Description of numerical algorithm for ACDR

This note describes the algorithm for the global solution of the model with a general demand function that satisfies the ACDR formulation, any distribution of productivities, and allows for fixed marketing costs.

1 Step 1: Inputs.

The user is asked to incorporate functions, or a corresponding vector for

a) The 'nonparametric' part of the demand, the function $d(v)$, whereby below we will abuse notation and follow the convention

$$d(v) \equiv d(\mu(v) - v) \tag{1}$$

(and possibly utility generating that, in the case of $\beta = 0$). The demand is given by

$$q\left(e^{v}, p_j^*/w_j\right) = e^{d(v)} e^{-\beta(\mu(v)-v)} \left(\frac{p_j^*}{w_j}\right)^{-\beta}. \tag{2}$$

b) Productivity distribution.

c) Choose whether they will be fixed costs or not.

d) Choose the rest of the parameters (population, trade costs).

2 Step 2: Markup function and other firm decisions

We now define a few more objects that we will use below. In particular, let p_j^* the choke-price which depends on the demand function and let z_{ij}^* the productivity of the firm that has zero profits. The price charged by marginal firm is $e^{\mu(v^*)} \frac{w_i \tau_{ij}}{z_{ij}^*}$. We define throughout the relative efficiency of firms,

$$v = \ln\left(p_j^* z / w_i \tau_{ij}\right) = \ln p_j^* - \ln c \tag{3}$$

i.e. v is the log difference of the cutoff price with the marginal cost of a certain firm,z. Notice that this relationship also implies

$$\frac{w_i \tau_{ij}}{p_j^*} \exp(v) = z \tag{4}$$

We will be using these definitions to change variables when appropriate.

The program computes a markup function, in logs, $\mu(v) = \ln m$ where $m \equiv p/c$ for $v \in [0, +\infty)$ by solving the implicit function as stated in the main paper

$$m - \left(\frac{\beta - d'(\ln(m) - v)}{\beta - 1 - d'(\ln(m) - v)}\right) = 0 \tag{5}$$

3 Step 3: Compute Initial and Counterfactual Equilibrium

We now develop an algorithm to compute the equilibrium for the model for a two different sets of parameters (initial and counterfactual equilibrium) so that we are able to make comparative static predictions with this model.

3a: Construct Bilateral Aggregates

We integrate over individual firm decisions to construct the following functions

a) *Aggregate Bilateral Fixed Costs.* We have that aggregate bilateral fixed costs are given by

$$F_{ij} = N_i w_j f_{ij} \int_{z_{ij}^*}^{\infty} g_i(z)\, dz$$

Using the change of variables (4) and the same relationship evaluated at v_{ij}^*,

$$z_{ij}^* \equiv w_i \tau_{ij} e^{v_{ij}^*} / p_j^*, \tag{6}$$

we then have

$$F_{ij} = N_i w_j f_{ij} \frac{w_i \tau_{ij}}{p_j^*} \int_{v_{ij}^*}^{\infty} g_i\left(\frac{w_i \tau_{ij}}{p_j^*} e^v\right) e^v dv$$

b) *Aggregate Bilateral Sales.* Turning to aggregate bilateral sales, and using again (6) and (4) we have

$$\frac{z}{z_{ij}^*} = e^{v - v_{ij}^*}, \tag{7}$$

and hence

$$e^v = \frac{z}{z_{ij}^*} e^{v_{ij}^*}, \tag{8}$$

so

$$X_{ij} = N_i L_j \int_{z_{ij}^*}^{\infty} e^{\mu\left(\ln z - \ln z_{ij}^* + v_{ij}^*\right)} \frac{w_i \tau_{ij}}{z} q\left(\frac{z}{z_{ij}^*} e^{v_{ij}^*}, p_j^*/w_j\right) g_i(z)\, dz$$

Using (7) then

$$\begin{aligned} X_{ij} &= N_i L_j \int_{v_j^*}^{\infty} e^{\mu(v)} \frac{w_i \tau_{ij}}{z_{ij}^* e^{v - v_{ij}^*}} q\left(e^v, p_j^*/w_j\right) g_i\left(z_{ij}^* e^{v - v_{ij}^*}\right) z_{ij}^* e^{v - v_{ij}^*} dv \\ &= N_i L_j w_i \tau_{ij} \int_{v_{ij}^*}^{\infty} e^{\mu(v)} q\left(e^v, p_j^*/w_j\right) g_i\left(e^v \frac{\tau_{ij}}{p_j^*/w_j} \frac{w_i}{w_j}\right) dv \end{aligned}$$

c) *Aggregate Bilateral Production Costs.* Also, using the same substitutions as before we can directly write aggregate bilateral productions costs

$$\begin{aligned} K_{ij} &= N_i L_j \int_{z_{ij}^*}^{\infty} \left(\frac{w_i \tau_{ij}}{z}\right) q\left(\frac{z}{z_{ij}^*} e^{v_{ij}^*}, p_j^*/w_j\right) g_i(z)\, dz \\ &= N_i L_j w_i \tau_{ij} \int_{v_{ij}^*}^{\infty} q\left(e^v, p_j^*/w_j\right) g_i\left(e^v \frac{\tau_{ij}}{p_j^*/w_j} \frac{w_i}{w_j}\right) dv \end{aligned}$$

d) *Aggregate Bilateral Profits.* Aggregate bilateral profits can be now written as

$$\Pi_{ij} = X_{ij} - K_{ij} - F_{ij}$$

so

$$\begin{aligned}
\frac{\Pi_{ij}}{N_i} &= L_j w_i \tau_{ij} \int_{v_{ij}^*}^{\infty} e^{\mu(v)} q\left(e^v, p_j^*/w_j\right) g_i \left(e^v \frac{\tau_{ij}}{p_j^*/w_j} \frac{w_i}{w_j} \right) dv \\
&\quad -L_j \times w_i \tau_{ij} \times \int_{v_{ij}^*}^{\infty} q\left(e^v, p_j^*/w_j\right) g_i \left(e^v \frac{\tau_{ij}}{p_j^*/w_j} \frac{w_i}{w_j} \right) dv \\
&\quad -f_j \times \frac{w_i \tau_{ij}}{p_j^*/w_j} \times \int_{v_{ij}^*}^{\infty} g_i \left(e^v \frac{\tau_{ij}}{p_j^*/w_j} \frac{w_i}{w_j} \right) e^v dv
\end{aligned}$$

Note that this is a function of wages, p_j^*/w_j, and v_{ij}^*.

3b: Use Bilateral Aggregate Functions to Compute the Equilibria

We compute the initial and the counterfactual equilibrium under two vectors of trade costs, $\{\tau_{ij}\}$ and $\{\tau_{ij}\}'$. The program computes the equilibrium for the variables p_j^*, v_j^*, w_j and N_j using the following four (4) equilibrium conditions : The zero profit condition for the cutoff firm for each market, the free entry condition, the labor market clearing condition, and the budget balance.

a) *Zero profit for exporters.* Profits are

$$\pi\left(z, z_{ij}^*, w_i\right) \equiv L_j \left(e^{\mu\left(\ln z - \ln z_{ij}^* + v_{ij}^*\right)} - 1 \right) \frac{w_i \tau_{ij}}{z} q \left(\frac{z}{z_{ij}^*} e^{v_{ij}^*}, p_j^*, w_j \right) \tag{9}$$

Now equate that with $w_j f_{ij}$ at $z = z_{ij}^*$ to get

$$L_j \left(e^{\mu\left(v_{ij}^*\right)} - 1 \right) \frac{w_i \tau_{ij}}{z_{ij}^*} q \left(e^{v_{ij}^*}, p_j^*, w_j \right) = \frac{w_j f_{ij}}{L_j}$$

or

$$\left(e^{\mu\left(v_{ij}^*\right)} - 1 \right) p_j^* e^{-v_{ij}^*} q \left(e^{v_{ij}^*}, p_j^*, w_j \right) = \frac{w_j f_{ij}}{L_j}$$

or

$$\frac{p_j^*}{w_j} \left(e^{\mu\left(v_{ij}^*\right)} - 1 \right) e^{-v_{ij}^*} q \left(e^{v_{ij}^*}, p_j^*, w_j \right) = \frac{f_{ij}}{L_j}$$

so

$$\begin{aligned}
\frac{p_j^*}{w_j} \left(e^{\mu\left(v_{ij}^*\right)} - 1 \right) e^{-v_{ij}^*} e^{d\left(v_{ij}^*\right)} e^{-\beta\left(\mu\left(v_{ij}^*\right) - v_{ij}^*\right)} \left(\frac{p_j^*}{w_j} \right)^{-\beta} &= \frac{f_{ij}}{L_j} \implies \\
\left(\frac{p_j^*}{w_j} \right)^{1-\beta} \left(e^{\mu\left(v_{ij}^*\right)} - 1 \right) e^{-v_{ij}^*} e^{d\left(v_{ij}^*\right)} e^{-\beta\left(\mu\left(v_{ij}^*\right) - v_{ij}^*\right)} &= \frac{f_{ij}}{L_j}
\end{aligned}$$

and hence the cut-off condition is

$$\left(\frac{p_j^*}{w_j} \right)^{1-\beta} \left(e^{\mu\left(v_{ij}^*\right)} - 1 \right) e^{d\left(v_{ij}^*\right)} e^{-v_{ij}^*} e^{-\beta\left(\mu\left(v_{ij}^*\right) - v_{ij}^*\right)} = \frac{f_{ij}}{L_j} \tag{10}$$

b) *Free entry condition.* The free entry condition is simply

$$\sum_j \frac{\Pi_{ij}}{N_i} = w_i f_e. \tag{11}$$

Note that this is a system in wages, p_j^*/w_j, and v_j^*.

c) *Labor market clearing.* Labor market clearing is given by

$$\begin{aligned}
\sum_j K_{ij} + \underbrace{w_i f_i \sum_v N_v \textstyle\int_{z_{vi}^*}^{\infty} dG_v(z)}_{\text{fixed cost labor}} + N_i w_i f_e &= w_i L_i \\
\sum_j \frac{K_{ij}}{w_i} + f_i \sum_k N_k \frac{\tau_{ki}}{p_i^*/w_i}\frac{w_k}{w_i} \textstyle\int_{v_{ki}^*}^{\infty} g_k(e^v \frac{\tau_{ki}}{p_i^*/w_i}\frac{w_k}{w_i}) e^v dv + N_i f^e &= L_i \\
\sum_j K_{ij} + \underbrace{\sum_j F_{ji}}_{\text{fixed cost labor}} + N_i w_i f_e &= w_i L_i
\end{aligned} \tag{12}$$

d) *Budget Balance.* Budget balance can also be written as

$$\begin{aligned}
\sum_i X_{ij} &= w_j L_j \implies \\
\sum_i N_i \textstyle\int_{v_{ij}^*}^{\infty} e^{\mu(v)} q\left(e^v, p_j^*, e_j\right) g_i \left(e^v \frac{\tau_{ij}}{p_j^*/w_j}\frac{w_i}{w_j}\right) \tau_{ij} w_i dv &= w_j
\end{aligned} \tag{13}$$

The implemented algorithm is as following

Step i: Guess a p_j^*/w_j and solve for v_{ij}^* using the cutoff condition, (10) (this step is necessary only if there is a fixed cost of selling to a market combined with a choke price)

Step ii: Using $\frac{p_j^*}{w_j}$ and v_{ij}^* we can solve for relative wages w_j using the free entry condition, (11) (if there is no fixed cost and Pareto $\frac{p_j^*}{w_j}$ is not needed to solve for the wages using the free entry condition)

Step iii: Using $\frac{p_j^*}{w_j}$, v_{ij}^* and w_j we can solve for N_i using the labor market clearing condition (12) -which should be equivalent to the current account balance here-

Step iv: Using $\frac{p_j^*}{w_j}$, v_{ij}^* and N_j we can see if budget balance, equation (13), is satisfied. If not, update $\frac{p_j^*}{w_j}$ and go to step 1 (with Pareto and CES or choke price the $\frac{p_j^*}{w_j}$ disappears from that relationship so we never need to deal with this step).

4 Step 4: Compute Equivalent Variation

We now compute the equivalent variation that would make the consumer equally better-off in the two equilibria. The input to this stage is the equilibrium outcomes that we computed in step 3, p_j^*, w_j, v_{ij}^* and N_j. We solve for the equivalent variation this for a) prespecified (separable-symmetric) utility function, b) using only the nonparametric demand estimates.

4.1 Case a) prespecified utility function, in the case of $\beta = 0$.

In this case $q\left(\frac{z}{z_{ij}^*}, p_j^*, e_j\right) = q\left(\frac{z}{z_{ij}^*}\right) = q\left(e^v\right)$

Step i: Computing Utility Level in the new ($'$) equilibrium

1. Given the prime equilibrium, we can compute:

$$\bar{u}_j' = \sum_i N_i' \textstyle\int_{\left(z_{ij}^*\right)'} u\left[q_{ij}(z, \left(p_j^*\right)', w_j')\right] g_i(z)dz \tag{14}$$

$$= \sum_i N_i' \textstyle\int_{\left(z_{ij}^*\right)'} u\left[\frac{x_{ij}(z, \left(p_j^*\right)', w_j')}{p_{ij}(z, \left(p_j^*\right)', w_j')}\right] g_i(z)dz \tag{15}$$

where $u(\cdot)$ is the utility function associated with $d(\cdot)$ above (i.e. $d(\cdot) = u'^{-1}(\cdot)$). Notice that with our specification and with fixed costs we can apply the standard change of variables. Notice that we can write

$$\bar{u}_j' = \sum_i N_i' \textstyle\int_{\left(v_{ij}^*\right)'} u\left(q\left(e^v\right)\right) g_i\left(e^v\left(\frac{w_i'\tau'_{ij}}{\left(p_j^*\right)'}\right)\right)\left(e^v \frac{w_i'\tau'_{ij}}{\left(p_j^*\right)'}\right) dv$$

$$= \sum_i N_i' \frac{w_i'\tau'_{ij}}{\left(p_j^*\right)'} \textstyle\int_{\left(v_{ij}^*\right)'} u\left(q\left(e^v\right)\right) e^v g_i\left(e^v w_i'\tau'_{ij}/\left(p_j^*\right)'\right) dv \tag{16}$$

2. By definition, $e(p\left(\tau'\right), u') = w'$, which we have normalized to one, $w' = 1$

Step ii: (Computing the expenditure level to achieve the same utility as in $'$ equilibrium)

$e(p\left(\tau\right), \bar{u})$, is given iby

$$e_j(\mathbf{N}, \mathbf{z}^*, \mathbf{w}, \bar{u}) = \min_{q_i(z)} \textstyle\sum_i N_i \int_{z_{ij}^*} p_{ij}\left(z, z_{ij}^*\right) q_{ij}\left(z, z_{ij}^*, w_j\right) g_i(z)dz \tag{17}$$

subject to:

$$\textstyle\sum_i N_i \displaystyle\int_{z_{ij}^*}^{\infty} u\left[q_{ij}\left(z, z_{ij}^*, w_j\right)\right] g_i(z)dz \geq \bar{u}_j', \tag{18}$$

First order conditions for this problem imply[1]

$$p_{ij}\left(z, z_{ij}^*\right) = \lambda_j u'\left[q_{ij}\left(z, \bar{u}_j'\right)\right] \implies$$
$$\ln p_{ij}\left(z, z_{ij}^*\right) - \ln \lambda_j = \ln u'\left[q_{ij}\left(z, \bar{u}_j'\right)\right]$$

which can be rearranged as

$$q_{ij}\left(z, \bar{u}\right) = u'^{-1}\left(e^{\ln p_{ij}\left(z, z_{ij}^*\right) - \ln \lambda_j}\right)$$
$$= \exp\left\{d\left(\ln p_{ij}\left(z, z_{ij}^*\right) - \ln \lambda_j\right)\right\}$$

[1] Notice that in this case, $\beta = 0$, we have $\lambda_j = p_j^*$ and using 20 we can write $\ln u'(q) = \ln p - \ln p^* = \mu(v) - v + \ln p_j^*$ for each v. We can compute the level of utility by integrating $\int_{v_j^*} u'\left(q\left(v\right)\right) g\left(v\right) dv = \int_{v_j^*}^{\infty} e^{\mu(v)-v} p_j^* g\left(v\right) dv$..

where $d = u'^{-1}$ and the quantity consumed is non zero only if $p_{ij}\left(z, z_{ij}^*, c_{ij}\right) < \lambda_i$ and

$$\begin{aligned}
\ln p_{ij}\left(z, z_{ij}^*\right) &= \mu\left(\ln z - \ln z_{ij}^* + v_j^*\right) + \ln\left(w_i \tau_{ij}/z\right) \\
&= \mu\left(\ln z - \ln z_{ij}^* + v_{ij}^*\right) + \ln\left(z_{ij}^* p_j^* e^{-v_{ij}^*}\right) - \ln z
\end{aligned}$$

so really we want

$$\ln p_{ij}\left(z, z_{ij}^*, p_j^*, v_{ij}^*\right) = \mu\left(\ln z - \ln z_{ij}^* + v_{ij}^*\right) + \ln z_{ij}^* - \ln z + \ln p_j^* - v_{ij}^*$$

We will now (finally) compute the expenditure required to achieve the same level of utility in the initial equilibrium in two steps.

We first need to solve for λ_j for each country. Using the constraint (18) assuming that it binds

$$\textstyle\sum_i N_i \displaystyle\int_{z_{ij}^*}^{\infty} u\left[d\left(\mu\left(\ln z - \ln z_{ij}^* + v_{ij}^*\right) + \ln z_{ij}^* - \ln z + \ln p_j^* - v_{ij}^* - \ln \lambda_j\right)\right] g_i(z) dz = \bar{u}_j'$$

subject to p_j^*, w_i, N_i computed in the initial equilibrium. With our notation and change of variables

$$\textstyle\sum_i N_i \displaystyle\int_{v_j^*}^{\infty} u\left[d\left(\mu\left(v\right) - v + \ln p_j^* - \ln \lambda_j\right)\right] g_i\left(\frac{w_i \tau_{ij}}{p_j^*} e^v\right) \frac{w_i \tau_{ij}}{p_j^*} e^v dv = \bar{u}_j' \tag{19}$$

This is a simple problem that given v_j^*, p_j^*, wages, and $\bar{u}_j'$ solves for λ_j equation by equation.

Second, once we know λ_j we can now compute $e(p\left(\tau\right), \bar{u}_j')$ since know the choices for $q_{ij}\left(z, \bar{u}\right)$ that we derived above satisfy the constraint minization problem so that

$$\begin{aligned}
e_j(\mathbf{N}, \mathbf{z}^*, \mathbf{w}, \bar{u}_j') &= \textstyle\sum_i N_i \int_{z_{ij}^*} p_{ij}\left(z, z_{ij}^*, p_j^*, v_{ij}^*\right) q_{ij}\left(z, \bar{u}'\right) g_i(z) dz \\
&= \textstyle\sum_i N_i \int_{z_{ij}^*} e^{\mu\left(\ln z - \ln z_{ij}^* + v_{ij}^*\right) + \ln z_{ij}^* - \ln z + \ln p_j^* - v_{ij}^*} \exp\left\{d\left(\mu\left(\ln z - \ln z_{ij}^* + v_{ij}^*\right) + \ln z_{ij}^* - \ln z + \ln p_j^* - v_j^* -\right.\right. \\
&= \textstyle\sum_i N_i \int_{z_{ij}^*} e^{\mu\left(\ln z - \ln z_{ij}^* + v_{ij}^*\right)} \dfrac{w_i \tau_{ij}}{z} \exp\left\{d\left(\mu\left(\ln z - \ln z_{ij}^* + v_{ij}^*\right) + \ln w_i \tau_{ij} - \ln z - \ln \lambda_j\right)\right\} g_i(z) dz
\end{aligned}$$

and with our notation ($z = \frac{w_i \tau_{ij}}{p_j^*} e^v$)

$$\begin{aligned}
e_j &= \textstyle\sum_i N_i \int_{v_j^*} e^{\mu(v)} w_i \tau_{ij} \exp\left\{d\left(\mu(v) - v + \ln p_j^* - \ln \lambda_j\right)\right\} g_i\left(e^v \dfrac{w_i \tau_{ij}}{p_j^*}\right) dv \\
&= \textstyle\sum_i N_i \times w_i \tau_{ij} \times \int_{v_j^*} e^{\mu(v)} \exp\left\{d\left(\mu\left(v\right) - v + \ln p_j^* - \ln \lambda_j\right)\right\} g_i\left(e^v \dfrac{w_i \tau_{ij}}{p_j^*}\right) dv
\end{aligned}$$

Notice that $\left(p_j^*\right)' = \lambda_j'$ hence

$$e_j' = \textstyle\sum_i N_i' \times w_i' \tau_{ij}' \times \int_{v_{ij}^{*\prime}} e^{\mu(v)} \exp\left\{d\left(\mu\left(v\right) - v\right)\right\} g_i\left(e^v \dfrac{w_i' \tau_{ij}'}{p_j^{*\prime}}\right) dv$$

Step iii: Computing the equivalent variation

1. (a) In the final step we need to compute the equivalent variation. Do so, notice that the equivalent variation is given by

$$EV_j = e\left(\mathbf{p}, u_j'\right) - w_j$$

and in percentage changes

$$\begin{aligned} \frac{EV_j}{w_j} &= \frac{e\left(\mathbf{p}, u_j'\right)}{w_j} - 1 \implies \\ \frac{EV_j}{w_j} &= \frac{w_j'}{w_j}\frac{e\left(\mathbf{p}, u_j'\right)}{w_j'} - 1 \end{aligned}$$

4.2 case b) Specifying a demand function with $\beta = 0$

In this second case we show how can we compute the equivalent variation when $\beta = 0$ and we only have estimated a demand funciton.

Step 0) From the first order conditions of our demand system $p(q) = u'(q)/\lambda$, i.e. from the simulated demand we can get the marginal utility ratio to the lagrange multiplier. Integrating this expression we obtain the transformed utility level for each given output level

$$\tilde{u}(\hat{q}) \equiv \frac{u(\hat{q}) - u(0)}{\lambda} = \int_0^q \frac{u'(v)}{\lambda} dv = \int_0^{\hat{q}} p(q(v))\, dv$$

this gives us the level of (normalized) utility for the counterfactual equilibrium. Notice that this is simple integration (i.e. there is no density) since we are trying to reconstuct the utility from its first derivative.

Step i) We want to find the equivalent variation that will generate the same utility in the initial equilibrium. We start by computing the level of utility in the counterfactual equilibrium.

$$\bar{\tilde{u}}_j' = \sum_i N_i' \frac{w_i'\tau_{ij}'}{\left(p_j^*\right)'} \int_{\left(v_{ij}^*\right)'} \tilde{u}\left(q\left(e^v\right)\right) e^v g_i\left(e^v w_i'\tau_{ij}' / \left(p_j^*\right)'\right) dv$$

Step ii) Compute λ_j using

$$\textstyle\sum_i N_i \displaystyle\int_{v_{ij}^*}^{\infty} \widetilde{u}\left[d\left(\mu(v) - v + \ln p_j^* - \ln\lambda_j\right)\right] g_i\left(\frac{w_i\tau_{ij}}{p_j^*} e^v\right) \frac{w_i\tau_{ij}}{p_j^*} e^v dv = \bar{\tilde{u}}_j'$$

and

$$e_j = \textstyle\sum_i N_i \times w_i\tau_{ij} \times \int_{v_{ij}^*} e^{\mu(v)} \exp\left\{d\left(\mu(v) - v + \ln p_j^* - \ln\lambda_j\right)\right\} g_i\left(e^v \frac{w_i\tau_{ij}}{p_j^*}\right) dv$$

and Step iii) is as before.

4.3 case c) Specifying a demand function with $\beta \geq 0$

In this third case we show how can we compute the equivalent variation when we do not have a utility function prespecified but simply a demand function. Notice that using Shepard's Lemma we can write the changes in the expenditure function as a result of changes in price changes, by taking the derivative of (17),

$$\frac{\partial e_j(\mathbf{p}, \bar{u})}{\partial p} = q\left(p, p_j^*\left(\mathbf{p}_j, \mathbf{e}_j\right), e_j\right) \ ,$$

where notice the dependence of the demand function on the cutoff price, p_j^*, which is implicitly solved from

$$\int p_{ij} q_{ij}\left(p_{ij}, p_j^*\left(\mathbf{p}_j, e_j\right), e_j\right) N_i g_i(z) d(z) = e_j$$

We use this to form a differential equation that will start from $e_j(\mathbf{p}_1, \bar{u})$ and take us all the way to $e_j(\mathbf{p}_1, \bar{u})$ adding up all the changes $\frac{\partial e_j(\mathbf{p}, \bar{u})}{\partial p}$ for each particular price change. We will do this in 5 steps.

Step i) We will be working on the product space. We need to create a mapping from productivities that correspond to each good, to the prices of the good. To do so, we first create a mapping from the relative efficiency space, v, to the prices. Notice that the price of a good with relative efficiency v can be written as

$$p(v; j) = e^{\mu(v) - v} p_j^* \ . \tag{20}$$

This way, since we have already created a v matrix, we can build a corresponding p matrix. The next step, is to construct the corresponding vector of relative efficiencies for the goods in the counterfactual equilibrium. Exploiting the definition $v = \ln p_j^* - \ln w_i \tau_{ij} + \ln z$ we can write

$$\begin{aligned}
v &= \ln p_j^* - \ln w_i \tau_{ij} + \ln z \implies \\
v &= \ln p_j^* - \ln w_i \tau_{ij} + \left(v' + \ln w_i' \tau_{ij}' / p_j^{*\prime}\right) \implies \\
v' &= v + \ln w_i \tau_{ij} / p_j^* - \ln w_i' \tau_{ij}' / p_j^{*\prime}
\end{aligned}$$

and thus the prices of the goods in the counterfactual equilibrium

$$p(v') = e^{\mu(v') - v'} p_j^{*\prime} \ .$$

Step ii) We now have to determine the number of goods corresponding to each v, p. We do that so that we can compute the density of points at a point of the grid v, $N_i g_i\left(e^v \frac{\tau_{ij}}{p_j^*/w_j} \frac{w_i}{w_j}\right) dv$ in the initial equilibrium and $N_i' g_i\left(e^v \frac{\tau_{ij}'}{p_j^{*\prime}/w_j'} \frac{w_i'}{w_j'}\right) dv$ in the final one. Now consider $\Omega_j \equiv \sum_i \sum_v N_i g_i\left(e^v \frac{\tau_{ij}'}{p_j^{*\prime}/w_j'} \frac{w_i'}{w_j'}\right) dv$ where v are all the grid points in our productivity grid and dv is the space across points. We can consider this as the total number of goods available (simply round Ω_j up to the nearest integer and multiply by a constant to adjust the total number of products that you want to have). We will be solving a differential equation for each one of the goods taking in account the entry and exit of goods.

Step iii) After we draw all the Ω goods we can construct.

$$\begin{aligned}
\sum_{\Omega} p(\omega) q\left(p(\omega), p_j^*, e_j\right) &= e_j \implies \\
\sum_{\Omega} p(\omega) q\left(p(\omega), p_j^*\left(\mathbf{p}_j, e_j\right), e_j\right) &= e_j
\end{aligned}$$

Then, for one ω at a time, the p_j^* can be solved as an implicit function of all prices $\mathbf{p}$ and e_j by solving the differential equations

$$\frac{\partial e_j\left(p, \mathbf{p}_{j,\omega'\neq\omega}, \bar{u}\right)}{\partial p} = q_{ij}\left(p, p_j^*\left(p\left(\omega\right), \mathbf{p}_{j,\omega'\neq\omega}, \mathbf{e}_j\right), e_j\right)$$

given the solution of p_j^* given from the budget constraint above rewritten as

$$p\left(\omega\right) q\left(p\left(\omega\right), p_j^*\left(p\left(\omega\right), \mathbf{p}_{j,\omega'\neq\omega}, e_j\right), e_j\right) + \sum_{\Omega/\omega} p\left(\omega'\right) q\left(p\left(\omega'\right), p_j^*\left(p\left(\omega\right), \mathbf{p}_{j,\omega'\neq\omega}, e_j\right), e_j\right) = e_j$$

where for the algorithm $p\left(\omega^{'}\right)$ is a given parameter for all $\omega' \neq \omega$

Step iv) Once we obtain $e_j\left(p_{ij}^{'}, \mathbf{p}_{-p_{ij}^{'}}, \bar{u}\right)$ we proceed iteratively to do this for every price in the price vector each good at a time to and update e_j starting all the way from the "effective" e_j we constructed in step iii) to the counterfactual e_j that corresponds to the new set of prices $e_j\left(\mathbf{p}', \bar{u}\right)$.

5 Appendix A

Current Account Balance

We want to double check that labor market clearing is equivalent to current account balance. We have that current account balance implies

$$\begin{aligned}
\underbrace{\sum_{i\neq j} X_{ij} - \sum_{j\neq i} X_{ji}}_{\text{trade balance}} + \underbrace{\sum_{j\neq i} F_{ji} - \sum_{i\neq j} F_{ij}}_{\text{payments to foreign labor}} &= 0 \iff \\
\sum_j X_{ij} - \sum_j X_{ji} + \sum_j F_{ji} - \sum_j F_{ij} &= 0 \iff \\
\sum_j X_{ij} + \sum_j F_{ji} - \sum_j F_{ij} &= \sum_j X_{ji} = w_i L_i
\end{aligned}$$

From free entry notice that

$$\begin{aligned}
\sum_j \Pi_{ij} &= N_i w_i f_e \implies \\
\sum_j \left(X_{ij} - K_{ij} - F_{ij}\right) &= N_i w_i f_e \implies
\end{aligned}$$

so that the Current account balance can be rewritten

$$\begin{aligned}
N_i w_i f_e + \sum_j K_{ij} + \sum_j F_{ij} + \sum_j F_{ji} - \sum_j F_{ij} &= w_i L_i \implies \\
N_i w_i f_e + \sum_j K_{ij} + \sum_j F_{ji} &= w_i L_i
\end{aligned}$$

which is equation (**??**) proving that, given free entry, labor market clearing and current account balance are equivalent

CES

We want to write the CES demand in a proper form so that it can be used in (9). We have

$$\begin{aligned} q(p) &= \frac{(p)^{-\sigma}}{P_j^{1-\sigma}} w_j \\ &= e^{-\sigma\left(\ln p - \ln P_j^{\frac{\sigma-1}{\sigma}}\right)} e^{\ln w_j} \end{aligned}$$

Notice that $p_j^* = \frac{\sigma}{\sigma-1}\frac{w_i \tau_{ij}}{z_{ij}^*}$ so thatXXX

6 Appendix B: Firm Statistics

We want to generate some firm statistics that will make it simpler to work with the data.

Firm sales distribution

Using the expression for firm quanity, equation (2), and the definition of prices, firm sales can be written as

$$\begin{aligned} y_j\left(e^v, p_j^*/w_j\right) &= L_j e^{\mu(v)-v} p_j^* e^{d(v)} e^{-\beta(\mu(v)-v)} \left(\frac{p_j^*}{w_j}\right)^{-\beta} \\ &= L_j e^{d(v)} e^{(1-\beta)(\mu(v)-v)} \left(p_j^*\right)^{1-\beta} w_j^{\beta}. \end{aligned}$$

The next is to relate firm relative productivity v to the cutoff v_j^* and ultimately to their actual fraction in the firm productivity. Notice that we have the CDF

$$\Pr = F\left(V \le v | V \ge v_j^*\right)$$

We numerically invert the probability that corresponds to each firm v, to compute the size of the firm as a function of its percentile. In fact a more relevant magnitude is sales divided by mean sales, where mean sales are given by (using also the definition of z_{ij}^*, equation 6)

$$\begin{aligned} \bar{X}_{ij} &= \frac{X_{ij}}{M_{ij}} = \frac{N_i L_j \int_{z_{ij}^*}^{\infty} e^{\mu\left(\ln z - \ln z_{ij}^* + v_{ij}^*\right)} \frac{w_i \tau_{ij}}{z} q\left(\frac{z}{z_{ij}^*} e^{v_j^*}, p_j^*/w_j\right) g_i(z)\, dz}{N_i \int_{z_{ij}^*}^{\infty} g_i(z)\, dz} \\ &= \frac{N_i L_j \int_{v_{ij}^*}^{\infty} e^{\mu(v)} \frac{w_i \tau_{ij}}{z_{ij}^* e^{v-v_{ij}^*}} q\left(e^v, p_j^*/w_j\right) g_i\left(z_{ij}^* e^{v-v_{ij}^*}\right) z_{ij}^* e^{v-v_{ij}^*}\, dv}{N_i \int_{v_{ij}^*}^{\infty} g_i\left(z_{ij}^* e^{v-v_{ij}^*}\right) z_{ij}^* e^{v-v_{ij}^*}\, dv} \\ &= \frac{N_i L_j w_i \tau_{ij} \int_{v_{ij}^*}^{\infty} e^{\mu(v)} q\left(e^v, p_j^*/w_j\right) g_i\left(z_{ij}^* e^{v-v_{ij}^*}\right) dv}{N_i w_i \tau_{ij} \int_{v_{ij}^*}^{\infty} g_i\left(z_{ij}^* e^{v-v_{ij}^*}\right) \frac{e^{v_{ij}^*}}{p_j^*} e^{v-v_{ij}^*}\, dv} \end{aligned}$$

and thus sales divided by mean sales

$$\frac{y_j\left(e^v, p_j^*/w_j\right)}{\bar{X}_{ij}} = \frac{L_j e^{d(v)} e^{(1-\beta)(\mu(v)-v)} \left(p_j^*\right)^{-\beta} w_j^{\beta}}{L_j \int_{v_j^*}^{\infty} e^{\mu(v)-v} q\left(e^v, p_j^*/w_j\right) \frac{g_i\left(z_{ij}^* e^{v-v_{ij}^*}\right) e^{v-v_{ij}^*}}{\int_{v_j^*}^{\infty} g_i\left(z_{ij}^* e^{v-v_{ij}^*}\right) e^{v-v_{ij}^*}\, dv} dv}$$

and remember

$$q\left(e^{v},p_{j}^{*}/w_{j}\right)=e^{d(v)}e^{-\beta(\mu(v)-v)}\left(\frac{p_{j}^{*}}{w_{j}}\right)^{-\beta}.$$

$$\frac{y_{j}\left(e^{v},p_{j}^{*}/w_{j}\right)}{\bar{X}_{ij}}=\frac{e^{d(v)}e^{(1-\beta)(\mu(v)-v)}}{\int_{v_{j}^{*}}^{\infty}e^{d(v)}e^{(1-\beta)(\mu(v)-v)}\frac{g_{i}\left(z_{ij}^{*}e^{v-v_{ij}^{*}}\right)e^{v-v_{ij}^{*}}}{\int_{v_{j}^{*}}^{\infty}g_{i}\left(z_{ij}^{*}e^{v-v_{ij}^{*}}\right)e^{v-v_{ij}^{*}}dv}dv}$$

Distribution of markups

We now need to plot the distribution of markups. Remember, that for any v we can solve for $\mu(v)$ using the implicit function (5) theorem. Then using the inversion as indicated above, we can compute the fraction of firms between any two productivity bins and plot the graph in BEJK.

XXXmean sales with Pareto

$$\begin{aligned}
\left(e^{v_{ij}^{*}}/p_{j}^{*}\right)^{\theta}L_{j}\left(w_{i}\tau_{ij}\right)^{1+\theta}\int_{v_{j}^{*}}^{\infty}e^{\mu(v)}q\left(e^{v},p_{j}^{*}/w_{j}\right)g_{i}\left(e^{v}\frac{\tau_{ij}}{p_{j}^{*}/w_{j}}\frac{w_{i}}{w_{j}}\right)dv &\implies\\
\left(e^{v_{ij}^{*}}/p_{j}^{*}\right)^{\theta}L_{j}\left(w_{i}\tau_{ij}\right)^{1+\theta}\left(\frac{\tau_{ij}}{p_{j}^{*}/w_{j}}\frac{w_{i}}{w_{j}}\right)^{-1-\theta}\int_{v_{j}^{*}}^{\infty}e^{\mu(v)}q\left(e^{v},p_{j}^{*}/w_{j}\right)g_{i}\left(e^{v}\right)dv &\implies\\
\left(e^{v_{ij}^{*}}\right)^{\theta}L_{j}p_{j}^{*}\int_{v_{ij}^{*}}^{\infty}e^{\mu(v)}q\left(e^{v},p_{j}^{*}/w_{j}\right)g_{i}\left(e^{v}\right)dv &\implies
\end{aligned}$$

XXX

Measured Markups

We want to compute measured markups from firms generated by the model. Consider a firm that sells in two different markets (this logic can be extended to more markets).

$$v=\ln\left(p_{j}^{*}z/w_{i}\tau_{ij}\right)$$

$$z_{ij}^{*}\equiv w_{i}\tau_{ij}e^{v_{ij}^{*}}/p_{j}^{*}$$

$$y_{j}\left(e^{v},p_{j}^{*}/w_{j}\right)=N_{i}L_{j}e^{\mu\left(\ln z-\ln z_{ij}^{*}+v_{ij}^{*}\right)}\frac{w_{i}\tau_{ij}}{z}q\left(\frac{z}{z_{ij}^{*}}e^{v_{ij}^{*}},p_{j}^{*}/w_{j}\right)$$

where

$$\begin{aligned}
q\left(e^{v},p_{j}^{*}/w_{j}\right) &= e^{d(v)}e^{-\beta(\mu(v)-v)}\left(\frac{p_{j}^{*}}{w_{j}}\right)^{-\beta}\\
&= e^{d\left(\ln\frac{p_{j}^{*}z}{w_{i}\tau_{ij}}\right)}e^{-\beta\left(\mu\left(\ln\frac{p_{j}^{*}z}{w_{i}\tau_{ij}}\right)-\ln\frac{p_{j}^{*}z}{w_{i}\tau_{ij}}\right)}\left(\frac{p_{j}^{*}}{w_{j}}\right)^{-\beta}\\
&= e^{d\left(\ln\frac{z}{z_{ij}^{*}}-v_{ij}^{*}\right)}e^{-\beta\left(\mu\left(\ln\frac{z}{z_{ij}^{*}}-\ln v_{ij}^{*}\right)-\ln\frac{z}{z_{ij}^{*}}+v_{ij}^{*}\right)}\left(\frac{p_{j}^{*}}{w_{j}}\right)^{-\beta}
\end{aligned}$$

Where sales can be written as

$$
\begin{aligned}
y_j\left(e^v, p_j^*/w_j\right) &= N_i L_j e^{\mu\left(\ln\frac{z}{z_{ij}^*}-v_{ij}^*\right)} \frac{w_i \tau_{ij}}{z} e^{d\left(\ln\frac{z}{z_{ij}^*}-v_{ij}^*\right)} e^{-\beta\left(\mu\left(\ln\frac{z}{z_{ij}^*}-\ln v_{ij}^*\right)-\ln\frac{z}{z_{ij}^*}+v_{ij}^*\right)} \left(\frac{p_j^*}{w_j}\right)^{-\beta} \\
&= w_j N_i L_j e^{d\left(\ln\frac{z}{z_{ij}^*}-v_{ij}^*\right)} e^{(1-\beta)\left(\mu\left(\ln\frac{z}{z_{ij}^*}-v_{ij}^*\right)-\ln\frac{z}{z_{ij}^*}+v_{ij}^*\right)} \left(\frac{p_j^*}{w_j}\right)^{1-\beta}
\end{aligned}
$$

In the same vain we can write sales in any other market as

$$
y_{j'}\left(e^v, p_{j'}^*/w_{j'}\right) = w_{j'} N_i L_{j'} e^{d\left(\ln\frac{z}{z_{ij}^*}\frac{z_{ij}^*}{z_{ij'}^*}-v_{ij'}^*\right)} e^{(1-\beta)\left(\mu\left(\ln\frac{z}{z_{ij}^*}\frac{z_{ij}^*}{z_{ij'}^*}-v_{ij'}^*\right)-\ln\frac{z}{z_{ij}^*}\frac{z_{ij}^*}{z_{ij'}^*}+v_{ij'}^*\right)} \left(\frac{p_{j'}^*}{w_{j'}}\right)^{1-\beta}
$$

and the ratio $\frac{z}{z_{ij}^*}$ can be directly linked to firm percentile in the domestic country.

Total spending for labor is given by

$$
\begin{aligned}
\tau_{ij}\frac{q\left(e^v, p_j^*/w_j\right)}{z} w_i &= \tau_{ij}\frac{w_i}{z} e^{d\left(\ln\frac{z}{z_{ij}^*}-v_{ij}^*\right)} e^{-\beta\left(\mu\left(\ln\frac{z}{z_{ij}^*}-\ln v_{ij}^*\right)-\ln\frac{z}{z_{ij}^*}+v_{ij}^*\right)} \left(\frac{p_j^*}{w_j}\right)^{-\beta} \\
&= \tau_{ij} w_i \frac{z_{ij}^*}{z} \frac{1}{w_i \tau_{ij} e^{v_{ij}^*}/p_j^*} e^{d\left(\ln\frac{z}{z_{ij}^*}-v_{ij}^*\right)} e^{-\beta\left(\mu\left(\ln\frac{z}{z_{ij}^*}-\ln v_{ij}^*\right)-\ln\frac{z}{z_{ij}^*}+v_{ij}^*\right)} \left(\frac{p_j^*}{w_j}\right)^{-\beta} \\
&= w_j e^{-\ln\frac{z}{z_{ij}^*}+v_{ij}^*} e^{d\left(\ln\frac{z}{z_{ij}^*}-v_{ij}^*\right)} e^{-\beta\left(\mu\left(\ln\frac{z}{z_{ij}^*}-\ln v_{ij}^*\right)-\ln\frac{z}{z_{ij}^*}+v_{ij}^*\right)} \left(\frac{p_j^*}{w_j}\right)^{1-\beta}
\end{aligned}
$$

and again if we consider a third country we can write

$$
\tau_{ij'}\frac{q\left(e^v, p_{j'}^*/w_{j'}\right)}{z} w_i = w_{j'} e^{-\ln\frac{z}{z_{ij}^*}\frac{z_{ij}^*}{z_{ij'}^*}+v_{ij'}^*} e^{d\left(\ln\frac{z}{z_{ij}^*}\frac{z_{ij}^*}{z_{ij'}^*}-v_{ij}^*\right)} e^{-\beta\left(\mu\left(\ln\frac{z}{z_{ij}^*}\frac{z_{ij}^*}{z_{ij'}^*}-\ln v_{ij'}^*\right)-\ln\frac{z}{z_{ij}^*}\frac{z_{ij}^*}{z_{ij'}^*}+v_{ij'}^*\right)} \left(\frac{p_{j'}^*}{w_{j'}}\right)^{1-\beta}
$$