Staggered Adjustment and Trade Dynamics*
(preliminary and incomplete)

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Abstract

We allow for the possibility that consumers are slow to adjust on the extensive margin by adding one new parameter to a Ricardian model of international trade. This extension provides a simple and parsimonious way to capture the low elasticity of trade to relative cost in the short run (as needed in open-economy macro models), together with the high elasticity in the long run (as needed trade models). We use the model to assess, at different horizons, how the global economy adjusts to rebalancing trade.

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1 Introduction

The quantitative analysis of international trade has advanced considerably in the last decade, yet it has made little contact with quantitative work in open-economy macroeconomics. A major tension between these two lines of work is their focus on very different frequencies. A concern of trade economists has been the long-term effects of various policies (such as protection), where change is typically infrequent. For such analysis the evidence suggests that trade responds quite elastically to changes in relative costs. A concern of open-economy macroeconomists has been the short-term effects of changes in relative costs due to swings in exchange rates. For this analysis the evidence points to an inelastic response of trade.

We seek to resolve this tension by developing a framework that encompasses the Armington (1969) model, commonly used in the open-economy macro literature (see for example Backus, Kehoe, and Kydland (1994), Heathcote and Perri (2002)), in which adjustment occurs at the intensive margin, and the multi-country Ricardian model of Eaton and Kortum (2002), commonly used in international trade (see for example Alvarez and Lucas (2007) and Caliendo and Parro (2010)), in which adjustment is at the extensive margin.

The framework we propose adds only one parameter to the model in Eaton and Kortum (2002) (henceforth EK). This parameter governs the speed with which buyers switch to lower-cost sources of supply as prices change. This source-adjustment friction is reminiscent of the price-adjustment friction in the Calvo (1983) model. At one extreme, if purchasers never switch sources, the framework reduces to an Armington model, in which a very low elasticity is appropriate. At the other extreme, if purchasers react immediately, we are back to the Ricardian model with a high elasticity. In between we capture a dynamic adjustment to changes in relative costs much like in the J-curve literature (see Bahmani-Oskooee and Ratha (2004)).

Our analysis is inspired by Ruhl (2005) and also has much in common with Drozd and Nosal (2011). Both incorporate more sophisticated dynamic optimizing behavior on the part of producers. In contrast, our suppliers are perfect competitors with no reason to look past the present. We introduce dynamic adjustment on the part of consumers. While we do so in a rather mechanical way, this simplicity keeps us within the tradition of existing static trade theory, allowing us to handle seamlessly bilateral trade among any number of heterogeneous countries separated by trade barriers.

The paper proceeds as follows. In Section 2, we describe our framework. In Section 3, we derive its implications for welfare and the terms of trade. In Section 4 we describe how to solve the model numerically, and then perform several exercises to reveal its properties. Finally, we develop a method of estimating the model in Section 5.

2 Model

Our point of departure is the EK trade model. The key elements are a world of I countries, indexed by n as importers and by i as exporters. There are a continuum of goods $j \in [0, 1]$ common to all countries. Each country has a continuum L_i of workers, who are also the only final consumers.

We adapt the dynamic model of technological change in Eaton and Kortum (2011). New techniques for producing any good j arrive independently to different countries. By date t the number of techniques in country i for producing good j with efficiency greater than z is distributed Poisson with parameter $T_i^t z^{-\theta}$. Here T_i^t reflects the accumulated arrival of techniques in i and $\theta > 0$ governs (inversely) the size of the efficiency gaps between techniques.

Preferences are standard constant elasticity of substitution (CES) with an elasticity $\sigma \geq 0$. Markets are perfectly competitive. We make the standard assumption of "iceberg" trade barriers: to deliver one unit of a good to country n requires shipping $d_{ni}^t \geq 1$ units in country i. We normalize $d_{ii}^t = 1$.

To make the analysis more consistent with the open-economy macro literature, we introduce nationwide productivity shocks A_i^t. The effective cost of labor in country i, with a wage w_i^t is thus w_i^t / A_i^t. Combining these terms, the unit cost in n of a good j produced in country i with a technique of efficiency z is

$$p_{ni}^t(j) = \frac{c_{ni}^t}{z},$$

where

$$c_{ni}^t = w_i^t d_{ni}^t / A_i^t. \tag{1}$$

Hence prices are affected by shocks to wages and productivity, which are nationwide, shocks to trade costs, which apply to individual exporter-importer pairs, and potentially by shocks to technology z which are specific to a particular good in a particular country.

Before turning to what is new in our model of staggered adjustment, we summarize useful results from the basic model in which consumers adjust immediately.

2.1 Full Adjustment

In the basic EK model (prior to introducing the adjustment friction) consumers in country n purchase good j produced with the low-cost technique.

If good j is purchased from country i it must be produced with the most efficient technique available there, whose efficiency we denote by $z_i^{a,t}(j)$ (here and in what follows, the a superscript denotes full adjustment). To derive its distribution, evaluate the Poisson distribution (for the arrival of techniques of efficiency greater than z) at 0, which yields the probability that no technique has arrived by date t to produce j with efficiency above z:

$$\Pr[z_i^{a,t}(j) \leq z] = e^{-T_i^t z^{-\theta}}, \tag{2}$$

the Fréchet distribution.

With full adjustment, the unit cost of a consumer in n buying good j from i is

$$p_{ni}^{a,t}(j) = \frac{c_{ni}^t}{z_i^{a,t}(j)}.$$

But, consumers will look for the lowest price across all potential source countries, resulting in a price

$$p_n^{a,t}(j) = \min_i \left\{ p_{ni}^{a,t}(j) \right\}.$$

The distribution of these prices is

$$G_n^{a,t}(p) = \Pr[p_n^{a,t}(j) \leq p] = 1 - e^{-\Phi_n^t p^\theta}, \tag{3}$$

where

$$\Phi_n^t = \sum_{k=1}^{I} T_k^t (c_{nk}^t)^{-\theta}. \tag{4}$$

The resulting CES price index is

$$P_n^{a,t} = \gamma \left(\Phi_n^t\right)^{-1/\theta}, \tag{5}$$

where γ is a parameter depending only on θ and σ.[1]

Also, in the full-adjustment model, the fraction of goods j that country n buys from i at date t is given by

$$\bar{\pi}_{ni}^{a,t} = \frac{T_i^t (c_{ni}^t)^{-\theta}}{\Phi_n^t}. \tag{6}$$

A property of the price distribution (3) is that it also applies to the subset of goods that country n purchases from any particular country i. Since the distribution of the lowest price

[1] To derive these results, we begin with the distribution of unit costs from a particular source i:

$$\begin{aligned} F_{ni}^t(p) &= \Pr\left[p_{ni}^{a,t}(j) \leq p\right] \\ &= \Pr\left[z_i^{a,t}(j) \geq \frac{c_{ni}^t}{p}\right] \\ &= 1 - e^{-T_i^t (c_{ni}^t)^{-\theta} p^\theta}. \end{aligned}$$

The distribution of prices in country n is therefore:

$$\begin{aligned} G_n^{a,t}(p) &= 1 - \Pr\left[\min_i \left\{p_{ni}^{a,t}(j)\right\} \geq p\right] \\ &= 1 - \prod_i \left[1 - F_{ni}^t(p)\right] \\ &= 1 - e^{-\Phi_n^t p^\theta}. \end{aligned}$$

The CES price index can then be derived by integrating over this price distribution:

$$\begin{aligned} \left(P_n^{a,t}\right)^{1-\sigma} &= \int_0^\infty p^{1-\sigma} dG_n^{a,t}(p) \\ &= \Gamma\left(\frac{\theta - (\sigma - 1)}{\theta}\right) \left(\Phi_n^t\right)^{(\sigma-1)/\theta}. \end{aligned}$$

in n is the same regardless of the source, country n's average spending per good is the same on goods from each source i. Thus, the trade share $\pi_{ni}^{a,t}$ (the share of n's spending devoted to goods produced in i) is exactly equal to $\bar{\pi}_{ni}^{a,t}$ (throughout, π denotes a spending share and $\bar{\pi}$ a share of goods), a result that no longer holds when consumers are slow to adjust.[2]

An implication is that, in the full adjustment model, the elasticity of the trade share with respect to a change in the wage in country i (relative to other wages) is

$$\frac{\partial \pi_{ni}^{a,t}}{\partial w_i^t} \frac{w_i^t}{\pi_{ni}^{a,t}} = -\theta(1 - \pi_{ni}^{a,t}).$$

Note that the elasticity of substitution σ plays no role. What matters is θ. Estimates vary, but a consensus is beginning to emerge from cross-sectional analysis of price and productivity data putting θ in the neighborhood of 4.[3]

As constructed, the model should apply equally to movements over time as well as to differences across country. A problem is that evidence on the effects of changes in the real

[2]We can use the results on price distributions described in the previous footnote to derive the probability that country i is the low cost supplier of a particular good to country n:

$$\begin{aligned} \bar{\pi}_{ni}^{a,t} &= \Pr\left[\min_{k \neq i}\left\{p_{nk}^{a,t}(j)\right\} \geq p_{ni}^{a,t}(j)\right] \\ &= \int_0^\infty e^{-\sum_{k\neq i} T_k^t \left(c_{nk}^t\right)^{-\theta} p^\theta} dF_{ni}^t(p) \\ &= \frac{T_i^t\left(c_{ni}^t\right)^{-\theta}}{\Phi_n^t} \int_0^\infty e^{-\Phi_n^t p^\theta} \Phi_n^t \theta p^{\theta-1} dp \\ &= \frac{T_i^t\left(c_{ni}^t\right)^{-\theta}}{\Phi_n^t}. \end{aligned}$$

Since there are a unit continuum of goods, $\bar{\pi}_{ni}^{a,t}$ is also the fraction of goods that n buys from i. We can also show that the distribution of the minimum price is the same conditional on the source country. Consider the probability that $p_{ni}^{a,t}(j)$ is less than p given that it is also less than $\min_{k\neq i}\left\{p_{nk}^{a,t}(j)\right\}$:

$$\begin{aligned} G_{ni}^{a,t}(p) &= \frac{\int_0^p e^{-\sum_{k\neq i} T_k^t \left(c_{nk}^t\right)^{-\theta} x^\theta} dF_{ni}^t(x)}{\int_0^\infty e^{-\sum_{k\neq i} T_k^t \left(c_{nk}^t\right)^{-\theta} x^\theta} dF_{ni}^t(x)} \\ &= \frac{= \frac{T_i^t\left(c_{ni}^t\right)^{-\theta}}{\Phi_n^t} \int_0^p e^{-\Phi_n^t x^\theta} \Phi_n^t \theta x^{\theta-1} dx}{= \frac{T_i^t\left(c_{ni}^t\right)^{-\theta}}{\Phi_n^t}} \\ &= G_n^{a,t}(p). \end{aligned}$$

It follows that $\pi_{ni}^{a,t} = \bar{\pi}_{ni}^{a,t}$.

[3]See Simonovska and Waugh (2009) who revisit the EK analysis of price data.

exchange rate (which are equivalent to wage changes in the model) over time, say at an annual frequency, is that they generate a much smaller response in trade shares. Our goal is to extend the model to reconcile a larger cross-sectional elasticity with a much smaller elasticity at an annual frequency in the time series.[4]

2.2 Staggered Adjustment

Our reconciliation introduces slow adjustment in the source from which consumers buy their goods. Consider an individual consumer's purchase of a particular product j. Each period she switches, with probability λ, her source of supply, looking across all potential suppliers of this good and choosing the one with lowest cost, as in the basic model. With probability $1-\lambda$, however, she buys the good from the same source of supply she bought from in the previous period, regardless of the lowest cost source. Various shocks may change all the prices the consumer faces, and she adjusts her purchases accordingly, but does not abandon this particular source for good j.

Since a consumer is locked into a source for an indefinite period, her choice of a new supplier may depend not only on the current price but her expectations of future prices from that source. We assume that consumers do not observe the location of the supplier, however, but only the price p. In this model the probability that a supplier is from country i is π_{ni} whatever p, so that p reveals no information about the source country i. Not knowing i, but only the set of available prices, the best forecast of the lowest price in any future period is the lowest price now.

This adjustment friction captures, in a mechanical way, the habitual nature of consumers. It is consistent with evidence, reported in the marketing literature, that half of a suppliers' customers remain loyal to it for at least five years (see Reichheld and Teal (1996)).[5]

[4] A related issue is that trade appears much more responsive to permanent changes in trade costs, such as the formation of a free trade agreement, than it is to short-run fluctuations in exchange rates. Ruhl (2005) and Kehoe and Ruhl (2008) reconcile the difference by assuming that individual exporters face a sunk cost of entering a foreign market which they won't bother to incur if they think a drop in trade costs is temporary. We pursue a different strategy, as theirs requires a complex, forward looking decision which would prove intractable, at least at this point, in quantitatively modeling bilateral trade across a large number of countries.

[5] We assume here that the switching rate λ is exogenous. Alternatively, we could allow the consumer to pay an increasing marginal cost to make the search intensity higher. This increasing marginal cost of search

There are a continuum of consumers each buying a continuum of products. We assume that the opportunity to switch is sufficiently independent across consumers and goods that, in the aggregate, a fraction λ of buyer-seller relationships are subject to switching. For ease of exposition we describe buyer behavior in terms of a representative consumer who switches sources independently across goods.

These assumptions deliver the following dynamics of where goods are purchased. Denote by $\bar{\pi}_{ni}^{t}$ the fraction of goods that n buys from i in period t. Denote by $\bar{\pi}_{ni}^{a,t}$ the fraction of goods that n buys from i among the goods for which n is choosing the low-cost supplier in period t, which is the same as the trade share $\pi_{ni}^{a,t}$ in EK and given by (6). The fraction $\bar{\pi}_{ni}^{t}$ thus evolves according to

$$\bar{\pi}_{ni}^{t} = \lambda\bar{\pi}_{ni}^{a,t} + (1-\lambda)\,\bar{\pi}_{ni}^{t-1}. \tag{7}$$

2.3 Price Distributions

What do these assumptions imply about the distribution of prices $G_{n}^{t}(p)$ that the consumer in n faces at date t? Let's start just with those that she buys from source i, calling their distribution $G_{ni}^{t}(p)$. This distribution evolves from the distribution at date $t-1$ as follows. Goods that she continues to buy from country i because she has not adjusted her source change price by a factor $\widehat{c}_{ni}^{t} = c_{ni}^{t}/c_{ni}^{t-1}$. (Throughout the paper we represent the ratio between the value of a variable in the current period x^{t} and the previous period x^{t-1} as $\widehat{x}^{t} = x^{t}/x^{t-1}$.) For those goods she buys from country i because she just switched to a lowest-cost source there, the distribution of prices $G_{n}^{a,t}(p)$ is as in EK and given by (3), the same for each source i.

Combining these effects we can write $G_{ni}^{t}(p)$ as a weighted average of these distributions, with weights reflecting the share of goods that adjusted

$$G_{ni}^{t}\left(p\right) = \lambda\frac{\bar{\pi}_{ni}^{a,t}}{\bar{\pi}_{ni}^{t}}G_{n}^{a,t}\left(p\right) + (1-\lambda)\,\frac{\bar{\pi}_{ni}^{t-1}}{\bar{\pi}_{ni}^{t}}G_{ni}^{t-1}\left(p/\widehat{c}_{ni}^{t}\right). \tag{8}$$

While different consumers may buy from producers selling at different prices, all consumers

intensity is the flip side of the increasing marginal cost of search for firms in Arkolakis (2010).

in a given country face the same distribution of prices.

2.4 Price Indices and Trade Shares

Symmetric CES preferences imply that a consumer in country n observing a price p for a good will spend on it an amount

$$x_n^t(p) = \left(\frac{p}{P_n^t}\right)^{-(\sigma-1)} X_n^t, \tag{9}$$

where X_n^t is her total spending and P_n^t is the Dixit-Stiglitz price index, which we now derive.

We can aggregate up purchases of individual goods from i to compute aggregate bilateral imports, X_{ni}^t, as

$$X_{ni}^t = \bar{\pi}_{ni}^t \int_0^\infty x_n^t(p)\, dG_{ni}^t(p). \tag{10}$$

Substituting (9), integrating, and dividing by X_n^t, the fraction of n's spending devoted to goods from country i is

$$\pi_{ni}^t = \bar{\pi}_{ni}^t \left(\frac{P_{ni}^t}{P_n^t}\right)^{-(\sigma-1)}, \tag{11}$$

where

$$P_{ni}^t = \left[\int_0^\infty p^{-(\sigma-1)} dG_{ni}^t(p)\right]^{-1/(\sigma-1)} \tag{12}$$

is the price index of goods that n purchases from i. Using (8) and integrating, a recursive expression for the source-destination specific price index emerges:

$$\left(P_{ni}^t\right)^{-(\sigma-1)} = \frac{\lambda \bar{\pi}_{ni}^{a,t}}{\bar{\pi}_{ni}^t}\left[\gamma\left(\Phi_n^t\right)^{-1/\theta}\right]^{-(\sigma-1)} + \frac{(1-\lambda)\,\bar{\pi}_{ni}^{t-1}}{\bar{\pi}_{ni}^t}\left(\hat{c}_{ni}^t\right)^{-(\sigma-1)}\left(P_{ni}^{t-1}\right)^{-(\sigma-1)}. \tag{13}$$

Weighting each $P_{ni}^{-(\sigma-1)}$ by $\bar{\pi}_{ni}^t$ and summing across i gives us a recursive expression for the overall price index

$$\left(P_n^t\right)^{-(\sigma-1)} = \lambda\left[\gamma\left(\Phi_n^t\right)^{-1/\theta}\right]^{-(\sigma-1)} + (1-\lambda)\sum_{i=1}^{I} \bar{\pi}_{ni}^{t-1}\left(\hat{c}_{ni}^t\right)^{-(\sigma-1)}\left(P_{ni}^{t-1}\right)^{-(\sigma-1)}, \tag{14}$$

a weighted average of the price index from the simple EK model (5), which applies to goods that are adjusted, and updated price indices for each source for the goods that are not adjusted.

2.5 Trade Share Dynamics

We will take the model to data on bilateral trade π^t_{ni} representing what each country spends on goods from each other country, as opposed to the share of goods purchased from them $\overline{\pi}^t_{ni}$.[6] We now derive what the model implies for the dynamics of π^t_{ni}. We define an auxiliary variable

$$\nu^t_{ni} \equiv \pi^t_{ni} \left[\frac{P^t_n}{\gamma \left(\Phi^t_n\right)^{-1/\theta}} \right]^{-(\sigma-1)}, \tag{15}$$

which adjusts the actual trade share by the ratio of the actual price index to the one that would prevail under perfect adjustment.[7] Adding it up over i we obtain

$$\nu^t_n \equiv \sum_k \nu^t_{nk} = \left[\frac{P^t_n}{\gamma \left(\Phi^t_n\right)^{-1/\theta}} \right]^{-(\sigma-1)}. \tag{16}$$

so that

$$\pi^t_{ni} = \nu^t_{ni} / \nu^t_n. \tag{17}$$

To derive a dynamic equation for ν^t_{ni} note that substituting (11) into (15) gives

$$\nu^t_{ni} = \overline{\pi}^t_{ni} \left(\frac{P^t_{ni}}{\gamma \left(\Phi^t_n\right)^{-1/\theta}} \right)^{-(\sigma-1)}. \tag{18}$$

Multiplying both sides of the dynamic price equation (13) by $\overline{\pi}^t_{ni} \left[\gamma \left(\Phi^t_n\right)^{-1/\theta}\right]^{(\sigma-1)}$ and exploiting (18) we get

$$\nu^t_{ni} = \lambda \overline{\pi}^{a,t}_{ni} + (1-\lambda) \left(\hat{c}^t_{ni}\right)^{-(\sigma-1)} \left(\widehat{\Phi}^t_n\right)^{-(\sigma-1)/\theta} \nu^{t-1}_{ni}. \tag{19}$$

[6] A mentioned above, in the static EK model, and for goods that are adjusted here, the two shares correspond since the distribution of prices is the same for goods regardless of source. For goods that are not adjusted, however, differences in the $\overline{c}^t_{ni}$ across i will make the price differences vary across sources.

[7] The ratio $P^t_n / \left[\gamma \left(\Phi^t_n\right)^{-1/\theta}\right]$ represents the welfare loss from a consumer's inability to adopt the frontier technologies instantaneously. Since $\gamma \left(\Phi^t_n\right)^{-1/\theta}$ represents the price index conditional on buying from only the lowest cost source, it must be that $P^t_n \geq \gamma \left(\Phi^t_n\right)^{-1/\theta}$. A country subject to frequent fluctuations of its exchange rate might face a much higher ratio P^t_n to $\gamma \left(\Phi^t_n\right)^{-1/\theta}$ (more so for low values of λ).

Note that the fraction of goods purchased from country i, among all goods whose source has adjusted, evolves according to

$$\hat{\bar{\pi}}_{ni}^{a,t} = \frac{\bar{\pi}_{ni}^{a,t}}{\bar{\pi}_{ni}^{a,t-1}} = \frac{\hat{T}_i^t \left(\hat{c}_{ni}^t\right)^{-\theta}}{\hat{\Phi}_n^t}. \tag{20}$$

Substituting this expression into (19), we obtain

$$\nu_{ni}^t = \lambda \bar{\pi}_{ni}^{a,t} + (1-\lambda) \left(\frac{\hat{\bar{\pi}}_{ni}^{a,t}}{\hat{T}_i^t} \right)^{\frac{\sigma-1}{\theta}} \nu_{ni}^{t-1}. \tag{21}$$

Given initial conditions, ν_{ni}^0 and $\bar{\pi}_{ni}^{a,0}$, we can simulate trade shares using this equation together with (17). The $\bar{\pi}_{ni}^{a,t}$ themselves evolve with changes in wages, productivity, and trade costs each period, according to:

$$\hat{\bar{\pi}}_{ni}^{a,t} = \frac{\hat{Z}_i^t \left(\hat{w}_i^t \hat{d}_{ni}^t \right)^{-\theta}}{\sum_k \bar{\pi}_{ni}^{a,t-1} \hat{Z}_k^t \left(\hat{w}_k^t \hat{d}_{nk}^t \right)^{-\theta}}, \tag{22}$$

where

$$Z_i^t = T_i^t \left(A_i^t\right)^{\theta}, \tag{23}$$

combines the two productivity terms as they matter for the full-adjustment trade shares.

How do trade shares π_{ni}^t respond to a change in the wage in some country i? The effects are more complicated than in the static model. Calculating the elasticity of the trade share with respect to w_i^t, and evaluating it at a static steady state in which all consumers are purchasing from low-cost suppliers, we get

$$\frac{\partial \pi_{ni}^t}{\partial w_i} \frac{w_i}{\pi_{ni}^t} = -\left[\lambda\theta + (1-\lambda)(\sigma-1)\right] (1-\pi_{ni}^t), \tag{24}$$

a weighted average of the elasticity with perfect adjustment and with no adjustment.

2.6 Equilibrium

In principle our model of slow adjustment could be embedded in a stochastic general equilibrium model in which trade deficits are determined by intertemporal choices. Here we simply show how it fits into a sequence of static equilibria, given a sequence of trade deficits D_n^t.

Each period t the labor-market equilibrium conditions are

$$w_i^t L_i^t = \sum_{n=1}^{I} \pi_{ni}^t \left(w_n^t L_n^t + D_n^t \right). \tag{25}$$

We can solve this system of equations for equilibrium wages w_i^t at date t, given labor forces L_i^t, deficits D_n^t, and any variables dated $t-1$ that enter π_{ni}^t using (17), (21), and (22), all into (25). Note that w_i^t enters (25) not only directly but through π_{ni}^t.

3 Implications

Having laid out the equilibrium of the model, we now turn to some key implications.

3.1 Real Wage

We first derive a measure of welfare (the real wage) which generalizes the expressions in EK and Arkolakis, Costinot, and Rodríguez-Clare (2012). Noting that $\bar{\pi}_{ni}^{a,t} = \pi_{ni}^{a,t}$, we can substitute (1) into (6), as it applies to purchases from domestic suppliers, to get

$$\bar{\pi}_{nn}^{a,t} = \frac{Z_n^t (w_n^t)^{-\theta}}{\Phi_n^t}.$$

We can then use (16) to replace Φ_n^t with P_n^t:

$$\bar{\pi}_{nn}^{a,t} = \gamma^{-\theta} Z_n^t \left(\frac{w_n^t}{P_n^t} \right)^{-\theta} \left(\nu_n^t \right)^{\theta/(\sigma-1)},$$

which can be rearranged to get an expression for the real wage:

$$\frac{w_n^t}{P_n^t} = \gamma^{-1} \left(Z_n^t \right)^{1/\theta} \left(\nu_n^t \right)^{1/(\sigma-1)} \left(\bar{\pi}_{nn}^{a,t} \right)^{-1/\theta}. \tag{26}$$

The first term (following the constant γ) captures the positive effect of productivity on the real wage, the second term captures the negative effect of the price level being above its full adjustment level, and the last term captures the gains from trade, which are larger when purchases from the domestic market are lower. This expression is not as convenient as its full-adjustment analog, however, since $\bar{\pi}_{nn}^{a,t}$ is not directly observable.

The new term in this expression, that appears due to the slow adjustment of consumers, is ν_n^t. It can be decomposed into the deviation of the source-specific price levels P_{ni}^t from their full-adjustment level, weighted by the share of goods purchased from i:

$$\nu_n^t = \sum_{k=1}^{I} \bar{\pi}_{ni}^t \left(\frac{P_{ni}^t}{\gamma \left(\Phi_n^t\right)^{-1/\theta}} \right)^{-(\sigma-1)}.$$

3.2 Terms of Trade

We will derive a formula for the terms of trade based on formulas for real bilateral import and export price indices as well as the relative price indices between a pair of countries, which can be interpreted as the real exchange rate.

Rearranging expression (11) gives a real bilateral importing price index of n from i:

$$\frac{P_{ni}^t}{P_n^t} = \left(\frac{\pi_{ni}^t}{\bar{\pi}_{ni}^t} \right)^{-1/(\sigma-1)}. \tag{27}$$

Notice that if $\lambda = 0$, so that $\bar{\pi}_{ni}^t$ is constant, import prices depend on current trade shares only. At the other extreme, if $\lambda = 1$, then $\bar{\pi}_{ni}^t = \pi_{ni}^t$ and the model implies that import price indices do not vary by source.

Next, we turn to the real exchange rate, P_i^t / P_n^t. Using the definition of ν_n^t from equation (16):

$$\begin{aligned} \frac{P_i^t}{P_n^t} &= \left(\frac{\nu_i^t}{\nu_n^t} \right)^{-1/(\sigma-1)} \left(\frac{\Phi_n^t}{\Phi_i^t} \right)^{1/\theta} \\ &= \left(\frac{\nu_i^t}{\nu_n^t} \right)^{-1/(\sigma-1)} \left(\frac{\left(d_{ni}^t\right)^{\theta} \bar{\pi}_{ni}^{a,t}}{\bar{\pi}_{ii}^{a,t}} \right)^{-1/\theta}. \end{aligned} \tag{28}$$

The first term captures the deviation of the price level in country i from its full-adjustment level (all relative to country n). The second term (on the second line) is high if adjusting consumers in country i purchase a lot from country n (relative to what consumers in country n would buy from producers in n), indicating that country n is a low-cost source. The last term captures the trade cost to i of taking advantage of low prices in country n.

We can also define a real bilateral exporter price index of n to i as $P^t_{in|n}/P^t_n$, where $P^t_{in|n} = P^t_{in}/d^t_{in}$ is the price index for n's exports to i. Thus, we have:

$$\frac{P^t_{in|n}}{P^t_n} \equiv \frac{P^t_{in}}{d^t_{in}}/P^t_n = \frac{P^t_{in}}{P^t_i}\frac{1}{d^t_{in}}\frac{P^t_i}{P^t_n}. \tag{29}$$

Combining the results from above, we can write the bilateral exporter price index as

$$\frac{P^t_{in|n}}{P^t_n} = \left(\frac{\pi^t_{in}}{\bar{\pi}^t_{in}}\right)^{-1/(\sigma-1)} \left(\frac{\nu^t_n}{\nu^t_i}\right)^{1/(\sigma-1)} \left(\frac{\bar{\pi}^{a,t}_{in}}{\bar{\pi}^{a,t}_{nn}}\right)^{1/\theta}. \tag{30}$$

These expressions allow us to generate predictions about the terms of trade from our simulations. In particular, we can express the terms of trade for country n, $\tilde{P}^t_n$, as the ratio of the weighted sum of the exporter price and import price indices

$$\tilde{P}^t_n = \frac{\sum_{i\neq n} \bar{\pi}^t_{in} \frac{P^t_{in|n}}{P^t_n}}{\sum_{k\neq n} \bar{\pi}^t_{kn}} \Bigg/ \frac{\sum_{i\neq n} \bar{\pi}^t_{ni} \frac{P^t_{ni}}{P^t_n}}{\sum_{k\neq n} \bar{\pi}^t_{nk}} \tag{31}$$

It is worth pointing out a key difference of our setup with a simple Armington setup, which is the workhorse framework in the international real business cycle literature (see Backus, Kehoe, and Kydland (1994)). In that model $P^t_{in|n} = P^t_{nn}$ whereas in our setup that relationship breaks down since only a subset of the goods sold domestically are exported. The implication for the prices arising from the interaction of this prediction with the staggered adjustment model, $\lambda > 0$, will be discussed in the simulation section.

4 Simulations

To simulate the model it is helpful to express all the equations in terms of changes. In particular, the labor market equilibrium condition becomes:

$$\hat{w}_i^t \hat{L}_i^t Y_i^{t-1} = \sum_{n=1}^{I} \frac{\hat{\nu}_{ni}^t \nu_{ni}^{t-1}}{\sum_{k=1}^{I} \hat{\nu}_{nk}^t \nu_{nk}^{t-1}} \left(\hat{w}_n^t \hat{L}_n^t Y_n^{t-1} + D_n^t \right), \tag{32}$$

where $Y_i^{t-1} = w_i^{t-1} L_i^{t-1}$. We can also express (21) more conveniently for simulation as

$$\hat{\nu}_{ni}^t = \lambda \hat{\bar{\pi}}_{ni}^{a,t} \frac{\bar{\pi}_{ni}^{a,t-1}}{\nu_{ni}^{t-1}} + (1-\lambda) \left(\frac{\hat{\bar{\pi}}_{ni}^{a,t}}{\hat{T}_i^t} \right)^{\frac{\sigma-1}{\theta}} \tag{33}$$

The core of the model is equations (32), (33), and, repeated here for convenience, (22):

$$\hat{\bar{\pi}}_{ni}^{a,t} = \frac{\hat{Z}_i^t \left(\hat{w}_i^t \hat{d}_{ni}^t \right)^{-\theta}}{\sum_k \bar{\pi}_{ni}^{a,t-1} \hat{Z}_k^t \left(\hat{w}_k^t \hat{d}_{nk}^t \right)^{-\theta}}.$$

The exogenous forcing variables are (for $t = 1, ..., T$) productivity changes $\hat{Z}_i^t$, technology changes $\hat{T}_i^t$, labor force changes $\hat{L}_i^t$, deficits D_n^t, and trade-cost shocks $\hat{d}_{ni}^t$. The initial conditions are both observables, Y_i^0 and π_{ni}^0, and unobservables $\bar{\pi}_{ni}^{a,0}$ and ν_n^0 (with $\nu_{ni}^0 = \pi_{ni}^0 \nu_n^0$). For simplicity, we assume that the system starts with full adjustment: $\bar{\pi}_{ni}^{a,0} = \pi_{ni}^0$ so that $\nu_n^0 = 1$.

Data and Calibration

We perform two numerical experiments in order to provide more intuition for the implications of the model. First, we document the effects on relative wages, real wages, terms of trade, and trade shares of a one-time permanent adjustment in trade deficits. Second, we feed the model a series of deficit and productivity shocks to evaluate its ability to replicate the elasticity of trade with respect to wages at different frequencies.

In these experiments we consider the manufacturing sectors of 17 countries (plus a Rest of World residual). The data on bilateral trade, production, wages, and prices, measured annually from 1970-2006, are described in the appendix.

In both exercises we treat trade barriers d_{ni}, technologies T_i, and labor forces L_i as constant. The values for deficits, D_i^t, and productivity shocks, $\hat{Z}_i^t$, will be specified differently in the two experiments, as described below. For initial conditions, we assume that in the first year of the simulation consumers have fully adjusted.

For our simulations we also need to specify the three parameters θ, σ, and λ. We set the long-run elasticity $\theta = 4$, consistent with Bernard, Eaton, Jensen, and Kortum (2003), Eaton, Kortum, and Kramarz (2011) and Simonovska and Waugh (2009), but lower than the estimates of Caliendo and Parro (2010) and EK. The elasticity of substitution, σ, is set to 0.5, slightly lower than the values of 0.9 to 1.5 used in the international real business cycle literature (see Heathcote and Perri (2002), Stockman and Tesar (1995), Backus, Kehoe, and Kydland (1994)) but within the range of empirical estimates of the short-run trade elasticity (see for example Reinert and Ronand-Holst (1992) and Blonigen and Wilson (1999)). Since our model can accommodate elasticities of substitution less than 1, we set σ to a low value to allow for the possibility that the value of exports initially declines after a fall in the exporter's relative wage, in the spirit of the J-curve literature. To explore the role of sluggish adjustment we consider a range of values for the adjustment parameter: $\lambda = 0.05$, 0.2, and 1. A switching rate in between the first and second of these values is consistent with what is reported in the marketing literature.[8]

Impulse Responses: Permanent Deficit Reduction

In the first exercise we set the initial trade shares and deficits in the model to their 2006 levels. Figure 1 shows manufacturing trade deficits (scaled by manufacturing gross production) in 2006. Note that Germany, Sweden, and Finland were running large trade surpluses, relative to production, while the Australia, Spain, Greece, Portugal and the United States all had large deficits. Our experiment then exogenously and permanently eliminates these trade deficits (or surpluses) in each of our 18 countries. We calculate the responses of the endogenous variables for each of the different values of λ. In Figures 2-4 we illustrate the impact on nominal wages, w_i^t, real wages, w_i^t/P_i^t, terms of trade, $\tilde{P}_i^t$, and trade shares (for

[8] Reichheld and Teal (1996), summarizing a number of case studies in the marketing literature, report that on average US corporations lose half their customers in five years, which corresponds to a $\lambda = 0.13$ in our model.

country i's top exporting destination), π_{ni}^t, for Austria, Canada, Germany, and the United States. Thus we show the response of a large trade-surplus country and a large trade-deficit country, together with smaller neighbors with more balanced trade.

Figure 2 shows the results for the full-adjustment case with $\lambda = 1$. The results are much like those in Dekle, Eaton, and Kortum (2008). The wage (relative to world GDP) rises in countries whose surplus is removed and falls in countries whose deficit is removed. Quantitatively, this wage effect is small. In terms of the real wage, the effect is miniscule. The reason is that price levels move largely in parallel to wage changes since most countries buy the majority of their goods from domestic producers. Trade shares, on the other hand, respond quite dramatically, by nearly 20 percent.

The response becomes increasingly more dramatic as we consider smaller values of the adjustment parameter λ, as shown in the next two figures. The inability of consumers to adjust their trade shares means that wages are initially very sensitive to changes in deficits as equation (25) implies. The wage response in the short run is many times larger than the long-run response. Furthermore, slow adjustment by consumers implies that the price level in the short run is much higher than in the long run so that the negative effect on the real wage is quite substantial in countries whose deficits are removed. We also see substantial short-run real wage gains in Germany, whose surplus is removed and, to a lesser extent, in Austria. In the long run, both price indices and wages fully adjust and the impact on real wages is minimal, exactly as in the $\lambda = 1$ case.

In the third panel of each figure we illustrate the movements of the terms of trade defined in equation (31) as the ratio of the price of exports to the price of imports. In the short-run because of the inability of the consumer to adjust, the terms of trade mainly follow the movements of relative wages, as is apparent from comparing the first and the third panels. Finally, notice that the large short-run movements in wages are amplified by the low parameter value for $\sigma = 0.5$. Because we set $\sigma < 1$ we sometimes see the trade share swing in opposite directions in the first and second period. The combination of a low λ and a low σ means that trade shares can temporarily increase with an increase in the relative wage (as in Germany) or decrease with a decrease in the relative wage (as in the United States).

The results in 4 indicate an effect of geographic proximity, at least in the short run.

While Austria and Canada display reasonably balanced trade to begin with, the response of Austrian wages is nearly as positive as Germany's while the response of Canadian wages is nearly as negative as for the United States. To examine these cross-country relationships more systematically, in Figure 5 we plot for each country the response of its wage to its initial trade imbalance (relative to production). The red labels indicate the short-run reponse while the blue indicate the long-run response. Three features stand out: (i) the slope is much larger slope in the short run, (ii) the relationship is much tighter in the long run, and (iii) the short-run response of wages in smaller countries appears to be often dictated by the response of their larger neighbors, most notably for Canada (influenced by the United States) and for Denmark (influenced by Germany).

Correlation of Trade and Wages

Next, we look at whether the model can generate data consistent with both long-run and short run estimates of trade elasticities. To do so, we run the model forward subjecting our 18 countries to two types of shocks: (i) the historical deficits D_i^t of the 18 countries over the period 1970-2006 and (ii) independent productivity shocks $\hat{Z}_i^t$ for each country drawn from a lognormal distribution with mean 0.01 and standard deviation 0.01. Starting in 1970 we solve the system of trade share and wage equations (described above) for each year $t = 1970, ..., 2006$.

We perform two different exercises on the model-generated data to give us some sense of how parameters might be identified from moments in actual data. The first considers how trade shares vary with relative wages. We run a simple panel regression of the model-generated trade shares on the model-generated wages (dropping the first five years to move away from our initial conditions in 1970) among 17 countries (we drop Rest-of-World to be in line with regressions we run on the actual data):

$$\ln \frac{\widehat{\pi}_{ni}^t}{\widehat{\pi}_{nn}^t} = \varepsilon_S \ln \frac{\widehat{w}_i^t}{\widehat{w}_n^t} + u_{ni}^t \text{ for all } t, n, i \neq n.$$

The first two rows of Table 1 report the mean (and standard deviation) of the OLS estimate of ε_S across 100 simulations: Notice that the coefficient falls with λ, switching signs between $\lambda = 0.05$ and $\lambda = 0.2$.

Table 1: Short-Run Regression Results

	$\lambda = 0.05$	$\lambda = 0.2$	$\lambda = 1$	Data
mean	0.383	-0.068	-2.07	0.21
std. deviation	0.001	0.044	0.076	0.021
model elasticity	0.275	-0.400	-4.00	

Equation (24) gives the theoretical impact elasticity of a change in wages on relative trade shares. The bottom row of the table reports the impact elasticity implied by the model (using a trade share approaching 0). There are several reasons why the regression coefficient does not correspond to the theoretical impact elasticity. One is that the trade share are actually positive, and vary over time and between pairs of countries. Another is that the theoretical elasticity is only exact at a steady state. Most importantly, only changes in deficits (assumed exogenous here) cause shocks to wages that should affect trade shares according to the impact elasticity. Productivity shocks have an independent effect on trade shares that goes in the opposite direction of their effect through wages, biasing the coefficient upward. More sluggish adjustment delivers a weaker connection between wages and trade shares, more in line with the short-run elasticities estimated in the open-economy macro literature (see, for example, Reinert and Ronand-Holst (1992) and Blonigen and Wilson (1999)).

Doing this exercise using the actual data (over the same period, $1976 - 2006$) yields a regression coefficient of 0.21. Notice that we come close to this coefficient with our model-generated data when $\lambda = 0.05$.

The second exercise suggests how we might nail down the long-run trade elasticity. Caliendo and Parro (2010) estimate a value of θ from data on trade shares and bilateral tariffs. We follow a similar procedure on data simulated from the model. We consider a hypothetical permanent elimination of ROW import tariffs in 1971, from a level of 10% in 1970. We simulate trade shares from the model using the procedure described above, imposing the values of $\hat{d}_{ni}^{1971}$ implied by the tariff reduction (with all other $\hat{d}_{ni}^{t} = 1$). From the simulated trade shares, we construct the Head and Ries (2001) index

$$H_{ni}^{t} = \frac{\pi_{ni}^{t}}{\pi_{nn}^{t}} \frac{\pi_{in}^{t}}{\pi_{ii}^{t}}.$$

The motivation is that if full-adjustment trade shares were used to construct this index then it would equal $(d_{ni}^t d_{in}^t)^{-\theta}$. We calculate how the index changes over the full period given the changes in trade costs implied by the hypothetical elimination of tariffs. Thus, we run the regression

$$\ln\left(\frac{H_{ni}^{2006}}{H_{ni}^{1970}}\right) = \varepsilon_L \ln\left(\hat{d}_{ni}^{1971}\hat{d}_{in}^{1971}\right) + u_{ni} \text{ for all } n, i \le n,$$

noting that in the full-adjustment model $\varepsilon_L = -\theta$. Across 100 simulations of the model with $\lambda = 0.20$, the mean OLS estimate of ε_L is -3.95 with a standard deviation of 0.014. Setting $\lambda = 0.05$ we get a mean estimate of -3.75 with a standard deviation 0.056. Thus, we get a fairly good handle on the true $\theta = 4$, even with a low λ, simply from the negative of this OLS slope coefficient.

5 Estimation

We now turn to a more formal procedure that could be used to estimate the parameters of the model. We consider two different specifications. The first treats productivity change as a random variable, and exploits data on wage changes, acknowledging that wages may change in response to productivity shocks. The second conditions on productivity and wage changes, absorbing a combination of them in source-country time-specific parameters. Both procedures work with the model in changes, as described in Section 4.

There are several elements common to both specifications. We treat changes in trade costs as a random variable, $\varepsilon_{ni}^t = \ln\left(\hat{d}_{ni}^t\right)^{-\theta}$ (with $\hat{d}_{ii}^t = 1$ for all i and t) with mean μ_ε and variance σ_ε^2. Also, to simplify, we restrict technology to grow at the same constant rate across countries, so that $\hat{T}_i^t = g_T$, giving us just one additional parameter.

Finally, in computing the model we assume that the system starts with full adjustment: $\bar{\pi}_{ni}^{a,0} = \pi_{ni}^0$ and $\nu_n^0 = 1$. In future iterations of our procedure we will attempt to relax this assumption.

5.1 Full Specification

Here we treat productivity change as a random variable $\ln \hat{Z}_i^t = \tau_i^t$ with mean μ_τ and variance σ_τ^2. These productivity shocks are assumed to be independent of deficits D_i^t, but we expect them to be correlated with wage changes $\hat{w}_i^t$ via the labor market equilibrium conditions (32). We will deal with this potential correlation by our choice of moment conditions.

Our approach here uses the data on wages and deficits. We will choose parameters so that, given the shocks, our model fits bilateral trade shares. Thus (32) plays no role in the estimation since it will hold as an identity.

Plugging in our specification, (22) becomes

$$\widehat{\bar{\pi}}_{ni}^{a,t} = \frac{\left(\hat{w}_i^t\right)^{-\theta} \exp\left(\tau_i^t + \varepsilon_{ni}^t\right)}{\sum_k \bar{\pi}_{nk}^{a,t-1} \left(\hat{w}_k^t\right)^{-\theta} \exp\left(\tau_k^t + \varepsilon_{nk}^t\right)}$$

and equation (33) becomes

$$\hat{\nu}_{ni}^t = \lambda \frac{\left(\hat{w}_i^t\right)^{-\theta} \exp\left(\tau_i^t + \varepsilon_{ni}^t\right)}{\sum_k \bar{\pi}_{nk}^{a,t-1} \left(\hat{w}_k^t\right)^{-\theta} \exp\left(\tau_k^t + \varepsilon_{nk}^t\right)} \frac{\bar{\pi}_{ni}^{a,t-1}}{\nu_{ni}^{t-1}} + (1-\lambda) \left(\frac{\left(\hat{w}_i^t\right)^{-\theta} \exp\left(\tau_i^t + \varepsilon_{ni}^t\right)}{g_T \sum_k \bar{\pi}_{nk}^{a,t-1} \left(\hat{w}_k^t\right)^{-\theta} \exp\left(\tau_k^t + \varepsilon_{nk}^t\right)} \right)^{(\sigma-1)/\theta}$$

Given initial conditions ν_n^0 and $\bar{\pi}_{ni}^{a,0}$, parameters, and a path of wages, we can iterate forward with these two equations updating $\bar{\pi}_{ni}^{a,t}$ and ν_{ni}^t one period at a time. We can then recover our prediction for trade shares as

$$\pi_{ni}^t = \frac{\nu_{ni}^t}{\sum_k \nu_{nk}^t}.$$

Thus, we can express the whole set of bilateral trade shares implied by the model as $\pi(\tau, \varepsilon; \Theta^1)$, where the parameter vector is

$$\Theta^1 = \left\{\theta, \sigma, \lambda, g_T, \mu_\varepsilon, \mu_\tau, \sigma_\varepsilon^2, \sigma_\tau^2\right\}.$$

With this specification the moment conditions are the sample analogs of the orthogonality between the shocks to productivity and deficits. We denote the stacked moment conditions by $f(\tau, \varepsilon)$.

5.2 Conditional Specification

Define the source-country fixed effect, as in EK, as $S_i^t = Z_i^t (w_i^t)^{-\theta}$ so that $\hat{S}_i^t = \hat{Z}_i^t (\hat{w}_i^t)^{-\theta}$. We are free to normalize by country 1 so that the model can be parameterized by

$$s_i^t = \hat{S}_i^t / \hat{S}_1^t$$

with $s_1^t = 1$. Here, we treat the set of s_i^t's as parameters to be estimated. An advantage of this approach is that we can remain agnostic about what drives w_i^t, since we absorb wage changes and productivity changes in the s_i^t. A cost of this approach is that we lose identification of the parameter θ, and hence of σ as well. Instead we identify the ratio, $\tilde{\theta} = \theta/(\sigma - 1)$.

From (22)

$$\widehat{\bar{\pi}}_{ni}^{a,t} = \frac{s_i^t \exp\left(\varepsilon_{ni}^t\right)}{\sum_k \bar{\pi}_{nk}^{a,t-1} s_k^t \exp\left(\varepsilon_{nk}^t\right)}. \tag{34}$$

Thus, we can write (33) as

$$\hat{\nu}_{ni}^t = \lambda \frac{s_i^t \exp\left(\varepsilon_{ni}^t\right)}{\sum_k \bar{\pi}_{nk}^{a,t-1} s_k^t \exp\left(\varepsilon_{nk}^t\right)} \frac{\bar{\pi}_{ni}^{a,t-1}}{\nu_{ni}^{t-1}} + (1 - \lambda) \left(\frac{s_i^t \exp\left(\varepsilon_{ni}^t\right)}{g_T \sum_k \bar{\pi}_{nk}^{a,t-1} s_k^t \exp\left(\varepsilon_{nk}^t\right)} \right)^{1/\tilde{\theta}}, \tag{35}$$

where, remember, $\tilde{\theta} = \theta/(\sigma - 1)$.

Given initial conditions ν_n^0 and $\bar{\pi}_{ni}^{a,0}$ and parameters (including the s_i^t's), we can iterate forward with these two equations as with the other specification, obtaining the model's predictions for trade shares $\pi(\varepsilon; \Theta^2)$. Here, the parameter vector is

$$\Theta^2 = \left\{ \tilde{\theta}, \lambda, g_T, \mu_\varepsilon, \mu_\tau, \sigma_\varepsilon^2, \sigma_\tau^2, \left\{ s_i^t \right\} \right\}.$$

With this specification the moment conditions are the sample analogs of the trade-cost shocks having mean μ_ε conditional on each source and destination country. We denote the stacked moment conditions by $f(\varepsilon)$.

5.3 Estimation Procedure

Since we fit our model to bilateral trade shares, our econometric setting is similar to Berry, Levinsohn, and Pakes (1995) who fit their model market shares of different automobiles. Their approach is nests an algorithm to invert the model's predictions for market shares to find the shocks that fit the actual shares, given parameter values. We follow the recent approach of Dubé, Fox, and Su (2009), itself an application of Su and Judd (2010), which replaces this nested inversion step by simply imposing the constraint that the model predict actual trade shares as part of the estimation routine. This procedure for estimating Θ employs Mathematical Programming with Equilibrium Constraints (MPEC).

For the first specification, our estimate of Θ^1, given a weighting matrix Ω^1 (say an identity matrix), solves

$$\begin{array}{rc} & \min_{\Theta^1,\tau,\varepsilon} f(\tau,\varepsilon)'\Omega^1 f(\tau,\varepsilon) \\ \text{subject to:} & \pi(\tau,\varepsilon;\Theta^1) = \Pi \end{array}$$

For the second specification, our estimate of Θ^2, given a weighting matrix Ω^2, solves

$$\begin{array}{rc} & \min_{\Theta^2,\varepsilon} f(\varepsilon)'\Omega^2 f(\varepsilon) \\ \text{subject to:} & \pi(\varepsilon;\Theta^2) = \Pi \end{array}$$

A Appendix

A.1 Data

We assemble annual data from 1970-2006 on bilateral trade, production, prices, and wages for 18 countries:

1	AUS	Australia
2	AUT	Austria
3	BNX	Belgium-Netherland-Luxembourg
4	CAN	Canada
5	DEU	Germany
6	DNK	Denmark
7	ESP	Spain
8	FIN	Finland
9	FRA	France
10	GBR	United Kingdom
11	GRC	Greece
12	ITA	Italy
13	JAP	Japan
14	KOR	Korea
15	PRT	Portugal
16	SWE	Sweden
17	USA	United States
18	ROW	Rest of World

Each of our measures covers the manufacturing sector. We describe them in turn.

A.1.1 Bilateral Trade

We construct our measure of bilateral trade from the World Bank's WITS (World Integrated Trade Solutions) bilateral trade database, available at http://wits.worldbank.org/wits/. All values are in current U.S. Dollars. To approximate the volume of trade in manufactured goods, we aggregate seven SITC 1-digit sectors:

	SITC Category
1	Beverages and Tobacco (1)
2	Crude Materials (2)
3	Animal and Vegetable oils and fats (4)
4	Chemicals (5)
5	Manufactured Goods classified chiefly by material (6)
6	Machinery and Transport Equipment (7)
7	Miscellaneous Manufactured Articles (8)

For a given bilateral flow, WITS provides both the value reported by the importer as well as the value reported by the exporter. We prefer the importer-reported value since the importer has a stronger incentive to measure a trade flow accurately to impose tariffs and duties. If the importer-reported flow is missing we substitute in the value reported by the exporter (we substitute 40 observations in this way). (One observation, Korea importing from Greece in 1970, was not reported by Korea or Greece, so we simply replaced the observation with the 1971 value.) We then checked the timeseries of each bilateral flow to identify suspiciously large swings up or down from one year to the next, which are likely the result of reporting errors. To address this volatility issue, we followed an algorithm. Let $x_{ni}^t(r)$ be country n's imports from i in year t as reported by either the importer $r = 1$, the exporter $r = 2$, or the geometric mean of the two $r = 3$. Let $\hat{x}_{ni}^t(r) = x_{ni}^t(r)/x_{ni}^{t-1}(r)$ be the gross growth rate of the given bilateral trade flow and let

$$y_{ni}^t(r) = \max\left\{\hat{x}_{ni}^t(r), 1/\hat{x}_{ni}^t(r)\right\}$$

be the size of the swing (up or down) in this trade flow. The algorithm proceeds in four steps:

1. If $y_{ni}^t(1) \leq 2$ for all t then use the importer-reported data.

2. Else if $y_{ni}^t(3) \leq 2$ for all t then use the geometric mean of importer and exporter-reported data.

3. Else if $y_{ni}^t(2) \leq 2$ for all t then use the exporter-reported data.

4. Else choose the reporter according to:

$$r = \arg \min_{r' \in \{1,2,3\}} \left\{ \max_{t \in \{1971,\ldots,2006\}} \left\{ y^t_{ni}(r') \right\} \right\},$$

thus minimizing the maximum swing in the timeseries for a given bilateral flow.

Note that the algorithm works on the entire timeseries of a given bilateral trade flow. We end up with the following results. 240 time series satisfy step 1, 23 satisfy step 2, 9 satisfy step 3, and the rest fall into step 4 with 29 yielding $r = 1$, 19 yielding $r = 3$, and 22 yielding $r = 2$.

A.1.2 Production

We measure the value of manufacturing production by scaling up manufacturing value added. The scaling factor is calculated as the ratio of gross manufacturing production to manufacturing value added, averaged over countries and years for which both measures are available. In future revisions, we will consider using the manufacturing production measures provided in the KLEMS database.

A.1.3 Prices

As a measure of manufacturing prices, we use the deflator for manufacturing production provided in the KLEMS dataset.

A.1.4 Wages

As a measure of wages, we use labor compensation in manufacturing divided by the number of manufacturing employees, as provided in the KLEMS dataset.

A.2 Endogenizing λ (preliminary)

We assume that the consumer has to incur a cost $h\left(\lambda^t\right)$ to achieve search intensity λ^t at a given time t. The function $h\left(\lambda^t\right)$ is increasing in λ^t, $h'\left(\lambda^t\right) > 0$, and we also assume that $h'(0) = 0$, $\lim_{\lambda \to 1} h\left(\lambda^t\right) = +\infty$. For simplicity we consider a consumer that is myopic in that choosing λ^t she ignores the impact of that choice to future consumption. If her income is I we can can express the optimization problem as

$$\max_{\lambda,\ C^{u,t},\ C^{a,t}} \left((1-\lambda)\left(C^{u,t}\right)^{\rho} + \lambda\left(C^{a,t}\right)^{\rho}\right)^{1/\rho}$$
$$s.t.\ \lambda P^{a,t} C^{a,t} + (1-\lambda) P^{u,t} C^{u,t} + h(\lambda) = I$$

where $C^{u,t}$ is the CES bundle of the goods for which the consumer will not adapt to the frontier technology this period and $C^{a,t}$ the CES bundle of goods for which the consumer will adapt and the elasticity of substitution is $\sigma = 1/(1-\rho)$. Here we assume that h is in the same units as income and henceforth we suppress time notation.

The above problem implies the following Lagrangian

$$[(1-\lambda)(C^u)^{\rho} + \lambda (C^a)^{\rho}]^{1/\rho} + \mu [\lambda P^a C^a + (1-\lambda) P^u C^u + h(\lambda) - I]\ ,$$

where μ is the Lagrangian multiplier (for simplicity we suppress the t notation throughout). Taking first order conditions (FOC) with respect to C^u, C^a, λ :

$$\begin{aligned} C^a &: [(1-\lambda)(C^u)^{\rho} + \lambda (C^a)^{\rho}]^{1/\rho-1} \lambda (C^a)^{\rho-1} = -\mu\lambda P^a\ , \\ C^u &: [(1-\lambda)(C^u)^{\rho} + \lambda (C^a)^{\rho}]^{1/\rho} (1-\lambda)(C^u)^{\rho-1} = -\mu (1-\lambda) P^u\ , \end{aligned}$$

which implies

$$\frac{(C^a)^{\rho}}{(C^u)^{\rho}} = \frac{P^a C^a}{P^u C^u}\ . \tag{36}$$

Finally, we also have the FOCs,

$$\lambda : \quad (1/\rho)[(1-\lambda)(C^u)^{\rho} + \lambda (C^a)^{\rho}]^{1/\rho-1}[-(C^u)^{\rho} + (C^a)^{\rho}] + \mu [P^a C^a - P^u C^u + h'(\lambda)] = 0$$

and of course

$$\lambda P^a C^a + (1-\lambda) P^u C^u + h(\lambda) = I\ .$$

Replacing for μ using the FOC for C^a we obtain

$$(1/\rho)\left[-(C^u)^\rho + (C^a)^\rho\right] - \frac{(C^a)^{\rho-1}}{P^a}\left[P^a C^a - P^u C^u + h'(\lambda)\right] = 0$$

Now using equation (36) in the above equation we have

$$\begin{aligned}(1/\rho)\left[-(C^a)^\rho \frac{P^u C^u}{P^a C^a} + (C^a)^\rho\right] - \frac{(C^a)^\rho}{P^a C^a}\left[P^a C^a - P^u C^u + h'(\lambda)\right] &= 0 \implies \\ (1/\rho - 1)\, P^a C^a \left[1 - \frac{P^u C^u}{P^a C^a}\right] &= h'(\lambda)\end{aligned}$$

Notice that using (36) we can write the above expression as

$$(1/\rho - 1)\, P^a C^a \left[1 - \left(\frac{P^u}{P^a}\right)^{\rho/(\rho-1)}\right] = h'(\lambda)$$

In the next section of this appendix we show that $P^u \geq P^a$. Thus, given that $\rho \in (0,1)$ the LHS of the above expression positive and by the properties of $h'(\lambda)$ there exists a unique λ that the consumer chooses. The larger the price difference between unadjusted and adjusted prices, the higher the λ chosen.

A.3 Welfare losses from non adjustment

We prove the following Lemma

Lemma 1 *Assume at time* 0 *the model is in a steady state,* $\sigma > 1$*, and that* $\hat{T}_i^t = 1$ *for all* i*. Then the price index in the model is larger or equal to the price index in the model with full adjustment,* $\lambda = 1$*:* $P_n^t \geq \gamma \left(\Phi_n^t\right)^{-1/\theta}$, $\forall t \geq 1$.

Proof. Since $P_n^t = \left[\sum_i \bar{\pi}_{ni}^t \left(P_{ni}^t\right)^{-(\sigma-1)}\right]^{-1/(\sigma-1)}$ and given formula (14) we simply need to show that

$$\gamma \left(\Phi_n^t\right)^{-1/\theta} \leq \left[\sum_i \bar{\pi}_{ni}^{t-1} \left(\hat{c}_{ni}^t\right)^{-(\sigma-1)} \left(P_{ni}^{t-1}\right)^{-(\sigma-1)}\right]^{-1/(\sigma-1)} \tag{37}$$

so that the price of adjusting goods is always lower than the price of non-adjusting goods. Intuitively, this relationship should be true by revealed preference and by the LLN.

Using the definition of $\hat{c}_{ni}^{t+1}$ we simply want to show that

$$\gamma \left(\sum T_k^t \left(\frac{w_k^t d_{nk}^t}{A_k^t} \right)^{-\theta} \right)^{-1/\theta} \leq \left[\sum_i \bar{\pi}_{ni}^{t-1} \left(\frac{w_i^t}{w_i^{t-1}} \frac{d_{ni}^t}{d_{ni}^{t-1}} \frac{A_{ni}^{t-1}}{A_{ni}^t} \right)^{-(\sigma-1)} \left(P_{ni}^{t-1} \right)^{-(\sigma-1)} \right]^{-1/(\sigma-1)} \tag{38}$$

We will prove it by induction starting by proving the result for the first period.

Period 1: For the first period the above formula implies

$$\left(\gamma \sum T_k^1 \left(\frac{w_k^1 d_{nk}^1}{A_k^1} \right)^{-\theta} \right)^{-\frac{1}{\theta}} \leq \left[\sum_i \bar{\pi}_{ni}^0 \left(\frac{w_i^1 d_{ni}^1 / A_i^1}{w_i^0 d_{ni}^0 / A_i^0} \right)^{-(\sigma-1)} \left(\gamma \sum_k T_k^0 \left(\frac{w_k^0 d_{nk}^0}{A_k^0} \right)^{-\theta} \right)^{\frac{(\sigma-1)}{\theta}} \right]^{-1/(\sigma-1)} \Longrightarrow$$

$$\left(\sum T_k^1 \left(\frac{w_k^1 d_{nk}^1}{A_k^1} \right)^{-\theta} \right)^{-\frac{1}{\theta}} \leq \left[\sum_i \left(T_i^0 \right)^{\frac{\sigma-1}{\theta}} \left(w_i^1 d_{ni}^1 / A_i^1 \right)^{-(\sigma-1)} \bar{\pi}_{ni}^0 \left(\sum_k \frac{T_k^0 \left(w_k^0 d_{nk}^0 / A_k^0 \right)^{-\theta}}{T_i^0 \left(w_i^0 d_{ni}^0 / A_i^0 \right)^{-\theta}} \right)^{\frac{(\sigma-1)}{\theta}} \right]^{-1/(\sigma-1)}$$

Thus we need to prove this formula. We consider the case $\sigma > 1$. Notice that in period 0 we are in steady state so that $\pi_{ni}^0 = \pi_{ni}^{a,0} = \bar{\pi}_{ni}^0 = \frac{T_i^0 \left(\frac{w_i^0 d_{ni}^0}{A_i^0} \right)^{-\theta}}{\sum_k T_k^0 \left(\frac{w_k^0 d_{nk}^0}{A_i^0} \right)^{-\theta}}$ and also that by assumption $T_i^0 = T_i^1$ and the above relationship can be written as

$$1 \geq \sum_i \frac{\left[T_i^1 \left(w_i^1 d_{ni}^1 \right)^{-\theta} \right]^{\frac{(\sigma-1)}{\theta}}}{\left[\sum T_k^1 \left(w_k^1 d_{nk}^1 \right)^{-\theta} \right]^{\frac{(\sigma-1)}{\theta}}} \bar{\pi}_{ni}^0 \left(\bar{\pi}_{ni}^0 \right)^{-\frac{(\sigma-1)}{\theta}} \Longleftrightarrow$$

$$1 \geq \sum_i \left(\bar{\pi}_{ni}^0 \right)^{1-\frac{(\sigma-1)}{\theta}} \left(\bar{\pi}_{ni}^{a,1} \right)^{\frac{(\sigma-1)}{\theta}} \tag{39}$$

Notice that $\sum_i \bar{\pi}_{ni}^0 = 1 = \sum_i \bar{\pi}_{ni}^{a,1}$ and we want to show that this inequality is true.

We use the Holder's inequality that states that:

Let $1/p + 1/q = 1$, with $p, q > 1$ then:

$$\sum_k |a_k b_k| \leq \left(\sum_k |a_k|^p \right)^{1/p} \left(\sum_k |b_k|^q \right)^{1/q}$$

Thus, we set $1/p = 1 - \frac{(\sigma-1)}{\theta}$, $1/q = \frac{(\sigma-1)}{\theta}$. We then have

$$\sum_k \left| \left(\bar{\pi}^0_{ni}\right)^{1-\frac{(\sigma-1)}{\theta}} \left(\bar{\pi}^{a,1}_{ni}\right)^{\frac{(\sigma-1)}{\theta}} \right| \leq \left[\sum_k \left| \left(\bar{\pi}^0_{ni}\right)^{1-\frac{(\sigma-1)}{\theta}} \right|^{\frac{1}{1-\frac{(\sigma-1)}{\theta}}}\right]^{1-\frac{(\sigma-1)}{\theta}} \left[\sum_k \left| \left(\bar{\pi}^{a,1}_{ni}\right)^{\frac{(\sigma-1)}{\theta}} \right|^{\frac{1}{\frac{(\sigma-1)}{\theta}}}\right]^{\frac{(\sigma-1)}{\theta}}$$

and since $\bar{\pi}^0_{ni}$, $\bar{\pi}^{a,1}_{ni}$ are always positive we finally have

$$\sum_k \left(\bar{\pi}^0_{ni}\right)^{1-\frac{(\sigma-1)}{\theta}} \left(\bar{\pi}^1_{ni}\right)^{\frac{(\sigma-1)}{\theta}} \leq \left[\sum_k \left(\bar{\pi}^0_{ni}\right)\right]^{1-\frac{(\sigma-1)}{\theta}} \left[\sum_k \left(\bar{\pi}^1_{ni}\right)\right]^{\frac{(\sigma-1)}{\theta}} = 1$$

which completes the proof for the first period.

Periods t, t+1: Now we need to show that equation (38) holds for all t. Notice that the equation can be rewritten as

$$\gamma \left(\sum T^t_k \left(w^t_k d^t_{nk}\right)^{-\theta}\right)^{(\sigma-1)/\theta} \geq \sum_i \bar{\pi}^{t-1}_{ni} \left[\left(\frac{T^t_i}{T^{t-1}_i}\right)^{-1/\theta} \frac{w^t_i d^t_{ni}/A^t_i}{w^{t-1}_i d^{t-1}_{ni}/A^{t-1}_i}\right]^{-(\sigma-1)} \left(P^{t-1}_{ni}\right)^{-(\sigma-1)}$$

The recursive formula for prices is given by equation (13) and substituting in this inequality we simply need to show

$$\left(\gamma \sum T^t_k \left(w^t_k d^t_{nk}\right)^{-\theta}\right)^{(\sigma-1)/\theta}$$

$$\geq \lambda^{t-1}_n \sum_i \bar{\pi}^{t-1}_{ni} \left[\left(\frac{T^t_i}{T^{t-1}_i}\right)^{-1/\theta} \frac{w^t_i}{w^{t-1}_i} \frac{d^t_{ni}}{d^{t-1}_{ni}}\right]^{-(\sigma-1)} \frac{\bar{\pi}^{a,t-1}_{ni}}{\bar{\pi}^{t-1}_{ni}} \left(\gamma \left(\Phi^{t-1}_n\right)^{-1/\theta}\right)^{-(\sigma-1)} \tag{40}$$

$$+ \left(1-\lambda^{t-1}_n\right) \sum_i \bar{\pi}^{t-1}_{ni} \left[\left(\frac{T^t_i}{T^{t-1}_i}\right)^{-1/\theta} \frac{w^t_i}{w^{t-1}_i} \frac{d^t_{ni}}{d^{t-1}_{ni}}\right]^{-(\sigma-1)} \frac{\bar{\pi}^{,t-2}_{ni}}{\bar{\pi}^{t-1}_{ni}} \left(\hat{c}^{t-1}_{ni}\right)^{-(\sigma-1)} \left(P^{t-2}_{ni}\right)^{-(\sigma-1)} \tag{41}$$

The logic of the proof is as follows

Step 1 First, notice that the terms on the RHS are a weighted sum of two terms with weights being λ^{t-1}_n. Thus, it suffices to show that one of the term in the RHS is larger than the other one and that the term in LHS is larger than that largest of the two terms in the RHS. We will show that the the first term on the RHS is the larger one.

Step 2 Notice that

$$\left[\gamma \sum T_k^t \left(w_k^t d_{nk}^t\right)^{-\theta}\right]^{(\sigma-1)/\theta} \geq \sum_i \left[\left(\frac{T_i^t}{T_i^{t-1}}\right)^{-1/\theta} \frac{w_i^t}{w_i^{t-1}} \frac{d_{ni}^t}{d_{ni}^{t-1}}\right]^{-(\sigma-1)} \bar{\pi}_{ni}^{a,t-1} \left[\gamma \left(\Phi_n^{t-1}\right)^{-1/\theta}\right]^{-(\sigma-1)}$$

the proof works exactly as in the period 1. In particular, we raise everything to the power of $-1/(\sigma-1)$ turn the inequality and use the definition of $\bar{\pi}_{ni}^{a,t-1}$ to arrive to the established inequality as in (39). This proves the relationship for all t

Step 3 We now simply need to show the relationship between the two terms in the RHS of equation (41) and in particular that

$$\begin{aligned} \sum_i \bar{\pi}_{ni}^{a,t-1} \left[\gamma \left(\Phi_n^{t-1}\right)^{-1/\theta}\right]^{-(\sigma-1)} &\geq \sum_i \bar{\pi}_{ni}^{t-2} \left(\hat{c}_{ni}^{t-1}\right)^{-(\sigma-1)} \left(P_{ni}^{t-2}\right)^{-(\sigma-1)} \implies \\ \left[\gamma \left(\Phi_n^{t-1}\right)^{-1/\theta}\right]^{-(\sigma-1)} &\geq \sum_i \bar{\pi}_{ni}^{t-2} \left(\hat{c}_{ni}^{t-1}\right)^{-(\sigma-1)} \left(P_{ni}^{t-2}\right)^{-(\sigma-1)} \end{aligned}$$

Notice however that we started from trying to establish the exact equation a period ahead (see equation (38)). Thus if the equation holds for time t it holds for time $t+1$. Since we know from the initial derivation that it holds for period 1 we use induction to prove our result. This argument completes the proof. ■

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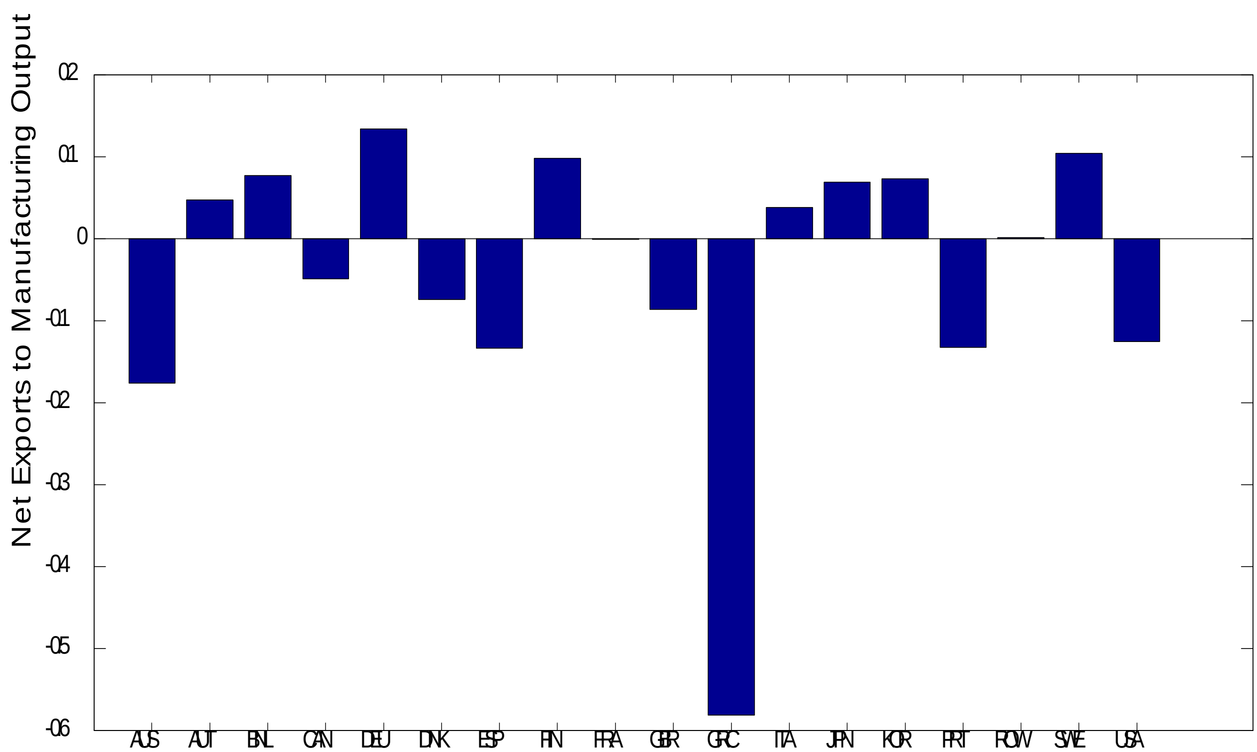

Figure 1: Manufacturing deficit to manufacturing output for G-6 countries in 2006

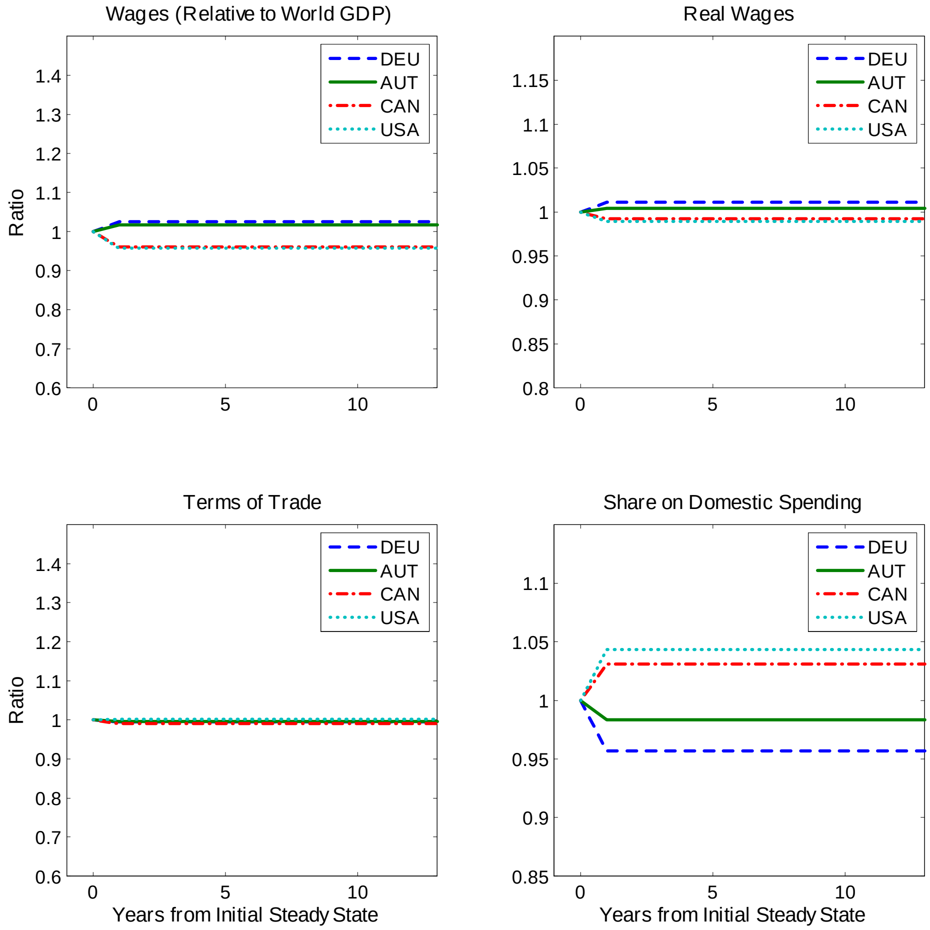

Figure 2: Impact of a permanent reduction in deficits, $\lambda = 1$

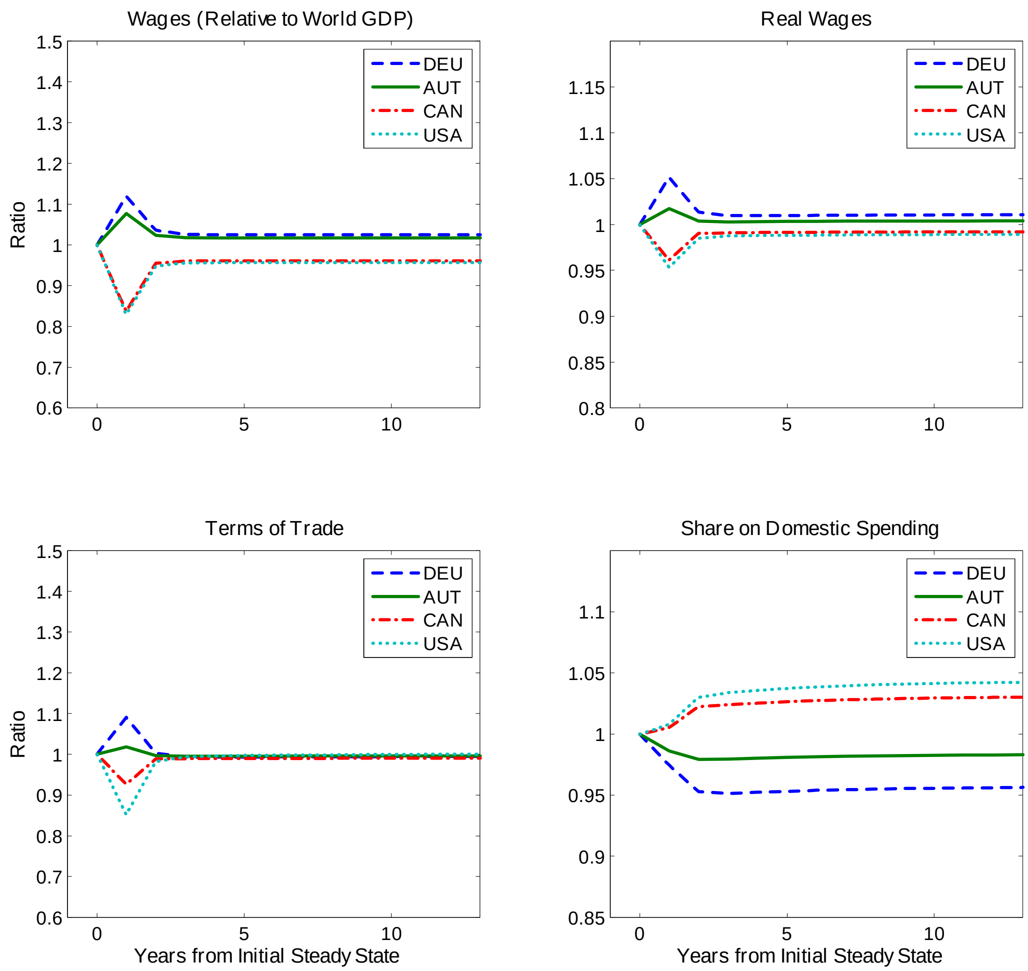

Figure 3: Impact of a permanent reduction in deficits, $\lambda = 0.2$

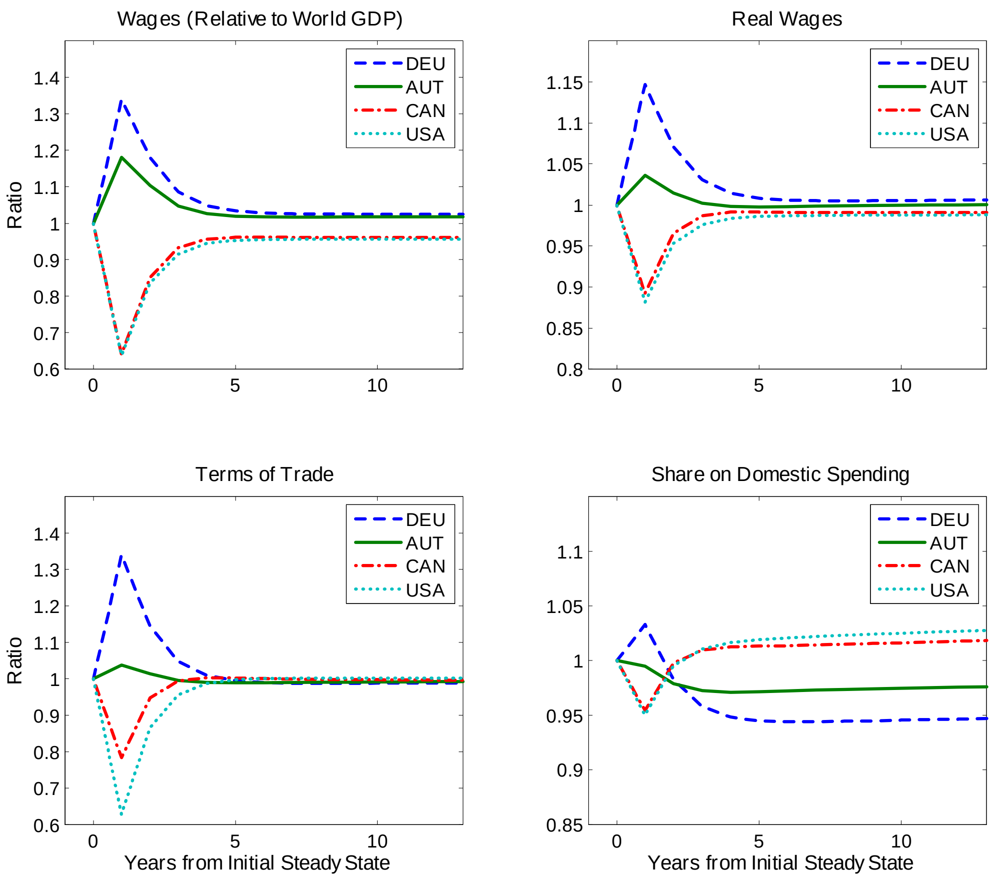

Figure 4: Impact of a permanent reduction in deficits, $\lambda = 0.05$

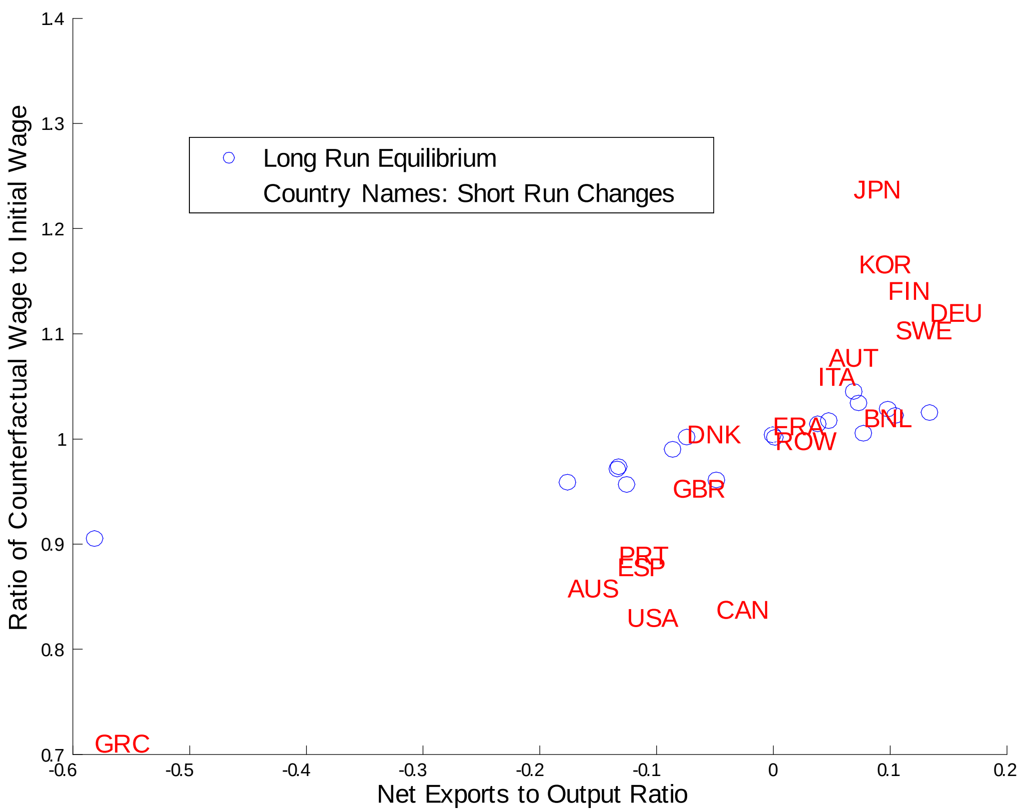

Figure 5: Wage Changes and Initial Net Export to Output Ratio. Short Run and Long Run Relationship for $\lambda = 0.2$.