Addendum to: New Trade Models, Same Old Gains?

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Abstract

This addendum provides generalizations of Proposition 1 for the cases of multiple sectors and tradable intermediate goods discussed in Section 5 of our main paper.

1 Extension (I): Multiple Sectors

1.1 Assumptions

Preferences, Technology, and Market Structure. It is standard to interpret models with Dixit-Stiglitz preferences, such as the one presented in Section 3 of our main paper, as "one-sector" models with a continuum of "varieties". Under this interpretation, our model can be extended to multiple sectors, $s = 1, ..., S$, by assuming that the representative agent has a two-tier utility function, with the upper-tier being Cobb-Douglas, with consumption shares $1 \geq \eta^s \geq 0$, and the lower-tier being Dixit-Stiglitz with elasticity of substitution $\sigma^s > 1$. Under this assumption, the consumer price index in country j formally becomes

$$P_j = \prod_{s=1}^{S} \left(P_j^s\right)^{\eta^s}, \tag{1}$$

where $P_j^s = \left(\int_{\omega \in \Omega^s} p_j^s(\omega)^{1-\sigma^s} d\omega\right)^{\frac{1}{1-\sigma^s}}$ is the Dixit-Stiglitz price index associated with varieties from sector s. For each sector s, primitive assumptions on technology and market structure are as described in Section 3 of our main paper. Superscripts s denote all sector-level variables.

Macro-level Restrictions. In this extension we use the following counterparts of R1-R3:

R1(MS) *For any country j and any sector s, $w_j L_j^s + \Pi_j^s - w_j N_j^s F_j^s = \sum_{i=1}^n X_{ji}^s$.*

In the one-sector case R1 states that $\sum_{i=1}^n X_{ij} = \sum_{i=1}^n X_{ji}$. This is equivalent to $w_j L_j + \Pi_j - w_j N_j F_j = \sum_{i=1}^n X_{ji}$ since $w_j L_j + \Pi_j - w_j N_j F_j = \sum_{i=1}^n X_{ij}$ by country j's representative agent's budget constraint. R1(MS) is simply the sector-level counterpart of the previous expression.

R2 (MS) *For any country j and any sector s, $\Pi_j^s = \zeta R_j^s$ with $\zeta \in [0, 1]$.*

Compared to R2 in the one-sector case, R2(MS) states that aggregate profits are a constant share of revenues in each sector, but also that the share of profits ζ is common across sectors.

R3 (MS) *The import demand system is such that for any sector s, any importer j, and any pair of exporters $i \neq j$ and $i' \neq j$, $\varepsilon_j^{sii'} = \varepsilon^s < 0$ if $i = i'$ and zero otherwise, with $\varepsilon_j^{sii'} \equiv \partial \ln\left(X_{ij}^s / X_{jj}^s\right) / \partial \ln \tau_{i'j}^s$.*

Note that R3(MS) allows the trade elasticities ε^s to vary across sectors.

1.2 Welfare Evaluation

Under the previous assumptions, Proposition 1 generalizes to:

Proposition 1 (MS) *Suppose that R1-R3(MS) hold. Then the change in real income associated with any foreign shock in country j can be computed as $\widehat{W}_j = \prod_{s=1}^{S} \left(\widehat{\lambda}_{jj}^{s}\right)^{\eta_j^s / \varepsilon^s}$, under perfect competition and monopolistic competition with restricted entry, and $\widehat{W}_j = \prod_{s=1}^{S} \left(\hat{\lambda}_{jj}^{s} \Big/ \widehat{L}_j^s\right)^{\eta_j^s / \varepsilon^s}$, under monopolistic competition with free entry.*

Proof. Like in our main paper, we consider separately the cases of perfect and monopolistic competition and use labor in country j as our numeraire, $w_j = 1$. For expositional purposes, we describe in detail the steps of the proof that are distinct from those in Appendix A of our main paper and omit others.

Case 1: Perfect competition

By the same logic as in Appendix A (Step 1), we have $d \ln Y_j = 0$. Combining this observation with Equation (1) and the definition of W_j, we obtain

$$d \ln W_j = -\sum_{s=1}^{S} \eta_j^s d \ln P_j^s.$$

Following the exact same reasoning as in Appendix A (Steps 2-4), one can easily check that R3(MS) implies

$$d \ln P_j^s = -\frac{d \ln \lambda_{jj}^s}{\varepsilon^s}.$$

Combining the two previous expressions and integrating (Step 5 in Appendix A), we get

$$\widehat{W}_j = \prod_{s=1}^{S} \left(\widehat{\lambda}_{jj}^{s}\right)^{\eta_j^s / \varepsilon^s},$$

which completes our proof under perfect competition.

Case 2: Monopolistic Competition

Using the same logic as in Appendix A (Step 1), we first show that $d \ln Y_j = 0$. Under free entry we know that $Y_j = L_j$, which immediately implies $d \ln Y_j = 0$. Under restricted entry, we know from the budget constraint of the representative agent in country j that $Y_j = L_j + \sum_{s=1}^{S} \Pi_j^s$.

Combining this observation with R1(MS), we get $Y_j = \sum_{s=1}^{S} R_j^s$. Since $\Pi_j^s = \zeta R_j^s$ for all $s = 1, ..., S$ by R2(MS), we thus have $Y_j = L_j + \zeta Y_j$. Totally differentiating the previous expression, we get $dY_j = 0$ and thus $d\ln Y_j = 0$. Since $d\ln Y_j = 0$ under monopolistic competition, the same reasoning as under perfect competition implies

$$d\ln W_j = -\sum_{s=1}^{S} \eta_j^s d\ln P_j^s.$$

Following the exact same reasoning as in Appendix A (Steps 2-4), one can also easily check that R3(MS) implies

$$d\ln P_j^s = -\frac{d\ln \lambda_{jj}^s}{\varepsilon^s} + \frac{d\ln N_j^s}{\varepsilon^s}.$$

Using the same logic as in Appendix A (Step 5), let us now show that $d\ln N_j^s = d\ln L_j^s$, under free entry, and $d\ln N_j^s = 0$, under restricted entry. Under free entry, we know that $\Pi_j^s = N_j^s F_j^s$, which implies $d\ln \Pi_j^s = d\ln N_j^s$. By R1(MS) and free entry, we know that $d\ln L_j^s = d\ln R_j^s$. By R2(MS), we also know that $d\ln R_j^s = d\ln \Pi_j^s$. Combining the previous series of equations, we obtain $d\ln N_j^s = d\ln L_j^s$. Finally, under restricted entry, $N_j^s = \overline{N}_j^s$ immediately implies $d\ln N_j^s = 0$. The last part of the proof is the same as under perfect competition and omitted. **QED** ■

2 Extension (II): Tradable Intermediate Goods

2.1 Assumptions

Preferences, Technology, and Market Structure. The primitive assumptions on preferences and market structure are the same as in Section 3 of our main paper. In terms of technology, however, we now allow goods $\omega \in \Omega$ to be used in the production of other goods. Formally, we assume that all goods can be aggregated into a unique intermediate good using the same Dixit-Stiglitz aggregator as for final consumption. Thus P_i now represents both the consumer price index in country i and the price of intermediate goods in this country. In this extension, the cost function for each good ω is given by

$$C_i(\boldsymbol{w}, \boldsymbol{P}, \boldsymbol{q}, \boldsymbol{t}, \omega) = \sum_{j=1}^{n} \left[c_{ij}(w_i, P_i, t_j, \omega) q_j + f_{ij}(w_i, P_i, w_j, P_j, t_j, \omega) \mathbb{I}(q_j > 0) \right],$$

where $\boldsymbol{P} \equiv \{P_i\}$ is the vector of intermediate good prices. In line with the previous literature we further assume that constant marginal costs and fixed exporting costs can be written as

$$\begin{aligned} c_{ij}(w_i, P_i, t_j, \omega) &\equiv \tau_{ij} \cdot w_i^{\beta} \cdot P_i^{1-\beta} \cdot \alpha_{ij}(\omega) \cdot t^{\frac{1}{1-\sigma}}, \\ f_{ij}(w_i, P_i, w_j, P_j, t_j, \omega) &\equiv \xi_{ij} \cdot h_{ij}(w_i^{\beta} P_i^{1-\beta}, w_j^{\beta} P_i^{1-\beta}) \cdot \phi_{ij}(\omega) \cdot m_{ij}(t), \end{aligned}$$

with $\beta \in [0,1]$ governing the share of intermediate goods in variable and fixed production costs. Similarly, we assume that fixed entry costs (if any) are given by $w_i^{\kappa} P_i^{1-\kappa} F_i$, with $\kappa \in [0,1]$ governing the share of intermediate goods in entry costs.

Macro-level Restrictions. In this extension our first two macro-level restrictions, R1(TI) and R2(TI), are exactly the same as in Section 3 of our main paper. R3(TI) still requires the import demand system to be such that for any importer j and any pair of exporters $i \neq j$ and $i' \neq j$, $\varepsilon_j^{ii'} = \varepsilon < 0$ if $i = i'$, and zero otherwise. The only difference with Section 3 of our main paper is that the import demand system now refers to the mapping from $(\boldsymbol{w}, \boldsymbol{P}, \boldsymbol{N}, \boldsymbol{\tau})$ into $\boldsymbol{X}$, and, so the partial elasticities $\varepsilon_j^{ii'}$ also hold fixed the price of intermediate goods, $\boldsymbol{P}$.[1]

2.2 Welfare Evaluation

Under the previous assumptions, Proposition 1 generalizes to:

Proposition 1 (TI) *Suppose that R1-R3(TI) hold. Then the change in real income associated with any foreign shock in country j can be computed as $\widehat{W}_j = \widehat{\lambda}_{jj}^{1/\varepsilon\beta}$, under perfect competition; $\widehat{W}_j = \widehat{\lambda}_{jj}^{1/\left[\varepsilon\beta - (1-\beta)\left(\frac{\varepsilon}{\sigma-1}+1\right)\right]}$, under monopolistic competition with restricted entry; and $\widehat{W}_j = \widehat{\lambda}_{jj}^{1/\left[\varepsilon\beta - (1-\beta)\left(\frac{\varepsilon}{\sigma-1}+1\right)+(1-\kappa)\right]}$, under monopolistic competition with free entry.*

Proof. Like in the previous proof, we consider separately the cases of perfect and monopolistic competition and use labor in country j as our numeraire, $w_j = 1$. For expositional purposes, we again describe in detail the steps of the proof that are distinct from those in Appendix A of our main paper and omit others. Throughout this proof we let $c_i \equiv w_i^{\beta} \cdot P_i^{1-\beta}$ and $c_{ij} \equiv \tau_{ij} c_i$.

Case 1: Perfect competition

[1] This generalization of the definition of the import demand system reflects the fact that there are now two inputs in production, labor and the aggregate intermediate goods, with prices given by $\boldsymbol{w}$ and $\boldsymbol{P}$, respectively.

By the same logic as in Appendix A (Step 1), we have $d\ln Y_j = 0$, which implies

$$d\ln W_j = -d\ln P_j. \tag{2}$$

Similarly, by the same logic as in Appendix A (Step 2), small changes in the consumer price index satisfy

$$d\ln P_j = \textstyle\sum_{i=1}^{n} \lambda_{ij} d\ln c_{ij}. \tag{3}$$

Finally, by the same logic as in Appendix A (Step 3), small changes in expenditure shares satisfy

$$d\ln\lambda_{ij} - d\ln\lambda_{jj} = \left(1-\sigma+\gamma_{ij}^{i}-\gamma_{jj}^{i}\right) d\ln c_{ij} + \textstyle\sum_{i'\neq i}^{n} \left(\gamma_{ij}^{i'}-\gamma_{jj}^{i'}\right) d\ln c_{i'j} - \left(1-\sigma-\gamma_{ij}^{j}+\gamma_{jj}^{j}\right) d\ln c_{jj}, \tag{4}$$

where $\gamma_{ij}^{i'}$ is given by the same expression as in Appendix A. Compared to Appendix A, the main difference is that we now have $d\ln c_{jj} \neq 0$. Combining Equations (3) and (4), we obtain

$$d\ln P_j = \textstyle\sum_{i=1}^{n} \lambda_{ij} \dfrac{d\ln\lambda_{ij} - d\ln\lambda_{jj} - \sum_{i'\neq i,j}^{n} \left(\gamma_{ij}^{i'}-\gamma_{jj}^{i'}\right) d\ln c_{i'j} + \left(1-\sigma-\gamma_{ij}^{j}+\gamma_{jj}^{j}\right) d\ln c_{jj}}{1-\sigma+\gamma_{ij}^{i}-\gamma_{jj}^{i}}. \tag{5}$$

Following the same logic as in Appendix A (Step 4), it is easy to check that Equation (4) and R3(TI) imply $1-\sigma+\gamma_{ij}^{i}-\gamma_{jj}^{i}=\varepsilon$, for all $i\neq j$, and $\gamma_{ij}^{i'}=\gamma_{jj}^{i'}$ for all $i'\neq i,j$. Combining this observation with Equation (5), we get

$$d\ln P_j = -\frac{\lambda_{jj}}{\varepsilon} + \textstyle\sum_{i=1}^{n} \lambda_{ij} \dfrac{\left(1-\sigma-\gamma_{ij}^{j}+\gamma_{jj}^{j}\right) d\ln c_{jj}}{\varepsilon}. \tag{6}$$

Using the definition of $\gamma_{ij}^{i'}$ in Appendix A, it is easy to check that $\gamma_{ij}^{j} = -\sum_{i'\neq j} \gamma_{ij}^{i'}$ and $\gamma_{jj}^{j} = -\sum_{i'\neq j} \gamma_{jj}^{i'}$, which implies

$$1-\sigma-\gamma_{ij}^{j}+\gamma_{jj}^{j} = 1-\sigma+\gamma_{ij}^{i}-\gamma_{jj}^{i}+\sum_{i'\neq i,j}\left(\gamma_{ij}^{i'}-\gamma_{jj}^{i'}\right)=\varepsilon.$$

Together with Equation (6), the previous expression further implies

$$d\ln P_j = -\frac{\lambda_{jj}}{\varepsilon} + d\ln c_{jj}. \tag{7}$$

By definition of c_{jj} and our choice of numeraire, we know that $d\ln c_{jj} = (1-\beta)\, d\ln P_j$. Thus small changes in the consumer price index satisfy

$$d\ln P_j = -\frac{d\ln \lambda_{jj}}{\varepsilon\beta}.$$

Combining the previous expression with Equation (2), we get

$$d\ln W_j = \frac{d\ln \lambda_{jj}}{\varepsilon\beta}.$$

The rest of the proof is the same as in Appendix A (Step 5).

Case 2: Monopolistic Competition

Using the same logic as in Appendix A (Step 1), we first show that $d\ln Y_j = d\ln R_j = 0$. Note that compared to Appendix A, the first of these two equalities is no longer a trivial implication of R1(TI): whereas total revenues are still $R_j = \sum_{i=1}^{n} X_{ji}$, the total expenditure of the representative agent in country j is now $Y_j \neq \sum_{i=1}^{n} X_{ij}$ since total imports also include expenditures on intermediate goods by firms from country j. Let us start with the case of free entry. Under free entry we know that $Y_j = L_j$, which immediately implies $d\ln Y_j = 0$. By R1(TI), R2(TI), and our Cobb-Douglas assumptions, we also know that total payments to labor are $\beta(1-\zeta)\, R_j + \kappa P_j^{1-\kappa} N_j F_j$, which must be equal to L_j. Since free entry and R2(TI) imply that $P_j^{1-\kappa} N_j F_j = \zeta R_j$, we then have $L_j = [\beta(1-\zeta) + \kappa\zeta]\, R_j$, hence $d\ln R_j = 0$ as well. Let us now turn to the case of restricted entry. Under restricted entry, R1(TI), R2(TI), and our Cobb-Douglas assumptions imply that total payments to labor are $\beta(1-\zeta)\, R_j$, which must be equal to L_j. This immediately implies $d\ln R_j = 0$. By R2(TI) and the budget constraint of the representative agent in country j, we also know that $Y_j = L_j + \zeta R_j$. Since $d\ln R_j = 0$, this implies that $d\ln Y_j = 0$. Like in Appendix A (Step 1), $d\ln Y_j = 0$ immediately implies

$$d\ln W_j = -d\ln P_j. \tag{8}$$

The next part of the proof follows closely Steps 2 through 4 in Appendix A. Compared to Appendix A, the main difference is that, like under perfect competition before, we now have $d\ln c_j \neq 0$. Using $\alpha_{ij}^{*} \equiv \sigma^{\frac{\sigma}{1-\sigma}}\,(\sigma-1)\,\frac{P_j}{c_{ij}}\left[\frac{\xi_{ij} h_{ij}(c_j, c_j)}{R_j}\right]^{1/(1-\sigma)}$ together with the fact that $d\ln R_j =$

0, and following the same logic as in Appendix A (Step 2), small changes in the consumer price index satisfy

$$d\ln P_j = \sum_{i=1}^{n}\left(\frac{\lambda_{ij}}{1-\sigma-\gamma_j}\right)\left[\left(1-\sigma-\gamma_{ij}\right)\left(d\ln\tau_{ij}+d\ln c_i\right)\right. \\ \left. +\gamma_{ij}\left[\frac{d\ln\xi_{ij}}{1-\sigma}+\frac{\partial\ln h_{ij}(c_i,c_j)}{\partial\ln c_i}\frac{d\ln c_i}{1-\sigma}+\frac{\partial\ln h_{ij}(c_i,c_j)}{\partial\ln c_j}\frac{d\ln c_j}{1-\sigma}\right]+d\ln N_i\right], \tag{9}$$

where γ_{ij} and γ_j are given by the same expressions as in Appendix A. Similarly, by the same logic as in Appendix A (Step 3), small changes in expenditure shares satisfy

$$d\ln\lambda_{ij}-d\ln\lambda_{jj} = \left(1-\sigma-\gamma_{ij}\right)\left(d\ln\tau_{ij}+d\ln c_i-d\ln c_j\right) \\ +\gamma_{ij}\left(\frac{\partial\ln h_{ij}(c_i,c_j)}{\partial\ln c_i}\frac{1}{1-\sigma}\right)\left(d\ln c_i-d\ln c_j\right) \\ +\frac{\gamma_{ij}d\ln\xi_{ij}}{1-\sigma}+\left(\gamma_{ij}-\gamma_{jj}\right)d\ln\alpha^*_{jj}+d\ln N_i-d\ln N_j. \tag{10}$$

Combining the previous expression with Equation (9) and noting that $\frac{\partial\ln h_{ij}(c_i,c_j)}{\partial\ln c_i}+\frac{\partial\ln h_{ij}(c_i,c_j)}{\partial\ln c_j}=1$, by the assumption that $h(\cdot)$ is homogeneous of degree 1, we then get

$$d\ln P_j = \sum_{i=1}^{n}\left(\frac{\lambda_{ij}}{1-\sigma-\gamma_j}\right)\left[d\ln\lambda_{ij}-d\ln\lambda_{jj}-\left(\gamma_{ij}-\gamma_{jj}\right)d\ln\alpha^*_{jj}\right. \\ \left. +\left(1-\sigma+\frac{\sigma\gamma_{ij}}{1-\sigma}\right)d\ln c_j+d\ln N_j\right]. \tag{11}$$

Following the same logic as in Appendix A (Step 4), it is easy to check that Equation (10) and R3(TI) imply $\gamma_{ij}=1-\sigma-\varepsilon$ for all i. Combining this observation with Equation (5), we obtain

$$d\ln P_j = \frac{-d\ln\lambda_{jj}+d\ln N_j}{\varepsilon}+\left[\frac{1-\sigma-\sigma\varepsilon}{\varepsilon\left(1-\sigma\right)}\right]d\ln c_j.$$

By definition of c_j and our choice of numeraire, we know that $d\ln c_j=(1-\beta)\,d\ln P_j$. Thus small changes in the consumer price index satisfy

$$d\ln P_j = \frac{-d\ln\lambda_{jj}+d\ln N_j}{\varepsilon\beta-(1-\beta)\left(\frac{\varepsilon}{\sigma-1}+1\right)}. \tag{12}$$

Finally, by the same logic as in Appendix A (Step 5), we must have $d\ln N_j = -(1-\kappa)\,d\ln P_j$ under free entry (since $d\ln R_j = 0$); and $d\ln N_j = 0$ under restricted entry (since $N_j = \overline{N}_j$). Combining these observations with Equations (8) and (12), we obtain

$$d\ln W_j = \frac{d\ln\lambda_{jj}}{\varepsilon\beta-(1-\beta)\left(\frac{\varepsilon}{\sigma-1}+1\right)}\text{, under monopolistic competition with restricted entry,}$$

$$d\ln W_j = \frac{d\ln\lambda_{jj}}{\varepsilon\beta-(1-\beta)\left(\frac{\varepsilon}{\sigma-1}+1\right)+(1-\kappa)}\text{, under monopolistic competition with free entry.}$$

The last part of the proof is the same as under perfect competition. **QED** ■