Theory Appendix for "Market Penetration Costs and the New Consumers Margin in International Trade"*

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Abstract

This is an online theoretical appendix supplementing the paper "Market Penetration Costs and the New Consumers Margin in International Trade."

Contents

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Part I

Theoretical Results

1 General Marketing Technology

In this section we derive a set of results for a marketing technology that is more general than the one presented in Arkolakis (2008). The purpose is to illustrate a general set of conditions under which you can have an equilibrium behavior that resembles the one considered in that paper. However, we will abstract from general equilibrium considerations and focus on the behavior of the individual firm.

1.1 The Setup

We will retain the same notation as in the main text of the paper. The entry cost $f(n_{ij})$ into a market depends on the fraction of the consumers $n_{ij} \in [0,1]$ reached. The following assumptions are made concerning $f(n_{ij})$ (also $f(0) = 0$ is an auxiliary assumption):

Assumption 1 $f'(n_{ij}) > 0 \; \forall n_{ij} \in [0,1]$

Assumption 2 $f''(n_{ij}) > 0 \; \forall n_{ij} \in [0,1]$

Assumption 3 $\lim_{n_{ij} \nearrow 1} f'(n_{ij}) = \infty$.

These assumptions are satisfied by the function presented in Arkolakis (2008), but also hold for other functions, e.g. $f(n) = \frac{1}{1-n} - 1$. Note that assumptions 1 and 2 imply the existence of one-to-one increasing functions $f^{-1}(\cdot)$ and $f'^{-1}(\cdot)$.

Following the setup in the main paper, which uses a Dixit-Stiglitz demand function, and using the optimal pricing rules of the firm, the profits for a firm with productivity ϕ reaching n_{ij} fraction of consumers in market j are given by

$$\pi_{ij}(\phi) = \underbrace{n_{ij} \frac{\left(\tilde{\sigma} \frac{w_i \tau_{ij}}{\phi}\right)^{1-\sigma}}{P_j^{1-\sigma}} \frac{X_j}{\sigma}}_{\text{variable profits}} - \underbrace{f(n_{ij})}_{\text{entry costs}} . \tag{1}$$

where $\tilde{\sigma} = \sigma / (\sigma - 1)$ is the constant markup, σ is the elasticity of substitution among varieties of the good, τ_{ij} is the iceberg transportation cost, w_i is the wage in country i (the exporter), X_j

is the market size of the (importing) country j and finally P_j is the Dixit-Stiglitz price index.

For an interior solution the FOC with respect to n_{ij} is:

$$\frac{\left(\tilde{\sigma}\frac{w_i \tau_{ij}}{\phi}\right)^{1-\sigma}}{P_j^{1-\sigma}} \frac{X_j}{\sigma} = f'\left(n_{ij}\right) \text{ if } n_{ij} \in (0,1) \ . \tag{2}$$

Notice that if

$$\frac{\left(\tilde{\sigma}\frac{w_i \tau_{ij}}{\phi}\right)^{1-\sigma}}{P_j^{1-\sigma}} \frac{X_j}{\sigma} \leq f'\left(0\right) \ , \tag{3}$$

then $n_{ij} = 0$ so that an interior solution does not exist for such ϕ's. Additionally, Assumption 3 guarantees there does not exist a $\phi \in [0, \infty)$ such that $n_{ij} \geq 1$; i.e. no firm will reach all consumers, regardless of how great its productivity.

1.2 Entry and Firm Size

Entry We can solve for n_{ij} by inverting equation (2). Notice that Assumptions 2 and 3 for the function f and the inverse function theorem imply that $f'^{-1}(x)$ increases and approaches 1 as x tends toward infinity. Since $\sigma > 1$, $\frac{\left(\tilde{\sigma}\frac{w_i \tau_{ij}}{\phi}\right)^{1-\sigma}}{P_j^{1-\sigma}} \frac{X_j}{\sigma}$ is increasing in ϕ, so $f'^{-1}\left(\frac{\left(\tilde{\sigma}\frac{w_i \tau_{ij}}{\phi}\right)^{1-\sigma}}{P_j^{1-\sigma}} \frac{X_j}{\sigma}\right)$ is increasing in ϕ.

From the above discussion it directly follows that there exists a threshold productivity ϕ_{ij}^* such that:

$$n_{ij}\left(\phi\right) = f'^{-1}\left(\left(\frac{\phi}{\phi_{ij}^*}\right)^{\sigma-1} f'\left(0\right)\right) \text{ if } \phi \in \left(\phi_{ij}^*, \infty\right) \tag{4a}$$

$$n_{ij}\left(\phi\right) = 0 \text{ if } \phi \in \left(0, \phi_{ij}^*\right] \ , \tag{4b}$$

where ϕ_{ij}^* is defined by:

$$\frac{\left(\tilde{\sigma}\frac{w_i \tau_{ij}}{\phi_{ij}^*}\right)^{1-\sigma}}{P_j^{1-\sigma}} \frac{X_j}{\sigma} = f'\left(0\right) \Leftrightarrow \left(\phi_{ij}^*\right)^{\sigma-1} = \frac{f'\left(0\right)}{\frac{X_j}{\sigma} \frac{\left(\tilde{\sigma} w_i \tau_{ij}\right)^{1-\sigma}}{P_j^{1-\sigma}}} \ . \tag{5}$$

Sales The sales of a firm with productivity ϕ are:

$$y_{ij}\left(\phi\right) = n_{ij}\left(\phi\right) \frac{\left(\tilde{\sigma}\frac{w_i \tau_{ij}}{\phi}\right)^{1-\sigma}}{P_j^{1-\sigma}} X_j \ . \tag{6}$$

Substituting into (5), we can rewrite the sales of the firms as:

$$y_{ij}(\phi) = f'^{-1}\left(\frac{\left(\tilde{\sigma}\frac{w_i\tau_{ij}}{\phi}\right)^{1-\sigma}}{P_j^{1-\sigma}}\frac{X_j}{\sigma}\right)\frac{\left(\tilde{\sigma}\frac{w_i\tau_{ij}}{\phi}\right)^{1-\sigma}}{P_j^{1-\sigma}}X_j \tag{7a}$$

$$= \sigma f'^{-1}\left(\left(\frac{\phi}{\phi_{ij}^*}\right)^{\sigma-1}f'(0)\right)\left(\frac{\phi}{\phi_{ij}^*}\right)^{\sigma-1}f'(0) \equiv h\left(\frac{\phi}{\phi_{ij}^*}\right)\ . \tag{7b}$$

Given Assumptions 1-3, $h\left(\frac{\phi}{\phi_{ij}^*}\right)$ has the following properties:

a) Sales tend to 0 as $\phi \to \phi_{ij}^*$. This can be seen from $(7b)$, noting that $h\left(\frac{\phi_{ij}^*}{\phi_{ij}^*}\right) = h(1) = 0$ since $f'^{-1}(f'(0)) = 0$. Since $f'^{-1}(\cdot)$ is increasing and continuous, $h\left(\frac{\phi}{\phi_{ij}^*}\right)$ is increasing and continuous in ϕ, so the result is immediate. Also, more productive firms sell more.

b) Sales tend to ∞ as $\phi \to \infty$. Notice that $\left(\frac{\phi}{\phi_{ij}^*}\right)^{\sigma-1}$ goes to infinity as ϕ goes to infinity. From Assumption 3, $f'^{-1}\left(\left(\frac{\phi}{\phi_{ij}^*}\right)^{\sigma-1}f'(0)\right)$ goes to 1 as ϕ goes to infinity, so the result is immediate from $(7b)$.

1.3 Growth of Sales

A more subtle question is how sales of firms of different productivities respond differentially to changes in the trade costs, which can be written purely as a function of changes in ϕ_{ij}^*. Specifically, do smaller firms increase their sales more than larger firms when trade costs decline? In what follows, we will show that they do, provided certain reasonable conditions are satisfied.

First note that as variable trade costs fall equation (5) implies that ϕ_{ij}^* falls; i.e. less productive firms can trade. For simplicity, we focus on the effect of a change in ϕ_{ij}^* on sales. Taking logs of $(7b)$, we have that:

$$\ln y_{ij} = \ln\sigma + \ln f'^{-1}\left(f'(0)\left(\frac{\phi}{\phi_{ij}^*}\right)^{\sigma-1}\right) + (\sigma-1)\ln\left(\frac{\phi}{\phi_{ij}^*}\right) + \ln f'(0)\ , \tag{8}$$

so that

$$\frac{\partial\ln y_{ij}}{\partial\ln\phi_{ij}^*} = \frac{\partial\ln f'^{-1}\left(f'(0)\left(\frac{\phi}{\phi_{ij}^*}\right)^{\sigma-1}\right)}{\partial\ln\phi_{ij}^*} - (\sigma-1)\ . \tag{9}$$

Consider $\phi \geq \phi_{ij}^*$. Using the chain rule and defining $x \equiv f'(0) \left(\frac{\phi}{\phi_{ij}^*} \right)^{\sigma - 1}$:

$$\begin{aligned} \frac{\partial \ln f'^{-1}(x)}{\partial \ln \phi_{ij}^*} &= \frac{\partial \ln f'^{-1}(x)}{\partial \ln x} \frac{\partial \ln x}{\partial \ln \phi_{ij}^*} && (10) \\ &= -\frac{\partial f'^{-1}(x)}{\partial x} \frac{x}{f'^{-1}(x)} (\sigma - 1) \ . && (11) \end{aligned}$$

By the inverse function theorem,

$$\frac{\partial f'^{-1}(x)}{\partial x} = \frac{1}{f''(f^{-1}(x))} \ . \tag{12}$$

Hence,

$$\frac{\partial \ln f'^{-1}(x)}{\partial \ln \phi_{ij}^*} = -\frac{1}{f''(f'^{-1}(x))} \frac{1}{f'^{-1}(x)} x (\sigma - 1) \ . \tag{13}$$

From Assumption 2, $f''(\cdot) > 0$ and $f'^{-1}(\phi) > 0$ if $\phi \geq \phi_{ij}^*$, so by (9), $\frac{\partial \ln y_{ij}}{\partial \ln \phi_{ij}^*} < 0$, i.e. a fall in ϕ_{ij}^* (such as from a fall in transportation costs) results in an increase in sales. How does such an effect change with firm size? Since x is monotonically increasing in ϕ, smaller firms' (i.e. firms with lower productivity ϕ) sales will increase more to a fall in ϕ_{ij}^* than larger firms' sales if and only if:

$$\frac{\partial}{\partial x} \left(\frac{x}{f''(f'^{-1}(x)) f'^{-1}(x)} \right) < 0 \ . \tag{14}$$

Note that:

$$\begin{aligned} &\frac{\partial}{\partial x} \left(\frac{x}{f''(f'^{-1}(x)) f'^{-1}(x)} \right) \\ &= \frac{1}{f''(f'^{-1}(x)) f'^{-1}(x)} - \frac{x \left(f'''(f'^{-1}(x)) f'^{-1}(x) f'^{-1\prime}(x) + f''(f'^{-1}(x)) f'^{-1\prime}(x) \right)}{\left[\left(f''(f'^{-1}(x)) f'^{-1}(x) \right) \right]^2} && (15a) \\ &= \frac{f''(f'^{-1}(x)) f'^{-1}(x) - x \left(f'''(f'^{-1}(x)) f'^{-1}(x) f'^{-1\prime}(x) + f''(f'^{-1}(x)) f'^{-1\prime}(x) \right)}{\left[\left(f''(f'^{-1}(x)) f'^{-1}(x) \right) \right]^2} \ . && (15b) \end{aligned}$$

Hence

$$\begin{aligned} \frac{\partial}{\partial x} \left(\frac{x}{f''(f'^{-1}(x)) f'^{-1}(x)} \right) &< 0 \Leftrightarrow && (16a) \\ x \left(f'''\left(f'^{-1}(x)\right) f'^{-1}(x) f'^{-1\prime}(x) + f''\left(f'^{-1}(x)\right) f'^{-1\prime}(x) \right) &> f''\left(f'^{-1}(x)\right) f'^{-1}(x) \Leftrightarrow && (16b) \\ x f'^{-1\prime}(x) \left(\frac{f'''(f'^{-1}(x))}{f''(f'^{-1}(x))} + \frac{1}{f'^{-1}(x)} \right) &> 1 \ . && (16c) \end{aligned}$$

Using equation (12) equation (16c) simplifies to:

$$x \left(\frac{f'''(f'^{-1}(x))}{\left[f''(f'^{-1}(x)) \right]^2} + \frac{1}{f'^{-1}(x) f''(f^{-1}(x))} \right) > 1 \ , \tag{17}$$

which is the condition so that the growth is higher for firms with smaller initial size.

Note that $\frac{1}{f'^{-1}(x) f''(f^{-1}(x))} > 0$ since $f'^{-1}(x) > 0$ and $f''(f^{-1}(x)) > 0$. Hence, a sufficient condition for (17) to be satisfied is:

$$x \frac{f'''(f'^{-1}(x))}{[f''(f'^{-1}(x))]^2} > 1 \Leftrightarrow \tag{18a}$$

$$\frac{\partial f''(f'^{-1}(x))}{\partial x} \frac{x}{f''(f'^{-1}(x))} > 1 \Leftrightarrow \tag{18b}$$

$$\frac{\partial \ln f''(f^{-1}(x))}{\partial \ln x} > 1 , \tag{18c}$$

i.e. the elasticity of the second derivative of the cost function with respect to productivity of the firm needs to be bigger than one. Notice that this condition requires $f''' > 0$. Notice also that

$$n = f'^{-1}(x) \implies f'(n) = x , \tag{19}$$

so that the above condition can be written as

$$\frac{\partial \ln f''(n)}{\partial \ln f'(n)} > 1 = \frac{f''' f'}{[f'']^2} > 1 . \tag{20}$$

Using (19) we can rewrite condition in equation (17) as

$$\frac{f'(n) f'''(n)}{[f''(n)]^2} + \frac{f'(n)}{n f''(n)} > 1 . \tag{21}$$

In the appendix of this section we show that an identical condition is required if we are trying to explain asymmetries in growth with a convex production cost function (in a perfect competition environment). A somewhat similar condition is necessary and sufficient for growth of younger firms in the Jovanovic (1982) model.

We will now illustrate some examples of particular functions. The function that is used in the main text of Arkolakis (2008) is $f(n) = \frac{1-(1-n)^{1-\beta}}{1-\beta}$ so that

$$\frac{f''' f'}{[f'']^2} = \frac{-\beta(-\beta-1)(1-n)^{-\beta-2}(1-n)^{-\beta}}{\beta^2 (1-n)^{-2\beta-2}} = \frac{\beta+1}{\beta} > 1 .$$

Notice that for the function n^α, using condition (17) we have

$$\begin{aligned} \frac{f'(n) f'''(n)}{[f''(n)]^2} + \frac{f'(n)}{n f''(n)} &= \frac{\alpha n^{\alpha-1} \alpha(\alpha-1)(\alpha-2) n^{\alpha-3}}{[\alpha(\alpha-1) n^{\alpha-2}]^2} + \frac{\alpha n^{\alpha-1}}{n\alpha(\alpha-1) n^{\alpha-2}} \\ &= \frac{(\alpha-2)}{(\alpha-1)} + \frac{1}{(\alpha-1)} = 1 . \end{aligned}$$

It can be verified by assuming $f(n) = n^a$ inside the framework of Arkolakis (2008) that this marketing function implies that, locally, all firms grow at the same rate. However, if n is bounded, due to e.g. saturation, the small firms (the ones with $n < 1$) will grow faster when trade costs decline even with that assumption.

1.4 Appendix to the Section

I now consider the condition for which a production cost function can generate asymmetries in the growth rates of firms of different size. Assume that $c(q)$ is an increasing and convex cost function (so that the production function is increasing and concave). ϕ is the productivity of the firm and p is the competitive price of the good. Assume that a firm takes prices as given and solves:

$$
\begin{aligned}
\max_q \left\{ pq - \frac{c(q)}{\phi} \right\} & \Longrightarrow \\
p &= \frac{c'(q)}{\phi} \Longrightarrow \\
c'^{-1}(p\phi) &= q \, .
\end{aligned}
$$

The inverse function theorem implies

$$
\left(c'^{-1}(p\phi)\right)' = \frac{\phi}{c''\left(c'^{-1}(p\phi)\right)} \, .
$$

The growth rate as a function of a reduction in price (e.g. trade liberalization) is given by

$$
\begin{aligned}
\frac{\partial \ln c'^{-1}(p\phi)}{\partial \ln(p)} &= \frac{p\phi}{c'^{-1}(p\phi)\, c''\left(c'^{-1}(p\phi)\right)} \\
&= \frac{c'(q)}{qc''(q)} \, .
\end{aligned}
$$

To see how this growth rate changes with ϕ we take the derivative of the negative of the above expression with respect to q (since q and ϕ are $1-1$)

$$
\begin{aligned}
-\frac{c''(q)\, qc''(q) - c'(q)\left[c''(q) + qc'''(q)\right]}{\left[qc''(q)\right]^2} &< 0 \Longrightarrow \\
\frac{qc'(q)\, c'''(q)}{c''(q)\, qc''(q)} + \frac{c'(q)\, c''(q)}{c''(q)\, qc''(q)} &> 1 \Longrightarrow \\
\frac{c'(q)\, c'''(q)}{\left[c''(q)\right]^2} + \frac{c'(q)}{qc''(q)} &> 1 \, .
\end{aligned}
$$

This is the same exact condition as for marketing cost function, equation (21), but now in terms of quantities and cost functions.

2 Idiosyncratic Shocks to Entry Costs

This section discusses the implications of adding random shocks to entry costs in a framework of monopolistic competition and heterogeneous firms. The main finding is that a random shock does not alter a number of the main predictions of the monopolistic competition model when the distribution of productivities is Pareto.

2.1 The Setup

Consumer's demand in country j is Constant Elasticity of Substitution (CES) with elasticity σ and with price index P_j and firms produce in a monopolistically competitive environment with constant marginal costs of production. There is a measure of J_i firms in country i. Firms pay a cost to reach foreign consumers that depends on the fraction of consumers reached n_{ij} where the specification of that cost follows the derivations in the main paper (Arkolakis (2008)). We assume that the firm has a separate random shock in its entry cost to each market, a shock which is i.i.d. across firms. Thus, a firm's fixed cost of entry to country j is represented as the product of a certain cost F_j, a random component f_j, and a component that depends on the fraction of consumers reached, n_{ij}. The probability distribution of f_j is the same across destination markets so that $f_j \sim G_f$. We will denote by $M_{ij}\left(f_j\right)$ the measure of firms from i with a shock f_j that enter in country j. Each firm in country i receives a productivity draw $\phi \sim G_{i,\phi}$, where it is assumed that $G_{i,\phi}$ is a Pareto distribution and where productivity determines the marginal cost of production. There is an iceberg transportation cost $\tau_{ij} \geq 1$ for a firm in country i to sell to country j.

These assumptions imply that the profits of a given firm ω from country i in country j charging a price p_{ij} and reaching a fraction of consumers n_{ij} and with random entry shock $f_j\left(\omega\right)$ and productivity ϕ are given by

$$\pi_{ij}\left(\phi\left(\omega\right), f_j\left(\omega\right)\right) = n_{ij}\frac{p_{ij}^{1-\sigma}}{P_j^{1-\sigma}}X_j - \frac{\tau_{ij}w_i}{\phi\left(\omega\right)}n_{ij}\frac{p_{ij}^{-\sigma}}{P_j^{1-\sigma}}X_j - F_j f_j\left(\omega\right)\frac{1-\left(1-n_{ij}\right)^{1-\beta}}{1-\beta} ,$$

where w_i is the wage in country i (the exporter) and X_j is the market size of the (importing) country j. Maximization of this expression with respect to p_{ij} gives us

$$\begin{aligned}(1-\sigma)\frac{p_{ij}^{-\sigma}}{P_j^{1-\sigma}}X_j+\sigma\frac{\tau_{ij}w_i}{\phi(\omega)}\frac{p_{ij}^{-\sigma-1}}{P_j^{1-\sigma}}X_j &= 0 \implies \\ p_{ij} &= \frac{\sigma}{\sigma-1}\frac{\tau_{ij}w_i}{\phi}.\end{aligned}$$

Substituting the optimal price to the profit function results in

$$\pi_{ij}(\phi(\omega),f_j(\omega)) = n_{ij}\frac{\left(\left(\frac{\sigma}{\sigma-1}\right)\frac{\tau_{ij}w_i}{\phi(\omega)}\right)^{1-\sigma}}{\sigma P_j^{1-\sigma}}X_j - F_jf_j(\omega)\frac{1-(1-n_{ij})^{1-\beta}}{1-\beta}.$$

Optimizing with respect to n_{ij} gives the entry cutoff rule for market j for firms with entry shock f_j,

$$\phi_{ij}^*(f_j) = \left(\frac{F_jf_j(\omega)}{\frac{\left(\frac{\sigma}{\sigma-1}\tau_{ij}w_i\right)^{1-\sigma}}{\sigma P_j^{1-\sigma}}X_j}\right)^{\frac{1}{\sigma-1}}, \tag{22a}$$

and optimal n_{ij},

$$n_{ij}(\phi,f_j) = 1-\left[\frac{F_jf_j(\omega)}{\frac{\left(\left(\frac{\sigma}{\sigma-1}\right)\frac{\tau_{ij}w_i}{\phi}\right)^{1-\sigma}}{\sigma P_j^{1-\sigma}}X_j}\right]^{1/\beta}. \tag{23}$$

Note that the threshold productivity depends on the realization of the random fixed cost shock f_j; to emphasize this fact, in what follows we refer to ϕ_{ij}^* as $\phi_{ij}^*(f_j)$. We can also calculate the sales of a firm in market j, $y_j(\phi(\omega),f_j(\omega))$ as a function of $\phi_{ij}^*(f_j)$:

$$\begin{aligned}y_j(\phi(\omega),f_j(\omega)) &= \left(1-\left[\frac{F_jf_j(\omega)}{\frac{\left(\left(\frac{\sigma}{\sigma-1}\right)\frac{\tau_{ij}w_i}{\phi}\right)^{1-\sigma}}{\sigma P_j^{1-\sigma}}X_j}\right]^{1/\beta}\right)\frac{\left(\frac{\sigma}{\sigma-1}\frac{\tau_{ij}w_i}{\phi(\omega)}\right)^{1-\sigma}}{P_j^{1-\sigma}}X_j \\ &= \sigma F_jf_j(\omega)\left[\frac{\phi(\omega)^{\sigma-1}}{\phi_{ij}^*(f_j)^{\sigma-1}}-\left(\frac{\phi(\omega)^{\sigma-1}}{\phi_{ij}^*(f_j)^{\sigma-1}}\right)^{(\beta-1)/\beta}\right].\end{aligned} \tag{24}$$

Notice that sales are increasing in productivity.

2.2 Entry and Aggregate Sales

We can calculate the average sales of firms from country i that sell to some country j. Since $\phi\sim G_{i,\phi}$ is Pareto distributed, the probability density for all the firms with a common entry shock f_j conditional on selling to country j can be written as $\mu_{ij}(\phi) = \frac{\theta\phi^{-\theta-1}}{\left(\phi^*_{ij}(f_j)\right)^{-\theta}}$. Average sales are then given by

$$\bar{X}_{ij} = \int\int_{\phi^*_{ij}(f)}^{\infty} y_j\left(\phi(\omega), f_j(\omega)\right)\theta\frac{\phi^{-\theta-1}}{\phi^*_{ij}(f_j)^{-\theta}}d\phi dG_f$$

$$\begin{aligned} &= \int\int_{\phi^*_{ij}(f)}^{\infty} \sigma F_j f_j\left[\frac{\phi^{\sigma-1}}{\phi^*_{ij}(f_j)^{\sigma-1}} - \left(\frac{\phi^{\sigma-1}}{\phi^*_{ij}(f_j)^{\sigma-1}}\right)^{\frac{\beta-1}{\beta}}\right]\theta\frac{\phi^{-\theta-1}}{\phi^*_{ij}(f_j)^{-\theta}}d\phi dG_f \\ &= \left[\frac{\sigma\theta}{\theta-(\sigma-1)} - \frac{\sigma\theta}{\theta-(\sigma-1)\frac{\beta-1}{\beta}}\right]F_j E(f)\ , \end{aligned} \tag{25}$$

where

$$E(f) = \int f dG_f\ .$$

Hence, average sales depend only on the expected value of f_j; they are unaffected by all other characteristics of the distribution. In the appendix for this section we show that the number of firms is related to market size of the destination market by

$$\frac{M_{ij}}{\lambda_{ij}} = \frac{X_j}{\left[\frac{\sigma\theta}{\theta-\sigma+1} - \frac{\sigma\theta}{\theta-(\sigma-1)\frac{\beta-1}{\beta}}\right]F_j E(f)}\ . \tag{26}$$

The immediate conclusion from these two expressions is that the magnitude of aggregate bilateral entry and average sales of these entrants are unaffected by random entry costs (Notice that specifying a bilateral fixed cost F_{ij} , as in Chaney (2008), instead of F_j will leave the results intact).

2.3 Exporters to Individual Destinations and Sales in France

A systematic finding in French trade data of Eaton, Kortum, and Kramarz (2010) is that firms that sell to a greater number of destinations have larger domestic sales on average. The theory outlined above is consistent with this finding since more productive firms export to more destinations and sell more in each destination. Formally, consider the domestic sales of all firms in country i that are sufficiently productive to export to country j; i.e. for each possible cost f_j, consider all firms with productivity at least $\phi^*_{ij}(f_j)$. As above, assume that the productivity distribution is Pareto, so that the probability density function conditional on exporting to country j for a given f_j is $\mu_{ij}(\phi; f_j) = \theta\phi^{-\theta-1} / \left(\phi_{ij}^*(f_j)\right)^{-\theta}$. Thus, average domestic sales conditional on selling to market j are

$$\int\int\int_{\phi_{ij}^*(f_j)}^{\infty} F_i f_i \sigma \left(\frac{\phi^{\sigma-1}}{\left(\phi_{ii}^*(f_i)\right)^{\sigma-1}} - \left(\frac{\phi^{\sigma-1}}{\left(\phi_{ij}^*(f_i)\right)^{\sigma-1}} \right)^{\frac{\beta-1}{\beta}} \right) \frac{\theta\phi^{-\theta-1}}{\left(\phi_{ij}^*(f_j)\right)^{-\theta}} d\phi dG_f dG_f$$

$$\frac{\sigma\theta F_i}{1-\sigma+\theta} \int\int f_i \left[\left(\frac{\phi_{ij}^*(f_j)}{\phi_{ii}^*(f_i)} \right)^{\sigma-1} - \left(\frac{\phi_{ij}^*(f_j)}{\phi_{ii}^*(f_i)} \right)^{\frac{\beta-1}{\beta}} \right] dG_f dG_f \ .$$

This expression involves both the entry thresholds for country j and country i, and these thresholds depend on the entry shocks f_j and f_i. In the appendix of this section we show that the ratio $\left(\frac{\phi_{ij}^*(f_j)}{\phi_{ii}^*(f_i)}\right)^{\sigma-1}$ is given by $\left(\frac{M_{ii}}{M_{ij}}\right)^{\frac{\sigma-1}{\theta}} \frac{f_j}{f_i}$, where M_{ij} is the measure of firms in country i exporting to country j, so that the expression above equals to

$$\begin{aligned}
& \frac{\sigma\theta F_i}{(\theta-\sigma+1)} \int\int f_i \left[\left(\frac{M_{ii}}{M_{ij}}\right)^{\frac{\sigma-1}{\theta}} - \left(\frac{M_{ii}}{M_{ij}}\right)^{\frac{\sigma-1}{\theta}\frac{\beta-1}{\beta}} \right] \frac{f_j}{f_i} dG_f dG_f \ . \\
= \ & \frac{\sigma\theta F_i}{(\theta-\sigma+1)} \left[\left(\frac{M_{ii}}{M_{ij}}\right)^{\frac{\sigma-1}{\theta}} - \left(\frac{M_{ii}}{M_{ij}}\right)^{\frac{\sigma-1}{\theta}\frac{\beta-1}{\beta}} \right] E(f) \\
= \ & \bar{X}_{ii} \left[\left(\frac{M_{ii}}{M_{ij}}\right)^{\frac{\sigma-1}{\theta}} - \left(\frac{M_{ii}}{M_{ij}}\right)^{\frac{\sigma-1}{\theta}\frac{\beta-1}{\beta}} \right],
\end{aligned}$$

where we used the expression for average sales given by (25). If $M_{ii} > M_{ij}$ (i.e. if more French firms produce domestically than export to country j), those firms that export to country j will have higher than average sales domestically. Notice that this expression is not influenced by idiosyncratic entry shocks at all.

2.4 Normalized Exporting Intensity

Define the normalized export intensity for a given firm as the ratio of its relative export sales to country j to its relative domestic sales. Normalized export intensity for a firm with productivity

ϕ and entry shocks f_i and f_j is given by

$$\begin{aligned}
\frac{y_{ij}\left(\phi\left(\omega\right),f_j\left(\omega\right)\right)/\bar{X}_{ij}}{y_{ii}\left(\phi\left(\omega\right),f_i\left(\omega\right)\right)/\bar{X}_{ii}} &= \frac{\dfrac{n_{ij}\frac{\left(\frac{\sigma}{\sigma-1}\frac{\tau_{ij}w_i}{\phi}\right)^{1-\sigma}}{P_j^{1-\sigma}}X_j}{\frac{\sigma\theta}{\theta-\sigma+1}F_jE(f)}}{\dfrac{n_{ij}\frac{\left(\frac{\sigma}{\sigma-1}\frac{w_i}{\phi}\right)^{1-\sigma}}{P_i^{1-\sigma}}X_i}{\frac{\sigma\theta}{\theta-\sigma+1}F_iE(f)}} \\
&= \frac{f_j}{f_i}\frac{\left(\phi_{ii}^*\left(f_i\right)\right)^{\sigma-1}}{\left(\phi_{ij}^*\left(f_j\right)\right)^{\sigma-1}}\frac{\left(1-\left(\frac{\phi^{\sigma-1}}{\left(\phi_{ij}^*(f_j)\right)^{\sigma-1}}\right)^{\frac{-1}{\beta}}\right)}{\left(1-\left(\frac{\phi^{\sigma-1}}{\left(\phi_{ii}^*(f_i)\right)^{\sigma-1}}\right)^{\frac{-1}{\beta}}\right)} \\
&= \left(\frac{M_{ij}}{M_{ii}}\right)^{\frac{\sigma-1}{\theta}}\frac{\left(1-\left(\frac{\phi^{\sigma-1}}{\left(\phi_{ij}^*(f_j)\right)^{\sigma-1}}\right)^{\frac{-1}{\beta}}\right)}{\left(1-\left(\frac{\phi^{\sigma-1}}{\left(\phi_{ii}^*(f_i)\right)^{\sigma-1}}\right)^{\frac{-1}{\beta}}\right)}.
\end{aligned}$$

Notice that we used the relationship (22a) and the fact that $\left(\frac{\phi_{ij}^*(f_j)}{\phi_{ii}^*(f_i)}\right)^{\sigma-1}$ is equal to $\left(\frac{M_{ii}}{M_{ij}}\right)^{\frac{\sigma-1}{\theta}}\frac{f_j}{f_i}$ (proven in the appendix of this section). Using this fact again,

$$\frac{y_{ij}\left(\phi\left(\omega\right),f_j\left(\omega\right)\right)/\bar{X}_{ij}}{y_{ii}\left(\phi\left(\omega\right),f_i\left(\omega\right)\right)/\bar{X}_{ii}} = \left(\frac{M_{ij}}{M_{ii}}\right)^{\frac{\sigma-1}{\theta}}\frac{1-\left(\frac{\phi^{\sigma-1}}{\left(\phi_{ij}^*(f_j)\right)^{\sigma-1}}\right)^{\frac{-1}{\beta}}}{1-\left(\frac{M_{ij}}{M_{ii}}\right)^{\frac{1}{\beta}\frac{\sigma-1}{\theta}}\left(\frac{f_j}{f_i}\frac{\phi^{\sigma-1}}{\left(\phi_{ij}^*(f_j)\right)^{\sigma-1}}\right)^{\frac{-1}{\beta}}}.$$

Notice that the distribution of f_j/f_i is by assumption independent of j. Notice also that whereas a given f_j implies a different $\phi_{ij}^*\left(f_j\right)$ for each country j, the distribution of $\phi^\theta/\left(\phi_{ij}^*\left(f_j\right)\right)^\theta$ is always Pareto. This analysis means that the distributions of both $\frac{f_j}{f_i}\frac{\phi^{\sigma-1}}{\left(\phi_{ij}^*(f_j)\right)^{\sigma-1}}$ and $\frac{\phi^{\sigma-1}}{\left(\phi_{ij}^*(f_j)\right)^{\sigma-1}}$ are independent of the destination country j. Of course, for different specification of the distributions of f's, the levels of $\frac{f_j}{f_i}\frac{\phi^{\sigma-1}}{\left(\phi_{ij}^*(f_j)\right)^{\sigma-1}}$, $\frac{\phi^{\sigma-1}}{\left(\phi_{ij}^*(f_j)\right)^{\sigma-1}}$ could be different for a given percentile of sales (but still the same across each destination country j, for a given percentile within the country). The calibration of Arkolakis (2008) is based on choosing a certain percentile of sales for each country j, and looking at the variations of normalized exporting intensity as the number of entrants, M_{ij}, changes for each country j.

The above imply that choosing different distributions for f's should only minorly affect the calibrated values of β obtained in Arkolakis (2008): The calibration is heavily based on variations of normalized exporting intensity (and normalized average sales, see above) across j's, caused entirely by changes in M_{ij}/M_{ii}, to identify β (and $\tilde{\theta}$). Thus, the fact that the level of $\frac{f_j}{f_i}\frac{\phi^{\sigma-1}}{\left(\phi^*_{ij}(f_j)\right)^{\sigma-1}}$ will be different for different distributions of f's means that little on that calibration hinges on the specification of the distribution of f's.

2.5 Appendix to the Section

We start by computing the market shares of firms from country i in country j. We use the measure of firms that would enter in market j if they had shock f_j,

$$M_{ij}\left(f_j\right) = J_i\left(b_i\right)^{\theta} / \left(\phi^*_{ij}\left(f_j\right)\right)^{\theta} \ . \tag{27}$$

Using equation (42) that gives the sales of an individual firm, we have that the market share of country i to country j are given by

$$\begin{aligned}
\lambda_{ij} &= \frac{\int M_{ij}\left(f_j\right)\int_{\phi^*_{ij}(f_j)}\sigma f_j\left[\frac{\phi^{\sigma-1}}{\left(\phi^*_{ij}(f_j)\right)^{\sigma-1}} - \left(\frac{\phi^{\sigma-1}}{\left(\phi^*_{ij}(f_j)\right)^{\sigma-1}}\right)^{\frac{\beta-1}{\beta}}\right]\frac{\theta\phi^{-\theta-1}}{\left(\phi^*_{ij}(f_j)\right)^{-\theta}}d\phi dG_f}{\sum_{\upsilon}\int M_{\upsilon j}\left(f_j\right)\int_{\phi^*_{\upsilon j}(f_j)}\sigma f_j\left[\frac{\phi^{\sigma-1}}{\left(\phi^*_{\upsilon j}(f_j)\right)^{\sigma-1}} - \left(\frac{\phi^{\sigma-1}}{\left(\phi^*_{\upsilon j}(f_j)\right)^{\sigma-1}}\right)^{\frac{\beta-1}{\beta}}\right]\frac{\theta\phi^{-\theta-1}}{\left(\phi^*_{\upsilon j}(f_j)\right)^{-\theta}}d\phi dG_f} \\
&= \frac{\int J_i\left(b_i\right)^{\theta} / \left(\phi^*_{ij}\left(f_j\right)\right)^{\theta} dG_f}{\sum_{\upsilon}\int J_{\upsilon}\left(b_{\upsilon}\right)^{\theta} / \left(\phi^*_{\upsilon j}\left(f_j\right)\right)^{\theta} dG_f} \\
&= \frac{J_i\left(b_i\right)^{\theta}\left(\tau_{ij}w_i\right)^{-\theta} E\left(f^{-\theta/(\sigma-1)}\right)}{\sum_{\upsilon} J_{\upsilon}\left(b_{\upsilon}\right)^{\theta}\left(\tau_{\upsilon j}w_{\upsilon}\right)^{-\theta} E\left(f^{-\theta/(\sigma-1)}\right)} \ .
\end{aligned}$$

cancelling out the expectations and letting $b_i = 1\ \forall i$ for the rest of our analysis we have

$$\lambda_{ij} = \frac{J_i\left(\tau_{ij}w_i\right)^{-\theta}}{\sum_{\upsilon} J_{\upsilon}\left(\tau_{\upsilon j}w_{\upsilon}\right)^{-\theta}} \ . \tag{28}$$

The total sales of country i to country j can be represented as[1]

$$X_{ij} = \lambda_{ij}X_j \implies$$

[1]Notice that these derivations imply that the total number of available varieties is given by

$$\begin{aligned}
N_j &= \sum_i N_{ij} \\
&= \sum_i \lambda_{ij}\frac{X_j}{\left[\frac{\sigma\theta}{\theta-\sigma+1} - \frac{\sigma\theta}{\theta-(\sigma-1)\frac{\beta-1}{\beta}}\right]F_j\int f_j dG_f} \\
&= \frac{X_j}{\left[\frac{\sigma\theta}{\theta-\sigma+1} - \frac{\sigma\theta}{\theta-(\sigma-1)\frac{\beta-1}{\beta}}\right]F_j\int f_j dG_f}
\end{aligned}$$

$$M_{ij} = \frac{\lambda_{ij} X_j}{\bar{X}_{ij}} \implies$$

$$M_{ij} = \frac{\lambda_{ij} X_j}{\left[\frac{\sigma\theta}{\theta-\sigma+1} - \frac{\sigma\theta}{\theta-(\sigma-1)\frac{\beta-1}{\beta}}\right] F_j E\left(f\right)} . \tag{29}$$

The price index in market j is:

$$\begin{aligned} P_j^{1-\sigma} &= \sum_i \int M_{ij}\left(f_j\right) \int p_{ij}\left(\phi\right)^{1-\sigma} n_{ij}\left(\phi, f_j\right) \frac{\theta \phi^{-\theta-1}}{\left(\phi_{ij}^*\left(f_j\right)\right)^{-\theta}} d\phi dG_f \\ &= \sum_i \left(\frac{\sigma}{\sigma-1}\tau_{ij} w_i\right)^{1-\sigma} \left[\frac{\sigma\theta}{\theta-\sigma+1} - \frac{\sigma\theta}{\theta-(\sigma-1)\frac{\beta-1}{\beta}}\right] \int M_{ij}\left(f_j\right) \left(\phi_{ij}^*\left(f_j\right)\right)^{\sigma-1} dG_f \ . \end{aligned}$$

where we have substituted for $n_{ij}\left(\phi, f_j\right)$ from equation (23) and for $p_{ij}\left(\phi\right)$ using the constant markup choice of the firm.

Notice that the total measure of entrants is $\int M_{ij}\left(f_j\right) dG_f$. Replacing for the measure of firms with entry cost f_j that would enter in market j, $M_{ij}\left(f_j\right) = J_i\left(b_i\right)^\theta / \left(\phi_{ij}^*\left(f_j\right)\right)^\theta$ (equation (27)) we have

$$P_j^{1-\sigma} = \sum_i J_i\left(b_i\right)^\theta \left(\frac{\sigma}{\sigma-1}\tau_{ij} w_i\right)^{1-\sigma} \bar{c} \int \left(\phi_{ij}^*\left(f_j\right)\right)^{\sigma-1-\theta} dG_f \ ,$$

where

$$\bar{c} = \frac{\theta}{1+\theta-\sigma} - \frac{\theta}{\theta-(\sigma-1)\frac{\beta-1}{\beta}} \ .$$

Replacing with the definition of $\phi_{ij}^*\left(f_j\right)$, equation (22a) we get

$$\left[\left(\frac{P_j}{\left(\frac{\sigma}{\sigma-1}\tau_{ij} w_i\right)}\right)^{1-\sigma} \frac{F_j}{\frac{1}{\sigma} X_j}\right]^{\frac{\theta}{\sigma-1}} = \bar{c}\left(\frac{F_j}{\frac{1}{\sigma} X_j}\right)\left(\int f_j^{1-\frac{\theta}{\sigma-1}} dG_f\right) \frac{\sum_i J_i\left(b_i\right)^\theta \left(\frac{\sigma}{\sigma-1}\tau_{ij} w_i\right)^{-\theta}}{\left(\frac{\sigma}{\sigma-1}\tau_{ij} w_i\right)^{-\theta}}$$

and using the cutoff rule from equation (22a) and the formula for the market shares, equation (28), we have:

$$\left(\phi_{ij}^*\left(f_j\right)\right)^\theta = \bar{c}\frac{F_j}{\frac{1}{\sigma} X_j}\left(\int f_j^{1-\frac{\theta}{\sigma-1}} dG_f\right) \frac{J_i\left(b_i\right)^\theta}{\lambda_{ij}} \left(f_j\right)^{\frac{\theta}{\sigma-1}} \ .$$

Notice that using (29) we have that

$$\left(\phi_{ij}^*\left(f_j\right)\right)^\theta = \frac{J_i\left(b_i\right)^\theta}{M_{ij}} \frac{\left(\int f_j^{1-\frac{\theta}{\sigma-1}} dG_f\right)}{E\left(f\right)} \left(f_j\right)^{\frac{\theta}{\sigma-1}} \ .$$

Using this last equation we get that

$$\left(\frac{\phi_{ij}^*\left(f_j\right)}{\phi_{ii}^*\left(f_j\right)}\right)^{\sigma-1} = \left(\frac{M_{ii}}{M_{ij}}\right)^{\frac{\sigma-1}{\theta}} \frac{f_j}{f_i} \ . \tag{30}$$

3 Free Entry

In this section we show that the model in Arkolakis (2008) with a predetermined number of potential entrants (following Chaney (2008)) gives identical solutions to a model where there is a free entry of new firms in each country (following Melitz (2003)). The only difference of the free entry setup with the Chaney (2008) setup is that all the profits are accrued to labor in order to pay the fixed costs of entry.

3.1 The Setup

All the assumptions except the one regarding entry of firms is as in the main paper. Consumer preferences are Dixit-Stiglitz with an elasticity σ. A firm in country i with productivity ϕ that is reaching fraction n_{ij} of the consumers of country j and charging price p_{ij} earns profit

$$\pi_{ij}(\phi) = n_{ij}\frac{p_{ij}^{1-\sigma}}{P_j^{1-\sigma}}w_jL_j - w_jL_j\tau_{ij}n_{ij}\frac{p_{ij}^{-\sigma}w_i}{P_j^{1-\sigma}\phi} - w_j^{\gamma}w_i^{1-\gamma}\frac{L_j^{\alpha}}{\psi}\frac{1-(1-n_{ij})^{-\beta+1}}{-\beta+1},$$

where τ_{ij} is the iceberg transportation cost, w_i is the wage in country i (the exporter), w_j is the wage rate and L_j the population of country j, and finally P_j is the Dixit-Stiglitz price index.

The firm's optimal choice of p_{ij}, which is independent of n_{ij}, is determined by the first order condition

$$p_{ij} = \tilde{\sigma}\frac{\tau_{ij}}{\phi}w_i,\ \tilde{\sigma} = \sigma/(\sigma-1).$$

Given this choice of p_{ij}, the firm's profits are

$$\pi_{ij}(\phi) = n_{ij}\frac{\left(\tilde{\sigma}\frac{\tau_{ij}}{\phi}w_i\right)^{1-\sigma}}{P_j^{1-\sigma}\sigma}w_jL_j - w_j^{\gamma}w_i^{1-\gamma}\frac{L_j^{\alpha}}{\psi}\frac{1-(1-n_{ij})^{-\beta+1}}{-\beta+1}.$$

The first order condition for the firm's market penetration choice is then given by

$$n_{ij} = 1-\left(\frac{\left(\tilde{\sigma}\frac{\tau_{ij}}{\phi}w_i\right)^{1-\sigma}}{P_j^{1-\sigma}\sigma}\frac{w_jL_j}{w_j^{\gamma}w_i^{1-\gamma}\frac{L_j^{\alpha}}{\psi}}\right)^{-1/\beta}. \tag{31}$$

Let $\phi_{ij}^* = \sup\{\phi : \pi(\phi) = 0\}$. Clearly, a firm with productivity ϕ_{ij}^* will choose $n_{ij} = 0$. The first order condition for market access thus implies

$$\left(\phi_{ij}^*\right)^{\sigma-1} = \frac{w_j^{\gamma}w_i^{1-\gamma}\frac{L_j^{\alpha}}{\psi}P_j^{1-\sigma}\sigma}{\left(\tilde{\sigma}\tau_{ij}w_i\right)^{1-\sigma}w_jL_j},$$

Substituting this back into (31) yields

$$n_{ij}(\phi) = 1 - \left(\frac{\phi^{\sigma-1}}{\left(\phi^*_{ij}\right)^{\sigma-1}} \right)^{-1/\beta}$$

as the firm's optimal market access, conditional on entering market j. The firm's profits from market j are

$$\begin{aligned} \pi_{ij}(\phi) = & \left[1 - \left(\frac{\phi^{\sigma-1}}{\left(\phi^*_{ij}\right)^{\sigma-1}} \right)^{-1/\beta} \right] \frac{\left(\tilde{\sigma} \frac{\tau_{ij}}{\phi} w_i\right)^{1-\sigma}}{P_j^{1-\sigma}\sigma} w_j L_j \\ & - w_j^{\gamma} w_i^{1-\gamma} \frac{L_j^{\alpha}}{\psi} \frac{\left[1 - \left(\frac{\phi^{\sigma-1}}{\left(\phi^*_{ij}\right)^{\sigma-1}} \right)^{\frac{-1+\beta}{\beta}} \right]}{-\beta+1}. \end{aligned}$$

Firms have to pay a fixed entry cost, f_e, in order to enter the market and receive a productivity draw. The productivity of a new entrant is assumed to be a Pareto random variable with shape parameter $\theta > \sigma - 1$ and location parameter b_i. The distribution function of a new entrant's productivity is therefore $G_i(\phi) = 1 - \frac{b_i^{\theta}}{\phi^{\theta}}$. A firm that receives a productivity draw lower than ϕ^*_{ii} immediately exits the market. In equilibrium, free entry implies that expected profits must be zero:

$$\begin{aligned} & \sum_v \int_{\phi^*_{iv}} \left[1 - \left(\frac{\phi}{\phi^*_{iv}} \right)^{-(\sigma-1)/\beta} \right] \frac{\left(\frac{\sigma}{\sigma-1} \frac{\tau_{iv} w_i}{\phi} \right)^{1-\sigma}}{P_v^{1-\sigma}\sigma} w_v L_v \theta \frac{\left(\phi^*_{iv}\right)^{\theta}}{\phi^{\theta+1}} \frac{\left(\phi^*_{ii}\right)^{\theta}}{\left(\phi^*_{iv}\right)^{\theta}} d\phi \\ & - \sum_v \int_{\phi^*_{iv}} w_v^{\gamma} w_i^{1-\gamma} \frac{L_v^{\alpha}}{\psi} \frac{1 - \left(\frac{\phi^{\sigma-1}}{\left(\phi^*_{iv}\right)^{\sigma-1}} \right)^{\frac{-1+\beta}{\beta}}}{-\beta+1} \theta \frac{\left(\phi^*_{iv}\right)^{\theta}}{\phi^{\theta+1}} \frac{\left(\phi^*_{ii}\right)^{\theta}}{\left(\phi^*_{iv}\right)^{\theta}} d\phi = \frac{w_i f_e}{\frac{b_i^{\theta}}{\left(\phi^*_{ii}\right)^{\theta}}} \Leftrightarrow \\ & \sum_v w_v^{\gamma} w_i^{1-\gamma} \frac{L_v^{\alpha}}{\psi} \left[\frac{\theta}{\theta - \sigma + 1} - \frac{\theta}{\theta - (\sigma-1)\frac{\beta-1}{\beta}} \right] \frac{\left(\phi^*_{ii}\right)^{\theta}}{\left(\phi^*_{iv}\right)^{\theta}} \\ & - \sum_v w_v^{\gamma} w_i^{1-\gamma} \frac{L_v^{\alpha}}{\psi} \frac{(\sigma-1)\frac{1}{\beta}}{\theta - (\sigma-1)\frac{\beta-1}{\beta}} \frac{\left(\phi^*_{ii}\right)^{\theta}}{\left(\phi^*_{iv}\right)^{\theta}} = \frac{w_i f_e}{\frac{b_i^{\theta}}{\left(\phi^*_{ii}\right)^{\theta}}} \Leftrightarrow \\ & \sum_v w_v^{\gamma} w_i^{1-\gamma} \frac{L_v^{\alpha}}{\psi} \left[\frac{(\sigma-1)^2 \frac{1}{\beta}}{\left(\theta - (\sigma-1)\frac{\beta-1}{\beta}\right)(\theta - \sigma + 1)} \right] \frac{\left(\phi^*_{ii}\right)^{\theta}}{\left(\phi^*_{iv}\right)^{\theta}} = \frac{w_i f_e}{\frac{b_i^{\theta}}{\left(\phi^*_{ii}\right)^{\theta}}}. \end{aligned} \tag{32}$$

3.2 Solving for the Equilibrium

The equilibrium number of firms producing in country i, N_i, is determined by the following labor market clearing condition:

$$
\begin{aligned}
& N_i \sum_{\upsilon} w_{\upsilon}^{\gamma} w_i^{-\gamma} \frac{L_{\upsilon}^{\alpha}}{\psi} (\sigma - 1) \left[\frac{\theta}{\theta - \sigma + 1} - \frac{\theta}{\theta - (\sigma - 1) \frac{\beta - 1}{\beta}} \right] \frac{(\phi_{ii}^*)^{\theta}}{(\phi_{i\upsilon}^*)^{\theta}} + N_i \frac{f_e}{1 - G_i(\phi_{ii}^*)} \\
& + \sum_{\upsilon} N_{\upsilon} \gamma \int_{\phi_{\upsilon j}^*} w_i^{1-\gamma} w_{\upsilon}^{\gamma - 1} \frac{L_i^{\alpha}}{\psi} \frac{1 - \left(\frac{\phi^{\sigma-1}}{\left(\phi_{\upsilon j}^*\right)^{\sigma-1}} \right)^{\frac{-1+\beta}{\beta}}}{-\beta + 1} \theta \frac{\left(\phi_{\upsilon j}^*\right)^{\theta}}{\phi^{\theta+1}} \frac{(\phi_{\upsilon\upsilon}^*)^{\theta}}{\left(\phi_{\upsilon j}^*\right)^{\theta}} d\phi \\
& + N_i \sum_{\upsilon} \int_{\phi_{i\upsilon}^*} (1 - \gamma) w_{\upsilon}^{\gamma} w_i^{-\gamma} \frac{L_{\upsilon}^{\alpha}}{\psi} \frac{1 - \left(\frac{\phi^{\sigma-1}}{(\phi_{i\upsilon}^*)^{\sigma-1}} \right)^{\frac{-1+\beta}{\beta}}}{-\beta + 1} \theta \frac{(\phi_{i\upsilon}^*)^{\theta}}{\phi^{\theta+1}} \frac{(\phi_{ii}^*)^{\theta}}{(\phi_{i\upsilon}^*)^{\theta}} d\phi = L_i \Leftrightarrow
\end{aligned}
$$

$$
\begin{aligned}
& N_i \sum_{\upsilon} \frac{w_{\upsilon}^{\gamma}}{w_i^{\gamma}} \frac{L_{\upsilon}^{\alpha}}{\psi} \left[\frac{\theta (\sigma - 1)^2 \frac{1}{\beta}}{(\theta - \sigma + 1) \left(\theta - (\sigma - 1) \frac{\beta - 1}{\beta} \right)} \right] \frac{(\phi_{ii}^*)^{\theta}}{(\phi_{i\upsilon}^*)^{\theta}} + N_i \frac{f_e}{1 - G_i(\phi_{ii}^*)} \\
& + \frac{(\sigma - 1) \frac{1}{\beta}}{\theta - (\sigma - 1) \frac{-1+\beta}{\beta}} \left[\gamma \sum_{\upsilon} N_{\upsilon} \frac{w_i^{\gamma-1}}{w_{\upsilon}^{\gamma-1}} \frac{L_i^{\alpha}}{\psi} \frac{(\phi_{\upsilon\upsilon}^*)^{\theta}}{(\phi_{\upsilon i}^*)^{\theta}} + (1 - \gamma) \sum_{\upsilon} N_i \frac{w_{\upsilon}^{\gamma}}{w_i^{\gamma}} \frac{L_{\upsilon}^{\alpha}}{\psi} \frac{(\phi_{ii}^*)^{\theta}}{(\phi_{i\upsilon}^*)^{\theta}} \right] = L_i, \qquad (33)
\end{aligned}
$$

where the last equivalence follows after substituting the free entry condition, (32). We will use the fact that marketing spending is a constant fraction, m, of sales (see the paper for details). Total income is composed of i) income other than fixed costs, ii) income from fixed costs from exporting activities, iii) income from exporting activities of foreign countries in country i, and the sum of all these equals total spending:

$$
\begin{aligned}
& (1 - m + \gamma m) \sum_{\upsilon} w_{\upsilon}^{\gamma} w_i^{1-\gamma} \frac{L_{\upsilon}^{\alpha}}{\psi} \left[\frac{\theta\sigma}{\theta - \sigma + 1} - \frac{\theta\sigma}{\theta - (\sigma - 1) \frac{\beta - 1}{\beta}} \right] N_i \frac{(\phi_{ii}^*)^{\theta}}{(\phi_{i\upsilon}^*)^{\theta}} + \\
& (1 - \gamma) m \sum_{\upsilon} w_i^{\gamma} w_{\upsilon}^{1-\gamma} \frac{L_i^{\alpha}}{\psi} \left[\frac{\theta\sigma}{\theta - \sigma + 1} - \frac{\theta\sigma}{\theta - (\sigma - 1) \frac{\beta - 1}{\beta}} \right] N_{\upsilon} \frac{(\phi_{\upsilon\upsilon}^*)^{\theta}}{(\phi_{\upsilon i}^*)^{\theta}} \\
& = \sum_{\upsilon} w_i^{\gamma} w_{\upsilon}^{1-\gamma} \frac{L_i^{\alpha}}{\psi} \left[\frac{\theta\sigma}{\theta - \sigma + 1} - \frac{\theta\sigma}{\theta - (\sigma - 1) \frac{\beta - 1}{\beta}} \right] N_{\upsilon} \frac{(\phi_{\upsilon\upsilon}^*)^{\theta}}{(\phi_{\upsilon i}^*)^{\theta}} \Leftrightarrow \\
& \sum_{\upsilon} w_{\upsilon}^{\gamma} w_i^{1-\gamma} \frac{L_{\upsilon}^{\alpha}}{\psi} N_i \frac{(\phi_{ii}^*)^{\theta}}{(\phi_{i\upsilon}^*)^{\theta}} = \sum_{\upsilon} w_i^{\gamma} w_{\upsilon}^{1-\gamma} \frac{L_i^{\alpha}}{\psi} N_{\upsilon} \frac{(\phi_{\upsilon\upsilon}^*)^{\theta}}{(\phi_{\upsilon i}^*)^{\theta}} ,
\end{aligned}
$$

which, substituting into (33) and using (32), yields

$$
\begin{aligned}
&N_i \sum_{\upsilon} w_{\upsilon}^{\gamma} w_i^{-\gamma} \frac{L_{\upsilon}^{\alpha}}{\psi} \left[\frac{\theta\left(\sigma-1\right)^2 \frac{1}{\beta}}{\left(\theta-\sigma+1\right)\left(\theta-\left(\sigma-1\right)\frac{\beta-1}{\beta}\right)} \right] \frac{\left(\phi_{ii}^*\right)^{\theta}}{\left(\phi_{i\upsilon}^*\right)^{\theta}} + N_i \frac{f_e}{1-G_i\left(\phi_{ii}^*\right)} \\
&+ N_i \frac{\left(\sigma-1\right)\frac{1}{\beta}}{\theta-\left(\sigma-1\right)\frac{-1+\beta}{\beta}} \left[\sum_{\upsilon} w_{\upsilon}^{\gamma} w_i^{-\gamma} \frac{L_{\upsilon}^{\alpha}}{\psi} \frac{\left(\phi_{ii}^*\right)^{\theta}}{\left(\phi_{i\upsilon}^*\right)^{\theta}} \right] = L_i \Leftrightarrow \\
&\left(\theta+1\right) N_i \frac{f_e}{\frac{b_i^{\theta}}{\left(\phi_{ii}^*\right)^{\theta}}} + N_i \frac{\theta-\sigma+1}{\sigma-1} \frac{f_e}{\frac{b_i^{\theta}}{\left(\phi_{ii}^*\right)^{\theta}}} = L_i \Leftrightarrow \\
&N_i \frac{\theta\sigma}{\sigma-1} \frac{f_e}{\frac{b_i^{\theta}}{\left(\phi_{ii}^*\right)^{\theta}}} = L_i,
\end{aligned}
$$

which yields the measure of operating firms N_i but also the number of equilibrium entrants $N_i\left(\phi_{ii}^*\right)^{\theta}/b_i^{\theta}$.

Notice that the share of income that goes to labor for the production of the entry cost, $\eta = \left(\sigma-1\right)/\left(\theta\sigma\right)$, equals exactly the share of total income that goes to profits in the model with no free entry in the main paper in Arkolakis (2008). All the remaining equations of the model are identical to the ones of the model with no free entry.

4 Departing from the CES Aggregator Assumption: The Linear Demand Case

In this section, we solve a version of the monopolistic competition model with heterogeneous firms and linear demand (Melitz and Ottaviano (2008)) and characterize firm entry and the distribution of firm sales in individual exporting destinations. We show that whereas the linear demand (a model that features departures from the CES aggregator and non-homothetic demand) has some qualitative properties that are aligned with the data, quantitatively it is not able to match the firm entry patterns and the size distribution of firms.

4.1 The Setup

Assume a measure L_j of identical consumers in each country, where each one of them is endowed with 1 unit of labor and does not value leisure. Preferences of a representative consumer over a continuum of products $\omega \in \Omega$ are given by

$$U_j = \alpha \int_{\Omega_j} q_j^c(\omega)\, d\omega - \frac{1}{2}\gamma \int_{\Omega_j} \left(q_j^c(\omega)\right)^2 d\omega - \frac{1}{2}\eta \left(\int_{\Omega_j} q_j^c(\omega)\, d\omega\right)^2$$

where α, η, γ are all positive and $q^c(\omega)$ is the quantity consumed. The consumer maximizes this utility function subject to the budget constraint

$$\int_{\Omega_j} q_j^c(\omega) p_j(\omega) d\omega = w_j,$$

where w_j is the unit wage and $p_j(\omega)$ is the price of good ω in country j.

The FOCs of the above problem yield $(\forall q^c(\omega) > 0)$:

$$\mu_j p_j(\omega) = \alpha - \gamma q_j^c(\omega) - \eta \int_{\Omega_j} q_j^c(\omega)\, d\omega. \tag{34}$$

where μ_j is the Lagrangian multipliers. Also, we can derive:

$$q_j^c(\omega) = \frac{1}{\gamma}\left(\alpha - \mu_j p_j(\omega) - \eta \int_{\Omega_j} q_j^c(\omega)\, d\omega\right). \tag{35}$$

Let $\Omega_j^* \subset \Omega_j$ represent consumed varieties in country j, and let M_j^c be the measure of this set. Defining:

$$\bar{q}_j^c \equiv \frac{1}{M_j^c}\int_{\Omega_j^*} q_j^c(\omega) d\omega, \qquad \bar{p}_j \equiv \frac{1}{M_j^c}\int_{\Omega_j^*} p_j(\omega) d\omega,$$

and integrating (34) over all $\omega \in \Omega^*$ yields:

$$\begin{aligned} \mu_j \bar{p}_j &= \alpha - \gamma \bar{q}_j^c - \eta M_j^c \bar{q}_j^c \implies \\ \bar{q}_j^c &= \frac{\alpha - \mu_j \bar{p}_j}{\gamma + \eta M_j^c}. \end{aligned}$$

Following (35), demand for variety ω for a country with a continuum of consumers of measure L_j is:

$$\frac{L_j}{\gamma}\left(\alpha - \mu_j p_j(\omega) - \eta \int_{\Omega_j} q_j^c(\omega)\, d\omega\right).$$

We will consider a symmetric equilibrium where all the firms from source country i with productivity ϕ choose the same equilibrium variables. It follows that $q_{ij}(\phi) = 0$ exactly when

$$\mu_j p_{ij}(\phi_{ij}^*) = \mu_j p_{ij}^* \equiv \alpha + \eta M_j^c \mu_j \frac{\bar{p}_j - \alpha}{\gamma + \eta M_j^c} = \frac{\gamma\alpha + \eta M_j^c \bar{p}_j \mu_j}{\gamma + \eta M_j^c}. \tag{36}$$

for some $\phi = \phi_{ij}^*$. Firm ϕ maximizes revenues minus production and shipping cost in each market j:

$$\begin{aligned} \pi_{ij}(\phi) &= \max_{q_{ij}(\phi), p_{ij}(\phi)} p_{ij}(\phi) q_{ij}(\phi) - \tau_{ij}\frac{w_i}{\phi} q_{ij}(\phi) \\ &= \max_{p_{ij}(\phi)} p_{ij}(\phi)\left(\frac{\alpha L_j}{\gamma + \eta M_j^c} - \frac{L_j}{\gamma}\mu_j p_{ij}(\phi) + \frac{\eta L_j}{\gamma}\frac{M_j^c \bar{p}_j \mu_j}{\gamma + \eta M_j^c}\right) \\ &\quad - \tau_{ij}\frac{w_i}{\phi}\left(\frac{\alpha L_j}{\gamma + \eta M_j^c} - \frac{L_j}{\gamma}\mu_j p_{ij}(\phi) + \frac{\eta L_j}{\gamma}\frac{M_j^c \bar{p}_j \mu_j}{\gamma + \eta M_j^c}\right). \end{aligned}$$

The above problem implies the FOC

$$\begin{aligned} \frac{\alpha L_j}{\gamma + \eta M_j^c} - 2\frac{L_j}{\gamma}\mu_j p_{ij}(\phi) + \frac{\eta L_j}{\gamma}\frac{M_j^c \bar{p}_j}{\gamma + \eta M_j^c} + \tau_{ij}\frac{w_i}{\phi}\frac{L_j}{\gamma}\mu_j &= 0 \implies \\ q_{ij}(\phi) &= \frac{L_j}{\gamma}\left(\mu_j p_{ij}(\phi) - \mu_j \tau_{ij}\frac{w_i}{\phi}\right) \end{aligned} \tag{37}$$

The FOCs also imply that:

$$\begin{aligned} \frac{L_j}{\gamma}\left(\mu_j p_{ij}(\phi_{ij}^*) - \mu_j \tau_{ij}\frac{w_i}{\phi_{ij}^*}\right) &= 0 \implies \\ p_{ij}(\phi^*) &= \tau_{ij}\frac{w_i}{\phi_{ij}^*}. \end{aligned}$$

From the FOC we also have

$$\begin{aligned} \frac{\alpha L_j}{\gamma + \eta M_j^c} + \frac{\eta L_j}{\gamma}\frac{M_j^c \bar{p}_j}{\gamma + \eta M_j^c} + \tau_{ij}\frac{w_i}{\phi^*}\frac{L_j}{\gamma}\mu_j &= 2\frac{L_j}{\gamma}\mu_j p_{ij}^* = 2\frac{L_j}{\gamma}\mu_j \tau_{ij}\frac{w_i}{\phi_{ij}^*} \implies \\ \frac{\alpha L_j}{\gamma + \eta M_j^c} + \frac{\eta L_j}{\gamma}\frac{M_j^c \bar{p}_j}{\gamma + \eta M_j^c} &= \frac{L_j}{\gamma}\mu_j \tau_{ij}\frac{w_i}{\phi_{ij}^*} \end{aligned}$$

which can be written as

$$\frac{1}{2}\frac{\gamma}{L_j}\left(\frac{\alpha L_j}{\gamma+\eta M_j^c}+\frac{\eta L_j}{\gamma}\frac{M_j^c\bar{p}_j}{\gamma+\eta M_j^c}+\tau_{ij}\frac{w_i}{\phi}\frac{L_j}{\gamma}\mu_j\right)=\mu_j p_{ij}(\phi)\implies$$

$$p_{ij}(\phi)=\frac{1}{2}\left(p_{ij}^*+\tau_{ij}\frac{w_i}{\phi}\right)\ , \tag{38}$$

and therefore using (39) and (38) the quantity is given by

$$q_{ij}(\phi)=\frac{L_j}{\gamma}\left(\frac{1}{2}\mu_j p_{ij}^*-\frac{1}{2}\mu_j\tau_{ij}\frac{w_i}{\phi}\right)\ . \tag{39}$$

From this point on we will assume the Pareto distribution of productivities of firms so that

$$G_i(\phi)=1-\frac{(b_i)^\theta}{\phi^\theta}\ .$$

The probability density conditional on $\phi\geq\phi_{ij}^*$ is given by

$$\mu_{ij}(\phi)=\theta\frac{\left(\phi_{ij}^*\right)^\theta}{\phi^{\theta+1}}\ , \tag{40}$$

and from now on we also maintain the assumption that

$$\eta=0\ ,$$

which implies that

$$\begin{aligned}\mu_j p_{ij}(\phi_{ij}^*)&=\alpha\implies\\ \mu_j w_i&=\frac{\alpha\phi_{ij}^*}{\tau_{ij}}\ .\end{aligned} \tag{41}$$

4.2 Distribution of Sales

The sales of the firm can be written as

$$p_{ij}(\phi)\,q_{ij}(\phi)=\left(\tau_{ij}\frac{w_i}{\phi_{ij}^*}+\tau_{ij}\frac{w_i}{\phi}\right)\frac{1}{2}\mu_j\frac{L_j}{\gamma}\frac{1}{2}\left(\tau_{ij}\frac{w_i}{\phi_{ij}^*}-\tau_{ij}\frac{w_i}{\phi}\right)\implies$$

$$p_{ij}(\phi)\,q_{ij}(\phi)=\mu_j\frac{L_j}{4\gamma}\left[\left(\tau_{ij}\frac{w_i}{\phi_{ij}^*}\right)^2-\left(\tau_{ij}\frac{w_i}{\phi}\right)^2\right]\ . \tag{42}$$

And using equation (41) in equation (42) we have

$$\begin{aligned}p_{ij}(\phi)\,q_{ij}(\phi)&=\frac{w_iL_j}{4\gamma}(\tau_{ij})^2\left(\frac{\alpha\phi_{ij}^*}{\tau_{ij}}\frac{1}{\left(\phi_{ij}^*\right)^2}-\left(\frac{\tau_{ij}}{\phi}\right)^2\frac{\alpha\phi_{ij}^*}{\tau_{ij}}\right)\\ &=\frac{w_iL_j}{4\gamma}\alpha\tau_{ij}\left(\frac{1}{\phi_{ij}^*}-\frac{\phi_{ij}^*}{\phi^2}\right)\ .\end{aligned}$$

With the use of the Pareto distribution assumption we get a stark prediction about the distribution of sales. In particular, using the Pareto distribution together with (42) we have that $1 - \Pr = \left(\phi_{ij}^{*}/\phi\right)^{\theta}$. Thus, using equation (41), the sales of a firm at a given percentile Pr are given by

$$y_{ij}\left(1-\Pr\right) = \frac{L_j \alpha^2}{4\gamma\mu_j}\left[1-\left(1-\Pr\right)^{2/\theta}\right] . \tag{43}$$

Combining (43) and the expression for average sales, $\bar{X}_{ij}$, equation (45) (derived right below) implies that the distribution of sales normalized by average sales is

$$\frac{y_{ij}\left(1-\Pr\right)}{\overline{X_{ij}}} = \frac{1-\left(1-\Pr\right)^{2/\theta}}{2\frac{1}{(\theta+2)}} , \tag{44}$$

which is independent of importing or exporting country characteristics just as in the model with CES demand.

However, one additional feature that should be pointed out is that as $\Pr \to 1$ the relative sales of the largest exporters do not go to infinity but rather to a constant number. Thus, arbitrary large increases in ϕ do not translate to arbitrarily large increases in relative sales as it would happen in the CES model. The reason for this result is the fact that the linear demand is asymptotically inelastic and not elastic as the CES demand. Nevertheless, when $\phi \to \phi_{ij}^{*}$, $y_{ij}\left(\phi_{ij}^{*}\right) \to 0$ which allows the model to generate firms with tiny sales as in the Arkolakis (2008) model.

4.3 Entry and Aggregate Sales

Average sales are given by integrating the above expression over the conditional pdf, equation (40),

$$\begin{aligned}
\overline{X_{ij}} &= \int_{\phi_{ij}^{*}} \frac{w_i L_j}{4\gamma}\alpha\tau_{ij}\left(\frac{1}{\phi_{ij}^{*}}-\frac{\phi_{ij}^{*}}{\phi^2}\right)\theta\frac{\left(\phi_{ij}^{*}\right)^{\theta}}{\phi^{\theta+1}}d\phi \\
&= \frac{\tau_{ij}}{\phi_{ij}^{*}}\theta\frac{w_i L_j}{4\gamma}\alpha\left(\frac{1}{\theta}-\frac{1}{(\theta+2)}\right) \\
&= \theta\alpha^2\frac{L_j}{\mu_j 4\gamma}\left(\frac{2}{\theta}\frac{1}{(\theta+2)}\right) .
\end{aligned} \tag{45}$$

Notice that the last line implies that average sales per firm, are not source country specific. The number of firms from source country i selling to country j is

$$M_{ij} = J_i \frac{b_i^\theta}{\left(\phi_{ij}^*\right)^\theta} \ ,$$

and therefore total sales are given

$$X_{ij} = J_i \frac{b_i^\theta}{\left(\phi_{ij}^*\right)^\theta} \theta \alpha^2 \frac{L_j}{\mu_j 4\gamma} \left(\frac{2}{\theta} \frac{1}{(\theta+2)} \right) \ , \tag{46}$$

J_i can be determined using free entry and labor market clearing conditions. In the appendix of this section we show that

$$J_i = \frac{L_i}{(\theta+1) f_e} \ , \tag{47}$$

in equilibrium.

We will start first by defining the market share of country i to country j :

$$\lambda_{ij} = \frac{X_{ij}}{\sum_\upsilon X_{\upsilon j}} \ ,$$

where we can use (46) and (47) to write the relationship as

$$\begin{aligned} \lambda_{ij} &= \frac{\frac{L_i}{(\theta+1)f_e} \frac{b_i^\theta}{\left(\phi_{ij}^*\right)^\theta} \theta \alpha^2 \frac{L_j}{\mu_j 4\gamma} \left(\frac{2}{\theta} \frac{1}{(\theta+2)} \right)}{\sum_\upsilon \frac{L_\upsilon}{(\theta+1)f_e} \frac{b_\upsilon^\theta}{\left(\phi_{\upsilon j}^*\right)^\theta} \theta \alpha^2 \frac{L_j}{\mu_j 4\gamma} \left(\frac{2}{\theta} \frac{1}{(\theta+2)} \right)} \\ &= \frac{L_i b_i^\theta \left(w_i \tau_{ij}\right)^{-\theta}}{\sum_\upsilon L_\upsilon b_\upsilon^\theta \left(w_\upsilon \tau_{\upsilon j}\right)^{-\theta}} \ . \end{aligned} \tag{48}$$

To examine the relationship between market share and number of firms, use the identity

$$\begin{aligned} \lambda_{ij} w_j L_j &= X_{ij} \implies \\ \lambda_{ij} w_j L_j &= M_{ij} \theta \alpha^2 \frac{L_j}{\mu_j 2\gamma} \frac{1}{\theta} \frac{1}{(\theta+2)} \implies \\ \frac{M_{ij}}{\lambda_{ij}} &= \frac{w_j \mu_j}{\frac{\alpha^2}{2\gamma} \frac{1}{(\theta+2)}} \ . \end{aligned} \tag{49}$$

From expression (41) this equation can be re-expressed as

$$\frac{M_{ij}}{\lambda_{ij}} = \frac{\phi_{jj}^*}{\frac{\alpha}{2\gamma} \frac{1}{(\theta+2)}} \ .$$

Therefore, we would expect to see more firms in a destination if the average producer there is more productive.

Notice that using the results of section (4.4) and equation (41) we can write

$$
\begin{aligned}
w_j &= \sum_{\upsilon} J_\upsilon \frac{b_\upsilon^\theta}{\left(\phi_{\upsilon j}^*\right)^\theta} \frac{1}{\mu_j} \frac{1}{2\gamma} \frac{\alpha^2}{(\theta+2)} \implies \\
\frac{1}{\mu_j} &= \frac{w_j}{\sum_\upsilon J_\upsilon \frac{b_\upsilon^\theta}{\left(\frac{\mu_j w_\upsilon \tau_{\upsilon j}}{\alpha}\right)^\theta} \frac{1}{2\gamma} \frac{\alpha^2}{(\theta+2)}} \implies \\
\left(\frac{1}{\mu_j}\right)^{\theta+1} &= \frac{(w_j)^{1+\theta}}{\frac{L_j}{(\theta+1)f_e} b_j^\theta \frac{1}{2\gamma} \frac{\alpha^{2+\theta}}{(\theta+2)}} \lambda_{jj} \implies \\
w_j \mu_j = &\left[b_j^\theta \frac{1}{2\gamma} \frac{\alpha^{2+\theta}}{(\theta+1) f_e (\theta+2)} \frac{L_j}{\lambda_{jj}} \right]^{1/(1+\theta)} .
\end{aligned} \tag{50}
$$

Replacing this expression into the equation (49) we see that the model implies more firms (normalized by market share) in markets with higher b_j and L_j conditional on λ_{jj},

$$
\frac{M_{ij}}{\lambda_{ij}} = b_j^{\theta/(1+\theta)} \left[\frac{L_j}{\lambda_{jj}}\right]^{1/(1+\theta)} \frac{\left[\frac{1}{2\gamma} \frac{\alpha^{2+\theta}}{(\theta+1) f_e (\theta+2)}\right]^{1/(1+\theta)}}{\frac{\alpha^2}{2\gamma} \frac{1}{(\theta+2)}} . \tag{51}
$$

Arkolakis (2008) using the French data by Eaton, Kortum, and Kramarz (2010) reports that normalized entry is positively related to both population and income per capita of the market. In the relationship above, b_j (which is positively related to income per capita, w_j) and L_j positively affect normalized entry, M_{ij}/λ_{ij}. This means that qualitatively the model has the ability to match the stylized fact that normalized entry is increasing in w_j and L_j. However, using the relationship (48) to replace for λ_{jj}, we then have

$$
\frac{M_{ij}}{\lambda_{ij}} = (w_j)^{\theta/(1+\theta)} \left[\sum_\upsilon L_\upsilon b_\upsilon^\theta \left(w_\upsilon \tau_{\upsilon j}\right)^{-\theta}\right]^{1/(1+\theta)} \frac{\left[\frac{1}{2\gamma} \frac{\alpha^{2+\theta}}{(\theta+1) f_e (\theta+2)}\right]^{1/(1+\theta)}}{\frac{\alpha^2}{2\gamma} \frac{1}{(\theta+2)}} . \tag{52}
$$

This relationship implies the following:

a) The relationship of normalized entry with respect to population is weakly positive since the population of country j is in the summation term and its changes are likely to affect the sum only by a little.

b) The relationship of normalized entry with respect to income per capita is positive since the term outside the summation has the correct coefficient. Notice that inside the summation there are the terms b_j and w_j which are both related to country income, but in opposite ways. Since both terms are in the summation they are likely to affect entry less than the term outside the summation.[2]

[2] Simonovska (2009) also assumes a non-homothetic demand system and points out a similar finding.

Similar derivations can be used to express average sales as

$$\begin{aligned}
\overline{X_{ij}} &= \frac{\lambda_{ij} w_j L_j}{M_{ij}} \\
&= \frac{w_j L_j}{\left[b_j^{\theta} \frac{1}{2\gamma} \frac{\alpha^{2+\theta}}{(\theta+1) f_e (\theta+2)} \frac{L_j}{\lambda_{jj}} \right]^{1/(1+\theta)}} \frac{\alpha^2}{2\gamma(\theta+2)} \\
&= w_j b_j^{-\theta/(1+\theta)} (L_j)^{\theta/(1+\theta)} \lambda_{jj}^{1/(1+\theta)} \frac{\frac{\alpha^2}{2\gamma(\theta+2)}}{\left[\frac{1}{2\gamma} \frac{\alpha^{2+\theta}}{(\theta+1) f_e (\theta+2)} \right]^{1/(1+\theta)}} \,.
\end{aligned} \tag{53}$$

The data of Eaton, Kortum, and Kramarz (2010) also indicate a positive relationship between average sales of French firms in a market with population and income per capita of the market. The positive relationship between average sales and population seems dubious for the same reason as for the case of normalized entry. The relationship of average sales with income per capita seems more likely to be satisfied.

4.4 Appendix to the Section

Aggregate Entry In particular, budget constraint (which is equivalent to labor market clearing) implies that

$$\begin{aligned}
w_i L_i &= \sum_j J_i \frac{b_i^{\theta}}{\left(\phi_{ij}^*\right)^{\theta}} \int_{\phi^*}^{\infty} p_{ij}(\phi) q_{ij}(\phi) \mu_{ij}(\phi) \, d\phi \implies \\
w_i L_i &= \sum_j J_i \frac{b_i^{\theta}}{\left(\phi_{ij}^*\right)^{\theta}} \int_{\phi^*}^{\infty} \mu_j \frac{L_j}{4\gamma} \left(\left(\tau_{ij} \frac{w_i}{\phi_{ij}^*} \right)^2 - \left(\tau_{ij} \frac{w_i}{\phi} \right)^2 \right) \mu_{ij}(\phi) \, d\phi \implies \\
w_i L_i &= \sum_j J_i \frac{b_i^{\theta}}{\left(\phi_{ij}^*\right)^{\theta}} \frac{L_j \theta}{4\gamma} \mu_j \frac{w_i^2}{(\phi_{ij}^*)^2} \tau_{ij}^2 \left(\frac{1}{\theta} - \frac{1}{\theta+2} \right) \,.
\end{aligned} \tag{54}$$

Using equation (42) profits can be written as

$$\pi_{ij}(\phi) = \mu_j \frac{L_j}{4\gamma} \left(\left(\tau_{ij} \frac{w_i}{\phi_{ij}^*} \right)^2 - \left(\tau_{ij} \frac{w_i}{\phi} \right)^2 \right) - \mu_j \tau_{ij} \frac{w_i}{\phi} \frac{L_j}{2\gamma} \left(\tau_{ij} \frac{w_i}{\phi_{ij}^*} - \tau_{ij} \frac{w_i}{\phi} \right) \implies$$

$$\pi_{ij}(\phi) = \mu_j \frac{L_j}{4\gamma} (\tau_{ij})^2 \left[\left(\frac{w_i}{\phi_{ij}^*} \right) - \left(\frac{w_i}{\phi} \right) \right]^2 \,. \tag{55}$$

Thus, expected profits are

$$\sum_{\upsilon}\left(\frac{\phi_{ii}^{*}}{\phi_{i\upsilon}^{*}}\right)^{\theta}\int_{\phi_{i\upsilon}^{*}}^{\infty}\pi_{i\upsilon}(\phi)\mu_{ij}(\phi)\,d\phi=\sum_{\upsilon}\int_{\phi^{*}}^{\infty}\pi_{i\upsilon}(\phi)\theta(\phi^{*})^{\theta}\phi^{-\theta-1}d\phi$$
$$=\theta(\phi_{ii}^{*})^{\theta}\sum_{\upsilon}\mu_{\upsilon}\frac{L_{\upsilon}}{4\gamma}(\tau_{i\upsilon})^{2}\int_{\phi^{*}}^{\infty}\left[\left(\frac{w_i}{\phi_{i\upsilon}^{*}}\right)-\left(\frac{w_i}{\phi}\right)\right]^{2}\phi^{-\theta-1}d\phi$$
$$=\theta w_i^{2}\sum_{\upsilon}\frac{\mu_{\upsilon}}{(\phi_{i\upsilon}^{*})^{2}}\frac{L_{\upsilon}}{4\gamma}\left(\frac{\phi_{ii}^{*}}{\phi_{i\upsilon}^{*}}\right)^{\theta}(\tau_{i\upsilon})^{2}\left[\frac{1}{\theta}+\frac{1}{\theta+2}-\frac{2}{\theta+1}\right].$$

Therefore the free entry condition implies

$$\frac{b_i^{\theta}}{(\phi_{ii}^{*})^{\theta}}\theta w_i^{2}\sum_{\upsilon}\frac{\mu_{\upsilon}}{(\phi_{i\upsilon}^{*})^{2}}\frac{L_{\upsilon}}{4\gamma}\left(\frac{\phi_{ii}^{*}}{\phi_{i\upsilon}^{*}}\right)^{\theta}(\tau_{i\upsilon})^{2}\left[\frac{1}{\theta}+\frac{1}{\theta+2}-\frac{2}{\theta+1}\right]=w_i f_e \implies$$
$$\frac{b_i^{\theta}}{(\phi_{ii}^{*})^{\theta}}\sum_{\upsilon}\left(\frac{\phi_{ii}^{*}}{\phi_{i\upsilon}^{*}}\right)^{\theta}\frac{L_{\upsilon}}{2\gamma}\mu_{\upsilon}\frac{w_i}{(\phi_{i\upsilon}^{*})^{2}}(\tau_{i\upsilon})^{2}\left[\frac{1}{(\theta+1)(\theta+2)}\right]=f_e\;.$$

now replacing the above equation inside (54)

$$w_iL_i=\sum_{\upsilon}J_i\frac{b_i^{\theta}}{(\phi_{i\upsilon}^{*})^{\theta}}\frac{L_{\upsilon}\theta}{4\gamma}\mu_{\upsilon}\frac{w_i^{2}}{(\phi_{i\upsilon}^{*})^{2}}\tau_{i\upsilon}^{2}\left(\frac{1}{\theta}-\frac{1}{\theta+2}\right)\implies$$
$$w_iL_i=J_i\sum_{\upsilon}\left(\frac{b_i}{\phi_{i\upsilon}^{*}}\right)^{\theta}\frac{L_{\upsilon}\theta}{4\gamma}\mu_{\upsilon}\frac{w_i^{2}}{(\phi_{i\upsilon}^{*})^{2}}\tau_{i\upsilon}^{2}\left(\frac{1}{\theta}-\frac{1}{\theta+2}\right)\implies$$

$$J_i=\frac{L_i}{(\theta+1)\,f_e} \tag{56}$$

Thus, the number of entrants is independent of variable trade costs and trade in general.

Trade Balance To complete the description of the model we discuss the implementation of the trade balance condition.

The budget constraint of the representative consumers implies

$$w_jL_j=\sum_{\upsilon}\underbrace{J_{\upsilon}\frac{b_{\upsilon}^{\theta}}{(\phi_{\upsilon j}^{*})^{\theta}}}_{\text{measure of entrants from }\upsilon}\times\text{av.sales in }j$$

Using equation (45) we can write this expression as

$$w_jL_j=\frac{1}{\mu_j}\sum_{\upsilon}J_{\upsilon}\frac{b_{\upsilon}^{\theta}}{(\phi_{\upsilon j}^{*})^{\theta}}\frac{L_j}{4\gamma}\frac{2}{\theta}\frac{\theta\alpha^{2}}{(\theta+2)}\;. \tag{57}$$

We can also ask where is the income of the consumers derived from. This income is derived from profits (which due to free entry equal entry costs) and production costs. Notice that both profits and production costs are paid by domestic firms to domestic consumers and profits + production costs =total sales of domestic firms. Therefore, we can also write the trade balance in the following form:

$$\begin{aligned} w_j L_j &= \sum_v \underbrace{J_j \frac{b_j^\theta}{\left(\phi_{jv}^*\right)^\theta}}_{\text{measure of entrants in } v} \times \text{av.sales in } v \\ &= \sum_v J_j \frac{b_j^\theta}{\left(\phi_{jv}^*\right)^\theta} \frac{L_v}{\mu_v 4\gamma} \frac{2}{\theta} \frac{\theta\alpha^2}{(\theta+2)} \ . \end{aligned} \tag{58}$$

As a result of the assumptions of the model, equations (57) and (58) imply that trade is balanced.

5 Market Penetration Costs: Alternative Interpretations

In this section we discuss alternative hypotheses for the informative advertising theory presented in Arkolakis (2008). The discussion is focused on developing mathematical isomorphisms to that framework.

5.1 Persuasive Advertisement

The purpose of this subsection is to show that there exist an isomorphism of the model of Arkolakis (2008), where firms pay a cost to reach more consumers, with a model where firms pay a market penetration cost to increase their sales per consumer.

The problem of the consumer is

$$\max_{x(\omega)} \left(\int u(\omega)^{1-\rho} x(\omega)^{\rho} d\omega \right)^{\frac{1}{\rho}}$$
$$\text{s.t.} \quad \int p(\omega) x(\omega) \leq w + \pi$$

where $p(\omega)$ represents the price of good ω, $x(\omega)$ the quantity demanded by the representative consumer, y is income per capita and $\sigma = 1/(1-\rho) > 1$. First order conditions give that the demand per consumer is

$$x(\omega) = u(\omega) \frac{p(\omega)^{-\sigma}}{P^{-\sigma}} (w + \pi) \ ,$$

where

$$P^{1-\sigma} = \int_{\omega \in \Omega} u(\omega) p(\omega)^{1-\sigma} d\omega \ . \tag{59}$$

There is a measure of L consumers. Looking at a symmetric equilibrium where all the firms with the same productivity face the same optimization problem we can write the problem of a firm with productivity ϕ as

$$\max_{u,p} u \frac{p^{1-\sigma}}{P^{1-\sigma}} (w + \pi) L - u \frac{w}{\phi} \frac{p^{-\sigma}}{P^{1-\sigma}} (w + \pi) L - wg(u(\phi)) \ , \tag{60}$$

thus

$$P^{1-\sigma} = M \int_{\phi^*} u(\phi) p(\phi)^{1-\sigma} \mu(\phi) d\phi \ , \tag{61}$$

where $\mu(\phi)$ is the probability density of firms conditional on operating ($\phi > \phi^*$) and M is the measure of operating firms.

FOC give us:

with respect to $p(\phi)$

$$p(\phi) = \frac{w}{\phi}\frac{\sigma}{\sigma-1} \ ,$$

with respect to $u(\phi)$

$$\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\frac{w}{\phi}\right)^{1-\sigma}\frac{(w+\pi)L}{P^{1-\sigma}}\frac{1}{\sigma} = g'(u(\phi)) \ .$$

Proposition 1 *Assume that the market penetration cost function is* $g(u(\phi)) = \frac{1}{\psi}\frac{1-(1-u)^{-\beta+1}}{-\beta+1}$. *Then the problem defined above is isomorphic to the one of Arkolakis (2008) where the market penetration cost increases as a function of the number of consumers reached.*

Proof: First notice that the cutoff ϕ of production, that determines the number of operating firms M, is given by

$$(\phi^*)^{\sigma-1} = \frac{\frac{1}{\psi}}{\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}(w)^{1-\sigma}\frac{(w+\pi)L}{P^{1-\sigma}}\frac{1}{\sigma}}.$$

In the case that the market penetration cost is a function of the fraction of consumers reached (I denote all equilibrium variables in this case with a tilde)

$$\left(\tilde{\phi}^*\right)^{\sigma-1} = \frac{\frac{1}{\psi}}{\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}(\tilde{w})^{1-\sigma}\frac{(\tilde{w}+\tilde{\pi})L}{\tilde{P}^{1-\sigma}}\frac{1}{\sigma}} \ .$$

Total labor required for market penetration costs is

$$l_m = \int_{\phi^*}\frac{1}{\psi}\frac{1-(1-u(\phi))^{-\beta}}{-\beta}\mu(\phi)\,d\phi \ ,$$

where for the case that the market penetration cost is a function of the fraction of consumers reached, $\tilde{n}(\phi)$,

$$\tilde{l}_m = \int_{\phi^*}\frac{1}{\psi}\frac{1-(1-\tilde{n}(\phi))^{-\beta}}{-\beta}\mu(\phi)\,d\phi \ .$$

Labor demand for production is

$$l_p = \int_{\phi^*} u(\phi)\frac{w}{\phi}\frac{p(\phi)^{-\sigma}}{P^{1-\sigma}}(w+\pi)L\mu(\phi)\,d\phi,$$

$$\tilde{l}_p = \int_{\phi^*}\tilde{n}(\phi)\frac{\tilde{w}}{\phi}\frac{\tilde{p}(\phi)^{-\sigma}}{\tilde{P}^{1-\sigma}}(\tilde{w}+\tilde{\pi})L\mu(\phi)\,d\phi \ ,$$

respectively and with

$$\tilde{P}^{1-\sigma} = \tilde{M} \int_{\phi^*} \tilde{n}(\phi)\, \tilde{p}(\phi)^{1-\sigma}\, \tilde{\mu}(\phi)\, d\phi \ .$$

We also have that the effective demand for a firm is

$$y(\phi) = u(\phi)\, L \frac{p(\phi)^{-\sigma}}{P^{1-\sigma}} (w + \pi) \ ,$$

where for the case that the market penetration cost is a function of the fraction of consumers reached we have

$$\tilde{y}(\phi) = \tilde{n}(\phi)\, L \frac{\tilde{p}(\phi)^{-\sigma}}{\tilde{P}^{1-\sigma}} (\tilde{w} + \tilde{\pi}) \ .$$

Finally, for the case of market penetration in terms of fraction of consumers reached we have

$$\tilde{x}(\phi) = \frac{\tilde{p}(\phi)^{-\sigma}}{\tilde{P}^{1-\sigma}} (\tilde{w} + \tilde{\pi}) \ .$$

Now, define the following variables $\tilde{n}(\phi) = u(\phi)$, $P = \tilde{P}$, $y(\phi) = \tilde{y}(\phi)$, $x(\phi) = \tilde{x}(\phi)$, $p(\phi) = \tilde{p}(\phi)$, $M = \tilde{M}$, $l_m(\phi) = \tilde{l}_m(\phi)$, $l_p(\phi) = \tilde{l}_p(\phi)$, $w = \tilde{w}$, $\pi = \tilde{\pi}$. Then, assuming Constant Elasticity of Substitution utility function with the same elasticity of substitution parameters, the same productivity distribution for firms as well as the same technology for producing the goods, the models of market penetration by reaching more consumers or selling more per consumer are isomorphic.

5.2 Random Evaluation

We now construct a case that is partially isomorphic to Arkolakis (2008). We assume that consumers have a random evaluation of each good while the firm has a constant marginal cost to reach additional consumers. Therefore, there are constant returns to scale in the marketing technology but decreasing revenues accrued from additional consumers. For simplicity we assume that there is a measure 1 of consumers in the market.

The notation is as in the previous subsection. Let the demand of a consumer for a specific good be

$$x(\omega) = a(\omega)^{\delta+1}\, y \frac{p(\omega)^{-\sigma}}{P^{1-\sigma}} \ , \ y = w + \pi$$

where $a(\omega)$ is an iid shock that each consumers gets for each good ω that is randomly drawn from an identical Pareto distribution with support $(0, \bar{a}]$ and $\delta > -1$. In particular,

$$\Pr[A < a|\bar{a}] = \left(\frac{a}{\bar{a}}\right)^{\gamma} ,$$

for $\gamma > 0$. In the cases where $\gamma > 0$ the probability density function is well defined in the interval $(0, \bar{a}]$,

$$\Pr[A = a|\bar{a}] = \gamma \frac{\bar{a}^{-\gamma}}{a^{-\gamma+1}} .$$

In fact, the mean is given by

$$\begin{aligned}\int_0^{\bar{a}} a\gamma \frac{a^{\gamma-1}}{\bar{a}^{\gamma}} da &= \int_0^{\bar{a}} \gamma \frac{a^{\gamma}}{\bar{a}^{\gamma}} da \\ &= \frac{\gamma}{\gamma+1}\bar{a} ,\end{aligned}$$

which means that it is well defined for $\gamma > 0$. To prove that $\Pr[A = a]$ is a pdf we have

$$\begin{aligned}\int_0^{\bar{a}} \gamma \frac{a^{\gamma-1}}{\bar{a}^{\gamma}} da &= \frac{a^{\gamma}}{\bar{a}^{\gamma}}\Big|_0^{\bar{a}} \\ (\text{if } \gamma &> 0) = 1 .\end{aligned}$$

The cases that we will analyze correspond to the theory of Arkolakis (2008) for $\beta \in [0, 1]$ as we will illustrate below.

Firm problem We assume that the firm has to pay a fixed cost f to reach each individual consumer (the problem can be easily extended to the case that Arkolakis considers where the cost is not linear but convex in the number of consumers).

The profits of a firm ϕ that charges a price p and reaches n fraction of the consumers are

$$\underbrace{p\int_{(1-n)^{1/\gamma}\bar{a}}^{\bar{a}} a^{\delta+1} y \frac{p^{-\sigma}}{P^{1-\sigma}} \gamma \frac{a^{\gamma-1}}{(\bar{a})^{\gamma}} da}_{\text{sales}} - \underbrace{\frac{w\tau}{\phi}\int_{(1-n)^{1/\gamma}\bar{a}}^{\bar{a}} a^{\delta+1} y \frac{p^{-\sigma}}{P^{1-\sigma}} \gamma \frac{a^{\gamma-1}}{(\bar{a})^{\gamma}} da}_{\text{production cost}} - \underbrace{nf}_{\text{marketing cost}}$$

The first term is the total sales to n fraction of the consumers (to calculate the lower bound of the distribution $(1-n)^{1/\gamma}\bar{a}$ that corresponds to reaching n fraction of the total population simply set $n = \Pr[A > a|\bar{a}]$ and use the cdf). The second term corresponds to the production and shipping costs (with marginal cost $\frac{w\tau}{\phi}$, where w is the wage and τ is the iceberg transportation cost). Simplifying the expression we get

$$\left(p-\frac{w\tau}{\phi}\right)\int_{(1-n)^{1/\gamma}\bar{a}}^{\bar{a}} a^{\delta+1}y\frac{p^{-\sigma}}{P^{1-\sigma}}\gamma\frac{a^{\gamma-1}}{\bar{a}^{\gamma}}da-nf \implies$$

$$\left(p-\frac{w\tau}{\phi}\right)y\frac{p^{-\sigma}}{P^{1-\sigma}}\frac{\gamma}{\gamma+\delta+1}\left(\frac{\bar{a}^{\gamma+\delta+1}}{(\bar{a})^{\gamma}}-\frac{\left[(1-n)^{1/\gamma}\bar{a}\right]^{\gamma+\delta+1}}{(\bar{a})^{\gamma}}\right)-nf \implies$$

$$\left(p-\frac{w\tau}{\phi}\right)y\frac{p^{-\sigma}}{P^{1-\sigma}}\frac{\gamma}{\gamma+\delta+1}\bar{a}^{\delta+1}\left(1-(1-n)^{(\gamma+\delta+1)/\gamma}\right)-nf\ .$$

Notice that the solution of the optimal pricing problem is a constant markup such that

$$p=\frac{\sigma}{\sigma-1}\frac{w\tau}{\phi}\ .$$

The choice of reaching an additional fraction of the consumers can be represented as the derivative with respect to n

$$\left(\frac{1}{\sigma}\right)y\frac{\left(\frac{\sigma}{\sigma-1}\frac{w\tau}{\phi}\right)^{1-\sigma}}{P^{1-\sigma}}\bar{a}\,(1-n)^{(\gamma+\delta+1)/\gamma-1}=f\ ,$$

which implies

$$n=1-\left(\frac{f}{\left(\frac{1}{\sigma}\right)y\frac{\left(\frac{\sigma}{\sigma-1}\frac{w\tau}{\phi}\right)^{1-\sigma}}{P^{1-\sigma}}\bar{a}}\right)^{\frac{\gamma}{\delta+1}}\ . \tag{62}$$

Notice that more productive firms reach more people (but no firms optimally reaches all of them).

Finally, notice that firms sell only if $\phi\geq\phi^{*}$

$$\frac{f}{\left(\frac{1}{\sigma}\right)y\frac{\left(\frac{\sigma}{\sigma-1}w\tau\right)^{1-\sigma}}{P^{1-\sigma}}\bar{a}} = (\phi^{*})^{\sigma-1} \implies \tag{63}$$

$$\phi^{*} = \left(\frac{f}{\left(\frac{1}{\sigma}\right)y\frac{\left(\frac{\sigma}{\sigma-1}w\tau\right)^{1-\sigma}}{P^{1-\sigma}}\bar{a}}\right)^{1/(\sigma-1)}\ . \tag{64}$$

This cutoff rule is exactly the same as in Arkolakis (if we normalize $\bar{a}=1$). Using (62) this cutoff rule implies that $n=1-\left(\left(\frac{\phi^{*}}{\phi}\right)^{\sigma-1}\right)^{\frac{\gamma}{\delta+1}} \implies (1-n)^{1/\gamma}=\left(\frac{\phi^{*}}{\phi}\right)^{\frac{\sigma-1}{\delta+1}}$.

We can now replace in the expression for sales given the optimal choices of the firm for p and n,

$$\begin{aligned} y(\phi) &= \left(p-\frac{w\tau}{\phi}\right)y\frac{p^{-\sigma}}{P^{1-\sigma}}\frac{\gamma}{\gamma+\delta+1}\bar{a}\left(1-(1-n)^{(\gamma+\delta+1)/\gamma}\right) \\ &= \sigma f\frac{\gamma}{\gamma+\delta+1}\left(\left(\frac{\phi}{\phi^{*}}\right)^{\sigma-1}-\left(\frac{\phi}{\phi^{*}}\right)^{-\frac{\gamma}{\delta+1}(\sigma-1)}\right)\ . \end{aligned} \tag{65}$$

Marketing costs as a fraction of total sales Notice that variable profits for each firm are given by

$$\pi^V(\phi) = f\frac{\gamma}{\gamma+\delta+1}\left(\left(\frac{\phi}{\phi^*}\right)^{\sigma-1} - \left(\frac{\phi}{\phi^*}\right)^{-\frac{\gamma}{\delta+1}(\sigma-1)}\right) .$$

Also average variable profits for operating firms are

$$\begin{aligned}\bar{\pi}^V &= f\frac{\gamma}{\gamma+\delta+1}\int\left(\left(\frac{\phi}{\phi^*}\right)^{\sigma-1} - \left(\frac{\phi}{\phi^*}\right)^{-\frac{\gamma}{\delta+1}(\sigma-1)}\right)\theta\frac{(\phi^*)^\theta}{\phi^{\theta+1}} \\ &= f\frac{\gamma}{\gamma+\delta+1}\left(\frac{\theta}{\theta-\sigma+1} - \frac{\theta}{\frac{\gamma}{\delta+1}(\sigma-1)+\theta}\right) .\end{aligned}$$

On the other hand we have that entry costs

$$n(\phi)f = \left(1-\left[\left(\frac{\phi^*}{\phi}\right)^{\sigma-1}\right]^{\frac{\gamma}{\delta+1}}\right)f ,$$

and that average entry costs

$$\begin{aligned}& f\left(1-\left[\left(\frac{\phi^*}{\phi}\right)^{\sigma-1}\right]^{\frac{\gamma}{\delta+1}}\right)\theta\frac{(\phi^*)^\theta}{\phi^{\theta+1}} \\ =\ & f\left(\frac{1}{\theta} - \frac{1}{\frac{\gamma}{\delta+1}(\sigma-1)+\theta}\right)\theta .\end{aligned}$$

This means that entry costs are a constant fraction of overall profits since

$$\frac{f\left(\frac{1}{\theta} - \frac{1}{\frac{\gamma}{\delta+1}(\sigma-1)+\theta}\right)\theta}{f\frac{\gamma}{\gamma+\delta+1}\left(\frac{\theta}{\theta-\sigma+1} - \frac{\theta}{\frac{\gamma}{\delta+1}(\sigma-1)+\theta}\right)} = \frac{\left(\frac{1}{\theta} - \frac{1}{\frac{\gamma}{\delta+1}(\sigma-1)+\theta}\right)}{\frac{\gamma}{\gamma+\delta+1}\left(\frac{1}{\theta-\sigma+1} - \frac{1}{\frac{\gamma}{\delta+1}(\sigma-1)+\theta}\right)} < 1 ,$$

as long as

$$\frac{1}{\theta} < \frac{1}{\theta-\sigma+1} ,$$

which, of course, holds.

The isomorphism We now discuss the isomorphism that the sales equation (65) with the corresponding one in Arkolakis (2008) exhibits. We consider the simple case where $\delta = 0$. The results follows by direct comparison of equation (65) to equation (14) in Arkolakis (2008) and noting that the two equations are the same if $\gamma + 1 = 1/\beta$. Cases with $\delta > 0$ yield the same results to the ones below by simply redefining notation. We distinguish the following two cases.

case a) If $\gamma > 1$ this case corresponds to the case $\beta \in (0, 1/2)$, $\frac{\beta-1}{\beta} \in (-\infty, -1)$ for the theory in Arkolakis (2008). In this case higher a implies higher density. In the limit where $\gamma \to +\infty$ we have $\frac{\gamma}{\gamma+1} \to 1$ and also $\left(\frac{\phi}{\phi^*}\right)^{-\gamma(\sigma-1)} \to 0$ for any $\phi > \phi^*$ which effectively means that $y(\phi) = \sigma f \left(\frac{\phi}{\phi^*}\right)^{\sigma-1}$ as in Melitz (2003). How should we interpret the result? If there is so much homogeneity of tastes that all the mass is concentrated in $\bar{a}$ then we effectively have the Melitz model.

case b) If $0 < \gamma < 1$ then the density decreases for higher a so that there many people that are not so fond of each good. This case corresponds to the case that $\beta \in (1/2, 1)$ so that $\frac{\beta-1}{\beta} \in (-1, 0)$. The limit $\gamma \to 0$ corresponds to the case $\beta \to 1$.

Part II

Robustness

In this section we illustrate some robustness of the results with respect to the parameters of the model.

6 Distribution of Sales

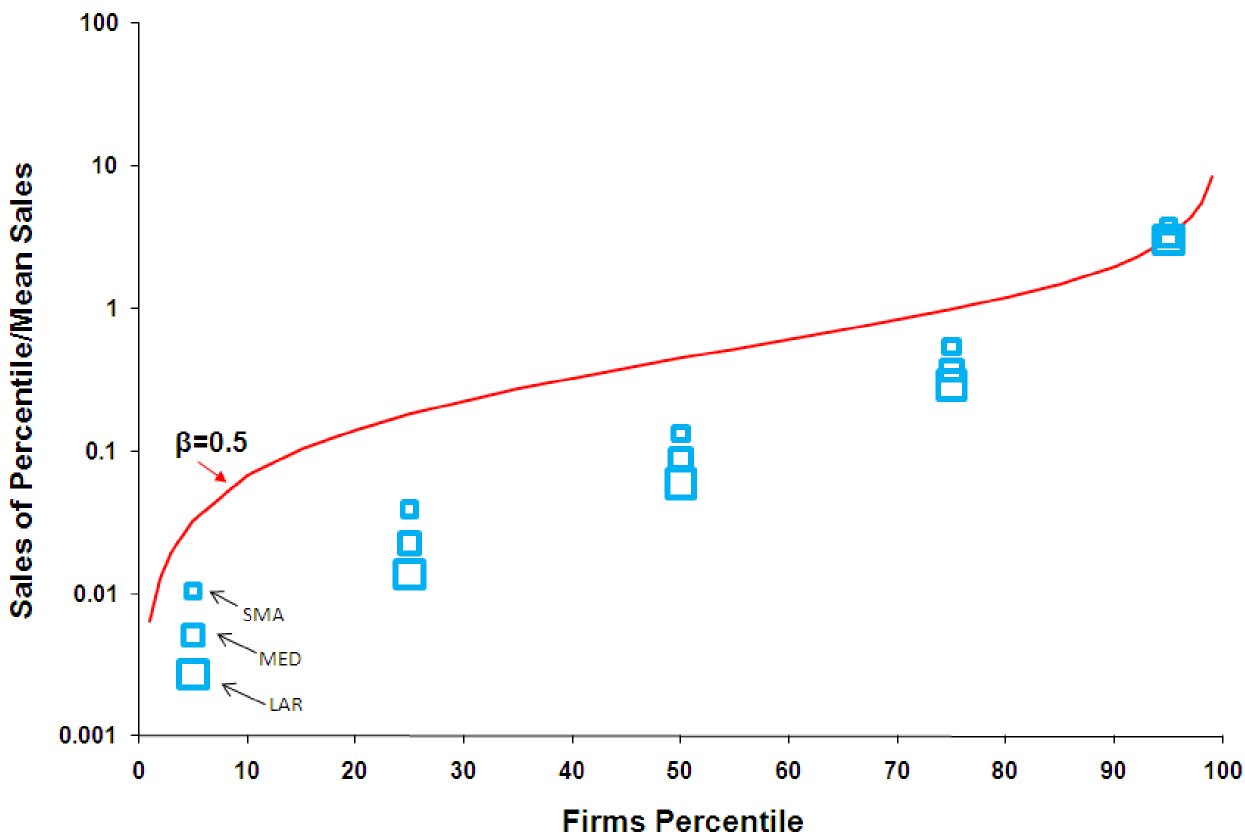

Figure 1: Distribution of sales relative to mean sales in model ($\beta = .5$, $\tilde{\theta} = 1.65$) and in the French data (for small, medium, and larger exporting destinations)

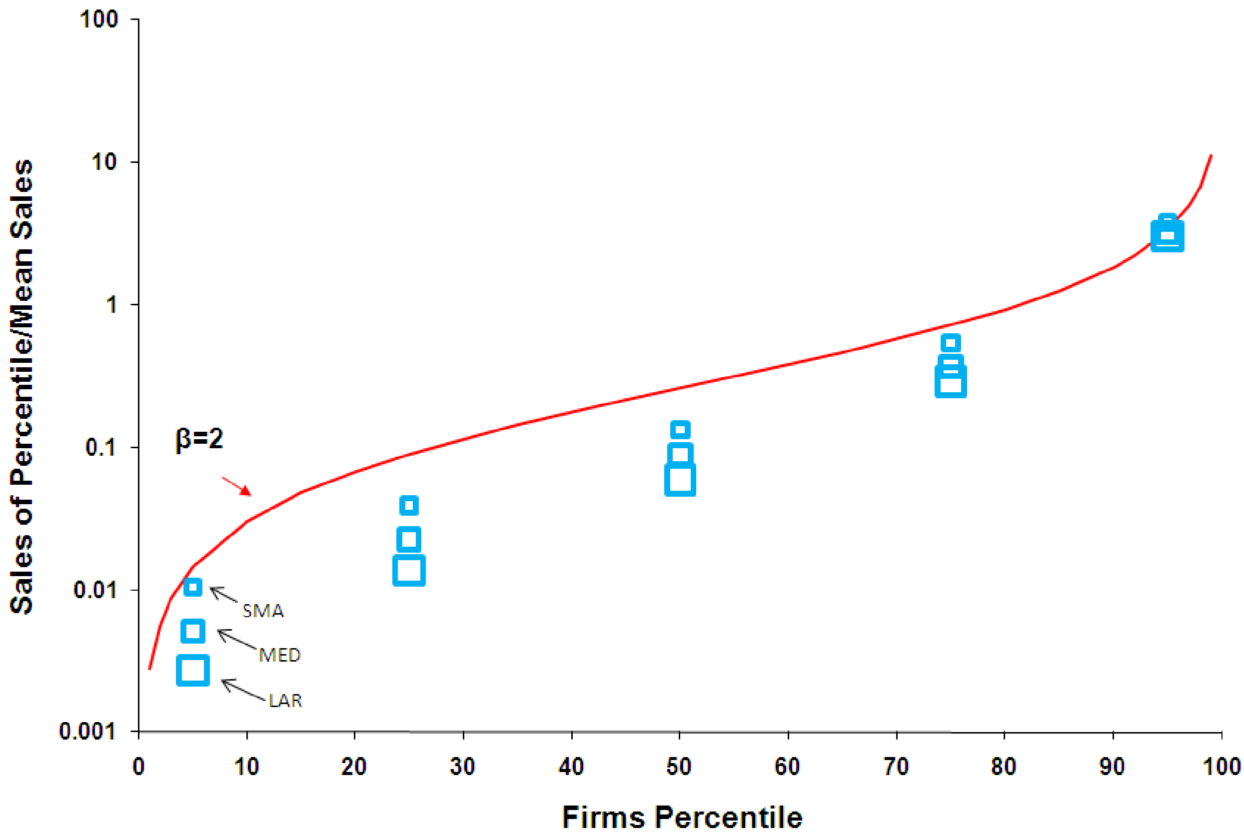

Figure 2: Distribution of sales relative to mean sales in model ($\beta = 2$, $\tilde{\theta} = 1.65$) and in the French data (for small, medium, and larger exporting destinations)

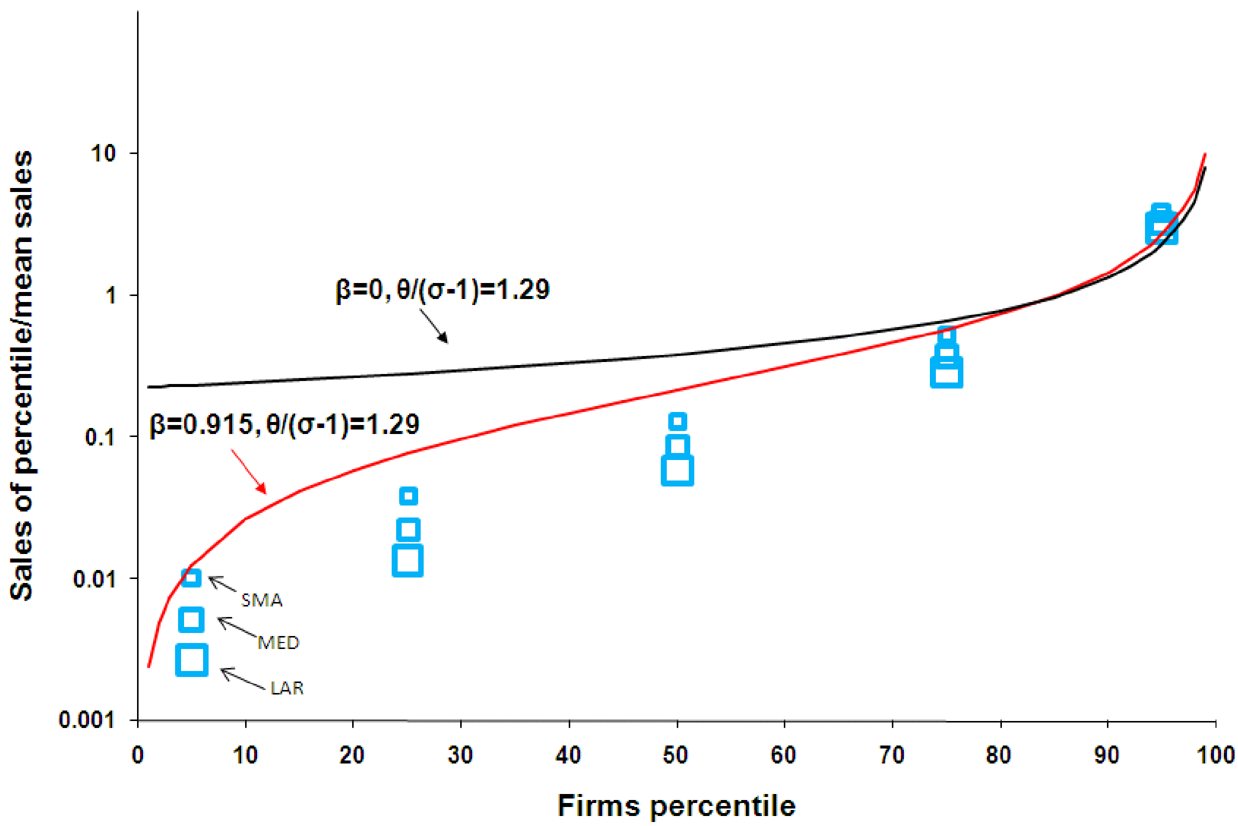

Figure 3: Distribution of sales relative to mean sales in model ($\beta = .915$, $\tilde{\theta} = 1.29$) and in the French data (for small, medium, and larger exporting destinations)

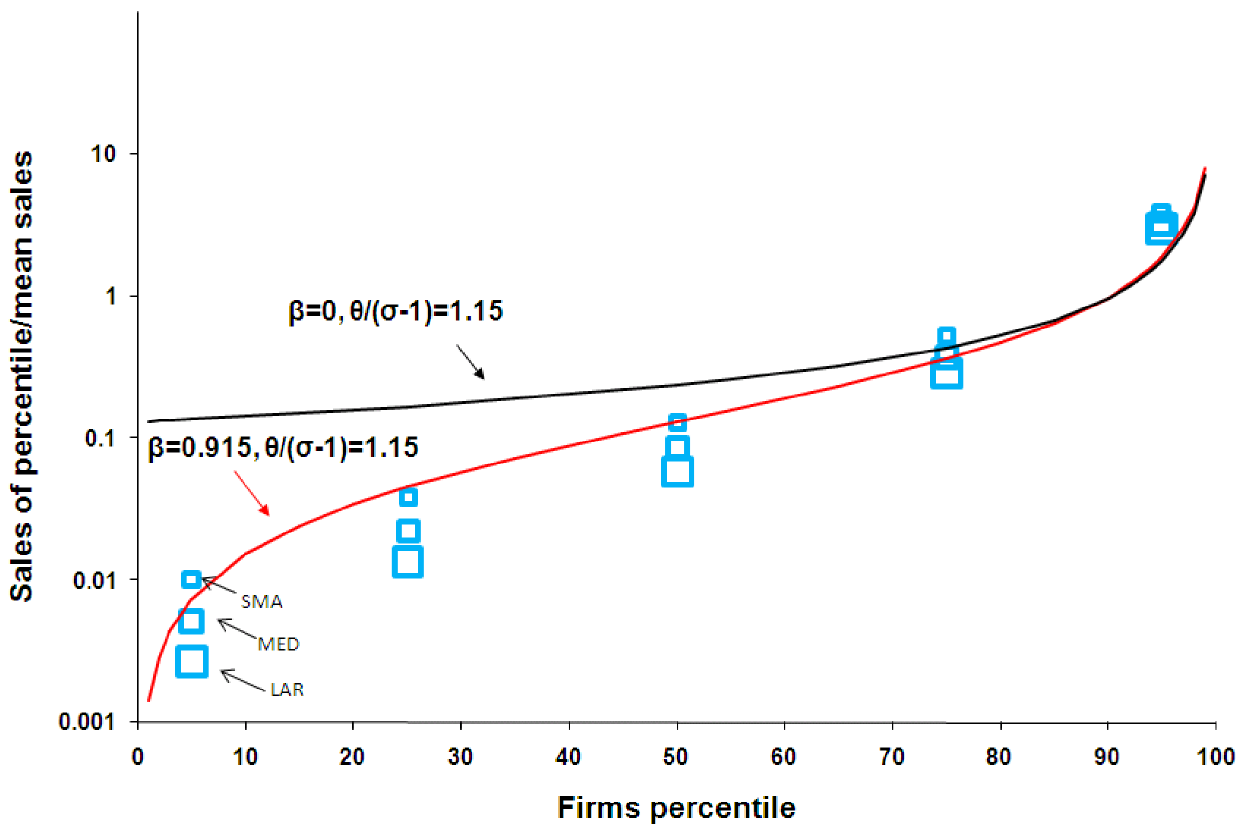

Figure 4: Distribution of sales relative to mean sales in model ($\beta = .915$, $\tilde{\theta} = 1.15$) and in the French data (for small, medium, and larger exporting destinations)

7 Normalized Average Sales

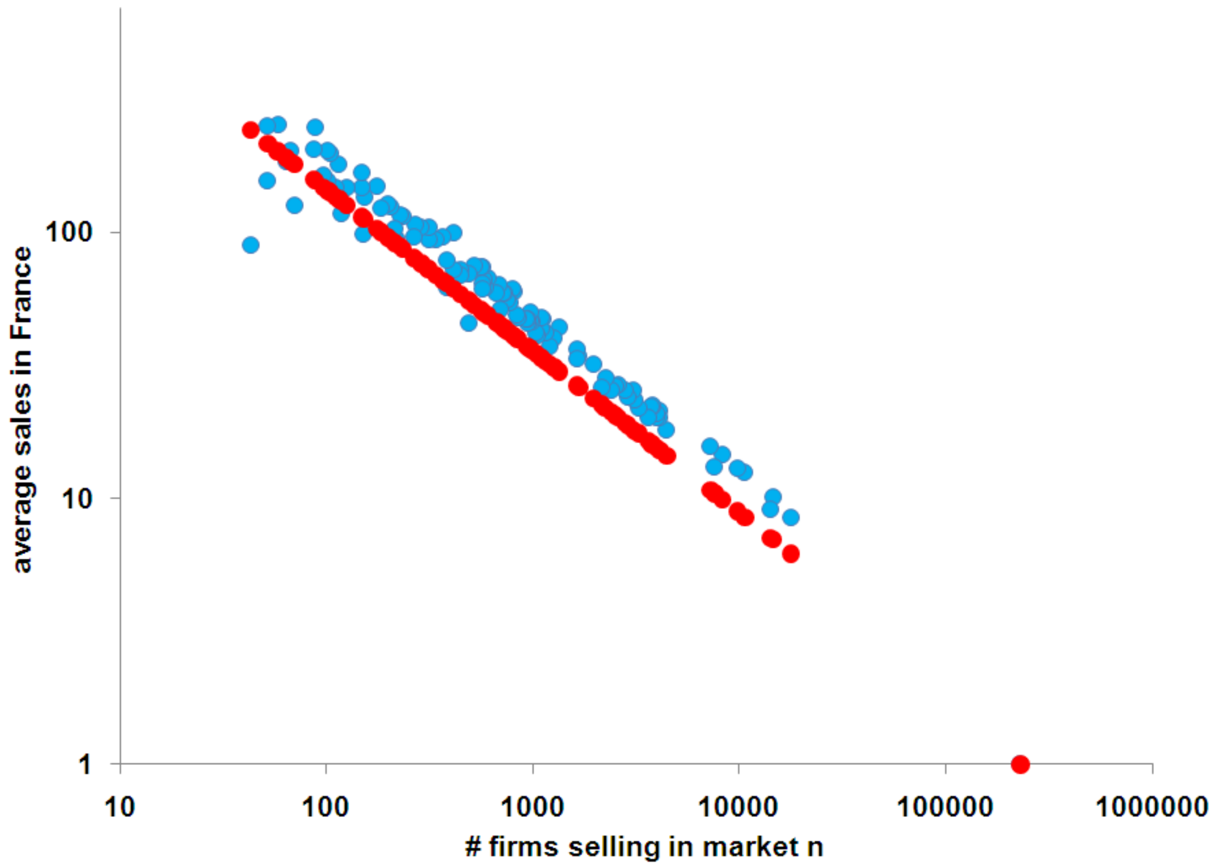

Figure 5: Normalized average sales in the French data and the model ($\beta = .5$, $\tilde{\theta} = 1.65$)

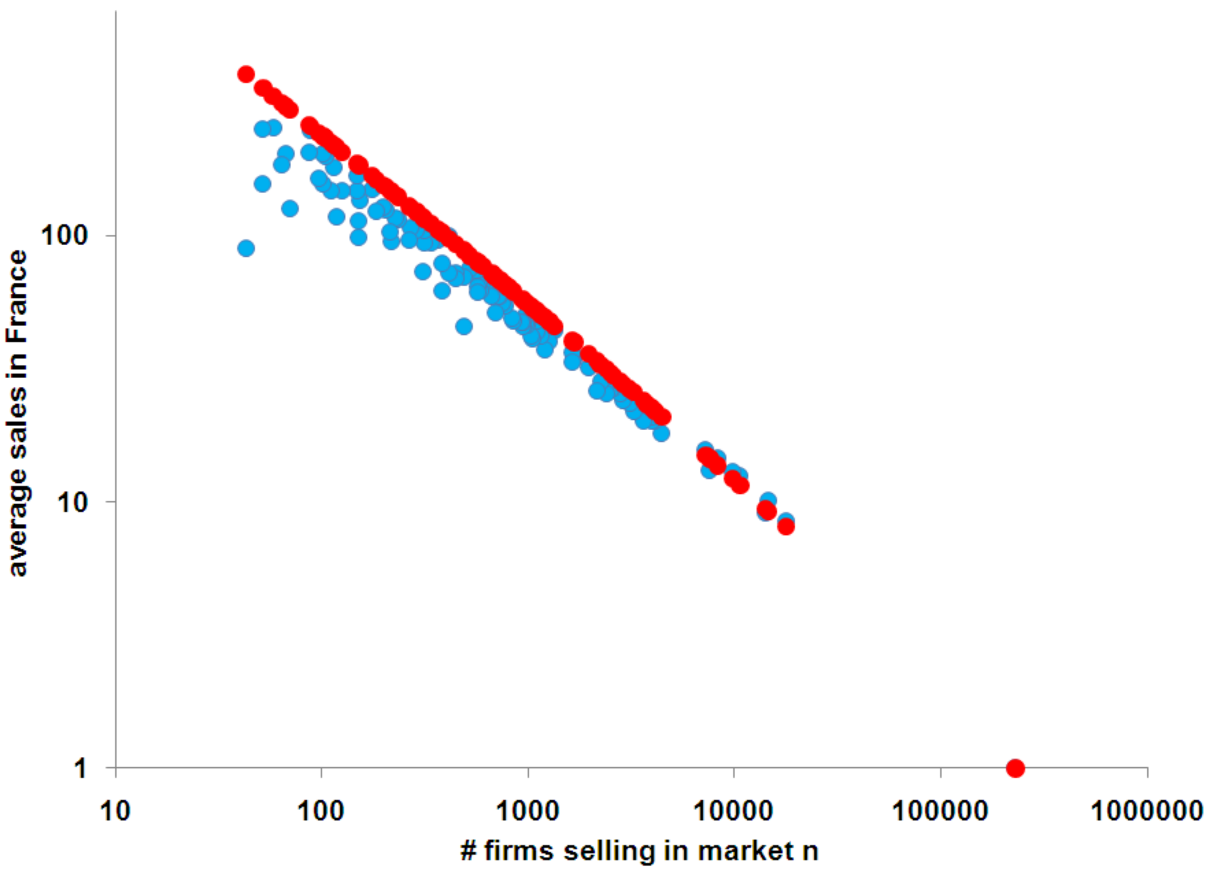

Figure 6: Normalized average sales in the French data and the model ($\beta = 2$, $\tilde{\theta} = 1.65$)

8 The Parameters a, γ and Normalized Firm Entry

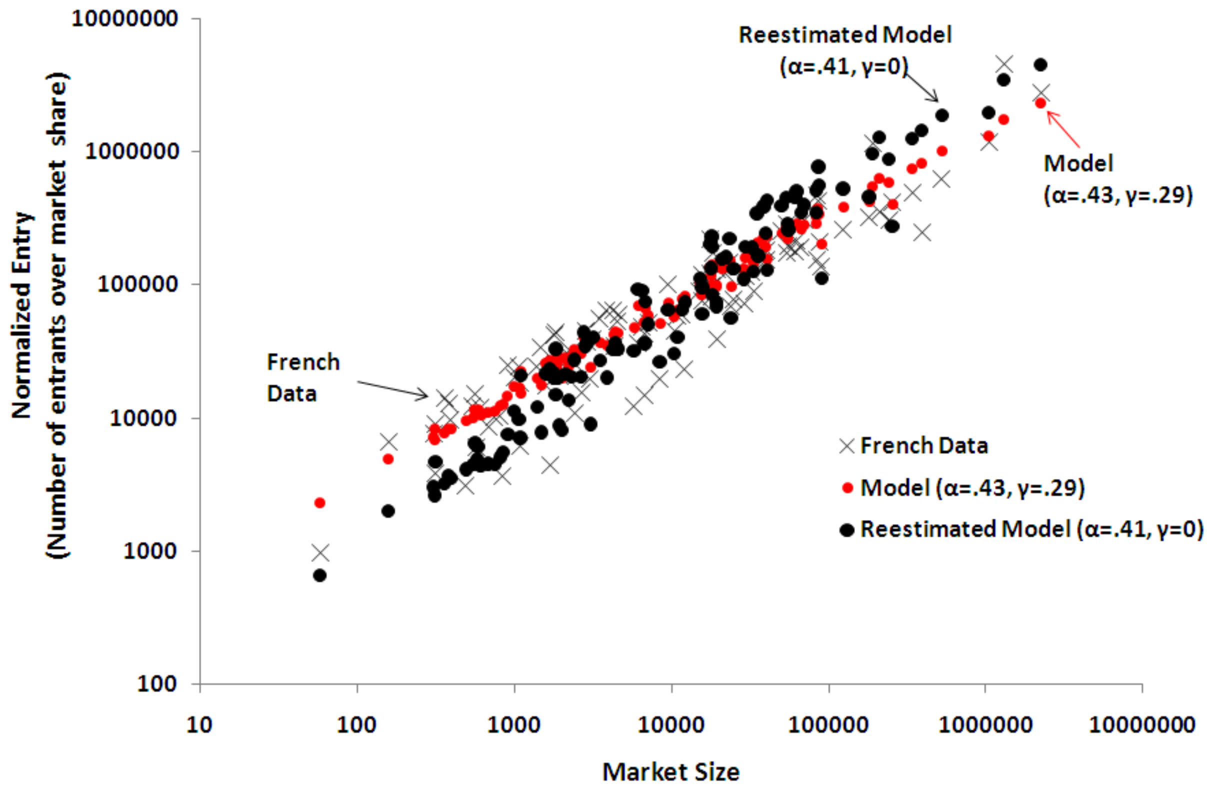

Figure 7: Normalized Entry (M_{ij}/λ_{ij}) in the data and the reestimated model (by assuming $\gamma = 0$ and reestimating α using the relationship between average sales and y_j,L_j)

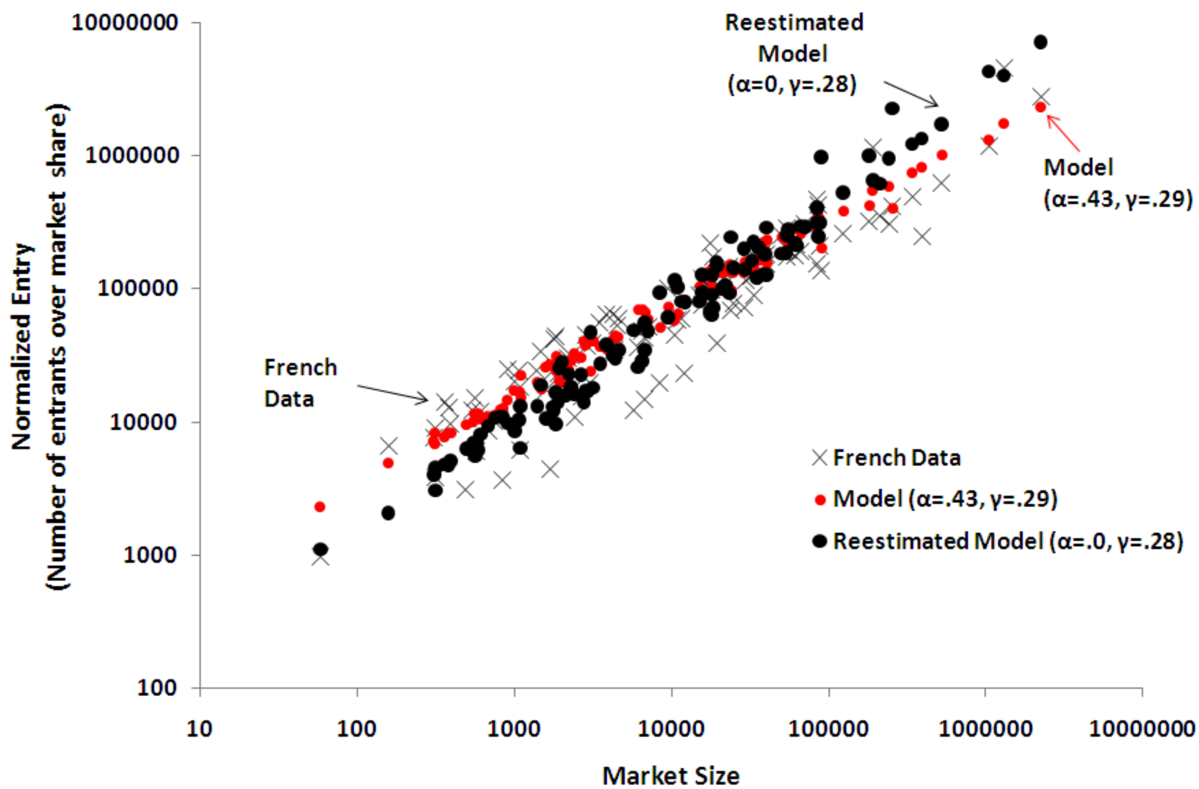

Figure 8: Normalized Entry (M_{ij}/λ_{ij}) in the data and the reestimated model (by assuming $\alpha = 0$ and reestimating γ using the relationship between average sales and y_j,L_j)

9 Export Growth and Marketing Convexity

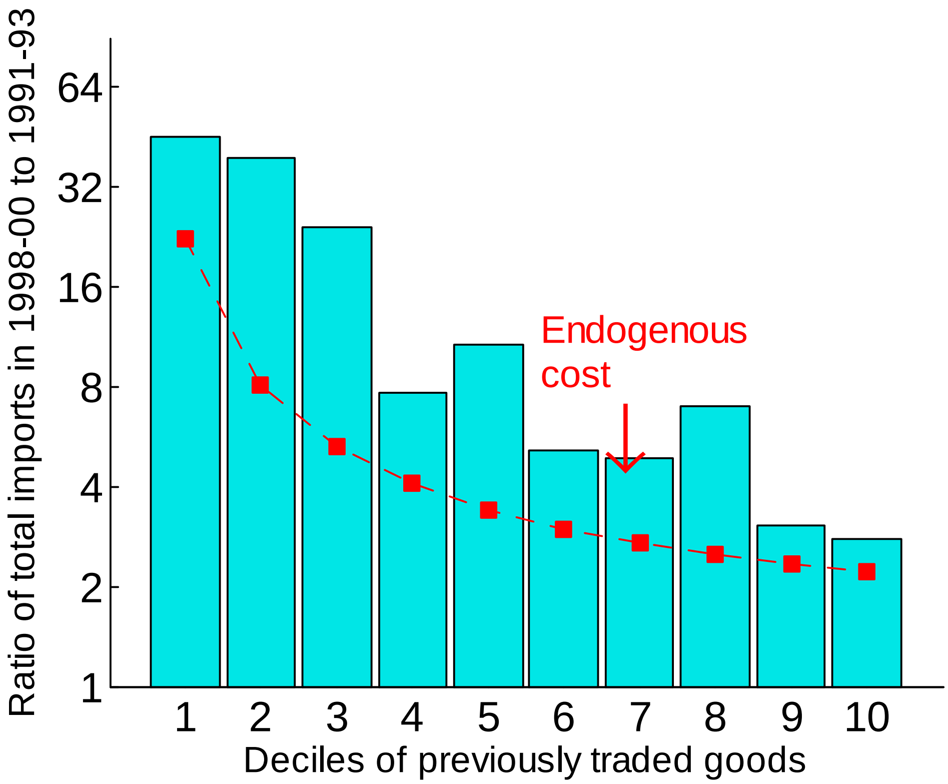

Figure 9: Growth by decile of previously traded goods, data and model ($\beta = .5$, $\tilde{\theta} = 1.65$)

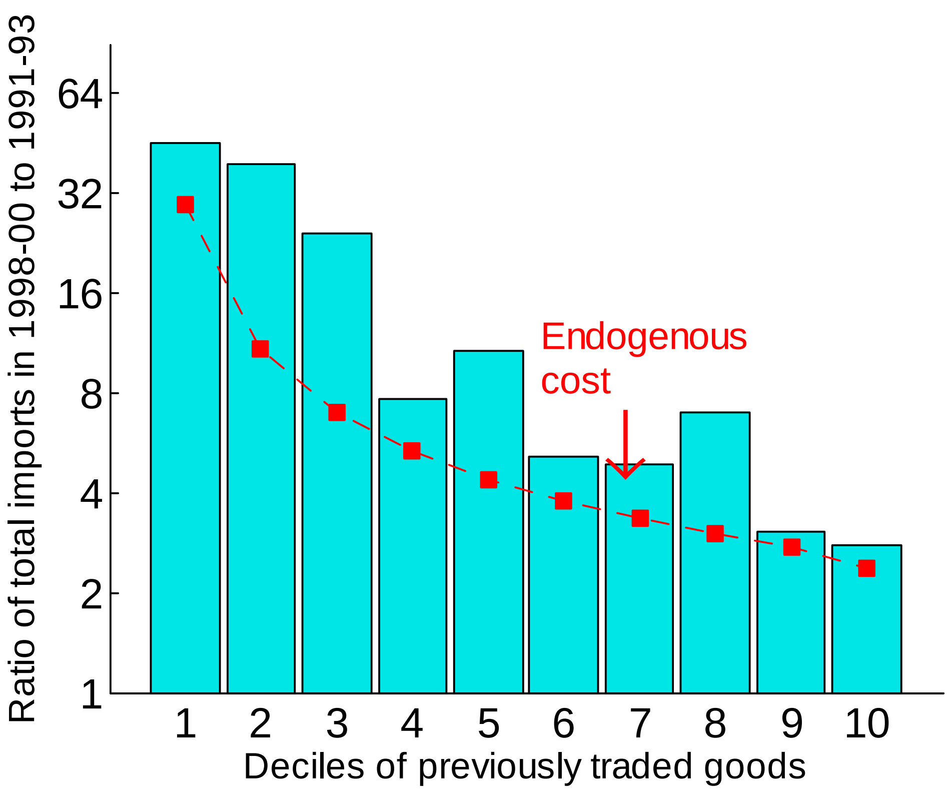

Figure 10: Growth by decile of previously traded goods, data and model ($\beta = 2$, $\tilde{\theta} = 1.65$)

10 Export Growth in the US-France,-Germany,-Mexico cases

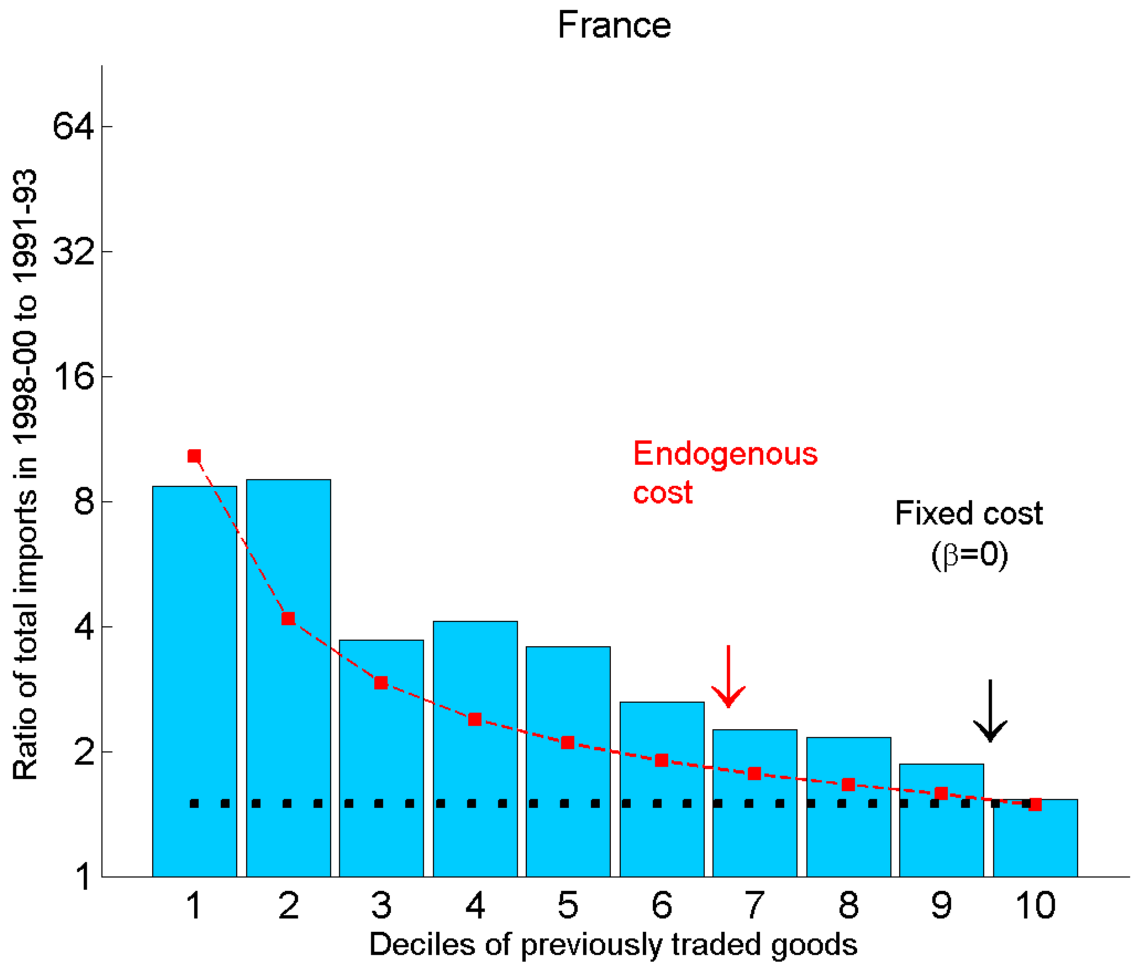

Figure 11: Growth by decile of previously traded goods, data for US-France and model calibrated to the US-France case under the two parameterizations.

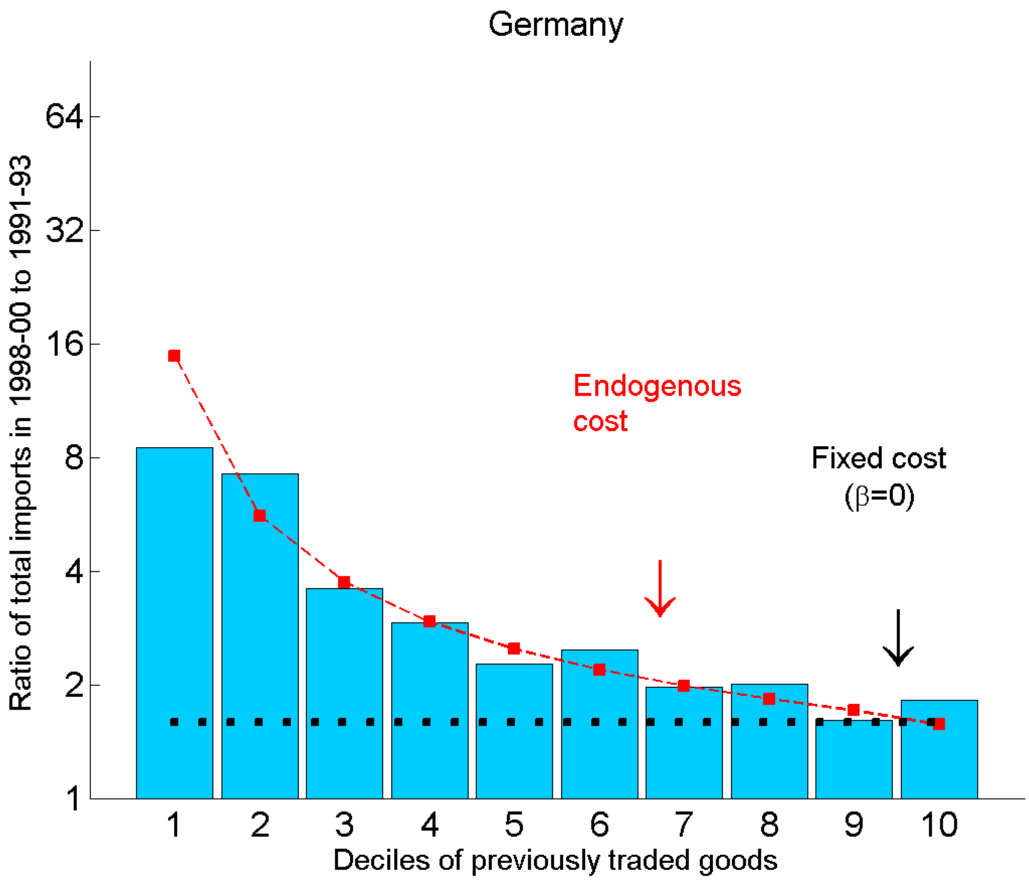

Figure 12: Growth by decile of previously traded goods, data for US-Germany and model calibrated to the US-Germany case under the two parameterizations.

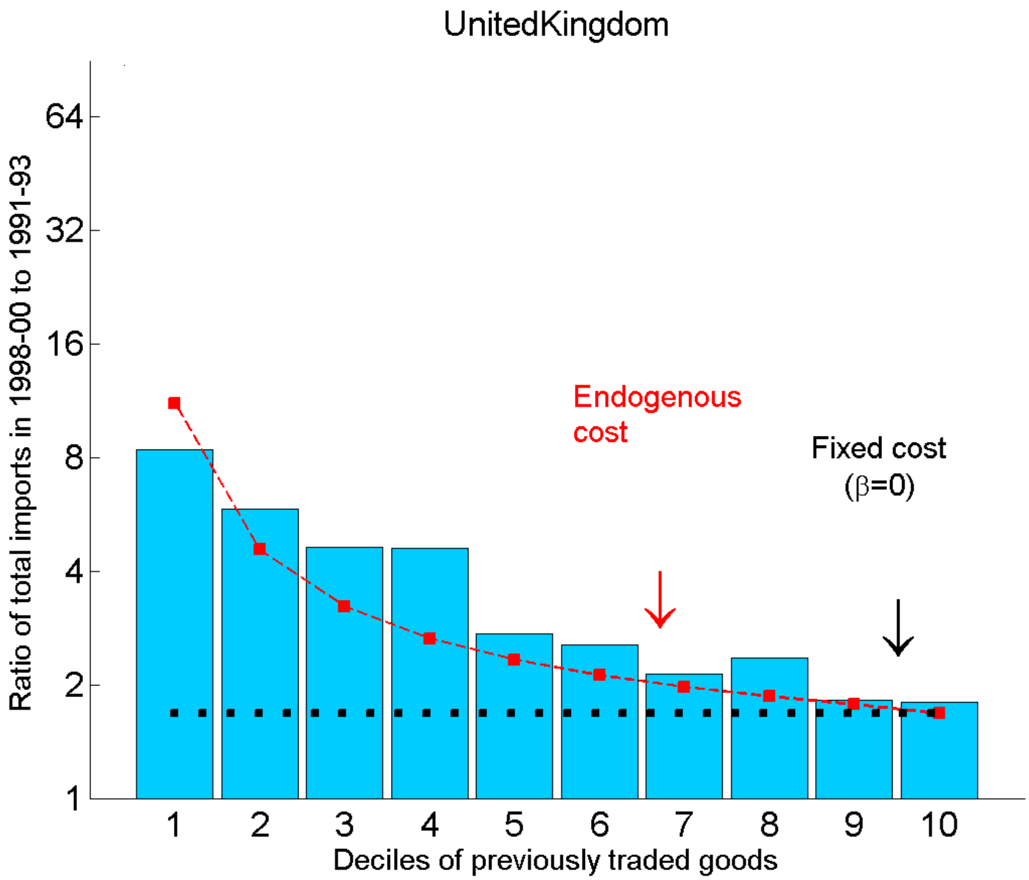

Figure 13: Growth by decile of previously traded goods, data for the United Kingdom and model calibrated to the US-United Kingdom case under the two parameterizations.

11 Trade Costs

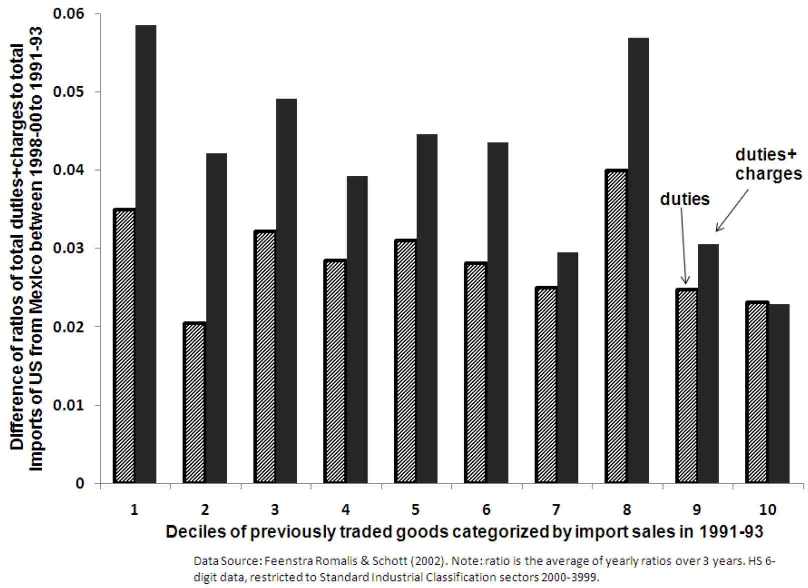

Figure 14: Trade costs changes in the US-Mexico NAFTA liberalization for previously traded goods at the Harmonized System 6-digit level categorized by initial trade. Restriction to manufacturing Standard Industrial Classification sectors.

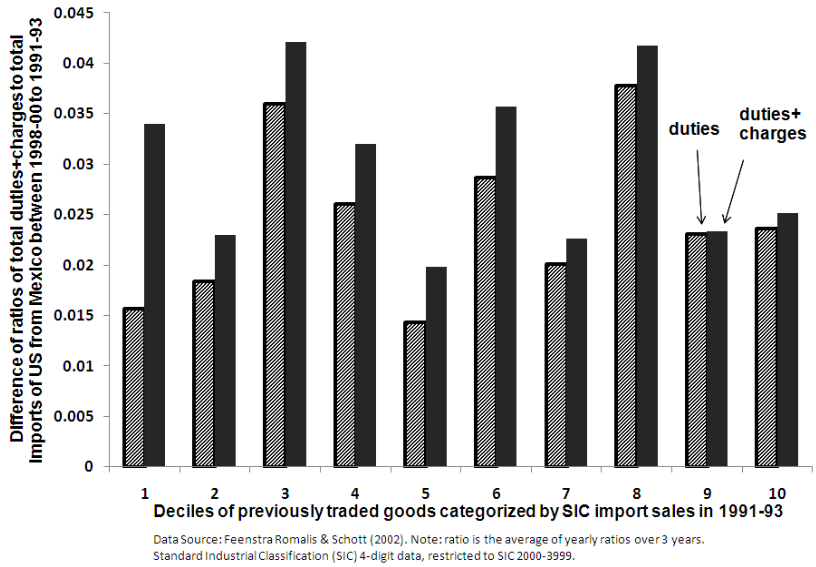

Figure 15: Trade costs changes in the US-Mexico NAFTA liberalization for previously traded goods at the Standard Industrial Classification 4-digit level categorized by initial trade. Restriction to manufacturing Standard Industrial Classification sectors.

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