More consumers or more per consumer?[1]

Costas Arkolakis

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Abstract

The purpose of this note is to show that there exist an isomorphism of the model of Arkolakis (Market Penetration Costs and the New Consumers Margin in International Trade) with a model where firms pay a market penetration cost to increase their sales per consumer.

1 Model

The problem of the consumer is

$$\max_{x(\omega)} \left(\int u(\omega)^{1-\rho} x(\omega)^{\rho} d\omega \right)^{\frac{1}{\rho}}$$
$$\text{s.t.} \int p(\omega) x(\omega) \leq w + \pi$$

where $p(\omega)$ represents the price of good ω, $x(\omega)$ the quantity demanded by the representative consumer, w wages, π profits and $\sigma = 1/(1-\rho) > 1$. First order conditions give that the demand per consumer is

$$x(\omega) = u(\omega) \frac{p(\omega)^{-\sigma}}{P^{-\sigma}} X$$

where

$$P^{1-\sigma} = \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \tag{1}$$

[1] I thank Marios Angeletos for suggesting this isomorphism. All remaining errors are mine.

There is a measure of L consumers. Looking at a symmetric equilibrium where all the firms with the same productivity face the same optimization problem we can write the problem of a firm with productivity ϕ as

$$\max_{u,p} u \frac{p^{1-\sigma}}{P^{1-\sigma}} (w+\pi) L - u \frac{w}{\phi} \frac{p^{-\sigma}}{P^{1-\sigma}} (w+\pi) L - wg(u(\phi)) \tag{2}$$

thus

$$P^{1-\sigma} = M \int_{\phi^*} u(\phi) p(\phi)^{1-\sigma} \mu(\phi) d\phi \tag{3}$$

where $\mu(\phi)$ is the density of firms conditional on operating $(\phi > \phi^*)$.

FOC conditions give us

wrt to $p(\phi)$

$$p(\phi) = \frac{w}{\phi} \frac{\sigma}{\sigma - 1}$$

wrt to $u(\phi)$

$$\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \left(\frac{w}{\phi}\right)^{1-\sigma} \frac{(w+\pi) L}{P^{1-\sigma}} \frac{1}{\sigma} = g'(u(\phi))$$

Proposition 1 *Assume that the market penetration cost function is* $g(u(\phi)) = \frac{1}{\psi} \frac{1-(1-u)^{-\beta+1}}{-\beta+1}$. *Then the problem defined above is isomorphic to the one of Arkolakis where the market penetration cost increases is a function of the number of consumers reached.*

Proof: First notice that the cutoff ϕ of production, that determines the number of operating terms M, is given by

$$(\phi^*)^{\sigma-1} = \frac{\frac{1}{\psi}}{\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} (w)^{1-\sigma} \frac{(w+\pi)L}{P^{1-\sigma}} \frac{1}{\sigma}}.$$

In the case that market penetration is by consumer (I denote all equilibrium variables in this case with a tilde)

$$\left(\tilde{\phi}^*\right)^{\sigma-1} = \frac{\frac{1}{\psi}}{\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} (\tilde{w})^{1-\sigma} \frac{(\tilde{w}+\tilde{\pi})L}{\tilde{P}^{1-\sigma}} \frac{1}{\sigma}}.$$

Total labor required for market penetration costs is

$$l_m = \int_{\phi^*} \frac{1}{\psi} \frac{1 - (1 - u(\phi))^{-\beta}}{-\beta} \mu(\phi)\, d\phi \ ,$$

where for the case of consumer reach is

$$\tilde{l}_m = \int_{\phi^*} \frac{1}{\psi} \frac{1 - (1 - n(\phi))^{-\beta}}{-\beta} \mu(\phi)\, d\phi \ .$$

Labor demand for production is

$$l_p = \int_{\phi^*} u(\phi) \frac{w}{\phi} \frac{p(\phi)^{-\sigma}}{P^{1-\sigma}} (w + \pi) L\mu(\phi)\, d\phi,$$

$$\tilde{l}_p = \int_{\phi^*} n(\phi) \frac{\tilde{w}}{\phi} \frac{\tilde{p}(\phi)^{-\sigma}}{\tilde{P}^{1-\sigma}} (\tilde{w} + \tilde{\pi}) L\mu(\phi)\, d\phi$$

respectively and with

$$\tilde{P}^{1-\sigma} = \tilde{M} \int_{\phi^*} \tilde{n}(\phi)\, \tilde{p}(\phi)^{1-\sigma}\, \tilde{\mu}(\phi)\, d\phi \ .$$

We also have that the effective demand for a firm is

$$y(\phi) = u(\phi)\, L \frac{p(\phi)^{-\sigma}}{P^{1-\sigma}} (w + \pi) \ ,$$

where for the case that market penetration is in terms of consumers we have

$$\tilde{y}(\phi) = n(\phi)\, L \frac{\tilde{p}(\phi)^{-\sigma}}{\tilde{P}^{1-\sigma}} (\tilde{w} + \tilde{\pi}) \ .$$

Finally, X is given by

$$X = \left(\int u(\omega)^{1-\rho}\, x(\omega)^{\rho}\, d\omega \right)^{\frac{1}{\rho}} = \left(M \int u(\phi) \left(\frac{p(\phi)^{-\sigma}}{P^{1-\sigma}} (w + \pi) \right)^{\rho} \mu(\phi)\, d\phi \right)^{\frac{1}{\rho}} \ ,$$

where in the case of market penetration in terms of consumers is

$$\tilde{X} = \left(\tilde{M} \int n(\phi)\, \tilde{x}(\phi)^{\rho}\, \tilde{\mu}(\phi)\, d\phi\right)^{\frac{1}{\rho}}$$

with

$$\tilde{x}(\phi) = \frac{\tilde{p}(\phi)^{-\sigma}}{\tilde{P}^{1-\sigma}} (\tilde{w} + \tilde{\pi}) \ .$$

Now, define the following variables $n(\phi) = u(\phi)$, $X = \tilde{X}$, $P = \tilde{P}$, $y(\phi) = \tilde{y}(\phi)$, $x(\phi) = \tilde{x}(\phi)$, $p(\phi) = \tilde{p}(\phi)$, $M = \tilde{M}$, $l_m(\phi) = \tilde{l}_m(\phi)$, $l_p(\phi) = l_p(\phi)$. Then assuming the same utility function (CES) the same parameters and the same productivity distribution for firms as well as the same technology for producing the good the models of market penetration by reaching more consumers or selling more per consumer are isomorphic.