

# Welfare Analysis of the Krugman Model of Trade

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## Abstract

In this appendix we solve a multi-country version of the monopolistic competition model with homogeneous firms by Krugman (1980). Our objective is to derive relationships in the model related to the elasticity of trade and the welfare gains from an increase in the trade to GDP ratio. The main result is that these relationships are comparable to the related expressions arising from the main quantitative heterogeneous firms models: when the models are calibrated to deliver a given change in trade from a change in tariffs, they also deliver the same welfare gains.

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# 1 Preliminaries

## 1.1 Consumer's problem

Let a potential variety  $\omega \in \Omega$ , where  $\Omega$  is the set of all potential varieties. There are  $i, j = 1, \dots, N$  countries, thus,  $\Omega = \bigcup_j \Omega_j$ . We assume that there is a measure  $L_j$  of consumers in each country  $j$ .

The problem of the representative consumer from country  $j$  is

$$\begin{aligned} \max & \left( \sum_{i=1}^N \int_{\Omega_i} q_{ij}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}, \\ \text{s.t.} & \sum_{i=1}^N \int_{\Omega_i} p_{ij}(\omega) q_{ij}(\omega) d\omega = w_j, \end{aligned}$$

where  $q_{ij}(\omega)$  is the quantity demanded of good  $\omega$ ,  $p_{ij}(\omega)$  is the price of that good,  $w_j$  is the wage that the consumer gets from inelastically supplying his unit of labor endowment, and  $\sigma > 1$  is the elasticity of substitution.

The above implies the following CES demand for the consumers in country  $j$ :

$$\begin{aligned} q_{ij}(\omega) &= \left( \frac{p_{ij}(\omega)}{P_j} \right)^{-\sigma} \frac{w_j L_j}{P_j}, \\ P_j &= \left( \int_{\Omega} p_{ij}(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}. \end{aligned}$$

### 1.1.1 Firm's problem

Each firm is a monopolist of a variety. All the firms from country  $i$  have a common productivity,  $\phi_i$ , and they produce one unit of the good using  $\frac{1}{\phi_i}$  units of labor. Firms have to pay a fixed cost of production in terms of domestic labor,  $f_i$ . They also incur an iceberg transportation cost,  $\tau_{ij}$ ,

to ship the good from country  $i$  to country  $j$ . Profit maximization implies that optimal pricing for a firm selling from country  $i$  to country  $j$  is

$$p_{ij}(\phi_i) = \frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{\phi_i}.$$

We will make the notation a bit cumbersome by carrying around the  $\phi_i$ 's in order to allow for direct comparison of our results with the heterogeneous firms example.

## 2 Equilibrium

In order to determine the equilibrium of the model, we have to consider the free entry condition and the labor market clearing. Free entry implies that firms will keep entering at each given country up to the point that expected profits are equal to zero for the firms of that country. This implies that in the equilibrium

$$\begin{aligned} \sum_j \frac{p_{ij}(\phi_i)^{1-\sigma}}{P_j^{1-\sigma}} \frac{w_j L_j}{\sigma} - w_i f_i &= 0 \implies \\ \sum_j \frac{\left(\frac{\sigma}{\sigma-1} \frac{\tau_{ij} w_i}{\phi_i}\right)^{1-\sigma}}{P_j^{1-\sigma}} \frac{w_j L_j}{\sigma} &= w_i f_i \implies \\ \frac{\sigma}{\sigma-1} \frac{1}{\sigma} \frac{w_i}{\phi_i} \sum_j \frac{\tau_{ij} \left(\frac{\sigma}{\sigma-1} \frac{\tau_{ij} w_i}{\phi_i}\right)^{-\sigma}}{P_j^{1-\sigma}} w_j L_j &= w_i f_i \implies \\ \frac{1}{\sigma-1} \frac{w_i}{\phi_i} q_i &= w_i f_i \implies \\ q_i &= f_i \phi_i (\sigma - 1), \end{aligned}$$

where by slightly abusing the notation we define  $q_i \equiv \sum_j \frac{\tau_{ij} \left(\frac{\sigma}{\sigma-1} \frac{\tau_{ij} w_i}{\phi_i}\right)^{-\sigma}}{P_j^{1-\sigma}} w_j L_j$ .

Using the labor market clearing implies that (where  $n_i$  is the mass of operating domestic firms)

$$\begin{aligned}
n_i \left( \sum_{v=1}^N \frac{\left( \frac{\sigma}{\sigma-1} \frac{\tau_{iv} w_i}{\phi_i} \right)^{-\sigma}}{P_v^{1-\sigma}} w_v L_v \frac{\tau_{iv}}{\phi_i} + f_i \right) &= L_i \implies \\
n_i \left( \sum_{v=1}^N \frac{\tau_{iv} \left( \frac{\sigma}{\sigma-1} \frac{\tau_{iv} w_i}{\phi_i} \right)^{-\sigma}}{P_v^{1-\sigma}} \frac{w_v L_v}{\phi_i} + f_i \right) &= L_i \implies \\
n_i \left( \frac{q_i}{\phi_i} + f_i \right) &= L_i \implies \\
n_i (f_i (\sigma - 1) + f_i) &= L_i \implies \\
n_i &= \frac{L_i}{\sigma f_i}. \tag{1}
\end{aligned}$$

Notice that the equilibrium measure of entrants is independent of variable trade costs.

Now we compute the fraction of total income in country  $j$  spent on goods from country  $i$ ,  $\lambda_{ij}$ ,

$$\begin{aligned}
\lambda_{ij} &= \frac{X_{ij}}{X_j} = \\
&= \frac{\frac{n_i \left( \frac{\sigma}{\sigma-1} \frac{\tau_{ij} w_i}{\phi_i} \right)^{1-\sigma}}{P_j^{1-\sigma}} w_j L_j}{\sum_{v=1}^N \frac{n_v \left( \frac{\sigma}{\sigma-1} \frac{\tau_{vj} w_v}{\phi_v} \right)^{1-\sigma}}{P_j^{1-\sigma}} w_j L_j}, \\
&= \frac{n_i \left( \frac{\tau_{ij} w_i}{\phi_i} \right)^{1-\sigma}}{\sum_{i=1}^N n_v \left( \frac{\tau_{vj} w_v}{\phi_v} \right)^{1-\sigma}},
\end{aligned}$$

and using equation (1), we have

$$\lambda_{ij} = \frac{\frac{L_i}{f_i} (\tau_{ij} w_i)^{1-\sigma} (\phi_i)^{\sigma-1}}{\sum_{v=1}^N \frac{L_v}{f_v} (\tau_{vj} w_v)^{1-\sigma} (\phi_v)^{\sigma-1}}. \tag{2}$$

Using a similar derivation, we also have

$$\begin{aligned}\lambda_{ij} &= \frac{X_{ij}}{X_j} = \\ &= \frac{n_i \left( \frac{\sigma}{\sigma-1} \frac{\tau_{ij} w_i}{\phi_i} \right)^{1-\sigma} w_j L_j}{P_j^{1-\sigma} w_j L_j},\end{aligned}$$

which implies that

$$P_j = (n_i)^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma-1} \frac{\tau_{ij} w_i}{\phi_i} (\lambda_{ij})^{\frac{1}{\sigma-1}}.$$

Looking at the domestic market share of  $j$ , we have

$$P_j = (n_j)^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma-1} \frac{w_j}{\phi_j} (\lambda_{jj})^{\frac{1}{\sigma-1}}. \quad (3)$$

Finally, the welfare is given by

$$\frac{w_j}{P_j} = \frac{\sigma-1}{\sigma} \phi_j \frac{(n_j)^{\frac{1}{\sigma-1}}}{(\lambda_{jj})^{\frac{1}{\sigma-1}}},$$

with  $n_j$  given by equation (1).

When looking at a trade liberalization episode, and in particular, the change of the trade to GDP ratio, which is essentially the change in  $\lambda_{jj}$ , the ratio of the welfare before and after the trade liberalization is given by

$$\frac{\frac{w'_j}{P'_j}}{\frac{w_j}{P_j}} = \left( \frac{\lambda'_{jj}}{\lambda_{jj}} \right)^{-\frac{1}{\sigma-1}}. \quad (4)$$

### 3 Discussion: Trade Liberalization

The expression (2) can be used to calibrate the parameters of the model in order to generate the elasticity of trade flows with respect to tariffs that we observe in the data. In fact, this

expression is similar to the one derived in other models with heterogeneous firms such as the ones of Eaton and Kortum (2002), the Chaney (2007) version of Melitz (2003) and Arkolakis (2006). The only difference is that in the latter cases  $\sigma - 1$  is replaced by the parameter that determines the heterogeneity of the productivities of the firms. In fact, the same thing holds for expression (4), which implies that the main quantitative models of trade with heterogeneous firms deliver exactly the same welfare predictions for the change of welfare in the case of a trade liberalization episode.

## 4 Bibliography

Arkolakis, Costas, 2006, “Market Penetration Costs and the New Consumers Margin in International Trade,” mimeo, Yale University.

Chaney, Thomas, 2007, “Distorted Gravity: The Intensive and the Extensive Margin of International Trade,” forthcoming, *American Economic Review*.

Eaton, Jonathan, and Samuel Kortum, 2002, “Technology, Geography, and Trade,” *Econometrica*, 70(5), pp. 1741-79.

Krugman, Paul, 1980, “Scale Economies, Product Differentiation, and the Pattern of Trade,” *American Economic Review*, 70(5), pp. 950-959.

Melitz, Marc, 2003, “The Impact of Trade on Intra-Industry Reallocations and Aggregate Productivity,” *Econometrica*, 71 (6), pp. 1695-1725.

## 5 Appendix

In order to compute wages across countries, we can make use of the following condition implied by balanced trade:

$$\begin{aligned}w_i L_i &= \sum_{v=1}^N \lambda_{iv} n_v p_v q_v \implies \\w_i L_i &= \sum_{v=1}^N \lambda_{iv} \frac{L_v}{\sigma f_v} \frac{\sigma}{\sigma - 1} \frac{w_v}{\phi_v} f_v \phi_v (\sigma - 1) \implies \\w_i L_i &= \sum_{v=1}^N \lambda_{iv} L_v w_v.\end{aligned}$$