against the unrestricted nonstationary model. More importantly, the parameters of the consumption and income equation are basically similar to those which we report below. Our findings are consistent with those of Kearl (1988) and Hause (1980), who found that relaxing stationarity improved the fit of their models of labour earnings but had little effect on key parameters.

Consequently, we have at least some evidence that our inferences about the form of the income process and the consumption equation are valid despite the fact that the stationary dynamic factor models are rejected against the nonstationary model.

Our reported standard errors of the parameter estimates are based on a modification of the formula provided in Chamberlain (1984). Chamberlain's formula is valid under the assumption that the discrepancy between the fitted covariances and the sample covariances arise only from sampling error in the covariances. Since the  $\chi^2$  goodness of fit tests discussed below indicate model misspecification, it seemed appropriate to scale the standard errors up by a factor equal to the square root of the mean square error of the estimated residuals of the models. The formula in Chamberlain assumes that the mean square error of the estimated residuals for the OMD estimator is one. It leads to standard error estimates for the model parameters which typically are about one-third smaller than the ones we report. To make our standard error estimates comparable to those of most other studies (e.g., Abowd and Card (1987, 1989), one may divide them by the square root of the mean square error reported in the Tables. We are, of course, on shaky ground in performing statistical inference in the presence of model misspecification, but this would seem to be an additional reason to prefer the conservative standard errors which we report.

### Data

For most of the analysis the data are from the 1976-1981 Panel Study of Income Dynamics (See Survey Research Center (1982)) and are limited to male household heads who responded in 1981. Consequently, in first differences, data are available for five years. For a given year, the sample contains male heads of household who were between the ages 18–60 inclusive, who had not retired, and who were employed, temporarily laid off, or unemployed at the time of the survey. We have limited the analysis to these years because the wage measure is unavailable for salaried workers prior to 1976. In the balanced sample, an individual is included only if he has complete observations on all the variables for all the years. The sample contains 1051 individuals. Because the wage measure is collected only if the individual is employed or on temporary layoff at the time of survey, the balanced sample is likely to consist of individuals with more stable employment histories than the sample at large. We experiment with unbalanced samples for 1976-1981 and for 1969-1981 as well.

A few of the variables require discussion.  $C_t^*$  is the sum of the family's food expenditures at home and outside of the home, deflated by the food component of the consumer price index. This is the consumption measure used in HM, Altonji (1986), Altonji and Siow (1987), and many other more recent studies of life cycle models based on the PSID. (See, for example, Pistaferri (1999), who provides references.) There appears to be considerable measurement error in the variable. We account for it with the error component  $u_{ct}$ .

The variable  $\Delta W_t^*$  is the change in the straight time wage at the time of the survey. Given our assumptions about measurement error, it is important to note that for both hourly workers and salary workers this wage variable is based upon survey questions which are independent of those used to construct the change in family income,  $\Delta Y_t^*$ . For salaried workers measurement error in  $\Delta W_t^*$  may be correlated with the true change in work hours, since the variable is usually imputed from information on salary per week, per month, or per year using a standard number of work hours (such as 40 hours per week). We ignore this potential problem.

As noted earlier, the consumption measure and the hourly wage measure refer to the time of the survey (typically in March) while family income and hours of unemployment,  $Z_t^*$ , refer to the calendar year which precedes the survey date.

For computational convenience, we followed HM and Hayashi (1985) and removed the effects of economy-wide disturbances and a variety of demographic characteristics from the variables used in the analysis of the dynamic factor models. We do so by first regressing the change in consumption, the change in income, and the income determinants against a set of year dummies, age,  $age^2$ ,  $age^3$ , education, the change in a dummy variable for marital status, current and lagged values of dummy variables for 8 Census regions, residence in an SMSA, and residence in a city with more than 500000 people, as well as variables for the level and squared value of the change of family size, the change in the number of children in the family unit, and the change in the number of children under age 6. The residuals from these regressions form the basis for the analysis below. Given the large samples which were used to form the residuals, the fact that the estimation was performed in two stages is of little consequence.

Finally, we have eliminated some outliers from the analysis.<sup>†</sup>

### 5. Estimates and Tests of the income and consumption models

We begin in Section 5.1 with the estimates of the stationary model (4.1) in logs and tests of the 0 restrictions on the stationary model that are implied by the income model and the RE-PIH. In Section 5.2 we report estimates of the income equations of the dynamic factor model as well as the consumption equations. In Section 5.3 we discuss a number of extensions, including the use of weighted least squares, experiments with alternative assumptions about measurement error, and estimates obtained when we extend the sample to years prior to 1976 and to individuals who are missing data for some years. In Section 5.4, we present estimates of the quarterly dynamic factor models.

5.1 ESTIMATES AND TESTS OF THE STATIONARY MODEL AND THE UNRESTRICTED DYNAMIC FACTOR MODELS

Table 1 presents OMD estimates of the stationary model (4.1) when the data are in logs. It consists of the covariances among the variables  $\Delta C_t^*$ ,  $\Delta Y_t^*$ ,  $\Delta W_t^*$ ,  $Z_t^*$  and  $\Delta N_t^*$  at 0, 1, and 2 lags. The model contains parameters for 65 distinct covariances that we estimate from 250 second moments. The signs of the contemporaneous covariances are reasonable. One distinguishing feature is that the covariances at the second lags are small for almost all of the variables. Out of 25 such covariances, only  $\text{Cov}(\Delta Y_t^*, \Delta Y_{t-2}^*)$  and  $\text{Cov}(Z_t^*, Z_{t-2}^*)$  are statistically significant at the 5% level. Furthermore, the estimates of the covariances are somewhat imprecise even though data on 1,051 individuals and between 3 and 5 sample moments are used to estimate them.‡

Table 2 presents  $\chi^2$  statistics, degrees of freedom, and p-values (marginal significance levels) of a series of restrictions on the sample moments. The row labels indicate the restrictions imposed under the null hypothesis of the test. The column labels indicate the restrictions maintained under the alternative hypothesis. The first column tests the stationary models against the unrestricted

 $<sup>\</sup>dagger$  Briefly, if the wage, hours, family income, or earnings showed an increase of 500% or a decline of 80% from the previous year, the observation was eliminated. Observations were also eliminated if consumption increased by 400% or decreased by 75%. Finally, we eliminated observations with an annual hours change of more than 3,000 hours, a level of hours above 5,000, or wage measures below \$0.50 per hour in 1972 dollars.

 $<sup>\</sup>ddagger$  3, 4 and 5 sample moments for the covariances involving second, first, and 0 lags, respectively.

		Consumption			Family Incom	e	Work Hours			
	$\Delta {C_t}^*$	$\Delta {C_{t-1}}^{\ast}$	$\Delta {C_{t-2}}^{\ast}$	$\Delta {Y_t}^*$	$\Delta Y_{t-1}{}^{\ast}$	$\Delta Y_{t-2}{}^{\ast}$	$\Delta N_t^*$	$\Delta {N_{t-1}}^{\ast}$	$\Delta {N_{t-2}}^{*}$	
$\Delta C_t^*$	$0.0880 \\ (0.00374)$	$-0.0395 \ (0.00253)$	$\begin{array}{c} 0.00404 \\ (0.00253) \end{array}$	0.00214 (0.00140)	$-0.00293 \\ (0.00149)$	$\begin{array}{c} -0.00017 \\ (0.00169) \end{array}$	0.000186 (0.00916)	0.000497 ( $0.00107$ )	$\begin{array}{c} -0.00163 \\ (0.00145) \end{array}$	
$\Delta \mathrm{C}_{\mathrm{t-1}}$ $\Delta \mathrm{C}_{\mathrm{t-2}}^{*}$				(0.00139) (0.00137) 0.00842 (0.00149)			(0.00986) (0.00982) 0.0000354 (0.00105)			
$\Delta {Y_t}^*$				0.0443 (0.00228)	$-0.0132 \\ (0.00133)$	$-0.00253 \\ (0.00125)$	0.00515 ( $0.000890$ )	-0.00182 (0.000792)	-0.000155 (0.000850)	
$\Delta {Y_{t-1}}^{\ast}$							-0.00250			
$\Delta Y_{t-2}{}^{\ast}$							0.000138) 0.0000425 (0.000850)			
$\Delta {N_t}^*$							0.0251 (0.00201)	$-0.00962 \\ (0.00101)$	$\begin{array}{c} -0.000801 \\ (0.000799) \end{array}$	
$\Delta N_{t-1}{}^{\ast}$										
$\Delta {N_{t-2}}^{\ast}$										
${\mathbf Z_t}^*$										
${\rm Z_{t-1}}^{*}$										
${\rm Z_{t-2}}^{*}$										
$\Delta W_t^*$										

 TABLE 1
 Optimal minimum distance estimate of covariances (data in logs)

(Continued overleaf)

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		Unemployment		Wage				
	$\mathbf{Z_{t}}^{*}$	${\rm Z_{t-1}}^*$	${\rm Z_{t-2}}^*$	$\Delta W_t^*$	$\Delta W_{t-1}^{*}$	$\Delta W_{t-2}{}^{\ast}$		
$\Delta {C_t}^*$	0.0000607	0.000137	0.000193	0.00331	-0.00091	0.000199		
. ~ *	(0.000166)	(0.000172)	(0.000197)	(0.000848)	(0.000875)	(0.000945)		
$\Delta \mathrm{C_{t-1}}^*$	0.000075			-0.00227				
*	(0.000147)			(0.00905)				
$\Delta C_{t-2}$	-0.000216			0.000467				
*	(0.000188)			(0.00104)				
$\Delta Y_t^*$	-0.000265	0.000560	0.0000648	0.00424	-0.000027	-0.000237		
	(0.000135)	(0.0001660)	(0.000154)	(0.000843)	(0.000802)	(0.000753)		
$\Delta Y_{t-1}^{*}$	0.0000332			-0.00265				
	(0.000159)			(0.000791)				
$\Delta { m Y_{t-2}}^*$	0.000104			0.000589				
	(0.000154)			(0.000802)				
$\Delta { m N_t}^*$	-0.000759	0.00102	0.000151	-0.000809	-0.000909	-0.000312		
	(0.000198)	(0.000214)	(0.000112)	(0.000534)	(0.000593)	(0.000504)		
$\Delta { m N_{t-1}}^*$	-0.000204			0.000293				
	(0.000140)			(0.000505)				
$\Delta { m N_{t-2}}^*$	-0.000096			-0.000025				
	(0.000122)			(0.000583)				
${\mathbf Z_t}^*$	0.00105	0.000417	0.000284	0.0000434	-0.000167	0.000067		
	(0.000160)	(0.0000959)	(0.000100)	(0.0000945)	(0.0000893)	(0.0000920)		
$Z_{t-1}^{*}$				0.0000398				
0 1				(0.0000972)				
$Z_{t-2}^{*}$				-0.000097				
t-2				(0.0000933)				
$\Delta W_{\star}^{*}$				0.0172	-0.00549	0.00113		
_ ···				(0.00142)	(0.000819)	(0.000552)		

 TABLE 1 (Continued)

(Standard errors in parentheses) Stationarity of covariances imposed. Balanced sample. See text for details.

	Maintained ass	sumptions of alternative l	hypothesis in test	
		Unrestricted nonstationary model	A: Stationarity	B: Stationarity Cov (wages, hours) $= 0$ Cov (wages, unempl.) $= 0$
Rest	rictions imposed under null hypothesis			
A:	1. Stationarity	$340{\cdot}5_{(185)}\ [0{\cdot}000]$		
B:	1. Stationarity 2. Cov (wages, unempl.) $= 0$ 3. Cov (wages, hours) $= 0$	$355{\cdot}6_{(185)}[0{\cdot}000]$	$15 \cdot 1_{(10)} [0 \cdot 127]$	
C:	<ol> <li>Stationarity</li> <li>O Cov between cons and lagged income determinants</li> </ol>	$345{\cdot}0_{(193)}[0{\cdot}000]$	$4 \cdot 6_{(8)}[0 \cdot 804]$	
D:	<ol> <li>Stationarity</li> <li>Cov (wages, unempl.) = 0</li> <li>Cov (wages, hours) = 0</li> <li>Cov (cons, lagged income determinants) = 0</li> </ol>	$359{\cdot}4_{(203)}[0{\cdot}000]$	$18.9_{(18)}[0.397]$	$3.8_{(8)}[0.878]$
E:	<ol> <li>Stationarity</li> <li>Factor model, unrestricted consumption equation</li> </ol>	$372 \cdot 6_{(208)}[0.000]$	$32 \cdot 1_{(23)} [0 \cdot 097]$	$17{\cdot}0_{(13)}[0{\cdot}199]$

TABLE 2Chi-square tests of restrictions on the covariance structure of consumption, hours, income, wages and<br/>unemployment (Data in logs)

Chi-Square<sub>(degrees of freedom)</sub> [p-values in brackets].

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nonstationary model (which fits the sample moments perfectly). The  $\chi^2$  statistic for the stationary model with no further restrictions is 340.5 with 185 degrees of freedom, which rejects stationarity with a p-value of less than 0.0001. We also tested separately for stationarity of the autocovariances of each of the five variables. We reject stationarity for all variables except for work hours. As we noted earlier, stationarity of the data is also strongly rejected for all models estimated using OMD in this paper (except in two cases discussed in Section 5.5). For reasons discussed in Section 4, we use the stationary model as a yardstick to assess the restricted models.

Because the income model with exogenous hours excludes the wage factor from the hours and unemployment equations, it implies the following zero restrictions on the stationary model.

RESTRICTIONS ON COV. STATIONARY MODEL IMPLIED BY INCOME EQUATIONS  $1.1c^\prime,\,1.1d,\,1.1e$ 

$$Cov(\Delta W^*_t, Z^*_{t+j}) = 0, \quad j = -2, -1, 0, 1, 2 \eqno(5.1)$$

$$Cov(\Delta W_t^*, \Delta N_{t+i}^*) = 0 \quad j = -2, -1, 0, 1, 2$$
(5.2)

We report tests of these restrictions in the second row of the table. Once stationarity is imposed, the restriction that wages do not vary with hours or unemployment passes at the 0.127 level. However, these restrictions fail when the data are in levels rather than logs. (Results not shown—see AMS).

In addition, since the RE-PIH model implies that past information does not cause a change in permanent income, it implies the following restrictions on the stationary model.

ZERO RESTRICTIONS ON COV. STATIONARY MODEL IMPLIED BY RE-PIH CONSUMPTION EQUATION

$$Cov(\Delta C_t^*, I_{t-i}^*) = 0, \quad I = \Delta Y, \Delta W, Z, \Delta N; j = 1, 2$$
(5.3)

The third row of the table tests these restrictions. They pass easily when stationarity is maintained.

The fourth row tests the zero restrictions on the income process and the 0 restrictions implied by RE-PIH for consumption. We obtained a p-value of 0.397 for these restrictions when testing them against the unrestricted stationarity model.

Finally, the table reports a test against the unrestricted stationary model (4.1) of the factor model consisting of the unrestricted consumption equation (l.la) and the income model with exogenous hours. The p-value to reject is 0.097, and so there

is only weak evidence against the factor structure once stationarity is maintained. The p-value for this dynamic factor model is 0.199when tested against the stationary model including 0 restrictions on the income process. The 0 restrictions implied by the RE-PIH model also pass easily.

In summary, we have strong evidence against stationarity, weaker evidence against the assumption that unemployment and hours do not vary with the wage change, little evidence against the zero restrictions on the relationship between consumption and lagged income determinants, and little evidence against the dynamic factor representation of the data.

# 5.2~ estimates of the dynamic factor models with exogenous income and hours

We now discuss estimates of the dynamic factor models with exogenous income and hours, and various consumption equations. We begin with the equations of the income model (l.lb, l.lc', l.ld, l.le). We then turn to the consumption equations. In estimating these models we have excluded the covariances between hours and wages and between unemployment and wages from the sample because the income model implies that these are 0.

### The income model

In Table 3 we present estimates of the equations of the income model with exogenous hours which are obtained when they are estimated jointly with the unrestricted consumption equation (1.1a). The estimates of the family income, wage, hours, and unemployment equations reported in the table are representative of the results which we obtained for the income equations when the restrictions associated with RE-PIH or the Keynesian model were imposed, although the precision of the coefficients on the income factor  $u_{yt}$ ,  $u_{yt-1}$ , and  $u_{yt-2}$  is higher in the latter cases. The long-run effect of a one-standard deviation innovation in a factor may be estimated by summing the factor loadings on that factor.

We will discuss the OMD estimates in column 1. The results indicate that most of the response of income to a wage innovation occurs in the initial period, and that most of the effect is permanent. A one-standard deviation increase in the wage factor, which is equal to 0.20 ( $\beta_{ww0}$ ), leads to an initial increase in family income of 0.021 and to a permanent increase in family income of about 0.02. This seems small in light of the fact that a one-standard deviation wage shock raises the wage level by about 0.07 in the long run and that the relationship between wages and work hours is weak.

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		OMD esti	imates	WLS est	timates
		Estimate	S.E.	Estimate	S.E.
Income	$(\Delta Y^*t)$				
	$^{\beta}$ yw0	0.0210	0.00682	0.0310	0.00930
	$\beta$ yw1	-0.000808	0.00392	0.00730	0.00727
	$^{\beta}$ yw2	-0.00108	0.00407	-0.00366	0.00623
	<sup>β</sup> yn0	0.0333	0.00870	0.0531	0.0125
	$\beta$ yn1	-0.00951	0.00564	-0.00735	0.00814
	$\beta yn2$	-0.0000868	0.00695	-0.00662	0.0103
	<sup>β</sup> yz0	-0.0169	0.00495	-0.0219	0.00580
	$^{\beta}$ yz1	0.0189	0.00627	0.0108	0.00707
	$^{\beta}$ yz2	0.00236	0.00609	0.00682	0.00758
	<sup>β</sup> yy0	0.0766	0.0737	0.119	0.0611
	$^{\beta}$ yy1	0.067	0.103	0.0217	0.075
	$^{\beta}yy2$	-0.0292	0.0326	-0.0294	0.0210
$\sigma^2_{y}$		0.0156	0.00218	0.0178	0.00539
Wage	$(\Delta W^*t)$				
	$^{\beta}$ ww0	0.200	0.0550	0.183	0.0536
	$^{\beta}$ ww1	-0.125	0.0541	-0.0877	0.0538
	$^{eta}ww2$	0.00357	0.00365	-0.00554	0.00607
$\sigma^2{}_{ m w}$		-0.0189	0.0176	-0.00649	0.0143
Hours	$(\Delta N^*t)$				
	<sup>β</sup> nn0	0.127	0.030	0.132	0.0268
	<sup>β</sup> nn1	-0.0575	0.0306	-0.0256	0.0254
	$^{\beta}$ nn2	-0.00272	0.00617	-0.00958	0.00919
	$^{\beta}$ nz0	-0.0374	0.00604	-0.0534	0.00871
	$^{\beta}$ nz1	0.0318	0.00563	0.0359	0.00680
	$^{\beta}$ nz2	0.00709	0.00405	0.0181	0.00737
$\sigma^2_{n}$		0.00173	0.00556	0.00857	0.00403
Unemploy.	$(Z^*t)$				
	$\beta$ zz0	0.0296	0.00254	0.0532	0.00362
	$\beta$ zz1	0.00951	0.00163	0.0171	0.00312
	$^{\beta}zz2$	0.00674	0.00247	0.00949	0.00327

TABLE 3 Equations of the income models (OMD and WLS estimates)(Data in Logs)

\*Both income equations were estimated jointly with their respective unrestricted consumption equations. Goodness of fit statistics are reported with the consumption equations in Table 5, Columns 1 and 4 respectively. MSE statistics are reported with the consumption equations in Table A3.

Inconsistency in the timing of wages and income is a possible explanation for the small response of family income to wages. We investigate this possibility below. It is also worth mentioning that the standard error on the wage measurement error variance  $\sigma_w^2$  is

large relative to the total variance in the wage change and that the point estimate is actually negative although not significant.

The value -0.0169 for  $\beta_{yz0}$  is the estimate of the short-run response of income to a one-standard deviation increase in the unemployment factor  $u_{zt}$ . This factor drives the log of (2000 + hours of unemployment) and the change in the log of annual hours. The effect on annual work hours is -0.037, while the effect on the unemployment variable is 0.0296. These results suggest that unemployment leads to a more than proportional reduction in work hours in the short run, perhaps through shorter work days, and to a less than proportional reduction in family income. In part, this reflects the fact that earnings of the head account for only about two-thirds of family income, and in part it may reflect the response of transfers to unemployment. In recent work Gruber (1997) has emphasized the importance of unemployment insurance in reducing the impact of a spell of unemployment on family income and consumption. The long-run effect of unemployment on family income is near 0.

The short-run effect of the work hours factor  $u_{nt}$  on the log of family income is only about one-quarter of its effect on work hours. The estimate implies that more than three-quarters of the effect on income is permanent. We obtain a small positive estimate of the variance of the measurement error component in hours,  $\sigma_n^2$ . These estimates come as a surprise, because Duncan and Hill (1984), Altonji (1986) and Altonji and Paxson (1986) and the recent survey by Bound *et al.* (1999) report strong evidence of substantial measurement error in the change in the log of annual hours. Below we obtain larger measurement error estimates when we use WLS and when we account for non-synchronization in the data.

The income factor  $u_{yt}$  has a strong effect on income. In all of the models that we estimated, it was the most important factor in the income model (after measurement error). The estimates imply that measurement error is responsible for 70.8 % of the variance of  $\Delta Y_t^*$ . Of the remaining 29.2%, 82.9% is due to  $u_{yt}$ , 8.6% is due to  $u_{nt}$ , 5.0% is due to  $u_{zt}$ , and 3.4% is due to  $u_{wt}$ . For the various models that we estimated, the contribution of the variances of wage, hours of work and unemployment innovations to the variance of the first difference of log income (after correcting for measurement error) is always less than 30%. It is possible that variations in bonuses, overtime premia, nonlabour income, and spouse's earnings are large enough to explain the importance of  $u_{yt}$ , although this runs counter to our priors.

In Table 4 we provide some evidence that the above decomposition of the variance of income reflects basic characteristics of the data rather than gross model misspecification or problems with the estimation procedures. In column 1 we present a regression of  $\Delta Y_t^*$  against  $\Delta Y_{t-1}^*$  and  $\Delta Y_{t-2}^*$ . The data are in logs. The R<sup>2</sup> is

	Variable	(1	.)	(2	2)	(3)		
		Parameter estimate	Standard error	Parameter estimate	Standard error	Parameter estimate	Standard error	
Fam Inc.	$\begin{array}{c} \text{Intercept} \\ \Delta Y^*_{t-1} \\ \Delta Y^*_{t-2} \end{array}$	$0.0022 \\ -0.3449 \\ -0.1494$	$0.0034 \\ 0.0152 \\ 0.0142$	$0.0059 \\ -0.3658 \\ -0.1642$	$0.0037 \\ 0.0152 \\ 0.0142$	$0.0069 \\ -0.4094 \\ -0.1806$	$0.0034 \\ 0.0152 \\ 0.0142$	
Wage	$\Delta W^*_t \ \Delta W^*_{t-1} \ \Delta W^*_{t-2}$			$0.2554 \\ 0.2304 \\ 0.1309$	$0.0213 \\ 0.0234 \\ 0.0221$	$0.2538 \\ 0.2590 \\ 0.1626$	$0.0198 \\ 0.0218 \\ 0.0207$	
Hours	$\Delta \mathbf{N}^{*}_{\mathrm{t}} \ \Delta \mathbf{N}^{*}_{\mathrm{t-1}} \ \Delta \mathbf{N}^{*}_{\mathrm{t-2}}$			$0.1759 \\ 0.1028 \\ 0.0794$	$0.0215 \\ 0.0233 \\ 0.0174$	$0.1886 \\ 0.1341 \\ 0.0986$	$0.0200 \\ 0.0217 \\ 0.0163$	
Unempl.	$egin{array}{c} \mathbf{Z}_{\mathrm{t}}^{*} \ \mathbf{Z}_{\mathrm{t}-1}^{*} \ \mathbf{Z}_{\mathrm{t}-2}^{*} \end{array}$			$-0.2106 \\ 0.2434 \\ -0.0155$	$0.0731 \\ 0.0754 \\ 0.0670$	$-0.2192 \\ 0.2479 \\ -0.0207$	$0.0678 \\ 0.0701 \\ 0.0622$	
Wife's	$\Delta NS_t^*$					0.3156	0.0128	
Hours	$\Delta \mathbf{NS}^*_{\mathrm{t-1}} \ \Delta \mathbf{NS}^*_{\mathrm{t-2}}$					$0.1741 \\ 0.0795$	$0.0138 \\ 0.0133$	
Wife's	$\mathbf{ZS}^*_{\mathrm{t}}$					-0.0298	0.0269	
Unempl.	$\frac{\text{ZS}_{\text{t}-1}^*}{\text{ZS}_{\text{t}-2}^*}$					$0.0116 \\ 0.0224$	$0.0265 \\ 0.0249$	
	${f R}^2$ MSE		$0.1154 \\ 0.0473$		$0.1822 \\ 0.0418$		$0.2970 \\ 0.0377$	

TABLE 4 Regression models for the change in log family income  $(\Delta Y_t^*)$ 

(Continued overleaf)

### TABLE 4 (Continued)

1000000000000000000000000000000000000	Regression models	for the change	in log earning	$(\Delta Y^*nt)$
---------------------------------------	-------------------	----------------	----------------	------------------

	Variable	(1	)	(2	)	(3)		
		Parameter estimate	Standard error	Parameter estimate	Standard error	Parameter estimate	Standard error	
Earnings	$\begin{array}{c} Intercept \\ \Delta Y^*_{t-1} \\ \Delta Y^*_{t-2} \end{array}$	$\begin{array}{c} 0.012811 \\ -0.412299 \\ -0.132896 \end{array}$	0.0032534 0.015327 0.013809	$0.019658 \\ -0.52459 \\ -0.196307$	0.003109 0.015191 0.013848	$0.019634 \\ -0.5245 \\ -0.196216$	0.003111 0.015197 0.013855	
Wage	$\begin{array}{c} \Delta W^*_t \\ \Delta W^*_{t-1} \\ \Delta W^*_{t-2} \end{array}$			$0.361598 \ 0.39457 \ 0.25144$	$0.01796 \\ 0.020287 \\ 0.019365$	$0.362132 \\ 0.393079 \\ 0.250672$	$0.017976 \\ 0.020309 \\ 0.01939$	
Hours	$\Delta \mathrm{N}^{*}_{\mathrm{t}} \ \Delta \mathrm{N}^{*}_{\mathrm{t}-1} \ \Delta \mathrm{N}^{*}_{\mathrm{t}-2}$			$0.371535 \ 0.297834 \ 0.154049$	0.018128 0.020333 0.016169	$0.370928 \\ 0.297084 \\ 0.152987$	$0.018155 \\ 0.020374 \\ 0.016193$	
Unempl.	$\begin{array}{c} \mathbf{Z}_t^* \\ \mathbf{Z}_{t-1}^* \\ \mathbf{Z}_{t-2}^* \end{array}$			$-0.516155 \ 0.410682 \ 0.133424$	$0.061596 \\ 0.063874 \\ 0.057101$	$-0.513953 \\ 0.409595 \\ 0.131799$	$0.06166 \\ 0.063981 \\ 0.057175$	
Wife's	$\Delta \mathbf{NS}^*_{\mathrm{t}}$					-0.010471	0.011673	
Hours	$\Delta \mathbf{NS}^*_{\mathrm{t-1}} \ \Delta \mathbf{NS}^*_{\mathrm{t-2}}$					$0.000374 \\ -0.017061$	$0.011723 \\ 0.01124$	
Wife's	$\mathbf{ZS}^*_{\mathrm{t}}$					-0.021584	0.024489	
Unempl.	$\begin{array}{c} \mathbf{ZS}^*_{t-1} \\ \mathbf{ZS}^*_{t-2} \\ \mathbf{R}^2 \end{array}$		0.1506		0.3871	$-0.006724 \\ 0.010421$	0.024121 0.022663 0.3877	
	MSE		0.0431		0.0312		0.0312	

\*Sample Size is 4085. All variables are residuals obtained from regressions of the original variable against a set of demographic variables and time dummies. See Page 23.

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0.115. In column 2 we add current and lagged wage, hours and unemployment changes. The  $R^2$  rises by 0.067 to 0.182. In column (3) we add the current value and two lags of  $\Delta NS_t^*$ , which is the change in the log of 1370 plus annual work hours of the spouse to the equation. We also add  $ZS_t^*$  and its lags, where  $ZS_t^*$  is the log of 1370 plus wife's hours of unemployment. The value 1370 is the mean of wife's work hours for wives who work positive hours. (We transformed the work hours and unemployment variables to reduce the influence of large percentage changes in hours worked by women working few hours on the log variables and to handle the fact that our sample includes unmarried men and men whose wives do not work in some years). These variables lead to an  $R^2$  of 0.297. We view these results as consistent with substantial measurement error in family income. They also suggest a relatively small role for variation in husband's work hours, unemployment, and wages in the variance of measured income. The results in Altonji and Siow for a similar sample of men suggest that adding the change in work hours lost due to illness and the interactions between wage changes and quits, layoffs, and promotions would result in only a small improvement in explanatory power. In light of the substantial explanatory power of wife's work hours and unemployment, it would be useful in future work to expand the dynamic factor model to include these variables.

### Results for consumption

Table 5 reports a series of consumption equations. The  $\chi^2$  statistic and the degrees of freedom reported at the bottom of each equation are for a test of the consumption equation and the associated equations for the income determinants against a stationary model (Model B in Table 2).

Column 1 presents the unrestricted log linear consumption equation (1.1a). None of the coefficients on lagged income determinants are significantly different from 0. This result is consistent with the tests for nonzero covariances between consumption and income determinants reported in Table 2.†  $\beta_{cw0}$ , the response of the consumption to  $u_{wt}$ , is estimated at 0.018 (0.0059) with a t-value of 3.08. The coefficient on  $u_{nt}$  is positive and the coefficient on  $u_{zt}$  is negative, but neither is statistically significant. The variable  $u_{yt}$  has a substantial positive but imprecisely estimated effect on consumption. The estimate of  $\beta_{cy0}$  is 0.0288 (0.0237).

<sup>&</sup>lt;sup>†</sup> When we use levels of consumption and income, we find the coefficients of lagged unemployment and lagged income both have fairly large coefficients with t values of 2.07 and 1.75 respectively.

		OM	D estima	ites			WI	LS estima	ates	
	1	2	3	4	5	6	7	8	9	10
Wage fac	tor: u <sub>w1</sub>									
$^{\beta}$ cw0	$0.018 \\ (0.00585)$	$0.0171 \\ (0.00597)$		$0.0226 \\ (0.00604)$		$0.0201 \\ (0.00755)$	0.0184 ( $0.00908$ )		$0.0185 \\ (0.00826)$	
$\beta$ cw1	$0.00122 \\ (0.00401)$					$0.00408 \\ (0.00678)$				
$^{\beta}cw2$	$0.00326 \\ (0.00499)$					0.00287 ( $0.00830$ )				
Unemploy	yment factor: u₂	st								
$^{\beta}cz0$	$-0.00 \\ (0.00625)$	0.00141 (0.00472)		0.00270 ( $0.00462$ )		$-0.00762 \\ (0.00659)$	-0.00456 (0.00554)		-0.00414 (0.00508)	
$^{\beta}cz1$	$-0.00101 \\ (0.00650)$					0.00663 (0.00756)				
$^{\beta}cz2$	0.00477 ( $0.00698$ )					$0.00508 \\ (0.00681)$				
Hours fac	ctor: u <sub>nt</sub>									
<sup>β</sup> cn0	$\begin{array}{c} 0.00422 \\ (0.00616) \end{array}$	0.00653 ( $0.00535$ )				$0.00542 \\ (0.00964)$	0.00878 (0.0113)			
$\beta$ cn 1	$-0.00547 \\ (0.00709)$					0.00494 ( $0.00982$ )				
<sup>β</sup> cn2	$-0.0109 \\ (0.0122)$					$\begin{array}{c} -0.00763 \\ (0.0158) \end{array}$				
Income fa	actor: u <sub>yt</sub>									
<sup>β</sup> cy0	$0.0288 \\ (0.0237)$	$0.0253 \\ (0.00655)$		$0.0242 \\ (0.00666)$		$0.0212 \\ (0.0186)$	$0.0196 \\ (0.0167)$		$0.0218 \\ (0.0142)$	

TABLE 5Consumption equations:  $\Delta C_t^*$ 

$^{\beta}$ cy1	-0.00341					-0.00911 (0.0182)				
$^{\beta}cy2$	-0.00476 (0.0266)					(0.0102) -0.00979 (0.0199)				
Consumption	n factor: u <sub>ct</sub>									
<sup>β</sup> cc0	$0.254 \ (0.00621)$	$0.255 \ (0.00582)$	$0.252 \\ (0.00606)$	$0.259 \ (0.00575)$	$0.256 \ (0.00628)$	$0.279 \\ (0.00743)$	$0.279 \\ (0.00807)$	$0.278 \\ (0.0124)$	$0.279 \ (0.00736)$	$0.277 \\ (0.0122)$
$\beta$ cc1	$-0.147 \ (0.00776)$	$-0.148 \\ (0.00717)$	$-0.147 \\ (0.00757)$	$-0.145 \ (0.00737)$	$-0.144 \\ (0.00813)$	$-0.153 \\ (0.0101)$	$-0.153 \ (0.0111)$	$0.154 \\ (0.0166)$	$-0.153 \ (0.0101)$	$-0.156 \ (0.0162)$
$^{\beta}cc2$	0.0171 (0.0105)	0.0177 ( $0.00984$ )	0.0187 ( $0.0105$ )	0.0169 (0.0100)	0.0186 ( $0.0122$ )	-0.00141 (0.0120)	-0.00116 (0.0129)	-0.000448 (0.0094)	-0.00116 (0.0117)	0.000166 ( $0.00895$ )
α			$0.239 \\ (0.0609)$		$0.232 \\ (0.0656)$			$0.269 \\ (0.0777)$		$0.355 \\ (0.0974)$
$\chi^2$	20.6	25.4	64.6	12.4	61.6					
DF	13	21	24	14	16	13	21	24	14	16
MSE	1.87	1.83	2.02	1.75	$2 \cdot 12$	1.52	1.48	1.51	1.31	1.35

\*Columns 1 and 6 are the unrestricted consumption equation in logs which is estimated with the log income model in Table 3. Col. 2 and 7 exclude lagged factors from the consumption equation. Col. 3 and 8 are the log Keynesian model. Col. 4 and 9 exclude lagged factors from the consumption equation and annual hours from all equations. Col. 5 and 10 are the log Keynesian model without annual hours. \*The  $\chi^2$  statistic and degrees of freedom are for a test of the consumption equation and the associated income model against the covariance stationary model. The cross covariances between hours and wages and unemployment: and wages are not used in estimation. 212 moments are used to estimate columns 1–3 and 6–8. 162 sample moments are used to estimate columns 4-5 and 9-10, which exclude annual hours.

\*Column 8 estimates did not satisfy standard convergence criterion.

In column 2 we impose the zero restrictions implied by the RE-PIH model if income and hours are exogenous. Specifically, consumption coefficients on all lagged income determinants are set to 0. This consumption equation and the associated income equations easily pass tests against the stationarity model and against the model with the unrestricted consumption equation. The coefficients on all of the income components except unemployment have the right sign, but only the wage and income factors are statistically significant. The fact that wage innovations are more important than unemployment is consistent with the evidence in table 4 that the unemployment effect on income is transitory. The coefficient  $\beta_{cw0}$  on the wage innovation is 0.0171 (0.0060), which is 68% of  $\beta_{cy0}$ . From the perspective of the RE-PIH model, this estimate of  $\beta_{cw0}$  seems a bit large given that the estimates of the associated income model (not reported) and the consumption parameters in column (2) together imply that the long-run effect on income of a one-standard deviation shock to u<sub>wt</sub> is only onequarter as large as a one-standard deviation shock to  $u_{vt}$ . The inconsistency in the timing of the wage and income data and the consumption and income data and a substitution effect of the wage on consumption are among possible explanations for this, in addition to sampling error. The discrepancy in the magnitudes of  $\beta_{cw0}$  and  $\beta_{cy0}$  is smaller in the quarterly models (Table 6) and when we estimate by weighted least squares (See Table 5, column 7).

Column 3 in Table 5 presents estimates for the five variable loglinear model with a Keynesian-type consumption function, which specifies that the change in consumption is proportional to the change in income. For this model the parameter  $\alpha$  is the response of consumption to a change in income. The point estimate is 0.239 (0.0609). The fact that this value is about four times larger than the coefficient of an OLS regression of the consumption change on the income change (not shown) is in part a reflection of the fact that the dynamic factor model accounts for measurement error in income. The Keynesian model is overwhelmingly rejected, although the estimates of the income process and the other processes do not differ much from those of the unrestricted factor model reported in Table 3. However, it should be mentioned that the estimates do not satisfy the standard convergence criterion.<sup>†</sup>

The consumption model in Column 4 corresponds to the model in column 2 after hours are eliminated from the income model. This has only a small effect on the estimate of  $\beta_{cw0}$  (which rises somewhat),  $\beta_{cz0}$ , and  $\beta_{cy0}$ . Column 5 reports the Keynesian consumption model after hours are eliminated from the income

<sup>&</sup>lt;sup>†</sup> We had difficulty getting the algorithm used to compute the optimal minimum distance estimator to converge even with various starting values.

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		1a Annual hours parameter estimates	1b Excluded standard error	2a Annual hours parameter estimates	2b Included standard error
Consumption e	quatio	n parameters			
$\beta$ cw0(u <sub>wt-1</sub> )	1	0.0118	0.00292	0.00897	0.00266
$\beta$ cz0(u <sub>zt-1</sub> )		0.00392	0.00250	0.000609	0.00240
$\beta$ cn0(u <sub>nt-1</sub> )				0.00366	0.00282
$^{\beta}$ cy0(u <sub>vt-1</sub> )		0.0114	0.00321	0.0103	0.00303
$\beta$ cc0(u <sub>ct</sub> )		0.258	0.00582	0.251	0.00564
$^{\beta}cc1(u_{ct-1})$		-0.145	0.00760	-0.145	0.00720
$^{\beta}cc2(u_{ct-2})$		0.0156	0.0102	0.0137	0.00973
Income equatio	n para	meters			
u <sub>wt:</sub>	$a_{0yw}$ $a_{1yw}$ $a_{2yw}$	$0.0279 \\ -0.0227 \\ 0.00313$	0.00587 0.00510 0.000716	$0.0216 \\ -0.0179 \\ 0.00251$	$0.00501 \\ 0.00442 \\ 0.000626$
u <sub>zt:</sub>	${f a}_{0yz} \ {f a}_{1yz} \ {f a}_{2yz}$	$-0.00703 \\ 0.00457 \\ -0.000510$	0·00392 0·00367 0·000532	$-0.0102 \\ 0.00783 \\ -0.00968$	$0.00356 \\ 0.00341 \\ 0.000499$
u <sub>nt:</sub>	$f{a}_{0yn} \ f{a}_{1yn} \ f{a}_{2yn}$			$0.0147 \\ -0.00944 \\ 0.00113$	$0.00382 \\ 0.00303 \\ 0.000427$
u <sub>yt</sub> :	$egin{array}{l} a_{0yy} \ a_{1yy} \ a_{2yy} \ \sigma^2{}_y \end{array}$	0.0172 0.00631 -0.00188 0.0103	$0.0421 \\ 0.0387 \\ 0.00535 \\ 0.00464$	$0.0302 \\ -0.00732 \\ 0.0000826 \\ 0.0112$	0.0414 0.0367 0.00499 0.00213
Wage equation	param	eters			
u <sub>wt:</sub>	$a_{0ww}$ $a_{1ww}$ $a_{2ww}$ $\sigma^2_w$	$0.0577 \\ -0.0359 \\ 0.00417 \\ -0.00608$	0.0121 0.00869 0.00106 0.00636	$0.0649 \\ -0.0417 \\ 0.00490 \\ -0.0109$	$0.0149 \\ 0.0107^* \\ 0.00129 \\ 0.00914$
Unemployment	equat	ion parameters			
u <sub>zt:</sub>	$a_{0zz} \ a_{1zz} \ a_{2zz}$	$0.00257 \\ -0.00113 \\ 0.000115$	0.000326 0.000247 0.0000348	$0.00252 \\ -0.00123 \\ 0.00138$	0.000243 0.000161 0.0000214
Annual hours e	quatio	n parameters			
u <sub>zt:</sub>	$a_{0nn}$ $a_{1nn}$ $a_{2nn}$ $a_{0nz}$ $a_{1nz}$ $a_{2nz}$ $\sigma^2$			$\begin{array}{c} 0.0134 \\ -0.00892 \\ 0.00108 \\ -0.00518 \\ 0.00369 \\ -0.000445 \\ 0.00313 \end{array}$	$\begin{array}{c} 0.00382\\ 0.00316\\ 0.00416\\ 0.00757\\ 0.00626\\ 0.0000861\\ 0.00387\end{array}$
$\chi^2$	- 11	47.1		60.4	
Degrees of free	dom	21		34	
MSE		1.87		1.86	

TABLE 6         Quarterly dynamic factor models (OMD estimat	es)
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 $<sup>\</sup>overline{{}^*\chi^2}$  statistic for test of the model against the stationary model. The degrees of freedom are the degrees of freedom of the test.

model, and the results are the same as those in column 3 for the full model.

The results for this section may be summarized as follows. First, we find little evidence that consumption is affected by lagged determinants of income. Second, the response of consumption to  $u_{wt}$  seems large and the response to  $u_{yt}$  seems small relative to the long-run effects of these variables on income. Fourth, the simple Keynesian model is rejected.

As for the family income process, the estimates are disappointing in terms of the fraction of the variance explained by the wage, unemployment and hours factors relative to the variance explained by the income factor  $u_{yt}$ . The regression analysis in Table 4 suggests that this finding reflects basic characteristics of our data. Inconsistency in the timing and time aggregation of some of the variables may also play a role. We turn to this issue below. A second explanation is that our assumptions about the properties of the measurement errors are invalid, leading to a misspecified income equation.

### 5.3 EXTENSIONS

#### Alternative assumptions about measurement error

We experimented with two alternative specifications of measurement error. First, we estimated most of our models assuming that the measurement error was zero for all equations except consumption. In all cases, these restricted models were handily rejected against their counterparts with measurement error. For example, the  $\chi^2$  statistic, with 3 degrees of freedom, for the model with no measurement error that corresponds to col. 2 of Table 5 is 13. Second, we also allowed for first-order moving average measurement errors (i.e.  $\varepsilon_{it} = \varepsilon_{it} + \tau_i \varepsilon_{it-1}$  for i = y,n,w). We cannot reject the null hypothesis that  $\tau_i$  is zero for all cases at the 5% significance level. Finally, we briefly experimented with cross-correlated measurement errors. However we were unsuccessful in our attempts to estimate these models.

### Weighted least squares

We report a set of weighted least squares (WLS) estimates of the stationary model in Table A1, the dynamic factor model of income in Table 3 (column 3), and the dynamic factor model of consumption in Table 5 (columns 6-10). Based on Newey's goodness of fit test, stationarity is still overwhelmingly rejected. (Results not reported). The estimates of the variances and covariances in Table A1 for all

models are larger in absolute values than those obtained with OMD. The disparity is even more dramatic when the variables are measured in levels, as reported in AMS. This pattern is consistent with the analysis in Altonji and Segal (1996) and in fact was part of the inspiration for that paper. The standard errors of the estimates are also larger.

The WLS estimates of the factor models in Table 3 and 5 are qualitatively similar to the corresponding OMD estimates in those tables. Measurement error explains a larger portion of the variance of the change in measured work hours. It is still insignificant and has the wrong sign for measured wages. The main substantive difference between the WLS and OMD estimates is that the absolute values of the factor loadings are typically larger when WLS is used.

### Unbalanced data

In an attempt to increase the precision of our estimates and to investigate whether the behaviour of persons in the balanced sample is different from that of persons with incomplete data, we estimated the model on two larger, unbalanced samples. In the unbalanced case the number of observations differs across sample covariances. In one sample, we start with the balanced sample for 1976-1981 and add individuals who did not have complete data on all variables for all the years. After this addition, between 1699 and 2877 observations are available to calculate each sample covariance. The OMD point estimates and standard errors obtained with this larger sample are bigger than before. The additional data does not improve the precision of our estimates because the sample covariances for those individuals with missing data are substantially larger than those of individuals with complete data. That is, the two sets of individuals appear to face different income processes. For example, the sample variances of income, wages and hours of work are about three times larger than for the balanced sample. The variance of unemployment is ten times larger. This is not surprising given that the wage measure is available in a given year only for persons who are employed or on temporary layoff at the time of the survey. As a result, the balanced sample is weighted toward individuals with relatively stable employment.

In the unbalanced sample, the response of consumption to a onestandard deviation shock to unemployment is negative, -0.00625(0.0046). The short-run effect on income is -0.0421 and the long-run effect is -0.0196, which implies that the response of consumption to a 1 unit permanent change in income arising from an unemployment shock is -0.319. (-0.319 = -0.00625/0.0196). The corresponding figures are 0.589 = 0.0254/0.043 for a wage shock, 0.154 = 0.0266/0.173 for an income shock, and 0.03 = 0.00162/0.054 for an hours shock. These estimated relative consumption effects are somewhat different from that using the balanced sample, adding to the evidence that the additional data in the unbalanced sample come from a different distribution of individuals.

In the second unbalanced sample, we also include data from before 1976 back to 1968. We extended the sample temporally in an attempt to improve precision. This led to an increase in the number of second moments used in estimation from 212 to 583.<sup>†</sup> We used the WLS estimator with this sample because it was computationally intractable to invert the full empirical fourth moment matrix. The estimates of the covariances of the stationary model are substantially larger than those obtained for the balanced sample. (Results not shown.) The parameters of the factor models also increase in absolute value and the relative consumption effects are similar to those reported for the first unbalanced sample.

# 5.4 TIME AGGREGATION AND NON-SYNCHRONOUS MEASUREMENTS: ESTIMATES OF THE QUARTERLY DYNAMIC FACTOR MODELS

Table 6 presents estimates of the quarterly dynamic factor model. (See equation (3.1-3.4)). Column 1a and b reports estimates and standard errors when annual hours are excluded from the analysis. In this model the response of consumption to the current wage, income, and unemployment innovations are unrestricted. We ignore the fact that the log of annual income is not equal to the sum of the logs of quarterly income.

The parameters of the income, wage and unemployment equations (the  $a_{mij}$ , m = 0, 1, 2) are the parameters of the polynomial distributed lag specifications in equation (3.4). In Figure A we plot the time pattern of the response of income  $Y_t^*$  (as opposed to  $\Delta Y_t^*$ ), the wage  $W_t^*$ , and unemployment  $Z_t^*$  to one-standard deviation innovations in  $u_{yt}$ ,  $u_{wt}$  and  $u_{zt}$ . The effect after seven periods is the long-run response of  $Y_t^*$ , etc, to the various shocks. The plots are based on the estimates of the  $a_{mij}$  in column 1.

 $Y_t^*$  initially increases by 0.0279 in response to a wage innovation of 0.0577. It rises above this level in the next period, declines almost to 0, before rising again. The long-run response is 0.028, so most of the wage effect is permanent, while the long-run effect of a wage innovation on the wage level is about 0.040. Since the mean of labour earnings is equal to about two-thirds of the mean of

<sup>&</sup>lt;sup>†</sup> We restricted our investigation to models with exogenous work hours. Certain sample moments are missing because the relevant questions were not asked in those survey years.  $C_{1973}$ ,  $W_{68}$ ,  $W_{69}$  are missing. The wage variable is unavailable for salary workers prior to 1976.



FIGURE A. Impulse response functions implied by quarterly dynamic factor model, annual hours excluded.

6 7

5

0

2

family income, the size of the response of income to the wage factor is sensible. These results are a substantial improvement over the results obtained using the annual model. However, the shape of the response is not entirely plausible and may well be an artifact of the quadratic polynomial imposed on the moving average coefficients used in estimation.

Unemployment shocks have essentially a 0 effect on unemployment and on income after about five-quarters. A one-standard deviation increase of 0.0025 reduces income by about -0.007 in the first quarter. (Recall that unemployment is the log of (2000 + hoursof unemployment), which means that an increase of 0.0025 corresponds to a decline in annual hours by about 0.25 percent from a base of about 2000 hours.). The income response is larger than one would expect given that the labour earnings of the head accounts for about two-thirds of the mean of family income, especially if unemployment benefits cushion part of the drop in earnings. The estimate in column 2a of Table 6 of the model including hours of work indicate that hours of work decline by about twice as much as hours of unemployment rise, which would explain much of the discrepancy.

Wages and unemployment explain 22.8% and 7.5% of the true variance in the quarterly change in income, while the income factor explains 69.6%. Wages and unemployment explain 15.3% and 1.8% of the steady state variance of change in the annual average of income, while the income factor explains 83.2%. The importance of measurement error in the annual average falls to 46.7%. The increase in the explanatory power of the wage in the quarterly models relative to the annual models is further evidence that treatment of timing is useful.

We now turn to the consumption coefficients. The parameter  $\beta_{cw0}$ is 0.0118 and  $\beta_{cy0}$  is 0.0114, which implies that the consumption response to a one-standard deviation wage innovation and a onestandard deviation income innovation are approximately equal. The response of consumption to the wage innovation seems large relative to the response to the income innovation. The long-run response of income to the wage innovation (0.028) is only 4/7th's as large as the response of income to the income innovation (0.048). The response of consumption to the unemployment innovation is small, statistically insignificant, and has the wrong sign. The small response of consumption to unemployment is consistent with the fact that the income parameters imply that the long-run response of income to a one-standard deviation innovation in unemployment is only 0.001. Once again, the size and permanence of the effect of unemployment hours on income is likely to be smaller for our sample of individuals who are working or on temporary layoff at the time of the survey in both year t and t - 1 than for the population as a whole.

Columns 2a presents a set of estimates of the quarterly models with annual hours included. As in column 1a, the response of consumption to the current wage, income, hours, and unemployment innovations are unrestricted. The implied time patterns of the responses of these variables to one-standard deviation innovations in the various factors are presented in Figures B.

In the long run, hours rise by about 1% in response to a onestandard deviation innovation in the hours factor. The long-run impact on income is slightly more than 1%. The income response seems a little large given the share of head's earnings in family income, although it is basically sensible. As noted above, the effect of unemployment on work hours is about double what one would expect if the shocks underlying the unemployment reports affected weeks worked but not hours/week. The long-run response of hours to unemployment innovations is basically zero, consistent with that of income. The income response is still larger than what one would expect if the only consequence of measured unemployment is to reduce annual hours of work.



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FIGURE B. Impulse response implied by quarterly dynamic factor model: annual hours included.

Measurement error explains about 25% of the variance in the change in measured hours of work. Measurement error now explains 53% of the variance of the change in measured income. Wages, hours of work, unemployment and income innovations explain 12%, 5%, 4%, and 79% of the variance of the change in true family income respectively. It is surprising that adding hours of work reduces the variance explained by wages and unemployment relative to that explained by the income factor.

In summary, accounting for non-synchronization in the data reduces the role assigned to measurement error in the variance of measured income. It also increases the explanatory power of the wage factor relative to the income factor in explaining the variation in true income. However our use of polynomial lag distributions limits the lag structures that can be considered.

### 6. Results for the life cycle consumption-labour supply model

Column 1 in Table 7 presents estimates of the dynamic factor model that only imposes the 0 restrictions implied by (2.2.6,2.2.7 and 2.2.8) on the general model (1.1). The p-value to reject the model against the covariance stationary alternative is 0.05. The covariance stationary model is analogous to (4.1), but is estimated using  $\Delta Y_t^{*n}$  in place of  $\Delta Y_t^*$ . Measurement error explains about three-quarters of the variance of  $\Delta Y^{n*}$ . The estimates of measurement error in wages and hours of work are imprecise. Variation in wage, hours of work, and unemployment innovations explain about two-thirds of the variation of  $\Delta Y^{n*}$  after correcting for measurement error, and about one-sixth of the total variance in  $\Delta Y^{n*}$ . The shortfall seems large even though labour earnings may contain bonuses, overtime wages or wages on second jobs, and these are not captured by our wage and hours variables. The regressions for labour earnings in Table 4 indicate that the marginal contribution to  $R^2$  of current and lagged wages, hours and unemployment is 0.237.

The variable  $\Delta C_t^*$  responds only to current innovations in the wage and earnings which suggests that  $\beta_{cn} = \beta_{nc} = 0$ . Unemployment is more transitory than wages in affecting the earnings process. An interesting finding is that innovations in the income factor lead to an increase in hours of work. It is possible that this represents the effect of overtime opportunities, which are not captured by our straight time wage measure.

In Column 2 we report estimates of a restricted labour supply model. Measurement error in nonlabour income is absorbed in  $u_{yt}$ , which explains the large increase in the value of  $\beta_{yy0}$  between column 1 and column 2. We attach no economic interpretation to it. In view of the results in Column 1 and the results for family income in Section 4, we do not impose restrictions on the earnings process to guard against misspecification of the earnings equation. Basically, we are using the unrestricted earnings process as an additional indicator to aid identification of the factor loadings in the other equations.  $\beta_{cn}$ ,  $\beta_{nc}$ , and  $\beta_{\eta y}$  are restricted to be zero.<sup>†</sup> The estimate of the intertemporal labour supply elasticity  $\beta_n$  is -0.117 with a standard error of 0.131. From equations (2.2.7) and (1.1d), we see that  $\beta_n$  can be identified

<sup>†</sup> It would have been preferable to include a dummy for the  $Cov(\Delta C_t, \Delta Y^n_t)$  rather than to restrict  $\beta_{\eta_y}$  to 0.

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	(1)	)	(2	)	(3	)
	Estimate	SE	Estimate	SE	Estimate	SE
Consumption	n ΔC					
$\beta$ cw0	0.0229	0.00582				
$\beta$ cw1	0.00216	0.00611				
$\beta$ cw2	0.00538	0.0078				
$\beta$ cz0	-0.000787	0.00496				
$\beta$ cn0	-0.00397	0.00591				
$^{\beta}$ cy0	0.0245	0.0127				
$\beta$ cc0	0.254	0.0058	0.252	0.00689	0.252	0.00829
$\beta$ cc1	-0.146	0.00713	-0.147	0.00765	-0.148	0.00826
$^{\beta}cc2$	0.0142	0.0097	0.0142	0.00974	0.0142	0.00979
Earnings $\Delta Y$	<sup>*</sup> nt					
<sup>β</sup> yw0	0.0459	0.00627	0.0503	0.0066	0.0495	0.00661
<sup>β</sup> yw1	0.0109	0.00617	0.0104	0.00656	0.0112	0.00662
<sup>β</sup> yw2	0.0031	0.00572	0.00373	0.00608	0.00391	0.00601
<sup>β</sup> yn0	-0.0285	0.00654	-0.0274	0.00659	-0.0272	0.00661
<sup>β</sup> yn1	0.0167	0.00581	0.0159	0.00585	0.0158	0.00585
$^{\beta}$ yn2	0.00866	0.00521	0.00827	0.0052	0.00816	0.00521
<sup>β</sup> yz0	0.0299	0.0362	0.0486	0.00706	0.00485	0.00697
$^{\beta}$ yz1	-0.0137	0.00764	-0.00416	0.00547	-0.0041	0.00549
$^{\beta}$ yz2	-0.0116	0.0154	-0.00204	0.00585	-0.00199	0.00587
$^{\beta}$ yy0	0.0158	0.0172	0.137	0.00537	0.137	0.00531
$^{\beta}$ yy1	0.014	0.0363	-0.0833	0.00622	-0.0834	0.00623
$^{eta}yy2$	0.045	0.034	0.00479	0.00677	0.00461	0.00677
$\sigma^2{}_{ m y}$	0.0127	0.00194				
Wage $\Delta W^*t$						
$\beta$ ww0	0.117	0.0137	0.107	0.012	0.108	0.0123
$^{\beta}$ ww1	-0.0457	0.012	-0.0377	0.0106	-0.0388	0.0108
$^{\beta}ww2$	0.00793	0.00518	0.00677	0.00557	0.0064	0.00547
$\sigma^2{}_{ m w}$	0.000879	0.00216	0.00233	0.00172	0.00214	0.00178
Unemployme	ent Z*t					
$\beta$ zz0	0.0264	0.0025	0.0282	0.0025	0.0282	0.0025
$^{\beta}$ zz1	0.00817	0.0017	0.00809	0.00172	0.00809	0.00172
$^{\beta}$ zz2	0.00507	0.00226	0.00491	0.00231	0.00496	0.00232
Hours $\Delta N^*t$						
<sup>β</sup> nn0	0.178	0.197	0.121	0.0169	0.121	0.0169
$^{\beta}$ nn1	-0.127	0.209	-0.0583	0.0169	-0.058	0.017
$^{\beta}$ nn2	-0.00181	0.00427	-0.00337	0.00573	-0.00319	0.00575
<sup>β</sup> nz0	-0.037	0.00558	-0.0353	0.00585	-0.0354	0.00574

 TABLE 7
 Estimates of the life cycle model

(Continued overleaf)

	(1)	)	(2	)	(3	)
	Estimate	SE	Estimate	SE	Estimate	SE
<sup>β</sup> nz1	0.0288	0.00533	0.0272	0.00539	0.0272	0.0054
$\beta$ nz2	0.00394	0.00386	0.00336	0.00386	0.00338	0.00386
<sup>β</sup> nw0	-0.00333	0.00332				
$\beta$ nw1	0.00395	0.00423				
$\beta$ nw2	-0.0044	0.00429				
<sup>β</sup> ny0	0.0386	0.0178				
$\sigma^2$ n	-0.0138	0.0619	0.00153	0.00282	0.00157	0.00281
βn			-0.117	0.131	-0.11	0.125
βc			-0.295	0.119	-0.37	0.288
<sup>β</sup> nc					-0.0195	0.0424
<sup>β</sup> ηw			-0.0841	0.0279	-0.0713	0.0384
<sup>β</sup> ηz			0.003	0.0168	0.00187	0.0112
$\sigma_n^2$			0.00893	0.0151	0.00552	0.01
$\chi^{2}$	31.7		43.6		43.2	
$\mathbf{DF}$	20		26		26	
MSE	1.81		1.82		1.82	

TABLE 7 (Continued)

\*Y\*t is the log of measured labour earnings. As in the other log models in the paper Z\*t is the log (2000 + Hours of Unemployment).

from  $Cov(\Delta N_t, \Delta W_{t-2})/Cov(\Delta W_t, \Delta W_{t-2})$ . From the point estimates in Table 1, one can see that the sign of the estimate of  $\beta_n$  is partially due to the insignificant and small negative estimate of  $Cov(\Delta N_t, \Delta W_{t-2})$ . The negative estimate of  $\beta_n$  has the wrong sign but is not significantly different from 0 or from the small positive values found in most micro data studies of intertemporal labour supply. The estimate of  $\beta_c$  is -0.295 with a standard error of 0.119. The negative estimate is predicted by the theory. The estimate of  $\beta_{nw}$ , the effect of wage innovations on the marginal utility of wealth, is -0.0841 with a standard error of 0.0279. The estimate of  $\beta_{\eta z}$ , which is the effect of unemployment innovations on the marginal utility of wealth and should be positive, is 0.0030 with a standard error of 0.0168. Given the finding that unemployment innovations have smaller and more transitory effects on earnings than wage innovations, the relative and absolute magnitudes of  $\beta_{nw}$ and  $\beta_{\eta z}$  are sensible. The variance of  $u_{\eta t}$  is imprecisely estimated at 0.00893. Wage and unemployment innovations explain 44% of the variance of the innovations in the marginal utility of income. We note that the point estimates of the variances of all the measurement errors are positive. The p-value to reject the model is 0.02.

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In Col. 3, we report estimates of the restricted labour supply model with  $\beta_{cn} = 4\beta_{nc}$ .  $\beta_{\eta y}$  is still restricted to be zero, and the earnings process is again unrestricted. We can get an estimate of  $\beta_{cn}$  from  $Cov(\Delta C_t, \Delta W_{t-2})/Cov(\Delta W_t, \Delta W_{t-2})$ . Since the estimate of  $Cov(\Delta C_t, \Delta W_{t-2})$  is imprecise (see Table 1b), the estimate of  $\beta_{cn}$ should be treated with caution. The estimate of  $\beta_{nc}$  is -0.019 with a large standard error of 0.042. The small negative point estimate suggests that consumption and leisure are weak complements but argues against explaining the excess response of consumption to the wage innovation in our RE-PIH model with hours treated as exogenous by appealing to a positive cross substitution effect of the wage on consumption. The point estimate for  $\beta_n$  remains negative, again with a large standard error. The point estimate for  $\sigma_{nt}$  is now 0.0055 which is one-third smaller than the previous estimate. Wage and unemployment innovations now explain about 48% of the variance of the innovations in the marginal utility of income. Estimates of the other parameters and the associated standard errors are about the same as before. Finally, we cannot reject the hypothesis that  $\beta_{nc} = 0$  (the  $\chi^2$  statistic for col. 2 against col. 3 is 0.4 with 1 degree of freedom).<sup>†</sup>

In summary, due to the large standard errors, we are not particularly successful in estimating the intertemporal labour supply elasticity nor the cross elasticity  $\beta_{cn}$ . However, we obtain economically sensible estimates of the effects of wage and unemployment innovations on the marginal utility of income. We find that these two components explain a little less than half of variance of innovations in the marginal utility of income.

### 7. Concluding remarks

Since the introduction contains a summary of the main results, we close by briefly discussing avenues for future research. Advances over the past 15 years in computational economics and computer speed would make it possible to examine a much richer class of models of consumption and labour supply than those discussed in the paper. Most of these models do not lead to closed form solutions and are unlikely to imply a linear MA process. Even the life cycle model that we estimate is only a log linear approximation. However, simulation based estimation methods, such as Gallant and Tauchen's (2001) efficient method of moments, provide a way

<sup>†</sup> In the Appendix, Table A2 presents a set of WLS estimates. The point estimates for  $\beta_n$ ,  $\beta_{nc}$  are still negative. Although consistent with earlier estimates, these results are surprising because in Table Al, we see that  $Cov (\Delta N_t, \Delta W_{t-2})/Cov (\Delta W_t, \Delta W_{t-2})$  and  $Cov (\Delta C_t, \Delta W_{t-2})/Cov (\Delta W_t, \Delta W_{t-2})$  are positive. The WLS results are basically similar to the OMD results, although the factor loadings and measurement error variance estimates are typically larger in absolute value.

to choose the parameters of more or less arbitrary models so that they are consistent with convenient descriptive models that summarize the data such as its dynamic factor representation. The dynamic factor representation is appealing because the dynamic relationships among the variables are transparent, and it is a very convenient framework with which to handle the most important forms of measurement error.

Over the past decade there has been a great deal of interest in the consumption and finance literatures in variation over time in uncertainty about income. Although we have not reported the results here we experimented extensively with specifications in which the variances of the innovations in components of income change over time and found fairly strong evidence that they do. The huge difference in variance estimates for the balanced and unbalanced samples provides an indication that there is substantial heterogeneity in income uncertainty. In a recent paper, Meghir and Pistaferri (2001) estimate ARCH models of earnings using PSID data and examine heterogeneity over time and across people in earnings uncertainty as well as the dynamics of uncertainty. It would be natural to extend this work to a multivariate moving average model of the type employed here so that the role of work hours, unemployment, wages, and other factors in heterogeneity in income risk could be examined.

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### Appendix: The Permanent Income Model When The Market Interest Rate And Subjective Discount Rate Differ

In this Appendix we examine the implications of the PIH type models for the behaviour of consumption, total income, savings, and assets. At the end we briefly summarize our experience in estimating the models.

The analysis builds on the developments in Flavin (1981) and Campbell and Deaton (1989). We extend those analyses to a more general case in which the market interest rate for borrowing and lending differs from the subjective discount rate used by consumers. The general model nests the PIH and Keynesian models as special cases. Here we present the main equations and results.<sup>†</sup>

We begin with the asset transition equation for individual i:

$$A_t = (1+r)(A_{t-1} + X_{t-1} - C_{t-1}) \tag{A.1}$$

 $A_t$ : Assets at time t.

 $X_t$ : Nonasset income at time t.

C<sub>t</sub>: Total consumption at time t.

r: The ex-post return on assets for individual i from t - 1 to t.

Individual subscripts are ignored for convenience. We have followed many previous researchers and imposed the simplifying assumption that the interest rate is constant across time and households. Note that  $X_t$  consists of all nonasset income, including labour earnings, transfers, gifts, etc. Income at time t,  $Y_t$ , consists of asset income and nonasset income.

$$Y_t = \frac{r}{1+r}A_t + X_t \tag{A.2}$$

Equations (A.1) and (A.2) must hold for all individuals, regardless of whether they obey the permanent income hypothesis, provided that they borrow and lend at a common interest rate  $r.\ddagger$ 

We work with the consumption equation

$$C_t = \frac{\delta}{1+\delta}A_t + \frac{\delta}{1+\delta}\sum_{k=0}^\infty \rho^k E_t X_{t+k} + V_t, \eqno(A.3)$$

where  $\delta$  is the subjective discount rate used by consumers to discount future nonasset income,  $\rho$  is  $1/(1+\delta)$ ,  $V_t$  is transitory consumption at time t and  $E_t$  is the expectations operator conditional on information available at t. Following Flavin, HM, Campbell and Deaton (1989), and many previous researchers, we consider (A.3) to be the PIH when  $\delta = r$  and the rate of return on assets is fixed. In this case (A.3) says that except for transitory departures that may be serially correlated, consumption at time t is equal to the annuity value of expected wealth at time t. If  $\delta$  is infinite, (A.3) is a Keynesian-type specification in which consumption is equal to the sum of current nonasset income and assets plus transitory consumption. One may show that when  $\delta$  is infinite,  $A_t$  is  $(1 + r)V_{t-1}$ , and so consumers spend all nonasset

 $<sup>\</sup>dagger\,$  The details of the derivations are in an unpublished Appendix that is available from the authors.

 $<sup>\</sup>ddagger$  The timing convention in equations (A.1) and (A.2) follows Flavin. Income and consumption are measured at the beginning of the period. Assets are measured at the end of the period. Interest is paid on assets over the period.

income as they receive it, and borrow and lend in response to transitory consumption.

To proceed further, one must specify a model for  $X_t$ . We specify that  $\Delta X_t$  is a moving average of a serially uncorrelated vector of factors  $u_{xt}$ . In keeping with the empirical specification used in the text we assume an MA(2) process here, although the analysis may easily be extended to an MA(K). In the MA(2) case  $\Delta X_t$  may be written as

$$\Delta \mathbf{X}_{t} = \beta_{\mathbf{x}0} \mathbf{u}_{\mathbf{x}t} + \beta_{\mathbf{x}1} \mathbf{u}_{\mathbf{x}t-1} + \beta_{\mathbf{x}2} \mathbf{u}_{\mathbf{x}t-2} \tag{A.4}$$

where  $\beta_{x0}$ ,  $\beta_{x1}$  and  $\beta_{x2}$  are row vectors of coefficients of the MA process. In our application the wage, unemployment, hours, and income factors are elements of the conformable column vector  $u_{xt}$ . Note, however, that the variables are in levels rather than logs and the income measure that should be used is nonasset income, not total income  $Y_t$ , which is used in the present paper and HM.

We make the key assumption that:

$$\begin{split} E_t u_{xt+k} &= 0, k > 0 \\ E_t u_{xt+k} &= u_{xt+k}, k \leq 0 \end{split}$$

This says that consumers do not have information that is useful in predicting wage changes other than the information in current and lagged values of  $u_{xt}$ . Since we do not have a direct measure of expectations, it is not possible to estimate the structural models considered below without a strong assumption about the consumer's information set. One could test this assumption by checking if consumption changes are correlated with future innovations  $u_{xt+1}$ . One would add the appropriate sample moments involving the change in consumption and future changes in wages, unemployment, etc to the model and test whether the elements of  $u_{xt+1}$ ,  $u_{xt+2}$  have 0 coefficients in the equation for  $\Delta C_t$ .

We now wish to solve for the relationship between  $\Delta C_t$ ,  $\Delta Y_t$ ,  $A_t$ , and the vector of income factors  $u_{xt}$ . We begin by solving for  $A_t$ . Using (A.4) and (A.3), one may show that  $C_{t-1}$  is

$$C_{t-1} = \frac{\delta}{1+\delta} A_{t-1} + X_{t-1} + \rho(\beta_{x1} + \rho\beta_{x2})u_{xt-1} + \rho\beta_{x2}u_{xt-2} + V_{t-1}.$$
(A.5)

Using the above equation to eliminate  $C_{t-1}$  from (A.1) leads to

$$\begin{split} A_{t} &= \frac{1+r}{1+\delta} A_{t-1} - \frac{1+r}{1+\delta} \left[ \left( \beta_{x1} + \rho \beta_{x2} \right) u_{xt-1} + \rho \beta_{x2} u_{xt-2} \right] \\ &- (1+r) V_{t-1} \end{split} \tag{A.6}$$

Solving backward for  $A_t$  assuming  $\delta > r$  and using (A.6) yields

$$\begin{split} \Delta A_t &= -\frac{r-\delta}{1+\delta} \sum_{k=1}^{\infty} \left(\frac{1+r}{1+\delta}\right)^k \left[ \left(\beta_{x1} + \rho \beta_{x2}\right) u_{xt-k-1} + \beta_{x2} u_{xt-2-k} \right] \\ &- \frac{r-\delta}{1+\delta} (1+r) \sum_{k=1}^{\infty} \left(\frac{1+r}{1+\delta}\right)^{k-1} V_{t-k-1}. \end{split}$$
(A.7)

After some straightforward but tedious algebra one may show that equations (A.4), (A.5), and (A.7) imply that is

$$\begin{split} \Delta C_{t} &= (\beta_{x0} + \rho \beta_{x1} + \rho^{2} \beta_{x2}) u_{xt} + \frac{\delta}{1+\delta} \frac{\delta - r}{1+\delta} \sum_{k=0}^{\infty} \left[ \frac{1+r}{1+\delta} \right]^{k} \\ &\times \left[ \left( \beta_{x1} + \rho \beta_{x2} \right) u_{xt-k-1} + \beta_{x2} u_{xt-k-2} \right] \\ &+ \frac{\delta}{1+\delta} \frac{\delta - r}{1+\delta} (1+r) \sum_{k=1}^{\infty} \left[ \frac{1+r}{1+\delta} \right]^{k-1} \\ &\times V_{t-k} + V_{t} - \left[ 1 + \delta \frac{1+r}{1+\delta} \right] V_{t-1} \end{split}$$
(A.8)

Next we modify (A.8) to take account of the fact that the principal consumption measure is family expenditure on food. We assume that the marginal propensity to consume food out of permanent income is  $\alpha$ . Let  $C_t$  denote food consumption for the rest of this section. Let  $V_{\rm ft}$  be transitory food consumption. Then (A.8) becomes

$$\begin{split} \Delta C_{t} &= \alpha (\beta_{x0} + \rho \beta_{x1} + \rho^{2} \beta_{x2})' u_{xt} + \alpha \frac{\delta}{1+\delta} \frac{\delta-r}{1+\delta} \\ &\times \sum_{k=0}^{\infty} \left[ \frac{1+r}{1+\delta} \right]^{k} \left[ \left( \beta_{x1} + \rho \beta_{x2} \right)' u_{xt-k-1} + \beta'_{x2} u_{xt-k-2} \right] \\ &+ \alpha \frac{\delta}{1+\delta} \frac{\delta-r}{1+\delta} (1+r) \sum_{k=1}^{\infty} \left[ \frac{1+r}{1+\delta} \right]^{k-1} V_{t-k} \\ &+ V_{ft} - V_{ft-1} - \alpha \left[ 1 + \delta \frac{1+r}{1+\delta} \right] V_{t-1} \end{split}$$
(A.9)

To solve for  $\Delta Y_t$ , one takes the difference of (A.2) and uses (A.7) to eliminate  $\Delta A_t$  and (A.3) to eliminate  $\Delta X_t$ . After some algebra one obtains

$$\Delta Y_t = \frac{r}{1+r} \frac{\delta - r}{1+\delta} \sum_{k=0}^{\infty} \left[ \frac{1+r}{1+\delta} \right]^k \left[ \left( \beta_{x1} + \rho \beta_{x2} \right) u_{xt-k-1} \right]^k$$

$$\begin{split} &+ \beta_{x2} u_{xt-k-2} \Big] + \beta_{x0} u_{xt} + \left[ \frac{1}{1+r} \beta_{x1} - \frac{r}{1+r} \rho \beta_{x2} \right] u_{xt-1} \\ &+ \frac{1}{1+r} \beta_{x2} u_{xt-2} + r \frac{\delta - r}{1+\delta} \sum_{k=1}^{\infty} \left[ \frac{1+r}{1+\delta} \right]^{k-1} V_{t-k-1} - r V_{t-1} \end{split}$$

$$(A.10)$$

In the RE-PIH case in which  $\delta = r$ , the equation for  $\Delta Y_t$  reduces to

$$\Delta Y_t = \beta_{x0} u_{xt} + \rho (\beta_{x1} - (1 - \rho)\beta_{x2}) u_{xt-1} + \rho \beta_{x2} u_{xt-2} - r V_{t-1}$$
 (A.10')

Equation (A.10') embodies all the restrictions imposed by the RE-PIH on the income process. It is interesting to note that the RE-PIH model can be estimated from data on nonasset income and total income. Consumption data are not needed. Note that lagged transitory consumption,  $V_{t-1}$ , is part of the income process. If transitory consumption is serially correlated, as is assumed by Hall and Mishkin and Bernanke and in the model above, the covariance of  $\Delta C_t$  and  $\Delta Y_{t-1}$  may be nonzero even if RE-PIH holds. The nonzero correlation comes from the fact that lagged savings enters the change in income as in equation (A.7).

Now consider the effect of the current innovation  $u_{xt}$  on consumption. In the RE-PIH case with  $\delta = r$  and thus  $\rho = 1/(1+\delta) = 1/(1+\delta)$ 1/(1 + r), the effect is the marginal propensity to consume food,  $\alpha$ , times the vector  $(\beta_{x0} + 1/(1+r)\beta_{x1} + (1/(1+r))^2\beta_{x2})$  consisting of the effects of the elements of  $u_{xt}$  on permanent income. The 0restrictions on the lagged innovations  $u_{xt-k}$ , k>0, in the RE-PIH case are a hallmark of the permanent income models under rational expectations. In the general case, the discount rate 1/(1 + r)is replaced by  $\rho = 1/(1+\delta)$ . In the general case, with  $\delta > r$ , one can see from (A.9) that lagged innovations matter. A sufficient condition for the coefficient on the lags to be positive (negative) is for  $\beta_{x1}$  and  $\beta_{x2}$  both to be positive (negative). If  $\beta_{x1}$  and  $\beta_{x2}$  are both positive, a positive value for  $u_{xt}$  implies that further increases in  $X_t$  will occur in t+1 and t+2. Consumption will respond too little to u<sub>xt</sub> from the perspective of the RE-PIH, but additional changes in consumption will occur in future periods. If  $\beta_{x1}$  and  $\beta_{x2}$  are both negative, with  $\beta_{x0}$  positive, then  $\Delta C_t$  will respond too much to a positive innovation in  $u_{xt}$  but the effect of  $u_{xt}$  on future consumption changes will be negative. Basically, in this case, consumers with high subjective discount rates do not place enough weight on the fact that an increase in u<sub>xt</sub> implies negative changes in X<sub>t</sub> in future periods. The general model is consistent with Flavin's finding in aggregate time series data that consumption responds too much to the current innovation in nonasset income if  $\beta_{x1} + \beta_{x2}/(1+r)$  is negative. It is consistent with Campbell and Deaton's (1989) analysis of whether consumption responds too little to the current innovation

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in income if  $\beta_{x1} + \beta_{x2}/(1+r)$  is positive. More generally it provides a framework with which to consider whether consumption is too smooth or too volatile relative to the process for labour income.

In AMS we report estimates of  $\rho$  using the change in consumption level, income level, and wage level that correspond to the results in the paper for logs. We do not focus on them in this paper because a log specification of the income process is more appropriate and the linear quadratic formulation of the consumption from problem has fallen from favor. Instead, we have informally compared the relative size of the effects of the unemployment, hours, wage, and income factors on consumption to the effects of these innovations on current and future income. While in a number of cases, particularly in the quarterly models that account for timing, we obtained point estimates of the discount factor  $\rho$  that are close to 1, in general the estimates have large standard errors and are sensitive to the details of the specification.

The structural estimates of the RE-PIH model in AMS are based on the use of family income  $\Delta Y_t$  rather than nonasset income  $\Delta X_t$  as the measure of income. As we have noted, the presence of feedback from transitory consumption to asset income leads to misspecification. We estimated versions of the model in which  $\Delta X_t$  is used as the income measure and tried various treatments for nonstationarity. We also estimated models in which the cross equation restrictions between the equation (A.10') for  $\Delta Y_t$  and the dynamic factor model for nonasset income, unemployment and wages are imposed, with and without imposing the cross equation restrictions between the consumption model and the nonasset income model. We obtained estimates of  $\rho$  of about 0.75 for both WLS and OMD when stationarity is imposed and OMD estimates near 1 when stationarity was relaxed by letting innovation variances differ across years for some variables. The use of cross equation restrictions between family income and nonasset income leads to a substantial improvement in precision over model estimates that rely exclusively on the restrictions between consumption and income to identify  $\rho$ . These results illustrate the theoretical point that one does not need consumption data to estimate the RE-PIH model.

Finally, we experimented with models consisting of the general consumption equation (A.9) and equation (A.10) for  $\Delta Y_t$  and the model for nonasset income, unemployment, wages, and hours. Using OMD and with stationarity imposed, we obtained an estimate of  $\rho = 1/(1 + \delta)$  of 0.62 with a standard error of 0.08 when we set 1/(1+r) to 0.9. We obtained estimates of 0.67 (0.24) for 1/(1+r) and 0.43 (0.23) for  $\rho$  when both are estimated freely.

There are too many problems and inconsistencies among the estimates of  $\rho$  to support conclusions about the rate at which consumers discount future income.

		Consumption	n	Family income		Work hours			
	$\Delta {C^*}_t$	$\Delta C^{\ast}{}_{t-1}$	$\Delta {C^*}_{t-2}$	$\Delta Y_{t}^{*}$	$\Delta Y^{*}{}_{t-1}$	$\Delta Y^{*}{}_{t-2}$	$\Delta N^{*}{}_{t}$	$\Delta N^{\ast}{}_{t-1}$	$\Delta N^*{}_{t-2}$
$\begin{array}{c} \Delta {C^*}_t \\ \Delta {C^*}_{t-1} \\ \Delta {C^*}_{t-2} \end{array}$	0·102 (0·00384)	$-0.0426 \\ (0.00261)$	-0.00324 (0.00264)	$\begin{array}{c} 0.00321\\ (0.00152)\\ 0.000899\\ (0.00158)\\ -0.000471\\ (0.00168)\end{array}$	$\begin{array}{c} -0.000521 \\ (0.00177) \end{array}$	0.00569 (0.00204)	$\begin{array}{c} 0.00132\\ (0.00108)\\ -0.000327\\ -(0.00118)\\ -0.00140\\ (0.00117)\end{array}$	0.000655 (0.00166)	-0.00131 (0.00170)
$\begin{array}{l} \Delta {Y^{*}}_{t} \\ \Delta {Y^{*}}_{t-1} \\ \Delta {Y^{*}}_{t-2} \end{array}$				0.0557 (0.00258)	-0.0163 (0.00167)	-0.00344 (0.00143)	$\begin{array}{c} (0.00117) \\ 0.00875 \\ (0.00109) \\ -0.00200 \\ (0.00930) \\ -0.00169 \\ (0.00103) \end{array}$	$\begin{array}{c} -0.00150 \\ (0.00952) \end{array}$	0.000503 (0.00110)
$egin{array}{l} \Delta {f N^*}_t \ \Delta {f N^*}_{t-1} \ \Delta {f N^*}_{t-2} \end{array}$							0.0401 (0.00265)	-0.0128 (0.00129)	-0.00183 (0.00102)
$\mathbf{Z}^{*}{}_{\mathrm{t}}$									
${Z^{\ast}}_{t-1}$									
${Z^{\ast}}_{t-2}$									
W <sup>*</sup> t								Contin	and overlage

 TABLE A1
 WLS estimate of covariances (Data in logs)

		Unemployment			Wages	
	${\mathbf Z^*}_{\mathsf t}$	${\rm Z^{*}}_{t-1}$	${\mathbf Z^*}_{\mathbf t-2}$	$\Delta \mathrm{W^*_t}$	$\Delta W^*_{t-1}$	$\Delta W^*_{t-2}$
$\Delta C_{t}^{*}$	-0.000245	0.000448	0.000272	0.00300	0.000755	-0.000701
	(0.000236)	(0.000310)	(0.000293)	(0.000951)	(0.00114)	(0.00123)
$\Delta C^*_{t-1}$	-0.0000851			-0.00266		
	(0.000283)			(0.000987)		
$\Delta C^*_{t-2}$	0.000186			0.00109		
0 2	(0.000363)			(0.00118)		
$\Delta Y_{t}^{*}$	-0.00101	0.00074	0.000375	0.00537	0.00186	-0.000573
C C	(0.000242)	(0.000274)	(0.000329)	(0.00102)	(0.00105)	(0.000926)
$\Delta Y_{t-1}^{*}$	-0.00265			-0.000215		
0 1	(0.000210)			(0.000906)		
$\Delta Y^{*}_{t-2}$	0.000453			0.000354		
0 2	(0.000256)			(0.000851)		
$\Delta N^*_t$	-0.00236	0.00232	0.000968	0.000321	0.000353	-0.000821
c .	(0.000392)	(0.000357)	(0.000341)	(0.000662)	(0.000786)	(0.000693)
$\Delta N^*_{t-1}$	-0.000565			0.000222		,
	(0.000248)			(0.000772)		
$\Delta N^*_{t=2}$	0.0000128			-0.000491		
t-2	(0.000204)			(0.000710)		
$Z^{*}$	0.00299	0.00114	0.000740	-0.000303	-0.000354	-0.0000901
- t	(0.000334)	(0.000206)	(0.000172)	(0.000178)	(0.000181)	(0.000199)
Z*+ 1	(************	(*****_***)	(*****=/	0.000208	(*****=*=)	(********
- t-1				(0.000163)		
Z*				0.000223		
<b>2</b> t-2				(0.000163)		
W*.				0.0281	-0.00914	-0.00123
t				(0.00211)	(0.00114)	(0.000845)

TABLE A1	(Continued)
IADDE III	(Continued)

(Standard errors in parentheses) Stationarity of Covariances Imposed: Balanced sample. See Text for Details.

	(1)		(2)		(3)	
	Estimate	SE	Estimate	SE	Estimate	SE
Consumption 4	∆C*t					
$\beta$ cw0	0.0251	0.00968				
$\beta$ cw1	0.00573	0.00845				
$\beta$ cw2	-0.00291	0.00903				
$\beta$ cz0	-0.00508	0.00654				
<sup>β</sup> cn0	0.00398	0.0263				
<sup>β</sup> cy0	0.00244	0.0436				
$\beta$ cc0	0.286	0.0118	0.267	0.0807	0.254	0.170
$\beta$ cc1	-0.147	0.0162	-0.160	0.0512	-0.169	0.116
$\beta$ cc2	-0.000883	0.00991	-0.00121	0.0168	-0.00138	0.0185
Earnings $\Delta Y^*$	t					
<sup>β</sup> yw0	0.0553	0.0162	0.0607	0.0155	0.0590	0.0155
$\beta$ yw1	0.0149	0.00987	0.0180	0.0141	0.0175	0.0139
$^{\beta}$ yw2	-0.0000738	0.00746	0.000136	0.0128	0.000339	0.0130
<sup>β</sup> yn0	-0.0424	0.00853 ·	-0.0425	0.0116	-0.0425	0.0122
<sup>β</sup> yn1	0.0197	0.00892	0.0195	0.00966	0.0195	0.0101
<sup>β</sup> yn2	0.0191	0.00759	0.0194	0.0104	0.0194	0.0109
$^{\beta}$ yz0	0.0281	0.178	0.0929	0.0192	0.0925	0.0201
$^{\beta}$ yz1	0.00808	0.722	-0.0103	0.0111	-0.0103	0.0115
$^{\beta}$ yz2	0.00189	0.0391	0.00339	0.0125	0.00337	0.0130
<sup>β</sup> yy0	0.161	2.29	0.165	0.0176	0.167	0.0173
<sup>β</sup> yy1	-0.0292	2.46	-0.0994	0.0193	-0.0983	0.0192
<sup>β</sup> yy2	0.00431	0.0684	0.00323	0.0143	0.00314	0.0149
$\sigma^2{}_{ m v}$	0.0142	0.458				
Wage $\Delta W^*t$						
<sup>p</sup> ww0	0.147	0.0372	0.136	0.0350	0.140	0.0360
<sup>p</sup> ww1	-0.0504	0.0281	-0.0438	0.0322	-0.0466	0.0337
<sup>p</sup> ww2	-0.00675	0.00706	-0.00703	0.0103	-0.00685	0.0104
$\sigma_{\rm w}^2$	0.00208	0.00646	0.00370	0.00599	0.00311	0.00635
Unemploymen	t Z*t					
<sup>p</sup> zz0	0.0534	0.00308	0.0533	0.00503	0.0533	0.00525
<sup>p</sup> zz1	0.0170	0.00250	0.0170	0.00428	0.0170	0.00447
<sup>p</sup> zz2	0.00925	0.00234	0.00921	0.00426	0.00921	0.00444
Hours $\Delta N^*t$						
<sup>p</sup> nn0	0.160	0.291	0.147	0.0285	0.148	0.0299
<sup>p</sup> nn1	-0.0627	0.462 ·	-0.0382	0.0266	-0.0383	0.0279
<sup>p</sup> nn2	-0.00499	0.0120	-0.00532	0.0108	0.00532	0.0113
<sup>p</sup> nz0	-0.0539	0.00911	-0.0527	0.0133	-0.0532	0.0129
<sup>p</sup> nz1	0.0361	0.0102	0.0361	0.00931	0.0361	0.00973
<sup>p</sup> nz2	0.0179	0.00759	0.0181	0.0102	0.0181	0.0106

 TABLE A2
 Estimates of the life cycle model (WLS)

### DYNAMIC FACTOR MODELS OF CONSUMPTION, HOURS AND INCOME 59

	(1)	(1)		(2)		)
	Estimate	SE	Estimate	SE	Estimate	SE
<sup>β</sup> nw0	0.00180	0.00568				
$\beta$ nw1	0.0000862	0.00599				
$\beta$ nw2	-0.00611	0.00584				
<sup>β</sup> ny0	0.0541	0.580				
$\sigma^2_n$	0.00204	0.0856	0.00610	0.00470	0.00605	0.00497
βn			-0.0253	0.216	-0.0255	0.208
$\beta_{\mathbf{c}}$			-0.119	0.427	-0.384	1.37
<sup>β</sup> nc					-0.0376	0.0738
βnw			-0.222	0.801	-0.0929	0.223
$\beta_{nz}$			0.0455	0.175	0.00996	0.0204
$\sigma_{\eta}{}^2$			0.322	3.88	0.0285	0.117
DFE	20		26		25	
MSE	1.77		1.60		1.60	

 TABLE A2
 (Continued)

 $\overline{{}^*Y^*_t}$  is the log of measured labour earnings. As in the other log models in the paper,  $Z^*_t$  is the log (2000 + Hours of Unemployment).