Additional Appendix to: "Market Penetration Costs and the New Consumers Margin in International Trade"

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Abstract

In this appendix I show that the model in Arkolakis (2008) with a fixed measure of potential entrants yields isomorphic results to a version of the model with free entry in the equilibrium.

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1 Model

A firm in country i that is reaching fraction n_{ij} of the market of country j and charging price p_{ij} earns profit

$$\pi_{ij}(\phi) = n_{ij}\frac{p_{ij}^{1-\sigma}}{P_j^{1-\sigma}}w_jL_j - w_jL_j\tau_{ij}n_{ij}\frac{p_{ij}^{-\sigma}w_i}{P_j^{1-\sigma}\phi} - w_j^{\gamma}w_i^{1-\gamma}\frac{L_j^{\alpha}}{\psi_{ij}}\frac{1-(1-n_{ij})^{-\beta+1}}{-\beta+1}.$$

The firm's optimal choice of p_{ij}, which is independent of n_{ij}, is determined by the first order condition

$$p_{ij} = \tilde{\sigma}\frac{\tau_{ij}}{\phi}w_i.$$

Given this choice of p_{ij}, the firm's profits are

$$\pi_{ij}(\phi) = n_{ij}\frac{\left(\tilde{\sigma}\frac{\tau_{ij}}{\phi}w_i\right)^{1-\sigma}}{P_j^{1-\sigma}\sigma}w_jL_j - w_j^{\gamma}w_i^{1-\gamma}\frac{L_j^{\alpha}}{\psi_{ij}}\frac{1-(1-n_{ij})^{-\beta+1}}{-\beta+1}.$$

The first order condition for the firm's market access choice is then given by

$$n_{ij} = 1-\left(\frac{\left(\tilde{\sigma}\frac{\tau_{ij}}{\phi}w_i\right)^{1-\sigma}}{P_j^{1-\sigma}\sigma}\frac{w_jL_j}{w_j^{\gamma}w_i^{1-\gamma}\frac{L_j^{\alpha}}{\psi_{ij}}}\right)^{-1/\beta}. \qquad (1)$$

Let $\phi_{ij}^{*} = \sup\{\phi : \pi(\phi) = 0\}$. Clearly a firm with productivity ϕ_{ij}^{*} will choose $n_{ij} = 0$. The first order condition for market access thus implies

$$\left(\phi_{ij}^{*}\right)^{\sigma-1} = \frac{w_j^{\gamma}w_i^{1-\gamma}\frac{L_j^{\alpha}}{\psi_{ij}}P_j^{1-\sigma}\sigma}{(\tilde{\sigma}\tau_{ij}w_i)^{1-\sigma}w_jL_j},$$

Substituting this back into (1) yields

$$n_{ij}(\phi) = 1-\left(\frac{\phi^{\sigma-1}}{\left(\phi_{ij}^{*}\right)^{\sigma-1}}\right)^{-1/\beta}$$

as the firm's optimal market access, conditional on entering market j. The firm's profits from market j are

$$\pi_{ij}(\phi) = \left[1 - \left(\frac{\phi^{\sigma-1}}{\left(\phi_{ij}^{*}\right)^{\sigma-1}}\right)^{-1/\beta}\right] \frac{\left(\tilde{\sigma}\frac{\tau_{ij}}{\phi}w_i\right)^{1-\sigma}}{P_j^{1-\sigma}\sigma} w_j L_j$$
$$- w_j^{\gamma} w_i^{1-\gamma} \frac{L_j^{\alpha}}{\psi_{ij}} \frac{\left[1 - \left(\frac{\phi^{\sigma-1}}{\left(\phi_{ij}^{*}\right)^{\sigma-1}}\right)^{\frac{-1+\beta}{\beta}}\right]}{-\beta+1}.$$

Firms have to pay a fixed entry cost, f_e, in order to enter the market and receive a productivity draw. The productivity of a new entrant is assumed to be a Pareto random variable with shape parameter $\theta > \sigma - 1$ and location parameter b_i. The distribution function of a new entrant's productivity is therefore $G(\phi; b_i) = 1 - \frac{b_i^{\theta}}{\phi^{\theta}}$. A firm that receives a productivity draw lower than ϕ_{ii}^{*} immediately exits the market. In equilibrium, free entry implies that expected profits must be zero:

$$\sum_{v} \int_{\phi_{iv}^{*}} \left[1 - \left(\frac{\phi}{\phi_{iv}^{*}}\right)^{-(\sigma-1)/\beta}\right] \frac{\left(\frac{\sigma}{\sigma-1}\frac{\tau_{iv}w_i}{\phi}\right)^{1-\sigma}}{P_v^{1-\sigma}\sigma} w_v L_v \theta \frac{\left(\phi_{iv}^{*}\right)^{\theta}}{\phi^{\theta+1}} \frac{\left(\phi_{ii}^{*}\right)^{\theta}}{\left(\phi_{iv}^{*}\right)^{\theta}} d\phi$$
$$- \sum_{v} \int_{\phi_{iv}^{*}} w_v^{\gamma} w_i^{1-\gamma} \frac{L_v^{\alpha}}{\psi_{iv}} \frac{1 - \left(\frac{\phi^{\sigma-1}}{\left(\phi_{iv}^{*}\right)^{\sigma-1}}\right)^{\frac{-1+\beta}{\beta}}}{-\beta+1} \theta \frac{\left(\phi_{iv}^{*}\right)^{\theta}}{\phi^{\theta+1}} \frac{\left(\phi_{ii}^{*}\right)^{\theta}}{\left(\phi_{iv}^{*}\right)^{\theta}} d\phi = \frac{w_i f_e}{\frac{b_i^{\theta}}{\left(\phi_{ii}^{*}\right)^{\theta}}} \Leftrightarrow$$
$$\sum_{v} w_v^{\gamma} w_i^{1-\gamma} \frac{L_v^{\alpha}}{\psi_{iv}} \left[\frac{\theta}{\theta-\sigma+1} - \frac{\theta}{\theta-(\sigma-1)\frac{\beta-1}{\beta}}\right] \frac{\left(\phi_{ii}^{*}\right)^{\theta}}{\left(\phi_{iv}^{*}\right)^{\theta}}$$
$$- \sum_{v} w_v^{\gamma} w_i^{1-\gamma} \frac{L_v^{\alpha}}{\psi_{iv}} \frac{(\sigma-1)\frac{1}{\beta}}{\theta-(\sigma-1)\frac{-1+\beta}{\beta}} \frac{\left(\phi_{ii}^{*}\right)^{\theta}}{\left(\phi_{iv}^{*}\right)^{\theta}} = \frac{w_i f_e}{\frac{b_i^{\theta}}{\left(\phi_{ii}^{*}\right)^{\theta}}} \Leftrightarrow$$
$$\sum_{v} w_v^{\gamma} w_i^{1-\gamma} \frac{L_j^{\alpha}}{\psi_{iv}} \left[\frac{(\sigma-1)^2\frac{1}{\beta}}{\left(\theta-(\sigma-1)\frac{\beta-1}{\beta}\right)(\theta-\sigma+1)}\right] \frac{\left(\phi_{ii}^{*}\right)^{\theta}}{\left(\phi_{iv}^{*}\right)^{\theta}} = \frac{w_i f_e}{\frac{b_i^{\theta}}{\left(\phi_{ii}^{*}\right)^{\theta}}}. \tag{2}$$

2 Solving for the equilibrium

The equilibrium number of firms producing in country i, N_i, is determined by the following labor market clearing condition:[1]

[1]The number of new firms is δN_i. This means that the number of firms that actually incur the fixed cost of entry is the ratio of δN_i to the survival rate of $1 - G\left(\phi_{ii}^{*}, b_i\right) = b^{\theta}/\left(\phi^{*}\right)^{\theta}$.

$$N_i \sum_{\upsilon} w_\upsilon^{\gamma} w_i^{-\gamma} \frac{L_j^{\alpha}}{\psi_{i\upsilon}} (\sigma-1) \left[\frac{\theta}{\theta-\sigma+1} - \frac{\theta}{\theta-(\sigma-1)\frac{\beta-1}{\beta}} \right] \frac{(\phi_{ii}^*)^{\theta}}{(\phi_{i\upsilon}^*)^{\theta}} + N_i \frac{f_e}{(1-G(\phi_{ii}^*, b_i))}$$

$$+ \sum_{\upsilon} N_\upsilon \gamma \int_{\phi_{\upsilon j}^*} w_\upsilon^{\gamma-1} w_i^{1-\gamma} \frac{L_j^{\alpha}}{\psi_{\upsilon j}} \frac{1-\left(\frac{\phi^{\sigma-1}}{(\phi_{\upsilon j}^*)^{\sigma-1}}\right)^{\frac{-1+\beta}{\beta}}}{-\beta+1} \theta \frac{(\phi_{\upsilon j}^*)^{\theta}}{\phi^{\theta+1}} \frac{(\phi_{\upsilon\upsilon}^*)^{\theta}}{(\phi_{\upsilon j}^*)^{\theta}} d\phi$$

$$+ N_i \sum_{\upsilon} \int_{\phi_{i\upsilon}^*} (1-\gamma) w_\upsilon^{\gamma} w_i^{-\gamma} \frac{L_j^{\alpha}}{\psi_{i\upsilon}} \frac{1-\left(\frac{\phi^{\sigma-1}}{(\phi_{i\upsilon}^*)^{\sigma-1}}\right)^{\frac{-1+\beta}{\beta}}}{-\beta+1} \theta \frac{(\phi_{i\upsilon}^*)^{\theta}}{\phi^{\theta+1}} \frac{(\phi_{ii}^*)^{\theta}}{(\phi_{i\upsilon}^*)^{\theta}} d\phi = L_i \Leftrightarrow$$

$$N_i \sum_{\upsilon} w_\upsilon^{\gamma} w_i^{-\gamma} \frac{L_\upsilon^{\alpha}}{\psi_{i\upsilon}} \left[\frac{\theta(\sigma-1)^2 \frac{1}{\beta}}{(\theta-\sigma+1)\left(\theta-(\sigma-1)\frac{\beta-1}{\beta}\right)} \right] \frac{(\phi_{ii}^*)^{\theta}}{(\phi_{i\upsilon}^*)^{\theta}} + N_i \frac{f_e}{(1-G(\phi_{ii}^*, b_i))}$$

$$+ \frac{(\sigma-1)\frac{1}{\beta}}{\theta-(\sigma-1)\frac{-1+\beta}{\beta}} \left[\gamma \sum_{\upsilon} N_\upsilon \frac{w_i^{\gamma-1}}{w_\upsilon^{\gamma-1}} \frac{L_i^{\alpha}}{\psi_{\upsilon i}} \frac{(\phi_{\upsilon\upsilon}^*)^{\theta}}{(\phi_{\upsilon i}^*)^{\theta}} + (1-\gamma) \sum_{\upsilon} N_i \frac{w_\upsilon^{\gamma}}{w_i^{\gamma}} \frac{L_\upsilon^{\alpha}}{\psi_{i\upsilon}} \frac{(\phi_{ii}^*)^{\theta}}{(\phi_{i\upsilon}^*)^{\theta}} \right] = L_i, \qquad (3)$$

where the last equivalence follows after substituting the free entry condition, (2). Balanced trade implies

$$\sum_{\upsilon} w_\upsilon^{\gamma} w_i^{1-\gamma} \frac{L_\upsilon^{\alpha}}{\psi_{i\upsilon}} \left[\frac{\theta\sigma}{\theta-\sigma+1} - \frac{\theta\sigma}{\theta-(\sigma-1)\frac{\beta-1}{\beta}} \right] N_i \frac{(\phi_{ii}^*)^{\theta}}{(\phi_{i\upsilon}^*)^{\theta}}$$

$$= \sum_{\upsilon} w_i^{\gamma} w_\upsilon^{1-\gamma} \frac{L_j^{\alpha}}{\psi_{\upsilon i}} \left[\frac{\theta\sigma}{\theta-\sigma+1} - \frac{\theta\sigma}{\theta-(\sigma-1)\frac{\beta-1}{\beta}} \right] N_\upsilon \frac{(\phi_{\upsilon\upsilon}^*)^{\theta}}{(\phi_{\upsilon i}^*)^{\theta}} \Leftrightarrow$$

$$\sum_{\upsilon} w_\upsilon^{\gamma} w_i^{1-\gamma} \frac{L_\upsilon^{\alpha}}{\psi_{i\upsilon}} N_i \frac{(\phi_{ii}^*)^{\theta}}{(\phi_{i\upsilon}^*)^{\theta}} = \sum_{\upsilon} w_i^{\gamma} w_\upsilon^{1-\gamma} \frac{L_j^{\alpha}}{\psi_{\upsilon i}} N_\upsilon \frac{(\phi_{\upsilon\upsilon}^*)^{\theta}}{(\phi_{\upsilon i}^*)^{\theta}},$$

which, substituting into (3), yields

$$
\begin{aligned}
&N_i \sum_{\upsilon} w_{\upsilon}^{\gamma} w_i^{-\gamma} \frac{L_{\upsilon}^{\alpha}}{\psi_{i\upsilon}} \left[\frac{\theta \left(\sigma - 1\right)^2 \frac{1}{\beta}}{\left(\theta - \sigma + 1\right) \left(\theta - \left(\sigma - 1\right) \frac{\beta - 1}{\beta}\right)} \right] \frac{\left(\phi_{ii}^*\right)^{\theta}}{\left(\phi_{i\upsilon}^*\right)^{\theta}} + N_i \frac{f_e}{\left(1 - G\left(\phi_{ii}^*, b_i\right)\right)} \\
&+ N_i \frac{\left(\sigma - 1\right) \frac{1}{\beta}}{\theta - \left(\sigma - 1\right) \frac{-1+\beta}{\beta}} \left[\sum_{\upsilon} w_{\upsilon}^{\gamma} w_i^{-\gamma} \frac{L_{\upsilon}^{\alpha}}{\psi_{i\upsilon}} \frac{\left(\phi_{ii}^*\right)^{\theta}}{\left(\phi_{i\upsilon}^*\right)^{\theta}} \right] = L_i \Leftrightarrow \\
&\left(\theta + 1\right) N_i \frac{f_e}{\left(1 - G\left(\phi_{ii}^*, b_i\right)\right)} + N_i \frac{\left(\sigma - 1\right) \frac{1}{\beta}}{\theta - \left(\sigma - 1\right) \frac{-1+\beta}{\beta}} \left[\sum_{\upsilon} w_{\upsilon}^{\gamma} w_i^{-\gamma} \frac{L_{\upsilon}^{\alpha}}{\psi_{i\upsilon}} \frac{\left(\phi_{ii}^*\right)^{\theta}}{\left(\phi_{i\upsilon}^*\right)^{\theta}} \right] = L_i \Leftrightarrow \\
&\left(\theta + 1\right) N_i \frac{f_e}{\left(1 - G\left(\phi_{ii}^*, b_i\right)\right)} + N_i \frac{\left(\theta - \sigma + 1\right)}{\left(\sigma - 1\right)} \frac{f_e}{\frac{b_i^{\theta}}{\left(\phi_{ii}^*\right)^{\theta}}} = L_i \Leftrightarrow \\
&N_i \left(\frac{\theta \sigma}{\left(\sigma - 1\right)} \right) \frac{f_e}{\frac{b_i^{\theta}}{\left(\phi_{ii}^*\right)^{\theta}}} = L_i,
\end{aligned}
$$

which yields the measure of operating firms.

Notice that the returns to labor for the production of the entry cost equal exactly the returns to profit in a model with no free entry of Arkolakis (2008). All the remaining equations of the model are identical to the ones of the model with no free entry.