Solving the standard model with Dixit Stiglitz preferences but non-standard iceberg transportation costs

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Abstract

This note invistigates whether allowing for non–standard iceberg transportation costs will make a difference in the standard CES Dixit-Stiglitz (D-S) monopolistic competition model with heterogeneous firms (Melitz '03).

1 Firm problem

I characterize the simplest example of transportation cost that are of iceberg form but not of the standard form (constant fraction of output). The firm takes as given the D-S demand of the consumer. The problem of the firm is

$$\max_p \frac{p^{1-\sigma}}{P^{1-\sigma}} wL - \frac{w}{\phi} t\left(\frac{p^{-\sigma}}{P^{1-\sigma}} wL\right) \frac{p^{-\sigma}}{P^{1-\sigma}} wL - f$$

where we assume

$$t(q) = \tau q^{a}$$

$\alpha < 0$. The unit cost of shipping the good is decreasing with the size of the shipment. As long as $\alpha > -1$ the total cost of the shipment increases and we dont get discontinuities in the decision of the firm. In fact, you need a bit tighter restriction which is $\sigma\alpha > -1$. In the case that $\alpha = 0$ we have the simple iceberg cost formulation

$$\max_p \frac{p^{1-\sigma}}{P^{1-\sigma}} wL - \frac{w}{\phi} \tau \left(\frac{p^{-\sigma}}{P^{1-\sigma}}\right)^{1+\alpha} wL - f$$

Solving the general formulation we have

$$
\begin{aligned}
(\sigma-1)\frac{p^{-\sigma}}{P^{1-\sigma}}wL &= (1+\alpha)\,\sigma\frac{w}{\phi}\tau\left(\frac{1}{P^{1-\sigma}}\right)^{(1+\alpha)} p^{-\sigma-\sigma\alpha-1}wL\frac{1}{P^{1-\sigma}} \implies \\
p &= (1+\alpha)\frac{\sigma}{(\sigma-1)}\frac{w}{\phi}\tau\left(\frac{p^{-\sigma}}{P^{1-\sigma}}\right)^{\alpha}(wL)^{\alpha} \implies \\
p &= \left[(1+\alpha)\frac{\sigma}{(\sigma-1)}w\tau\left(\frac{1}{P^{1-\sigma}}\right)^{\alpha}(wL)^{\alpha}\right]^{1/(1+\sigma\alpha)}\phi^{-1/(1+\sigma\alpha)} \implies \\
p &= \frac{(1+\alpha)\,\sigma}{(\sigma-1)}\frac{w}{\phi}\tau q^{\alpha}(wL)^{\alpha} \implies \\
p &= \frac{(1+\alpha)\,\sigma}{(\sigma-1)}\frac{w}{\phi}t(q)(wL)^{\alpha} \implies
\end{aligned}
$$

What are the relative demands among two firms? We have

$$
\begin{aligned}
\frac{p(\phi_1)\,q(\phi_1)}{p(\phi_2)\,q(\phi_2)} &= \frac{\frac{p(\phi_1)^{1-\sigma}}{P^{1-\sigma}}wL}{\frac{p(\phi_2)^{1-\sigma}}{P^{1-\sigma}}wL} \\
&= \left(\frac{\left[(1+\alpha)\frac{\sigma}{(\sigma-1)}w\tau\left(\frac{1}{P^{1-\sigma}}\right)^{\alpha}(wL)^{\alpha}\right]^{1/(1+\sigma\alpha)}\phi_1^{-1/(1+\sigma\alpha)}}{\left[(1+\alpha)\frac{\sigma}{(\sigma-1)}w\tau\left(\frac{1}{P^{1-\sigma}}\right)^{\alpha}(wL)^{\alpha}\right]^{1/(1+\sigma\alpha)}\phi_2^{-1/(1+\sigma\alpha)}}\right) \\
&= \frac{\phi_1^{-1/(1+\sigma\alpha)}}{\phi_2^{-1/(1+\sigma\alpha)}}
\end{aligned}
$$

2 Results

From the last equation we can observe that if the ϕ's are Pareto distributed then the distribution of sales is simply Pareto but with a different coefficient. Additionally, the elasticity of demand of the consumer with respect to prices is constant in this case, and actually the same as in the no iceberg cost case. The D-S demand is given. Thus, any changes in the pricing of the firm will be reflected into the elasticity of demand in the same way as before. There is no other margin of response to the prices of the consumer except than the D-S margin. Thus, the non-iceberg cost formulation looks isomorphic in terms of responses of consumer demand to price changes to the iceberg cost case.

But what do the transportation costs look like as a function of quantities shipped? Someone can look at the data for that, for example for different goods to look at the quantities shipped and the associated c.i.f. and f.o.b. prices. To my knowledge and to this date, there is no conclusive work on this subject.