The Extensive Margin of Exporting Products: The Continuum Case*

Costas Arkolakis[‡]
Yale University, CESifo and NBER

Marc-Andreas Muendler[¶]
UC San Diego, CESifo and NBER

First version: November 13, 2007
This version: August 3, 2011

Abstract

We present a heterogeneous-firm model with product choice in continuous space. The continuum model yields similar estimation equations to a companion paper in discrete space and also generates the documented empirical regularities within and across firms (Arkolakis and Muendler 2010). In a generalization, we consider a cannibalization effect, by which a firm loses infra-marginal sales as it adds additional products in a market. To fit empirical relationships more closely, we allow for product sales that drop off faster than log linearly as the firm adds additional products. Estimates of the key entry cost parameters in the generalized model confirm those in our more parsimonious model: firms face a strong decline in product sales with scope but market-specific entry costs drop fast so that wide-scope exporters sell their lowest-selling products in minor amounts.

Keywords: International trade; heterogeneous firms; multi-product firms; firm and product panel data; Brazil

JEL Classification: F12, L11, F14

*We thank David Atkin, Thomas Chaney, Arnaud Costinot, Don Davis, Gilles Duranton, Jonathan Eaton, Elhanan Helpman, Kalina Manova, Gordon Hanson, Lorenzo Caliendo, Sam Kortum, Giovanni Maggi, Marc Melitz, Peter Neary, Jim Rauch, Steve Redding, Kim Ruhl, Peter Schott, Daniel Trefler and Jon Vogel as well as several seminar and conference participants for helpful comments and discussions. Roberto Álvarez kindly shared Chilean exporter and product data for the year 2000, for which we report comparable results to our Brazilian findings in an online Data Appendix at URL *econ.ucsd.edu/muendler/research*. Oana Hirakawa and Olga Timoshenko provided excellent research assistance. Muendler and Arkolakis acknowledge NSF support (SES-0550699 and SES-0921673) with gratitude. A part of this paper was written while Arkolakis visited the University of Chicago, whose hospitality is gratefully acknowledged.

[‡]costas.arkolakis@yale.edu (*www.econ.yale.edu/∼ka265*).

[¶]muendler@ucsd.edu (*www.econ.ucsd.edu/muendler*). Ph: +1 (858) 534-4799.

1 Introduction

Market entry costs are a key component in recent trade theory with heterogeneous firms. To assess the relative magnitudes and relevance of entry costs components, we study multi-product exporters. Multi-product exporters face different entry cost components when they enter markets and when they widen their local product scope. In this paper, we present a generalized model of multi-product exporters set in continuous product space. We allow for two additional components of market entry costs compared to a companion paper in discrete product space (Arkolakis and Muendler 2010). We consider a cannibalization effect, by which additional products reduce the sales of a firm's infra-marginal products; and we model product sales that drop with scope faster than log linearly to fit empirical relationships more closely. We show that continuous product space delivers compact theoretical relationships for empirical work, even though the number of products is countable in the data.

Our model has heterogeneous firms and products, where firms face destination-specific local entry costs for each of their products. Replicating our model for countably many products (Arkolakis and Muendler 2010), the model in this paper rests on a single source of firm heterogeneity: productivity. Firms face declining efficiency in producing their less successful products, similar to Eckel and Neary (2010) and Mayer, Melitz and Ottaviano (2011). Our setup extends the Melitz (2003) framework to the multi-product setting where the firm decides along three export margins: its entry into export destinations at a first extensive margin, its *exporter scope* (the number of products) in a destination at a second extensive margin, and its individual product sales in a destination at the residual intensive margin. Our generalized model of multi-product exporters continues to preserve predictions of previous trade theory. At the firm level, the model reproduces predictions of Melitz (2003) and Chaney (2008) for the upper tail of the total sales distribution. At the country level, our model generates a gravity equation for bilateral trade flows where the Pareto shape parameter plays a similar role as the Fréchet shape parameter in Eaton, Kortum and Kramarz (2010), and fixed market entry costs affect bilateral trade.

A parametric specification of our model generates a relationship between a product's sales, exporter scope and the product's local sales rank, where product sales drop off faster than log linearly in the product's sales rank. When estimated, this relationship yields coefficient estimates for the relevant market entry cost components. We use three-dimensional panel data of Brazilian manufacturing producers, their destination markets and their products at the Harmonized System

(HS) 6-digit level as in Arkolakis and Muendler (2010). We then compare the results for this sample to evidence from a sample of Chilean manufacturing producers and a sample of Brazilian commercial intermediaries selling manufactures.

For Brazilian manufacturing producers, our non-linear least squares estimates confirm the more restrictive ones from Arkolakis and Muendler (2010), and we find estimates for Chilean manufacturing producers to be close to the ones for Brazilian producers. Our non-linear estimates suggest that incremental local entry costs drop at an elasticity of around -1.4 when producers introduce additional products in a market where they are present. But firm-product efficiency drops off even faster with an elasticity of around 2.6 at the sample mean, as producers introduce products further away from their core competency. Adding the two fixed scope cost components, there are net overall diseconomies of scope with a scope elasticity of around 1.2, similar to what we find in our companion paper (Arkolakis and Muendler 2010).[1]

Our generalized estimates show that the simpler model in Arkolakis and Muendler (2010) approximates main parameters reasonably well for manufacturing producers. Manufacturers are the focus of much of the related empirical literature (Eaton, Kortum and Kramarz 2004, Bernard, Redding and Schott 2011, e.g.). For commercial intermediaries, however, the non-log-linearity in product introduction costs matters considerably: the according scope elasticity estimate would be downward biased by almost 40 percent in the more restrictive log-linear estimation model. Our evidence suggests that the relevant curvature in product introduction costs mainly stems from products with minor sales. The existence of products with minor sales is, in turn, consistent with the main mechanism of our theory. In our model, highly productive wide-scope firms overcome local entry costs for their lowest-selling products relatively easily but they face fast increasing marginal production costs away from core competency. Together, these two features explain why sales of the lowest-selling products are minor at wide-scope firms.

Our nested demand system is based on conventional constant elasticities of substitution (CES) in continuous product space and can provide an alternative multi-product framework compared to the influential work by Bernard, Redding and Schott (2011 and 2011), who also use continuous product space. Similar to Feenstra and Ma (2008), our model allows for a cannibalization effect. In contrast to our CES framework with monopolistic competition, however, Feenstra and Ma (2008) consider a countable number of firms and find a varying cannibalization effect to yield

[1] The cannibalization effect augments the costs of a wider exporter scope in addition, but we do not attempt to estimate this demand-side aspect using firm-level supply data.

a well defined exporter scope optimum in the absence of diseconomies of scope on the cost side. A countable number of firms with product-market power, however, does not lead to closed-form estimation relationships. Similar to recent work by Mayer et al. (2011), the parametric specification of our model generates a non-log-linear relationship between firm-product sales and the product rank. Beyond that similarity, the Mayer et al. (2011) framework complements our model in demonstrating how variable markups further improve specific predictions.

The remainder of this paper is organized as follows. We introduce the general model in Section 2. In Section 3 we derive predictions for bilateral trade under a Pareto distribution and adopt parametric functional forms for firm-level predictions. We obtain structural estimates of entry cost parameters in Section 4. Section 5 concludes. Some details on derivations and data sources are relegated to the Appendix.

2 A Model of Exporter Scope and Product Scale

Our model has heterogeneous firms that sell a finite measure of products in the markets where they enter. Our continuum model rests on a single source of firm heterogeneity and there are three key ingredients, just as in the corresponding model with countably many discrete products: a firm's productivity explains total sales of the firm's products worldwide; firm-product specific efficiency determines individual product sales worldwide; and a firm repeatedly incurs fixed entry costs per product destination by destination. Beyond the benchmark specification in Arkolakis and Muendler (2010), we consider a cannibalization effect by which additional products reduce the sales of a firm's infra-marginal products and we allow for product sales that drop off faster than log linearly as a firm adds products.

2.1 Consumers

There are N countries. We label the source country of an export shipment with s and the export destination with d. There is a measure of L_d consumers at destination d. Consumers have symmetric preferences with a constant elasticity of substitution σ over a continuum of varieties. A conventional "variety" becomes the product composite

$$X_{sd}(\omega) \equiv \left[\int_{\underline{g}}^{G_{sd}(\omega)} x_{sd}(\omega; g)^{\frac{\varepsilon-1}{\varepsilon}} \, \mathrm{d}g \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

in continuous product space, where $G_{sd}(\omega)$ is the measure of products that firm ω sells in country d, and $x_{sd}(\omega; g)$ is the quantity of product g that consumers consume. In marketing terminology, the product composite is a firm's product line or product mix. The index numbers g start at some initial index $\underline{g}$.[2] The elasticity of substitution ε is constant across products within the product line and possibly different from σ. We assume that every product line is uniquely offered by a single firm, but a firm may ship different product lines to different destinations.

The set of product lines shipped from source country s to destination d for consumption is $\mathbf{\Omega}_{sd}$. So the consumer's utility at destination d is

$$\left(\sum_{s=1}^{N} \int_{\omega \in \mathbf{\Omega}_{sd}} \left[\int_{\underline{g}}^{G_{sd}(\omega)} x_{sd}(\omega; g)^{\frac{\varepsilon-1}{\varepsilon}} \, \mathrm{d}g \right]^{\frac{\varepsilon}{\varepsilon-1} \frac{\sigma-1}{\sigma}} \mathrm{d}\omega \right)^{\frac{\sigma}{\sigma-1}} \quad \text{where } \varepsilon > 1,\ \sigma > 1, \varepsilon \neq \sigma. \qquad (1)$$

Allanson and Montagna (2005) use a similar nested utility to study implications of the product life-cycle for market structure. Atkeson and Burstein (2008) adopt a similar nested CES form in a heterogeneous-firms model of trade but do not consider multi-product firms. The specification generalizes monopolistic-competition models of trade (such as Krugman 1980).[3] For preferences to be well defined, we require that $\varepsilon > 1$ and $\sigma > 1$. Subsequent derivations do not materially differ if we assume that products within a product line are more substitutable among each other than with outside products ($\varepsilon > \sigma > 1$), or less substitutable ($\sigma > \varepsilon > 1$). Evidence in Broda and Weinstein (2007) suggests that products are stronger substitutes within brands than across brands so we consider $\varepsilon > \sigma > 1$ our main case.[4]

The consumer receives per-capita income $y_d = w_d + \pi_d$, where w_d is the wage for labor, inelastically supplied to producers in country d, and $\pi_d \equiv \int_{\omega=1}^{N} \pi_{sd\omega}/L_d$ is the consumer's share in total profits earned by the country's producers. Country d's total income is $y_d L_d$. The consumer's

[2] When we turn to parametric estimation equations we set $\underline{g} = 1$ similar to discrete product space.

[3] The counterpart to (1) in discrete product space are consumer preferences at d

$$\left(\sum_{s=1}^{N} \int_{\omega \in \mathbf{\Omega}_{sd}} \left[\sum_{g=1}^{G_{sd}(\omega)} x_{sd}(\omega; g)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1} \frac{\sigma-1}{\sigma}} \mathrm{d}\omega \right)^{\frac{\sigma}{\sigma-1}} \quad \text{where} \quad \varepsilon > 1,\ \sigma > 1, \varepsilon \neq \sigma,$$

where $\mathbf{\Omega}_{sd}$ is the set of product lines shipped from s to d. We analyze that case in Arkolakis and Muendler (2010).

We could make the elasticities of substitution country-specific (ε_d and σ_d), and all our results would continue to apply. We keep the elasticities the same across destinations to emphasize that our results do not depend on preference assumptions.

[4] Their preferred estimates for ε and σ within and across domestic U.S. brand modules are 11.5 and 7.5.

first-order conditions of utility maximization imply a product demand

$$x_{sd}(\omega; g) = \left(\frac{p_{sd}(\omega; g)}{P_{sd}(\omega; G_{sd})} \right)^{-\varepsilon} X_{sd}(\omega; G_{sd}), \tag{2}$$

given the firm's product line $X_{sd}(\omega; G_{sd})$ and the product-line price $P_{sd}(\omega; G_{sd})$:

$$X_{sd}(\omega; G_{sd}) \equiv \left[\int_{\underline{g}}^{G_{sd}(\omega)} x_{sd}(\omega; g)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}g \right]^{\frac{\varepsilon}{\varepsilon-1}} \text{ and } P_{sd}(\omega; G_{sd}) \equiv \left[\int_{\underline{g}}^{G_{sd}(\omega)} p_{sd}(\omega; g)^{-(\varepsilon-1)} \mathrm{d}g \right]^{-\frac{1}{\varepsilon-1}}.$$

The first-order conditions also imply a product-line demand

$$X_{sd}(\omega; G_{sd}) = \left(\frac{P_{sd}(\omega; G_{sd})}{P_d} \right)^{-\sigma} X_d \qquad \text{with} \quad X_d \equiv \left[\sum_{s=1}^{N} \int_{\omega \in \mathbf{\Omega}_{sd}} X_{sd}(\omega; G_{sd})^{\frac{\sigma-1}{\sigma}} \, \mathrm{d}\omega \right]^{\frac{\sigma}{\sigma-1}} \tag{3}$$

and the consumer price index

$$P_d \equiv \left[\sum_{s=1}^{N} \int_{\omega \in \mathbf{\Omega}_{sd}} P_{sd}(\omega; G_{sd})^{-(\sigma-1)} \, \mathrm{d}\omega \right]^{-\frac{1}{\sigma-1}}. \tag{4}$$

So, the demand for firm ω's product g, produced in source country s and sold to destination country d, is

$$x_{sd}(\omega; g) = \left(\frac{p_{sd}(\omega; g)}{P_{sd}(\omega; G_{sd})} \right)^{-\varepsilon} \left(\frac{P_{sd}(\omega; G_{sd})}{P_d} \right)^{-\sigma} \frac{y_d L_d}{P_d} \tag{5}$$

with $P_d \, X_d = y_d L_d$.

This relationship gives rise to a cannibalization effect for $\varepsilon > \sigma$. Since $P_{sd}(\omega; G_{sd})$ strictly decreases in exporter scope for $\varepsilon > 1$, wider exporter scope diminishes infra-marginal sales and reduces $x_{sd}(\omega; g)$ for $\varepsilon > \sigma$. In other words, if products within a product line are more substitutable among each other than with outside products, widening exporter scope is costly to the exporter because it diminishes the sales of infra-marginal products.[5]

2.2 Firms

Following Chaney (2008), we assume that there is a continuum of potential producers of measure J_s in each source country s. Productivity is the single source of firm heterogeneity so that, under the

[5] Under constant elasticities of substitution in our model, the cannibalization effect does not vary by scope. So the cannibalization effect does not limit exporter scope at a finite level, and we have to require local entry costs for an optimum to exist. Feenstra and Ma (2008) show in a related framework that a countable number of firms with product-market power implies that widening exporter scope more than proportionally cannibalizes the sales of infra-marginal products. A countable number of firms as in Feenstra and Ma (2008), however, does not lead to a closed-form equilibrium.

model assumptions below, firms of the same type ϕ from country s face an identical optimization problem in every destination d. Since all firms with productivity ϕ will make identical decisions in equilibrium, it is convenient to name them by their common characteristic ϕ from now on.

A firm of type ϕ chooses the measure of products $G_{sd}(\phi)$ to sell to a given market d. We call an exporter's measure of products G_{sd} shipped to destination d the *exporter scope* at destination d. A firm produces each product $g \in [\underline{g}, G_{sd}(\phi)]$ with a linear production technology $q_{sdg}(\phi_g) = \phi_g \, \ell$ where ϕ_g is the efficiency of the g-th product of a firm ϕ and ℓ is employment contracted at the source country's wage w_s. When exported, a product incurs a standard iceberg trade cost so that $\tau_{sd} > 1$ units must be shipped from s for one unit to arrive at destination d. We normalize $\tau_{ss} = 1$ for domestic sales. Note that this iceberg trade cost is common to all firms and to all firm-products shipping from s to d. A firm's sales of its individual product g at destination d are $p_{sd}(g)\, x_{sdg}$, where $p_{sd}(g)$ is the product price.

Without loss of generality we order each firm's products in terms of their efficiency so that $\partial\phi_g/\partial g < 0$ for all g. Under the convention that a firm's lowest-index product is its most productive product, we write the efficiency of the g-th product of a firm ϕ as[6]

$$\phi_g \equiv \frac{\phi}{h(g)} \qquad \text{with} \quad h'(g) > 0. \tag{6}$$

An implication of a strictly increasing marginal-cost schedule $h'(g) > 0$ is that a firm will enter export market d with the most efficient product first and then expand its scope moving up the marginal-cost ladder product by product. We normalize $h(\underline{g}) = 1$ so that $\phi_{\underline{g}} = \phi$. Varying firm-product efficiencies will support the result that some products systematically sell higher across markets, consistent with empirical evidence (Arkolakis and Muendler 2010). The assumption that the firm faces a drop in efficiency for each additional product as its exporter scope widens is a common assumption in multi-product models of exporters. Related models include Eckel and Neary (2010), who call the lowest-index product the firm's core competency, and Nocke and Yeaple (2006) and Mayer et al. (2011). Bernard et al. (2011) offer a stochastic generalization of this framework, in which firms endogenously adopt products of different competency levels.

Related to the marginal-cost schedule $h(g)$ we define firm ϕ's product efficiency index as

$$H(G_{sd}) \equiv \left(\int_{\underline{g}}^{G_{sd}} h(g)^{-(\varepsilon-1)} \, \mathrm{d}g \right)^{-\frac{1}{\varepsilon-1}}. \tag{7}$$

[6] The function $h(g)$ could be considered destination specific but such generality would introduce degrees of freedom that are not relevant for empirical regularities in three-dimensional firm-product-destination data.

This unweighted efficiency index will play an important role in the firm's optimal scope choice. Since the marginal-cost schedule strictly increases in exporter scope, a firm's product efficiency index strictly falls as its exporter scope widens. Similar to the stochastic setting with endogenous choice in Bernard et al. (2011), a wide-scope firm tolerates less efficient products in its product line.

As the firm widens its exporter scope, it also incurs a destination-specific *fixed local entry cost* that is non-zero at the minimum scope and weakly increases in exporter scope:

$$F_{sd}(G_{sd}) \qquad \text{with} \quad F_{sd}(\underline{g}) > 0 \text{ and } F'_{sd}(G_{sd}) \in (0, +\infty)\,. \tag{8}$$

There is a non-zero beachhead cost $F_{sd}(\underline{g})$ to enter a foreign market with at least one incremental product because a firm would otherwise export to all destinations worldwide.[7] For any additional product, marginal local entry cost must be strictly positive and finite for a well-defined optimum to exist.

To summarize, there are two scope-dependent cost components in our model. There is the marginal-cost schedule $h(g)$, which rises as the product index g increases. And there is the incremental local entry cost $F'_{sd}(g)$, which may fall with the product index g.[8] Suppose the incremental local entry cost is constant and independent of g with $F'_{sd}(g) = f_{sd}$. Then a firm in our model exhibits diseconomies of scope because the marginal-cost schedule $h(g)$ strictly increases with the product index g (the firm's total cost function violates subadditivity). Multi-product firms may still arise in equilibrium because a highly productive firm ϕ may produce products far from its core more cheaply than a single-product firm with low productivity.

A firm ϕ's profit in a given destination market d is

$$\pi_{sd}(\phi) = \max_{G_{sd}, P_{sd}, \{p_{sd}(g)\}_{g\in[\underline{g}, G_{sd}]}} \int_{\underline{g}}^{G_{sd}} \left(p_{sd}(g) - \frac{\tau_{sd}\, w_s}{\phi/h(g)} \right) p_{sd}(g)^{-\varepsilon} \frac{(P_{sd})^{\varepsilon-\sigma}}{(P_d)^{-(\sigma-1)}}\, y_d L_d \,\mathrm{d}g - F_{sd}(G_{sd}) \tag{9}$$

by demand (5), where $\pi_{sd}(\phi)$ denotes maximized profits.

[7] No fixed local entry cost is needed in the discrete product-space version of our model where the firm's first product causes a nontrivial fixed local entry cost.

[8] A definition of economies of scope at the firm level is that a firm's total cost function is subadditive. A firm's total cost function is subadditive if the firm can produce an output vector more cheaply than when it is broken up into two or more firms. Formally, a firm ϕ's total cost function $C_\phi(\cdot)$ is subadditive at the infinite-dimensional output vector $\mathbf{x} = (x_{\underline{g}} \cdots x_g \cdots x_{G_{sd}})$ if and only if

$$C_\phi(\mathbf{x}) < \textstyle\sum_{i=1}^n C_\phi(\mathbf{x}^i),$$

where the non-negative vectors $\mathbf{x}^i$ satisfy $\sum_{i=1}^n \mathbf{x}^i = \mathbf{x}$ for all $\mathbf{x}^i$ and for at least two $\mathbf{x}^i$ non-zero.

The firm's profit maximization problem can be broken into two steps. First is the choice of the price index P_{sd} along with the individual product prices $p_{sd}(g)$ to maximize (9), given G_{sd} and subject to the constraint that $(P_{sd})^{-(\varepsilon-1)} = \int (p_{sd}(g))^{-(\varepsilon-1)} \mathrm{d}g$ by (2).[9] The first-order conditions with respect to the product-line price P_{sd} and individual prices $p_{sd}(g)$ imply product prices

$$p_{sd}(\phi; g) = \tilde{\sigma}\, \tau_{sd}\, w_s\, h(g)/\phi \tag{10}$$

with an identical markup over marginal cost $\tilde{\sigma} \equiv \sigma/(\sigma-1) > 1$ for $\sigma > 1$ (see Appendix A). The optimal product-line price

$$P_{sd}(\phi; G_{sd}) = [\tilde{\sigma}\, \tau_{sd}\, w_s/\phi]\, H(G_{sd})$$

strictly decreases in G_{sd} (because the firm's product efficiency index strictly falls). The optimal markup, however, does not depend on exporter scope for constant elasticities of substitution under monopolistic competition.[10]

A firm's choice of optimal prices implies optimal sales for product g:[11]

$$p_{sd}(\phi; g)\, x_{sdg}(\phi; g) = y_d L_d \left(\frac{\phi\, P_d}{\tilde{\sigma}\, \tau_{sd}\, w_s} \right)^{\sigma-1} H(G_{sd})^{\varepsilon-\sigma}\, h(g)^{-(\varepsilon-1)}. \tag{11}$$

Integrating (11) over the firm's products at destination d, firm ϕ's optimal total exports to destination d are

$$t_{sd}(\phi) = \int_{\underline{g}}^{G_{sd}} p_{sd}(\phi; g)\, x_{sdg}(\phi; g)\, \mathrm{d}g = y_d L_d \left(\frac{\phi\, P_d}{\tilde{\sigma}\, \tau_{sd}\, w_s} \right)^{\sigma-1} H(G_{sd})^{-(\sigma-1)} \tag{12}$$

and, given constant markups over marginal cost, profits from exporting to destination d become

$$\pi_{sd}(\phi) = \max_{G_{sd}} \frac{t_{sd}(\phi)}{\sigma} - F_{sd}(G_{sd}) = \max_{G_{sd}} \frac{y_d L_d}{\sigma} \left(\frac{\phi\, P_d}{\tilde{\sigma}\, \tau_{sd}\, w_s} \right)^{\sigma-1} H(G_{sd})^{-(\sigma-1)} - F_{sd}(G_{sd}).$$

Second is the choice of exporter scope G_{sd}, given the optimal product-line price $P_{sd}(\phi; G_{sd})$ and optimal individual product prices $p_{sd}(\phi; g)$. For optimal exporter scope G_{sd} to be well defined, we make the following assumption.

[9] We thank Elhanan Helpman for pointing out the Lagrangian solution strategy.

[10] In discrete product space, too, the optimal markup does not vary with exporter scope for constant elasticities of substitution under monopolistic competition.

[11] This relationship is consistent with an important empirical regularity in the data: within firms and destinations, exports are concentrated in few top-selling products (Fact 1 in Arkolakis and Muendler 2010). Note that $h(g)$ increases with rank g so exports drop with a product's rank in the firm's sales. Adequate restrictions on $h(g)$ can be used to vary the degree of concentration of exports in few products. In fact, the existence of $H(G)$ in definition (7) requires a sufficient degree of convexity in $h(g)$ already, which in turn gives rise to some concentration of exports in few products.

Assumption 1 (Strictly increasing combined incremental scope costs). *Combined incremental scope costs* $Z_{sd}(G) \equiv F'_{sd}(G)h(G)^{\varepsilon-1}/H(G)^{\varepsilon-\sigma}$ *strictly increase in exporter scope* G *at any* $G \in [\underline{g}, \infty)$.

Assumption 1 is a global condition and implies that the second-order condition is not only satisfied at the local optimum $G_{sd}(\phi)$, but globally, so that the combination of variable product introduction costs and fixed local entry costs is globally strictly convex in exporter scope.[12] Assumption 1 guarantees that a firm will choose finite exporter scope.

Taking the first derivative of the profit function with respect to G_{sd} and setting it to zero, optimal exporter scope $G_{sd}(\phi)$ must satisfy the first-order condition

$$\frac{\bar{\sigma}\, y_d L_d}{\sigma}\left(\frac{\phi\, P_d}{\tilde{\sigma}\,\tau_{sd}\, w_s}\right)^{\sigma-1} = F'_{sd}\left(G_{sd}(\phi)\right) h\left(G_{sd}(\phi)\right)^{\varepsilon-1} H\left(G_{sd}(\phi)\right)^{-(\varepsilon-\sigma)} \equiv Z_{sd}(G_{sd}(\phi)), \quad (13)$$

where the composite elasticity term $\bar{\sigma} \equiv (\sigma-1)/(\varepsilon-1)$ is strictly less than one if and only if products within the product line are closer substitutes among each other than with outside products ($\varepsilon > \sigma > 1$) so that widening scope cannibalizes inframarginal sales. Note that the left-hand side is constant. So, for a well-defined optimum to exist *combined marginal scope costs* $Z_{sd}(G) \equiv F'_{sd}(G)h(G)^{\varepsilon-1}/H(G)^{\varepsilon-\sigma}$ must strictly increase in exporter scope at the optimal $G_{sd}(\phi)$ (that is profits must be concave in G_{sd} at the optimal $G_{sd}(\phi)$).

The combined incremental scope costs $Z_{sd}(G) \equiv F'_{sd}(G)h(G)^{\varepsilon-1}/H(G)^{\varepsilon-\sigma}$ consist of three intuitive effects. The first term $F'_{sd}(G)$ captures the incremental local entry cost as a firm widens its scope. The second term is the marginal cost level that a firm faces for the incremental product as it widens its scope, where the firm considers the related revenue impact $h(G)^{\varepsilon-1}$ by (11). The third term reflects the cannibalization effect: as a firm widens its scope, it reduces the sales of its infra-marginal products if products within a product line are more substitutable among each other than with outside products ($\varepsilon > \sigma$).

[12]Evaluated at an extremum, the second-order condition for optimal product scope is equivalent to

$$\frac{F''_d(G_{sd}(\phi))}{F'_{sd}\left(G_{sd}(\phi)\right)} + (\varepsilon-1)\frac{h'(G_{sd}(\phi))}{h\left(G_{sd}(\phi)\right)} + (1-\bar{\sigma})\frac{h\left(G_{sd}(\phi)\right)^{-(\varepsilon-1)}}{H\left(G_{sd}(\phi)\right)^{-(\varepsilon-1)}} > 0,$$

where the composite elasticity term $\bar{\sigma} \equiv (\sigma-1)/(\varepsilon-1)$ satisfies $\bar{\sigma} < 1$ if and only if $\varepsilon > \sigma > 1$ (that is if and only if products within the product line are closer substitutes among each other than with outside products so that widening exporter scope cannibalizes inframarginal sales). The first term in the condition reflects changes in incremental local entry cost, the second term changes in the marginal cost of an incremental product, and the third term changes in the cannibalization effect.

We seek to express the first-order condition for optimal scope in terms of firm productivity ϕ. Firm ϕ exports from s to d if and only if $\pi_{sd}(\phi) \geq 0$. This break-even condition is equivalent to the condition that exporter scope weakly exceed a minimum profitable scope $G_{sd}(\phi) \geq G^*_{sd}$ and to the condition that the firm's productivity weakly exceed a productivity threshold $\phi \geq \phi^*_{sd}$. Using first-order condition (13) in the profit function and restricting profits to zero, the minimum profitable scope G^*_{sd} is implicitly given by

$$\frac{F'_{sd}(G^*_{sd})}{F_{sd}(G^*_{sd})} = \bar{\sigma}\frac{h(G^*_{sd})^{-(\varepsilon-1)}}{H(G^*_{sd})^{-(\varepsilon-1)}}. \tag{14}$$

Equivalently, reformulating the break-even condition and using the above expression for minimum profitable scope, the productivity threshold for exporting from s to d is

$$\phi^*_{sd} \equiv \left(\frac{\sigma}{\bar{\sigma}}\frac{Z_{sd}(G^*_{sd})}{y_d L_d}\right)^{\frac{1}{\sigma-1}} \frac{\tilde{\sigma}\,\tau_{sd}\,w_s}{P_d}. \tag{15}$$

Using the definitions of minimum profitable scope (14) and minimum productivity (15), we restate the first-order condition for optimal scope (13) more succinctly as

$$Z_{sd}(G_{sd}(\phi)) = Z_{sd}(G^*_{sd})\left(\frac{\phi}{\phi^*_{sd}}\right)^{\sigma-1}. \tag{16}$$

Note that the left-hand side of (16) must strictly increase in $G_{sd}(\phi)$ by the second-order condition (Assumption 1) and that the right-hand side of (16) strictly increases in ϕ. So, $G_{sd}(\phi)$ strictly increases in ϕ.

Using (13) in (11), we can rewrite individual product sales (11) and total sales (12) as

$$p_{sd}(\phi; g)\,x_{sdg}(\phi; g) = \frac{\sigma}{\bar{\sigma}}\,F'_{sd}\left(G_{sd}(\phi)\right) h\left(G_{sd}(\phi)\right)^{\varepsilon-1}\ h(g)^{-(\varepsilon-1)} \quad \text{for } g \leq G_{sd}(\phi) \tag{17}$$

and

$$t_{sd}(\phi) = \frac{\sigma}{\bar{\sigma}}\,Z_{sd}(G_{sd}(\phi))\,H\left(G_{sd}(\phi)\right)^{-(\sigma-1)}. \tag{18}$$

The following proposition summarizes the findings.

Proposition 1 *If Assumption 1 holds (combined incremental scope costs strictly increase in exporter scope), then for all $s, d \in \{1, \ldots, N\}$*

- *exporter scope $G_{sd}(\phi)$ is positive and strictly increases in ϕ for $\phi \geq \phi^*_{sd}$;*
- *total firm exports $t_{sd}(\phi)$ are positive and strictly increase in ϕ for $\phi \geq \phi^*_{sd}$.*

Proof. The first statement follows from (16) because combined incremental scope costs $Z_{sd}(G)$ strictly increase in exporter scope G by Assumption 1 and $\sigma > 1$. The second statement follows from (18) because combined incremental scope costs $Z_{sd}(G)$ strictly increase in exporter scope G by Assumption 1 and $G_{sd}(\phi)$ strictly increases in ϕ so that $H(\cdot)^{-(\sigma-1)}$ strictly increases in ϕ. ■

We define *exporter scale* in market d as

$$a_{sd}(\phi) \equiv \frac{t_{sd}(\phi)}{G_{sd}(\phi)} = \frac{\sigma}{\bar{\sigma}} \frac{Z_{sd}(G_{sd}(\phi))}{G_{sd}(\phi)} H(G_{sd}(\phi))^{-(\sigma-1)}, \tag{19}$$

where both numerator and denominator are conditional on exporting from s to d. In (19), both $Z_{sd}(\cdot)$ and $H(\cdot)^{-(\sigma-1)}$ increase in exporter scope. So, to generate exporter scale that increases in exporter scope (as our data suggest), it is sufficient that either $Z_{sd}(\cdot)$ or $H(\cdot)^{-(\sigma-1)}$ increase in scope more than proportionally. We turn to $Z_{sd}(G)/G$ and choose to impose a restriction that is more stringent than Assumption 1.[13]

Case C1 (Strong diseconomies of scope). *The combined incremental scope costs* $Z_{sd}(G) \equiv F'_{sd}(G)H(G)^{-(\varepsilon-\sigma)}h(G)^{\varepsilon-1}$ *strictly increase in G at any* $G \in [\underline{g}, \infty)$ *with an elasticity*

$$\frac{\partial \ln Z_{sd}(G)}{\partial \ln G} = \frac{G\, Z'_{sd}(G)}{Z_{sd}(G)} > 1.$$

The restriction requires that Z_{sd} increases more than proportionally with G, that is the scope elasticity of combined incremental scope costs exceeds one. So it implies Assumption 1 by which Z_{sd} increases with G.

Proposition 2 *If* $Z_{sd}(G)$ *satisfies Case C1 (if combined incremental scope costs exhibit strong diseconomies), then exporter scale* $a_{sd}(\phi)$ *strictly increases in* ϕ *for* $\phi \geq \phi^*_{sd}$.

Proof. Recall that exporter scope $G_{sd}(\phi)$ strictly increases in ϕ (Proposition 1). So by (19), $a_{sd}(G(\phi))$ strictly increases in ϕ if and only if $\partial \ln a_{sd}(G)/\partial G = \partial \ln Z(G)/\partial G - 1/G - (\sigma - 1)\partial \ln H(G)/\partial G > 0$. Since $\partial \ln H/\partial G < 0$ by definition (7), the inequality is satisfied if $\partial \ln Z(G)/\partial G > 1/G$ or, equivalently, if $\partial \ln Z(G)/\partial \ln G > 1$ (Case C1). ■

In summary, if combined incremental scope costs strictly increase then exporter scope $G_{sd}(\phi)$ and total exports $t_{sd}(\phi)$ increase with the firm's productivity (Proposition 1). But exporter scale

[13] Our later parametrization implies that the alternative sufficient condition that $H(G)^{-(\sigma-1)}/G$ strictly increases in G (in addition to Assumption 1) is equivalent to $G^{-\bar{\alpha}}/(1-G^{-\bar{\alpha}}) > \bar{\sigma}/\bar{\alpha}$ for some $\bar{\alpha} > 0$ to be derived. This condition may or may not be satisfied as scope G varies. In contrast, Case C1 is equivalent to a simple parameter restriction and independent of G in our later parametrization.

$a_{sd}(\phi) = t_{sd}(\phi)/G_{sd}(\phi)$ increases in productivity only if combined incremental scope costs increase more than proportionally in scope (Proposition 2). So strong diseconomies in combined incremental scope costs generate the empirical regularity that, within destinations, mean exporter scope and mean exporter scale are positively associated (Fact 3 in Arkolakis and Muendler 2010).[14] The intuition for this result is that if $Z_{sd}(G)$ increases fast enough a firm will introduce additional products only if its productivity and thus the profitability of its products is sufficiently high. Therefore only highly productive firms will be wide scope firms and those firms have a high scale per product.

3 Model Predictions

To derive clear predictions for the model equilibrium we specify a Pareto distribution of firm productivity following Helpman, Melitz and Yeaple (2004) and Chaney (2008). To match regularities in the cross section of firms, we adopt parametric functional forms in addition to specifying a Pareto distribution. These assumptions generate an important regularity observed in the data: within destinations, there are few wide-scope and large-sales firms but many narrow-scope and small-sales firms (Fact 2 in Arkolakis and Muendler 2010).

Consider a firm's productivity ϕ to be drawn from a Pareto distribution with a source-country dependent location parameter b_s and a worldwide shape parameter θ over the support $[b_s, +\infty)$ for $s = 1, \ldots, N$. So the cumulative distribution function of ϕ is $\Pr = 1 - (b_s)^\theta/\phi^\theta$ and the probability density function is $\theta(b_s)^\theta/\phi^{\theta+1}$, where more advanced countries are thought to have a higher location parameter b_s. Therefore the measure of firms selling to country d, that is the measure of firms with productivity above the threshold ϕ^*_{sd}, is

$$M_{sd} = J_s(b_s)^\theta/(\phi^*_{sd})^\theta. \tag{20}$$

[14]To be precise, we document in Arkolakis and Muendler (2010) that scope-weighted mean exporter sale at any given percentile strictly increases with mean exporter scope. It is easy to work with exporter scale $a_{sd}(\phi)$ and to characterize its properties such as in Proposition 2. Those properties carry over to scope-weighted mean exporter scale. The reason is that exporter scale is tightly related to scope-weighted mean exporter scale since $a_{sd}(\phi)$ is a monotonic function of productivity if and only if scope-weighted mean exporter scale is a monotonic function of productivity. To see this note that $(t(\phi) + x) \,/\, (G(\phi) + y) \le x/y \iff t(\phi)/G(\phi) \le x/y$. Thus, including lower percentiles in the mean exporter scale will lead to declines in value only if t/G declines.

The probability density function of the conditional distribution of entrants is given by

$$\mu_{sd}(\phi;\theta) = \begin{cases} \theta(\phi^*_{sd})^{\theta}/\phi^{\theta+1} & \text{if } \phi \geq \phi^*_{sd}, \\ 0 & \text{otherwise.} \end{cases} \tag{21}$$

3.1 Aggregate properties and the gravity equation

Under the Pareto assumption we can compute several aggregate statistics for the model. We denote aggregate bilateral sales of firms from s to country d with T_{sd}. The corresponding average sales per firm are defined as $\bar{T}_{sd}$, so that $T_{sd} = M_{sd}\bar{T}_{sd}$ and

$$\begin{aligned} \bar{T}_{sd} &= \int_{\phi^*_{sd}} t_{sd}(\phi)\,\mu_{sd}(\phi;\theta)\,\mathrm{d}\phi \qquad (22) \\ &= \frac{\sigma}{\bar{\sigma}} \int_{\phi^*_{sd}}^{\infty} F'_{sd}(G_{sd}(\phi))\,\frac{h(G_{sd}(\phi))^{\varepsilon-1}}{H(G_{sd}(\phi))^{\varepsilon-1}}\,\mu_{sd}(\phi;\theta)\,\mathrm{d}\phi. \end{aligned}$$

The latter equality follows by substituting in optimal total exports (18) and using expression (13) for optimal scope. To guarantee that average sales per firm $\bar{T}_{sd}$ are finite and positive we require two additional necessary assumptions.

Assumption 2 (Pareto probability mass in low tail). *The Pareto shape parameter satisfies* $\theta > \sigma-1$.

Assumption 3 (Bounded local fixed costs and product efficiency). *Incremental local entry costs and product efficiency satisfy*

$$\tilde{F}_{sd}(\underline{g}) \equiv \int_{\underline{g}}^{\infty} \frac{F'_{sd}(G)}{G}\,\frac{h(G)^{\varepsilon-1}}{H(G)^{\varepsilon-1}}\,Z_{sd}(G)^{-\tilde{\theta}}\,\frac{G\,Z'_{sd}(G)}{Z_{sd}(G)}\,dG \in (0,+\infty)\,,$$

where $\tilde{\theta} \equiv \theta/(\sigma-1)$.

To express average sales (22) as a function of fundamentals, recall that exporter scope $G_{sd}(\phi)$ strictly increases in ϕ (Proposition 1) so we can change variables from ϕ to G. The first-order condition for optimal scope (16) implies that

$$\frac{\phi}{\phi^*_{sd}} = \left(\frac{Z_{sd}(G)}{Z_{sd}(G^*_{sd})}\right)^{\frac{1}{\sigma-1}} \quad \text{and} \quad \frac{\mathrm{d}\phi}{\phi} = \frac{1}{\sigma-1}\,\frac{Z'_{sd}(G)}{Z_{sd}(G)}\,\mathrm{d}G$$

so that using (21)

$$\mu_{sd}(\phi;\theta)\,\mathrm{d}\phi = \tilde{\theta}\,\frac{Z_{sd}(G^*_{sd})^{\tilde{\theta}}}{Z_{sd}(G)^{\tilde{\theta}}}\,\frac{Z'_{sd}(G)}{Z_{sd}(G)}\,\mathrm{d}G \equiv \mu_{sd}(G;\tilde{\theta})\,\mathrm{d}G, \tag{23}$$

where $\tilde{\theta} \equiv \theta/(\sigma-1)$. Using this expression in (22), average sales can be rewritten as

$$\bar{T}_{sd} = \frac{\sigma}{\bar{\sigma}}\,\tilde{\theta}\,Z_{sd}(G^*_{sd})^{\tilde{\theta}}\,\tilde{F}_{sd}(G^*_{sd}), \tag{24}$$

where $\tilde{F}_{sd}(G^*_{sd})$ is defined in Assumption 3. So, despite the choice of multiple products, bilateral average sales can be summarized as a function of the model's parameters θ, σ, ε, $Z_{sd}(G^*_{sd})$ and the properties of $\tilde{F}_{sd}(G^*_{sd})$. Assumptions 2 and 3 guarantee that average sales per firm are positive and finite.

Finally, we can use definition (20) of M_{sd} together with definition (15) of ϕ^*_{sd} and expression (24) for average sales to derive bilateral expenditure shares of country d on products from country s

$$\lambda_{sd} = \frac{M_{sd}\bar{T}_{sd}}{\sum_k M_{kd}\bar{T}_{kd}} = \frac{J_s(b_s)^{\theta}(\tau_{sd}w_s)^{-\theta}\,\tilde{F}_{sd}(G^*_{sd})}{\sum_k J_k(b_k)^{\theta}(\tau_{kd}w_k)^{-\theta}\,\tilde{F}_{kd}(G^*_{kd})}. \tag{25}$$

Remarkably, the elasticity of trade with respect to variable trade costs is $-\theta$, as in Eaton and Kortum (2002) and Chaney (2008).[15] Thus, our framework is consistent with bilateral gravity. The difference between our model, in terms of aggregate bilateral trade flows, and the framework of Eaton and Kortum (2002) is that fixed costs affect bilateral trade as in Chaney (2008). Beyond previous work, we provide a micro-foundation of how changes in fixed costs affect aggregate bilateral trade through the effects of the term $\tilde{F}_{sd}(G^*_{sd})$ on λ_{sd}. So the model offers a tool to consider the responsiveness of overall trade to changes in fixed cost components (for a simulation see Arkolakis and Muendler 2010).

We can also compute mean exporter scope in a destination. For the average number of products to be a well defined and finite we will need the necessary assumption that

Assumption 4 (Strongly increasing combined incremental scope costs). *Combined incremental scope costs satisfy*

$$\int_{\underline{g}}^{\infty} Z_{sd}(G)^{-\tilde{\theta}} \frac{G\,Z'_{sd}(G)}{Z_{sd}(G)}\,dG \in (0,+\infty)\,,$$

where $\tilde{\theta} \equiv \theta/(\sigma-1)$.

Then average exporter scope of firms from s selling to destination d is

$$\bar{G}_{sd} = \int_{\phi^*_{sd}} G_{sd}(\phi)\,\mu_{sd}(\phi;\theta)\,\mathrm{d}\phi = \tilde{\theta}\;Z_{sd}(G^*_{sd})^{\tilde{\theta}} \int_{G^*_{sd}}^{\infty} Z_{sd}(G)^{-\tilde{\theta}} \frac{G\,Z'_{sd}(G)}{Z_{sd}(G)}\,\mathrm{d}G, \tag{26}$$

[15] In our model, the elasticity of trade with respect to trade costs is the (negative) Pareto shape parameter, whereas it is the (negative) Fréchet shape parameter in Eaton and Kortum (2002).

where the second equality follows from (23). Expression (26) implies that mean exporter scope is invariant to destination market characteristics other than local entry costs if G^*_{sd} is constant across destinations. A priori there is no reason why mean exporter scope $\bar{G}_{sd}$ should be strongly related to variables such as bilateral distance between s and d or to the size of the destination market. As we will show shortly for a specific parametrization, we can make G^*_{sd} destination invariant while making local entry costs source and destination specific. This resonates with the evidence of highly robust scope distributions across destinations as presented in Arkolakis and Muendler (2010).

3.2 Predictions for the cross section of firms

We pick specific functional forms for empirical predictions. Our parametrization serves a double purpose: the parametrization yields convenient relationships for estimation and simulation, and it accounts for non-log-linearity in the relationship between firm-product sales and product rank. Evidence in Arkolakis and Muendler (2010, Figure 1) suggests that product sales drop off faster than log linearly for a firm's lowest-selling products.

Guided by the regularities documented in Arkolakis and Muendler (2010, Facts 1 through 3), we set local entry cost $F_{sd}(G_{sd})$ and the product cost schedule $h(g)$ to[16]

$$\begin{aligned} F_{sd}(G_{sd}) &= \gamma_{sd}\, w_d\, \kappa + \gamma_{sd}\, w_d\, (G_{sd})^{\delta+1}/(\delta+1) \quad \text{for } \delta \in (-\infty, +\infty), \\ h(g) &= (g+\beta)^{\alpha} \qquad\qquad\qquad\qquad\qquad\quad \text{for } \beta > 0. \end{aligned} \tag{27}$$

Local entry cost $F_{sd}(G_{sd})$ are defined in terms of destination-country labor units. The beachhead cost $\gamma_{sd}\, w_d\, \kappa$ for presence in a market is proportional to the local entry cost for additional products with a constant factor of κ. This assumption will result in constant minimum scope worldwide, independent of source and destination countries. Note that incremental local entry cost $F'_{sd}(G)$ are strictly positive irrespective of the value of δ. In particular, $\delta < -1$ is possible (for sufficiently high $\gamma_{sd}\kappa$ so that $F_{sd}(G_{sd}) > 0$ for all $G_{sd} \in [\underline{g}, +\infty)$).

Corresponding to the product cost schedule $h(g)$, there is the firm-product efficiency index (7). For consistence with empirical evidence, we impose an additional restriction.[17]

[16]For estimation, we will consider the generalization to industry-specific local entry costs

$$F_{isd}(G_{isd}) = \gamma_i \gamma_{sd} w_d\, \kappa + \gamma_i \gamma_{sd}\, w_d\, (G_{isd})^{\delta+1}/(\delta+1).$$

For it is straightforward to derive the so generalized estimation relationships, we suppress the industry index to save on notation.

[17]The alternative parametrization of $h(g) = g^{\alpha}$ for $\alpha < 1/(\varepsilon-1)$ and some $\underline{g} < 1$ would result in a simpler

Case C2 (Bounded firm-product efficiency). *The scope elasticity of the marginal cost schedule $h(g)$ satisfies $\alpha > 1/(\varepsilon-1)$ and the initial product $\underline{g} = 1$.*

Then the firm-product efficiency index (7) becomes

$$H(G_{sd})^{-(\varepsilon-1)} = \int_1^{G_{sd}} (g+\beta)^{-\alpha(\varepsilon-1)} \mathrm{d}g = \frac{(1+\beta)^{-\bar{\alpha}} - (G_{sd}+\beta)^{-\bar{\alpha}}}{\bar{\alpha}} \text{ with } \bar{\alpha} \equiv \alpha(\varepsilon-1)-1 > 0. \quad (28)$$

Parametrization (27) implies for combined incremental scope costs (13) that their scope elasticity becomes

$$\frac{\partial \ln Z_{sd}(G)}{\partial \ln G} = \underbrace{\delta}_{\text{local entry cost}} + \underbrace{\alpha(\varepsilon-1)\frac{G_{sd}}{G_{sd}+\beta}}_{\text{prod. efficiency}} + \underbrace{\frac{(1-\bar{\sigma})\bar{\alpha}}{(1+\beta)^{-\bar{\alpha}}(G_{sd}+\beta)^{\bar{\alpha}} - 1}\frac{G_{sd}}{G_{sd}+\beta}}_{\text{cannibalization effect}}. \quad (29)$$

Assumption 1 requires that this expression is strictly positive at any scope. Case C1 requires in addition that the expression strictly exceeds one at any scope. Note that the cannibalization effect becomes negligibly small as scope grows. So, as scope becomes arbitrarily large, Assumption 1 requires that $\delta + \alpha(\varepsilon-1) > 0$ and Case C1 requires that $\delta + \alpha(\varepsilon-1) > 1$.[18]

We can restate a firm ϕ's optimal choices in terms of parameters. We express the optimal sales (17) of a firm ϕ for its g-th product in market d as a function of exporter scope in d and the product's rank in d:

$$p_{sdg}(\phi)x_{sdg}(\phi) = \frac{\sigma}{\bar{\sigma}}\gamma_{sd}\, w_d\, G_{sd}(\phi)^{\delta}[G_{sd}(\phi)+\beta]^{\alpha(\varepsilon-1)}\,[g+\beta]^{-\alpha(\varepsilon-1)} \quad \text{for } g \le G_{sd}(\phi). \quad (30)$$

Exporter scope (16) at destination d is implicitly given by

$$\left(\frac{G_{sd}(\phi)}{G^*}\right)^{\delta}\left(\frac{G_{sd}(\phi)+\beta}{G^*+\beta}\right)^{1+\bar{\alpha}}\left(\frac{(1+\beta)^{-\bar{\alpha}} - [G_{sd}(\phi)+\beta]^{-\bar{\alpha}}}{(1+\beta)^{-\bar{\alpha}} - [G^*+\beta]^{-\bar{\alpha}}}\right)^{1-\bar{\sigma}} = \left(\frac{\phi}{\phi^*_{sd}}\right)^{\sigma-1}, \quad (31)$$

where the productivity threshold (15) for exporting from s to d is

$$\phi^*_{sd} \equiv \left(\frac{\sigma}{\bar{\sigma}}\frac{\gamma_{sd}\, w_d}{y_d L_d}(G^*)^{\delta}(G^*+\beta)^{1+\bar{\sigma}\bar{\alpha}}\left[\frac{(1+\beta)^{-\bar{\alpha}}(G^*+\beta)^{\bar{\alpha}} - 1}{\bar{\alpha}}\right]^{1-\bar{\sigma}}\right)^{\frac{1}{\sigma-1}} \frac{\tilde{\sigma}\,\tau_{sd}\, w_s}{P_d} \quad (32)$$

and minimum profitable scope (14) is implicitly given by

$$(G^*)^{\delta}(G^*+\beta)\left[(1+\beta)^{-\bar{\alpha}}(G^*+\beta)^{\bar{\alpha}} - 1 - \frac{\bar{\sigma}\bar{\alpha}}{\delta+1}\frac{G^*}{G^*+\beta}\right] = \bar{\sigma}\bar{\alpha}\,\kappa. \quad (33)$$

firm-product efficiency index $H(G_{sd})$ but would violate the empirical evidence that $\alpha(\varepsilon-1)$ significantly exceeds one.

[18] In the absence of a cannibalization effect ($\bar{\sigma} = 1$) and for log-linear product sales in product rank ($\beta = 0$), expression (29) simplifies to $\delta + \alpha(\varepsilon-1)$ for any scope, as in our model with countably many products (Arkolakis and Muendler 2010).

Table 1: Pareto Shape Parameters in Upper Tail

Variable	Scope elasticity in limit	Pareto shape parameter in upper tail
Exporter scope $G_{sd}(\phi)$	1	$\tilde{\theta}\,[\delta+\alpha(\varepsilon-1)]$
Total exports $t_{sd}(\phi)$	$\delta+\alpha(\varepsilon-1)$	$\tilde{\theta}$
Exporter scale $a_{sd}(\phi)$	$\delta+\alpha(\varepsilon-1)-1$	$\tilde{\theta}\,[\delta+\alpha(\varepsilon-1)]/[\delta+\alpha(\varepsilon-1)-1]$

Empirically, minimum profitable scope is constant across markets. Using the limiting case of $G^*=1$ in (14) shows that a minimum scope of one would be consistent with κ arbitrarily close to $\kappa=-1/(\delta+1)$, which is strictly positive for $\delta<-1$ as our estimation (in Section 4) will confirm empirically.[19]

Total firm exports (18) to destination d are

$$t_{sd}(\phi)=\frac{\sigma}{\bar{\sigma}}\frac{\gamma_{sd}\,w_d}{\bar{\alpha}}\,G_{sd}(\phi)^{\delta}\left(G_{sd}(\phi)+\beta\right)\left[(1+\beta)^{-\bar{\alpha}}\left(G_{sd}(\phi)+\beta\right)^{\bar{\alpha}}-1\right] \tag{34}$$

and exporter scale at destination d is therefore

$$a_{sd}(\phi)\equiv\frac{t_{sd}(\phi)}{G_{sd}(\phi)}=\frac{\sigma}{\bar{\sigma}}\frac{\gamma_{sd}\,w_d}{\bar{\alpha}}\,G_{sd}(\phi)^{\delta-1}\left(G_{sd}(\phi)+\beta\right)\left[(1+\beta)^{-\bar{\alpha}}\left(G_{sd}(\phi)+\beta\right)^{\bar{\alpha}}-1\right]. \tag{35}$$

The above derivations have distributional implications for the upper tail of exporter scope, total exports and exporter scale. With our assumption that a firm's productivity ϕ is drawn from a Pareto distribution with a worldwide shape parameter θ, we can infer the shape parameters in the upper tail. We present the results in Table 1.

Consider exporter scope first. The left-hand side of the optimal exporter scope condition (31) changes in scope with an elasticity given by expression (29), and the right-hand side changes in productivity with an elasticity of $\sigma-1$. So, as scope becomes arbitrarily large, the left-hand side of the optimal exporter scope condition (31) simplifies to $\delta+\alpha(\varepsilon-1)$. As a result, exporter scope approximates a Pareto distributed variable in the upper tail with a shape parameter $\tilde{\theta}\,[\delta+\alpha(\varepsilon-1)]$, just as in our model with countably many discrete products (Arkolakis and Muendler 2010).[20] Note

[19] So a compact and empirically admissible parametrization for incremental local entry cost would be

$$F_{sd}(G_{sd})=-(\gamma_{sd}\,w_d+\epsilon_{sd})/(\delta+1)+\gamma_{sd}\,w_d\,(G_{sd})^{\delta+1}/(\delta+1)$$

for $\delta<-1$ and some $\epsilon_{sd}>0$ arbitrarily close to zero.

[20] For a Pareto distributed random variable ϕ with shape parameter θ and location parameter ϕ^*_{sd}, the transformed

that in the absence of a cannibalization effect ($\bar{\sigma} = 1$) and for log-linear product sales in product rank ($\beta = 0$), expression (29) simplifies to $\delta + \alpha(\varepsilon - 1)$ for any scope so that exporter scope is a Pareto distributed variable at any level.[21]

Now turn to total exports (34). Total exports change in exporter scope with an elasticity of

$$\delta + \frac{G}{G+\beta} + \frac{\bar{\alpha}}{1 - (1+\beta)^{\bar{\alpha}}(G+\beta)^{-\bar{\alpha}}} \frac{G}{G+\beta},$$

which approaches $\delta + 1 + \bar{\alpha} = \delta + \alpha(\varepsilon – 1)$ as G grows arbitrarily large. So, total exports approximate a Pareto distributed variable in the upper tail with a shape parameter $\tilde{\theta}$, which is reminiscent of results in the Chaney (2008) and Eaton et al. (2010) models. Finally, by its definition, exporter scale changes in exporter scope with the above elasticity less one. So, exporter scale is approximately Pareto distributed in the upper tail with a shape parameter $\tilde{\theta}\,[\delta + \alpha(\varepsilon - 1)]/[\delta + \alpha(\varepsilon - 1) - 1]$.

In Appendix B we provide a comparison to the model with countably many products (Arkolakis and Muendler 2010).

4 Estimation

Equation (30) is the basis for our estimation of the continuum model. The most important generalization compared to our estimation in Arkolakis and Muendler (2010) is that this estimation equation accounts for a possible non-log-linearity in the relationship between firm-product sales and product rank.[22] We augment the equation by a multiplicative error term ϵ_{sdg} and estimate it in its log form. In addition, we make local entry costs industry specific with parameters $\gamma_i \gamma_{sd}$ instead of just γ_{sd}.

random variable $x = A\,(\phi)^B$ is Pareto distributed with shape θ/B and location $A\,(\phi^*_{sd})^B$. To see this, apply the change of variables theorem to $\phi(x) = (x/A)^{1/B}$ and the Pareto probability density function $\mu(\phi)$ to find that $\int_a^b \mu(\phi)\,\mathrm{d}\phi = \int_{x(a)}^{x(b)} \mu(\phi(x))\phi'(x)\,\mathrm{d}x = \int_{x(a)}^{x(b)} (\theta/B)\,[A(\phi^*_{sd})^B]^{\theta/B}/(x)^{\theta/B+1}\,\mathrm{d}x$.

[21] $G_{sd}(\phi)$ approaches a Pareto distribution in the upper tail because the second and third factors on the left-hand side of (31) converge to a constant as $G_{sd}(\phi)$ grows. If $\bar{\sigma}$ approaches one, that is in the absence of a cannibalization effect, the third factor on the left-hand side of (31) vanishes. If β approaches zero, that is if product sales are log-linear in product rank, the first and second factor collapse to a single factor. So, in the absence of a cannibalization effect and for log-linear product sales in product rank, exporter scope is Pareto distributed at any level.

[22] In contrast, the presence or absence of a cannibalization effect, if products within a product line are more substitutable among each other than with outside products ($\bar{\sigma} < 1$), is of no consequence for estimation because the resulting term gets absorbed in the regression constant.

So our estimation equation becomes

$$\ln p_{sdg}(\phi)\, x_{isdg}(\phi) \;=\; \delta \ln G_{isd}(\phi) + \alpha(\varepsilon{-}1)\log\left\{G_{isd}(\phi)+\beta\right\} - \alpha(\varepsilon{-}1)\log\left\{g+\beta\right\} + \ln \sigma\gamma_i/\bar{\sigma} + \ln \gamma_{sd} w_d \;+\; \ln \epsilon_{sdg}. \tag{36}$$

The relationship is not log linear and we estimate equation (36) with non-linear least squares. Using data on firm-level product sales, estimation of (36) identifies the three relevant market entry cost parameters: the scope elasticity of incremental local entry costs δ, the scope elasticity of product introduction costs $\alpha(\varepsilon{-}1)$, and the curvature parameter β of product introduction costs.

Firm-level sales data appear less suitable (than consumption data say) to estimate the composite elasticity term $\bar{\sigma} \equiv (\sigma{-}1)/(\varepsilon{-}1)$, which determines the cannibalization effect. We therefore do not attempt to quantify the cannibalization effect, which augments the costs of a wider exporter scope.[23]

We use the same data on Brazilian manufacturing exporters as in Arkolakis and Muendler (2010), which come from the universe of customs declarations for merchandize exports during the year 2000 by any firm. From the pristine Brazilian customs records, we constructed a three-dimensional panel of Brazilian exporters, their respective destination countries, and their export products at the Harmonized System (HS) 6-digit level. On the product side, we restrict our data to manufactured products only (excluding agricultural and mining products). Beyond evidence on Brazilian manufacturing producers, we also obtain estimates for two further samples in this paper. First, we obtain estimates for Chile, a neighboring Latin American country. We construct a comparable three-dimensional panel of Chilean manufacturing exporters derived from the universe of customs declarations by Chilean manufacturing producers in 2000 (Álvarez, Faruq and López 2007, aggregated to the HS 6-digit level for comparison). Second, we seek evidence on local entry costs and incremental scope costs for commercial intermediaries that ship manufactured products.[24] We report additional detail on our data and summary statistics in Appendix C. In Arkolakis and Muendler (2010), we present further stylized facts that robustly emerge from tabulating and plotting our data. Our continuum model in this paper is fully consistent with those regularities.

[23] Evidence in Broda and Weinstein (2007) suggests that products are stronger substitutes within firms than across firms so we consider a cannibalization effect with $\bar{\sigma} < 1$ the relevant case. Their preferred estimates for ε and σ within and across domestic U.S. brand modules are 11.5 and 7.5.

[24] In 2000, our *SECEX* data for manufactured merchandize sold by Brazilian firms from any sector, including commercial intermediaries, covers 95.9 percent of Brazilian exports recorded in the World Trade Flow data (Feenstra, Lipsey, Deng, Ma and Mo 2005). Exports of manufactured merchandize shipped directly by Brazilian manufactur-

Table 2: Non-linear Estimates of Individual Product Sales: Continuum Case

Log Exp./prod.	sample: Any sales			Sales $\geq$ US$100		
controls		Dest.	Dest.&Ind.		Dest.	Dest.&Ind.
	(1)	(2)	(3)	(4)	(5)	(6)
Brazilian Producers exporting Manufactures						
Scope elasticity of						
incr. local entry cost (δ)	-1.453	-1.426	-1.332	-1.072	-1.069	-1.057
	(.005)	(.005)	(.005)	(.005)	(.005)	(.006)
prod. efficiency ($\alpha(\varepsilon-1)$)	2.985	2.965	2.951	1.872	1.945	2.079
	(.015)	(.014)	(.012)	(.013)	(.013)	(.011)
Curvature of						
prod. efficiency (β)	.833	.753	.623	-.065	-.031	.014
	(.025)	(.023)	(.019)	(.017)	(.017)	(.015)
Obs.	162,570	162,570	162,570	141,163	141,163	141,163
R^2	.496	.541	.646	.370	.433	.562
Chilean Producers exporting Manufactures						
Scope elasticity of						
incr. local entry cost (δ)	-1.495	-1.394	-1.281	-1.195	-1.074	-1.009
	(.011)	(.012)	(.011)	(.012)	(.012)	(.012)
prod. efficiency ($\alpha(\varepsilon-1)$)	2.628	2.552	2.634	1.743	1.701	1.860
	(.038)	(.036)	(.033)	(.033)	(.031)	(.029)
Curvature of						
prod. efficiency (β)	.590	.430	.562	-.109	-.195	-.028
	(.054)	(.048)	(.047)	(.040)	(.035)	(.037)
Obs.	37,172	37,172	37,172	34,024	34,024	34,024
R^2	.419	.451	.557	.329	.375	.496
Brazilian Commercial Intermediaries exporting Manufactures						
Scope elasticity of						
incr. local entry cost (δ)	-1.281	-1.247	-1.107	-1.079	-1.015	-.913
	(.008)	(.009)	(.009)	(.009)	(.009)	(.009)
prod. efficiency ($\alpha(\varepsilon-1)$)	4.023	3.561	3.352	3.044	2.548	2.408
	(.047)	(.039)	(.034)	(.042)	(.033)	(.029)
Curvature of						
prod. efficiency (β)	10.450	7.165	5.740	9.062	4.795	3.420
	(.306)	(.226)	(.182)	(.313)	(.192)	(.142)
Obs.	35,960	35,960	35,960	31,326	31,326	31,326
R^2	.489	.536	.599	.409	.469	.539

Source: Brazilian *SECEX* 2000, manufacturing firms as well as commercial intermediaries shipping manufactured products, and Chilean customs data 2000 (Álvarez et al. 2007).
Note: Products at the Harmonized-System 6-digit level. Industry fixed effects at the *CNAE* two-digit level for Brazil and at the most frequent HS 2-digit level for Chile (highest-sale or, if tie, mode HS 2-digit product group). Constant and destination fixed effects not reported. Standard errors in parentheses. Regression equation (for $g \leq G_{sd\omega}$):

$$\ln p_{sdg\omega} x_{isdg\omega} = \ln \gamma_{sd} w_d + \ln \sigma \gamma_i / \bar{\sigma} + \delta \ln G_{isd\omega} + \alpha(\varepsilon-1) \log\{G_{isd\omega} + \beta\} - \alpha(\varepsilon-1) \log\{g + \beta\} + \ln \epsilon_{isdg\omega},$$

where i indexes the industry, s the source country, d the destination, and ω the firm.

Table 2 presents results from estimating (36) with non-linear least squares (NLLS). To assess robustness we estimate equation (36) with and without destination and industry fixed effects, and separately for the subsample of firm-product sales observations of at least US\$100.

Estimates for scope elasticity coefficients are robust across different specifications for the full sales sample (columns 1 through 3). For Brazilian manufacturing producers, NLLS estimates for the scope elasticity of incremental local entry costs δ fall in the range from -1.33 to -1.45. NLLS estimates for the scope elasticity of product introduction costs $\alpha(\varepsilon-1)$ range in a narrow band between 2.95 and 2.98. These estimates are close to the preferred OLS estimates in our companion paper (Arkolakis and Muendler 2010, Table 3, column 4) of -1.38 for δ and 2.66 for $\alpha(\sigma-1)$.[25] The NLLS estimates for Chilean manufacturing producers are close too (middle panel of Table 2), with δ estimates ranging between -1.28 and -1.50 and $\alpha(\varepsilon-1)$ estimates ranging between 2.55 and 2.63. All these estimates suggest that both Case C1 and C2 are satisfied in the data. The magnitude of the δ estimate implies that incremental local entry costs drop at an elasticity of around -1.4 when producers introduce additional products in a market where they are present. But, as producers introduce products further away from their core competency, firm-product efficiency drops off even faster with an elasticity of around 2.6 in NLLS estimation at the sample mean (that is $3.0 \cdot 3.5/(3.5 + .6)$ by the elasticity of product introduction cost in equation (29) for an exporter scope of 3.5 at the sample mean and $\alpha(\varepsilon-1) = 3.0$ as well as $\beta = .6$). Combining the two fixed scope cost components, there are net overall diseconomies of scope with a scope elasticity of around 1.2.

The curvature parameter β in product introduction costs is statistically significantly different from zero for the full sales sample (columns 1 through 3). This curvature parameter seems to become relevant only because wide-scope firms introduce products with minor sales. When we restrict the sample to products with sales of at least US\$100 (columns 4 through 6), for instance, then the curvature parameter β becomes statistically indistinguishable from zero for Brazilian manufacturers in two out of three specifications (column 5 and 6), and for Chilean manufacturers in the main specification with destination and industry fixed effects (column 6). This evidence suggests that the relevant curvature in product introduction costs mainly stems from products with

---

ers accounts for 81.7 percent of the total manufactures exports, and manufactured merchandize shipped indirectly by Brazilian commercial intermediaries accounts for 14.2 percent.

[25] In Arkolakis and Muendler (2010), we restrict the relationship between firm-product sales and product rank to be log-linear, and account for residual firm-destination effects because of discrete product space.

minor sales. It would be misguided, however, to drop sales observations below US$100 from the sample when estimating scope elasticities. Precisely these products with minor sales provide the identifying information that wide-scope firms relatively easily overcome local entry costs for their lowest-selling products, although firms must sell these minor products at fast dropping scale (in our theory because of high marginal cost away from core competency). Estimates of scope elasticity coefficients would not be informative in a sample with sales restricted to a minimum threshold.

The lower-most panel in Table 2 allows us to compare estimates for Brazilian commercial intermediaries to the upper estimates for manufacturing producers. (Products in both samples are manufactures only.) The comparison provides us with a sense of plausibility. Compared to Brazilian manufacturing firms, Brazilian intermediaries face a less favorable scope elasticity of incremental local entry costs δ, in a range from -1.11 to -1.28. Put differently, intermediaries do not encounter as favorable economies of scope in local entry costs as do producers. A consistent interpretation of this finding is that intermediaries with no production of their own do not benefit from as rapidly declining incremental local entry costs when they introduce lower-ranked products in a market, perhaps because they cannot count on producer reputation or brand recognition to the same degree as a producer can. In addition, intermediaries face faster increases in product introduction costs than producers, with $\alpha(\varepsilon-1)$ estimates ranging between 3.35 and 4.02. In addition, intermediaries face considerably more strongly curved product introduction costs, with β estimates between 5.7 and 10.5, compared to only between $.62$ and $.83$ for producers. One interpretation of the latter findings is that intermediaries cannot introduce products for global sales more easily than producers, who might also adopt products for resale, perhaps again because intermediaries cannot rely as much on brand reputation or brand recognition. Together, these findings of more adverse entry cost elasticities for intermediaries are consistent with the overall finding that intermediaries account for only about a seventh of Brazilian manufacturing-product exports.

How strongly biased would a linear estimator be? For an empirical answer, we set the curvature parameter β to zero and estimate equation (36) with OLS. Table 3 reports the results for four alternative specifications of fixed effects. The coefficient magnitudes are similar across specifications, which gradually augment OLS with destination, industry and firm fixed effects. For Brazilian manufacturing producers, log-linear estimates for the scope elasticity of incremental local entry costs δ fall in the range from -1.17 to -1.56 (embracing the NLLS estimates). Log-linear estimates for the scope elasticity of product introduction costs $\alpha(\varepsilon-1)$ range in a band between 2.51 and

Table 3: Linear Estimates of Individual Product Sales: Continuum Case

Log Exp./prod.		Firm-destination-product data		
estimator	OLS	OLS	Ind. FE	Firm FE
controls		Dest.	Dest.	Dest.
	(1)	(2)	(3)	(4)
Brazilian Producers exporting Manufactures				
Log # Products	1.168	1.204	1.319	1.557
	(.007)	(.007)	(.007)	(.008)
Log Product Rank	-2.508	-2.525	-2.574	-2.624
	(.007)	(.007)	(.007)	(.008)
Obs.	162,570	162,570	162,570	162,570
Panels			259	10,215
R^2 (within)	.493	.538	.510	.582
Scope elast. of incr. local entry cost (δ)	-1.340	-1.321	-1.256	-1.067
Scope elast. of prod. efficiency ($\alpha(\varepsilon-1)$)	2.508	2.525	2.574	2.624
Chilean Producers exporting Manufactures				
Log # Products	.826	.929	1.058	1.177
	(.017)	(.017)	(.015)	(.017)
Log Product Rank	-2.239	-2.258	-2.259	-2.349
	(.017)	(.017)	(.015)	(.017)
Obs.	37,172	37,172	37,172	37,172
Panels			91	4,099
R^2 (within)	.418	.450	.429	.543
Scope elast. of incr. local entry cost (δ)	-1.412	-1.330	-1.201	-1.172
Scope elast. of prod. efficiency ($\alpha(\varepsilon-1)$)	2.239	2.258	2.259	2.349
Brazilian Commercial Intermediaries exporting Manufactures				
Log # Products	1.048	1.047	1.160	1.311
	(.013)	(.013)	(.013)	(.016)
Log Product Rank	-1.974	-1.999	-2.012	-2.090
	(.013)	(.013)	(.013)	(.016)
Obs.	35,960	35,960	35,960	35,960
Panels			70	2,627
R^2 (within)	.456	.513	.506	.652
Scope elast. of incr. local entry cost (δ)	-.927	-.951	-.853	-.779
Scope elast. of prod. efficiency ($\alpha(\varepsilon-1)$)	1.974	1.999	2.012	2.090

Sources: Brazilian *SECEX* 2000, manufacturing firms as well as commercial intermediaries shipping manufactured products, and Chilean customs data 2000 (Álvarez et al. 2007).
Note: Products at the Harmonized-System 6-digit level. Industry fixed effects at the *CNAE* two-digit level for Brazil and at the *ISIC* two-digit level for Chile. Constant and destination fixed effects not reported. R^2 is within fit for FE regressions (columns 3 and 4). Standard errors in parentheses. F tests on implied scope elasticities (δ and α) show significance at the 1-percent level in all cases. Regression equation (for $g \leq G_{sd\omega}$ and $\beta = 0$):

$$\ln p_{sdg\omega}\, x_{isdg\omega} = \ln \gamma_{sd} w_d + \ln \sigma\gamma_i/\bar{\sigma} + [\delta + \alpha(\varepsilon-1)] \ln G_{isd\omega} - \alpha(\varepsilon-1) \ln g \;+\; \ln \epsilon_{isdg\omega},$$

where i indexes the industry, s the source country, d the destination, and ω the firm.

2.62 (somewhat below the NLLS estimates). These estimates too are close to the preferred OLS estimates in our companion paper (Arkolakis and Muendler 2010, Table 3, column 4) of -1.38 for δ and 2.66 for $\alpha(\sigma-1)$.

Their similarity notwithstanding, the variation in coefficient magnitudes across specifications confirms an expected underlying bias from log-linear estimation. For a zero curvature parameter β, the theoretically most adequate specification contains industry and destination fixed effects (column 3). We know from NLLS estimation, however, that the curvature parameter is non-zero. We expect the omission of the non-linear curvature component to introduce a negative bias into OLS estimation. The reason is that the curvature parameter β matters most for narrow-scope firms with low productivity, because their exports per product are pushed relatively higher ($\beta > 0$) than product sales by the mean firm, while the curvature parameter matters little for wide-scope firms with high productivity, because their scope is large compared to β so that their exports per product are pushed up less than product sales by the mean firm. A firm fixed effect is a simple non-parametric way to control for omitted firm characteristics such as unobserved productivity.[26] Indeed, inclusion of a firm fixed effect raises the regression coefficient on the number of products in confirmation of a negative bias (comparing columns 3 and 4).

In economic terms, however, the bias from omitting the non-linear curvature term is small for manufacturing producers. In the Brazilian producer sample, the scope elasticity of incremental local entry costs δ is estimated to be -1.26 under OLS (column 3) instead of -1.33 under NLLS (Table 2 column 3), and the scope elasticity of product introduction costs $\alpha(\varepsilon-1)$ is estimated as 2.57 under OLS instead of 2.95 under NLLS. In the Chilean producer sample, the scope elasticity of incremental local entry costs δ is estimated to be -1.20 under OLS instead of -1.28 under NLLS, and the scope elasticity of product introduction costs $\alpha(\varepsilon-1)$ is estimated as 2.26 under OLS instead of 2.63 under NLLS.

These small differences justify our restriction to log-linearity of firm-product sales and OLS estimation in our companion paper (Arkolakis and Muendler 2010). For commercial intermediaries, however, the non-log-linearity in product introduction costs matters considerably for product introduction cost estimation. In the Brazilian intermediary sample, the scope elasticity of incremental local entry costs δ is estimated to be -1.16 under OLS and still close to the estimate of

[26] Our model with countably many products (Arkolakis and Muendler 2010) also calls for a firm fixed effect to control for incrementally changing exports per product as a firm becomes more productive, while its exporter scope remains unchanged between discrete thresholds.

-1.11 under NLLS, but the scope elasticity of product introduction costs $\alpha(\varepsilon-1)$ is estimated as only 2.01 under OLS instead of 3.35 under NLLS, a downward bias of forty percent under OLS.

5 Conclusion

Market entry costs are a crucial component in recent trade theory. Combined with firm heterogeneity, fixed market entry costs can provide novel predictions of exporter behavior and aggregate exports. This companion paper to Arkolakis and Muendler (2010) presents a generalized model of multi-product exporters that allows for two additional components of market entry costs. In continuous product space, we consider a cannibalization effect by which additional products reduce the sales of a firm's infra-marginal products, and we allow for product sales that drop off faster than log linearly as a firm adds products that sell for ever smaller amounts. The model preserves main predictions of prior heterogeneous-firm models of trade, such as a single elasticity of exporting and a power-law distribution of exports in the upper tail. Our model generates in addition estimable equations to identify several relevant market entry cost parameters.

Our evidence suggests that there is relevant curvature in product introduction costs, and that this curvature mainly stems from products with minor sales. The existence of products with minor sales is, in turn, consistent with the main mechanism of our theory: highly productive wide-scope firms overcome local entry costs for their lowest-selling products relatively easily but they face fast increasing marginal production costs away from core competency. For Brazilian manufacturing producers, our non-linear least squares estimates confirm the more restrictive ones from earlier work. The estimates continue to be consistent with the hypothesis that producers encounter economies of scope in fixed local entry costs but even stronger diseconomies of scope in product introduction costs so that the firms face overall diseconomies of scope in every destination market.

Appendix

A Optimal markup choice

Maximizing the firm's constrained objective function

$$\max_{P_{sd},\{p_{sd}(g)\}_{g\in[\underline{g},G_{sd}]}} \pi_{sd}(\phi)+\lambda\left(P_{sd}-\left[\int_{\underline{g}}^{G_{sd}} p_{sd}(g)^{-(\varepsilon-1)}\mathrm{d}g\right]^{-\frac{1}{\varepsilon-1}}\right),$$

where profit $\pi_{sd}(\phi)$ is given by (9), yields the first order conditions

$$p_{sd}(\phi;g)=\frac{\varepsilon\, y_d L_d}{(\varepsilon-1)\, y_d L_d+\lambda(\phi)\ (P_d)^{-(\sigma-1)}\ P_{sd}(\phi;G_{sd})^{\sigma}}\frac{\tau_{sd}\, w_s}{\phi/h(g)} \tag{A.1}$$

for individual product prices $p_{sd}(g)$ and the first-order condition

$$\lambda(\phi)=-(\varepsilon-\sigma)\frac{P_{sd}(\phi;G_{sd})^{\varepsilon-\sigma-1}y_d L_d}{(P_d)^{-(\sigma-1)}}\int_{\underline{g}}^{G_{sd}}\left(p_{sd}(g)-\frac{\tau_{sd}\, w_s}{\phi/h(g)}\right)p_{sd}(g)^{-\varepsilon}\,\mathrm{d}g \tag{A.2}$$

for the product-line price P_{sd}, where λ is the Lagrange multiplier on the constraint ($\lambda < 0$ if and only if $\varepsilon > \sigma$) and G_{sd} is given. Using (A.1) in (A.2), the latter first-order condition simplifies to

$$\lambda(\phi)\ (P_d)^{-(\sigma-1)}\ P_{sd}(\phi;G_{sd})^{\sigma}=-(\varepsilon-\sigma)y_d L_d/\sigma.$$

Using that expression in (A.1) yields optimal product prices[27]

$$p_{sd}(\phi;g)=\tilde{\sigma}\,\frac{\tau_{sd}\, w_s}{\phi/h(g)},$$

with a product-independent markup over marginal cost $\tilde{\sigma}\equiv\sigma/(\sigma-1)>1$ for $\sigma>1$, and the optimal product-line price

$$P_{sd}(\phi;G_{sd})=[\tilde{\sigma}\,\tau_{sd}\, w_s/\phi]\,H(G_{sd}).$$

B Comparison to discrete product space

Under our parametrization (27), we compare our continuum model to the model with countably many discrete products (Arkolakis and Muendler 2010). We do not consider a cannibalization

[27] Allanson and Montagna (2005) seem to erroneously state the optimal markup in terms of the inner nest's elasticity of substitution ε.

effect in Arkolakis and Muendler (2010), so we also set $\bar{\sigma} = 1$ here. Similarly, in Arkolakis and Muendler (2010) we impose log-linear product sales in product rank, so we set $\beta = 0$ for the comparison here.[28]

We turn to aggregate exports first. Using the parametric expression for a firm's total exports (34) in the definition of average sales per firm (22), and changing the variable of integration from ϕ to G as in (23), yields

$$\begin{aligned}\bar{T}_{sd} &= \frac{\sigma}{\bar{\sigma}}\,\tilde{\theta}\,Z_{sd}(G^*_{sd})^{\tilde{\theta}} \cdot (\gamma_{sd}\,w_d)^{-(\tilde{\theta}-1)}[\delta + \alpha(\varepsilon-1)] \int_{G^*} G^{-[\delta+\alpha(\varepsilon-1)](\tilde{\theta}-1)-1}\,\mathrm{d}G \\ &= \gamma_{sd}\,w_d\,\sigma\,\frac{\tilde{\theta}}{\tilde{\theta}-1}(G^*)^{\delta+\alpha(\varepsilon-1)} \qquad \text{(B.3)}\end{aligned}$$

for $\varepsilon = \sigma$ (that is for $\bar{\sigma} = 1$) and $\beta = 0$, where $\tilde{\theta} \equiv \theta/(\sigma-1)$. The integral exists if Assumption 1 ($\delta + \alpha(\varepsilon-1) > 0$) and Assumption 2 ($\tilde{\theta} > 1$) hold, just as in discrete product space (Arkolakis and Muendler 2010, Table 2).

The entry-cost related parameter $\tilde{F}_{sd}(G^*)$, as defined in Assumption 3, can be recovered from the above expression using the fact that $\bar{T}_{sd} = (\sigma/\bar{\sigma})\,\tilde{\theta}\,Z_{sd}(G^*)^{\tilde{\theta}}\,\tilde{F}_{sd}(G^*)$ by (24):

$$\tilde{F}_{sd}(G^*) = \frac{(\gamma_{sd}\,w_d)^{-(\tilde{\theta}-1)}}{\tilde{\theta}-1}\,(G^*)^{-[\delta+\alpha(\varepsilon-1)](\tilde{\theta}-1)}. \qquad \text{(B.4)}$$

Note that $\tilde{F}_{sd}(G^*)$ is not proportional to average local entry costs $\bar{F}_{sd}$, contrary to the model in discrete product space. Average local entry costs for firms from country s at destination d are $\bar{F}_{sd} \equiv \int_{G^*} F_{sd}(G)\mu_{sd}(G;\tilde{\theta})\,\mathrm{d}G$. By our parametrization (27) and the definition of $\mu_{sd}(G;\tilde{\theta})$ in (23), average local entry costs become

$$\bar{F}_{sd} = \gamma_{sd}\,w_d\,\kappa + \frac{\gamma_{sd}\,w_d}{\delta+1}\,\frac{[\delta + \alpha(\varepsilon-1)]\tilde{\theta}}{[\delta + \alpha(\varepsilon-1)]\tilde{\theta} - (\delta+1)}\,(G^*)^{\delta+1}$$

for $\varepsilon = \sigma$ and $\beta = 0$. The term $\bar{F}_{sd}$ is well defined (the integral converges) if local entry costs are bounded with $\delta + \alpha(\varepsilon-1) > (\delta+1)/\tilde{\theta}$, just as in discrete product space (Arkolakis and Muendler 2010, Table 2). But there need to be beachhead entry costs in the continuum model ($\kappa > 0$), otherwise every firm would sell to every market. As a result, average local entry cost $\bar{F}_{sd}$ are not proportional to $\tilde{F}_{sd}(G^*)$ in the continuum model.

[28] In the general case with a cannibalization effect and non-log-linear product sales in product rank the following integrals have no parametric solution in closed form. In the absence of a cannibalization effect but under non-log-linear product sales in product rank the solution to the following integrals involves a combination of the Euler gamma function and the hypergeometric function.

In our parametrization of local entry costs, the beachhead cost $\gamma_{sd}\, w_d\, \kappa$ is proportional to the local entry cost for additional products with a constant factor of κ. As a result, the minimum exporter scope G^* is constant worldwide (independent of both source and destination country). This parametric choice preserves the feature from discrete product space that average local entry costs are a constant fraction of average sales per firm: $\bar{F}_{sd}/\bar{T}_{sd} = \textit{const.}$ (Arkolakis and Muendler 2010, Proposition 3). That property of the discrete model does not generally carry over to the continuum model, however, if the beachhead cost component κ_{sd} is source-country or destination-country specific, or both. Naturally, in discrete product space minimum exporter scope is one (any firm enters with at least one product). There is no such natural lower bound in the continuum model.

Using expression (B.4) for the entry-cost related parameter $\tilde{F}_{sd}(G^*)$ in the bilateral expenditure shares (25) of country d on products from country s yields the gravity relationship

$$\lambda_{sd} = \frac{J_s(b_s)^{\theta}(\tau_{sd}w_s)^{-\theta}\, \tilde{F}_{sd}(G^*)}{\sum_k J_k(b_k)^{\theta}(\tau_{kd}w_k)^{-\theta}\, \tilde{F}_{kd}(G^*)} = \frac{J_s(b_s)^{\theta}(\tau_{sd}w_s)^{-\theta}\, (\gamma_{sd}\, w_d)^{-(\tilde{\theta}-1)}}{\sum_k J_k(b_k)^{\theta}(\tau_{kd}w_k)^{-\theta}\, (\gamma_{kd}\, w_d)^{-(\tilde{\theta}-1)}}$$

in the absence of a cannibalization effect and for log-linear product sales in product rank. In discrete product space, in contrast, the entry-cost related parameter $\tilde{F}_{sd}(G^*)$ is a Riemann zeta function so that only the curvature of combined incremental scope costs matters, but not entry costs levels such as the parameter γ_{sd} here.

Finally, average exporter scope (26) of firms from s selling to destination d becomes

$$\begin{aligned}
\bar{G}_{sd} &\equiv \int_{G^*} F_{sd}(G)\mu_{sd}(G;\tilde{\theta})\, \mathrm{d}G \\
&= \tilde{\theta}\, Z_{sd}(G^*_{sd})^{\tilde{\theta}} \cdot (\gamma_{sd}\, w_d)^{-\tilde{\theta}}[\delta + \alpha(\varepsilon - 1)] \int_{G^*} G^{-[\delta+\alpha(\varepsilon-1)]\tilde{\theta}}\, \mathrm{d}G \\
&= \frac{[\delta + \alpha(\varepsilon - 1)]\tilde{\theta}}{[\delta + \alpha(\varepsilon - 1)]\tilde{\theta} - 1}\, G^*
\end{aligned}$$

for $\varepsilon = \sigma$ and $\beta = 0$. The integral exists if Assumption 4 ($\delta + \alpha(\sigma - 1) > 1/\tilde{\theta}$) holds, just as in discrete product space (Arkolakis and Muendler 2010, Table 2).

C Data

In our pristine exports data from SECEX (*Secretaria de Comércio Exterior*), product codes are 8-digit numbers (under the common Mercosur nomenclature), of which the first six digits coincide

Table C.1: Sample Characteristics by Source and Destination

From source s	Brazil Manufacturers				Chile Manf.	Brazil Intm.
to destination d	USA	Argentina	OECD	World	World	World
	(1)	(2)	(3)	(4)	(5)	(6)
# of Firms (M)	3,083	4,590	5,041	10,215	4,099	2,627
# of Destinations (N)	1	1	23	170	140	132
# of HS-6 products (G)	2,144	2,814	2,772	3,717	3,199	2,777
# of Observations	10,775	21,623	36,359	162,570	37,183	35,960
Destination share in Total exp.	.257	.144	.559	1	1	1
Firm shares in Total exports						
Single-prod. firms	.123	.086	.142	.090	.041	.086
Multi-prod. firms' top product	.662	.555	.625	.597	.715	.595
Multi-prod. firms' other prod.	.215	.359	.233	.313	.243	.319
Median Total exports ($T_d(m)$)	.120	.068	.137	.089	.038	.041
Median Exporter scope ($G_d(m)$)	1	2	2	2	2	2
Median Avg. Exp. scale ($a_d(m)$)	.068	.031	.070	.037	.014	.012
Mean Total exports ($\bar{t}_d$)	3.170	1.192	4.217	3.720	2.779	1.101
Mean Exporter scope ($\bar{G}_d$)	3.495	4.711	3.933	5.278	5.454	9.426
Mean Avg. Exp. scale ($\bar{a}_d$)	.907	.253	1.072	.705	.510	.117

Sources: *SECEX* 2000 for Brazil, manufacturing firms and their manufactured products as well as commercial intermediaries and their manufactured products; Chilean customs data 2000 (Álvarez et al. 2007) for manufacturing firms. *Note*: Aggregate regions (world, OECD) treated as single destinations, collapsing product shipments to different countries into single product shipment. The worldwide average number of products across destination countries is 3.518 among Brazilian manufacturers, for instance, but 5.278 for the world as single destination; it is 2.909 across destination countries worldwide among Chilean manufacturers but 5.454 for the world; and it is 5.740 across destination countries worldwide among Brazilian intermediaries but 9.426 for the world as single destination. Products at the Harmonized-System 6-digit level. Exports in US$ million fob. OECD includes all OECD members in 1990. The U.S. is Brazil's top export destination in 2000, Argentina second to top. Firms' average exporter scale (a_d in US$ million fob) is the scope-weighted arithmetic mean of exporters' average exporter scales.

with the first six HS digits. We aggregate the pristine monthly exports data to the HS 6-digit product, firm and year level. We apply the same level of aggregation to the Chilean data from Álvarez et al. (2007).

On the product side, we restrict our data to manufactured products only as mentioned. On the firm side, we focus on manufacturing firms and their direct exports of manufactures.[29] The restriction to manufacturing firms and their manufactured products makes our findings closely comparable to our companion paper Arkolakis and Muendler (2010) as well as to earlier research by Eaton et al. (2004) and Bernard et al. (2011), for example. For a broader interpretation of estimates, we also report findings from the complementary group of Brazilian commercial intermediary firms

[29]We obtain the firm's *CNAE* four-digit industry (for 654 industries across all sectors of the economy) from the administrative RAIS records (*Relação Anual de Informações Sociais*) at the Brazilian labor ministry. The level of detail in *CNAE* is roughly comparable to the *NAICS 2007* five-digit level.

and their exports of manufactures. We remove export records with zero value from the Brazilian data, which include shipments of commercial samples but also potential reporting errors, and lose 408 of initially 162,978 exporter-destination-product observations. Our results on exporter scope do not materially change when including or excluding zero-shipment products from the product count. There are no reported shipments with zero value in the Chilean data.

Our Brazilian manufacturer sample includes 10,215 firms with shipments of 3,717 manufacturing products at the 6-digit Harmonized System level to 170 foreign destinations, and a total of 162,570 exporter-destination-product observations. The Chilean manufacturer sample is considerably smaller; it has 4,099 firms with shipments of 3,199 manufacturing products at the 6-digit Harmonized System level to 140 foreign destinations, and a total of 37,183 exporter-destination-product observations. Our Brazilian intermediary sample includes 2,627 firms in the commercial sector with shipments of 2,777 manufacturing products at the 6-digit Harmonized System level to 132 foreign markets, and a total of 35,960 exporter-destination-product observations. Exporters shipping multiple products dominate. They ship more than ninety percent of all exports both from Brazil and Chile, and their single top-selling products account for almost sixty percent of all Brazilian exports and more than seventy percent of all Chilean exports.

To analyze export behavior, we decompose a firm ϕ's total exports $t_{d\phi}$ from Brazil or Chile to destination market d into the firm's number of products sold at d (the exporter scope) $G_{d\phi}$ and the firm's average sales per export product in d (the average exporter scale) $a_{d\phi} \equiv t_{d\phi}/G_{d\phi}$:

$$t_{d\phi} = \sum_{g=1}^{G_{d\phi}} p_{dg\phi} x_{dg\phi} = G_{d\phi}\, a_{d\phi},$$

where $p_{dg\phi}$ is the price of product g and $x_{dg\phi}$ its export quantity. To calculate summary medians and means of these variables for regional aggregates and the world as a whole in Table C.1 (columns 3 to 6), we treat the aggregate as if it were a single destination and collapse all product shipments to different countries into a single product shipment. For estimation in the text, in contrast, we analyze these variables country by country.

The median exporter is a relatively small exporter, with sales to the rest of the world totalling around US\$ 90,000 from Brazil (column 4) and US\$ 40,000 from Chile (column 5). The mean Brazilian (Chilean) exporter, in contrast, sells around US\$ 3.7 (2.8) million abroad, more than 40 (70) times as much as the median manufacturer. Exporter scope and average exporter scale exhibit similarly stark differences between mean and median. The median Brazilian manufacturer sells two products worldwide, but the mean scope per firm is 5.3 products. The median Brazilian manufacturer has sales of around US$ 40,000 per product, but the mean exporter scale per exporter is US$ 700,000, or around 20 times as high as that for the median firm.[30]

The importance of the top-selling product at multi-product exporters and the mean-median ratios repeat across destinations. To investigate the robustness across countries, we select Brazil's top two export destinations (United States and Argentina), as well as the OECD aggregate. Our theory emphasizes the importance of exporting behavior within destinations. Within single countries, the mean manufacturer's exports exceed the median manufacturer's exports by similarly large factors as in the aggregate, between 14 (in Argentina, column 2) and 26 (in the United States, column 1). In the OECD aggregate (column 3), exports of the mean firm exceed the exports of the median firm by a factor of about 30. Interestingly, the same mean-median ratio of about 30 prevails in the non-OECD aggregate (not reported).

We also investigate Brazil's commercial intermediaries and their manufacturing-product shipments. The shares of single- and multi-product firms are strikingly similar for intermediaries selling manufactured products (column 6) and the manufacturers shipping directly (column 4). The mean-median ratios vary more, however. While the mean Brazilian manufacturer sells more than 40 times as much as the median manufacturer, the mean Brazilian commercial intermediary sells only around 30 times as much as the median intermediary. This differences between Brazilian manufacturers and intermediaries is due to a smaller mean-median ratio for average exporter scale at intermediaries than at manufacturers (10 times versus 20 times), whereas the mean intermediary sells more than nine products worldwide compared to the mean manufacturer with only five.

[30] The means in Table C.1 are calculated as follows. A source country's total exports T_d are decomposed into $T_d = M_d\,\bar{G}_d\,\bar{a}_d$, where M_d is the number of exporters to destination d, $\bar{G}_d \equiv \sum_{\phi=1}^{M_d} G_d(\phi)/M_d$ is the exporters' mean exporter scope, and $\bar{a}_d \equiv \bar{t}_d/\bar{G}_d$ is the average exporter scale (the products' mean sales at those firms). Equivalently, $\bar{a}_d$ is the weighted arithmetic mean of $a_d(\phi)$ over all ϕ, with weights $G_d(\phi)$: $\bar{a}_d = \sum_{\phi=1}^{M_d} G_d(\phi)\,a_d(\phi)/[\sum_{\phi=1}^{M_d} G_d(\phi)] = \bar{t}_d/\bar{G}_d$. As the decomposition shows, scope weighting is necessary for the mean scope and the mean exporter scale to yield total exports when multiplied.

References

Allanson, Paul and Catia Montagna, “Multiproduct Firms and Market Structure: An Explorative Application to the Product Life Cycle,” *International Journal of Industrial Organization*, September 2005, *23* (7-8), 587–597.

Álvarez, Roberto, Hasan Faruq, and Ricardo A. López, “New Products in Export Markets: Learning from Experience and Learning from Others,” August 2007. Indiana University, Bloomington, unpublished manuscript.

Arkolakis, Costas and Marc-Andreas Muendler, “The Extensive Margin of Exporting Products: A Firm-level Analysis,” *NBER Working Paper*, December 2010, *16641.*

Atkeson, Andrew and Ariel T. Burstein, “Pricing-to-Market, Trade Costs, and International Relative Prices,” *American Economic Review*, December 2008, *98* (5), 1998–2031.

Bernard, Andrew B., Stephen J. Redding, and Peter K. Schott, “Multi-product Firms and Trade Liberalization,” *Quarterly Journal of Economics*, 2011. advance access July 19, 2011.

Broda, Christian and David E. Weinstein, “Product Creation and Destruction: Evidence and Price Implications,” *NBER Working Paper*, 2007, *13041.*

Chaney, Thomas, “Distorted Gravity: The Intensive and Extensive Margins of International Trade,” *American Economic Review*, September 2008, *98* (4), 1707–21.

Eaton, Jonathan and Samuel Kortum, “Technology, Geography, and Trade,” *Econometrica*, September 2002, *70* (5), 1741–79.

__ , __ , and Francis Kramarz, “Dissecting Trade: Firms, Industries, and Export Destinations,” *American Economic Review: Papers and Proceedings*, May 2004, *94* (2), 150–54.

__ , __ , and __ , “An Anatomy of International Trade: Evidence from French Firms,” February 2010. University of Chicago, unpublished manuscript.

Eckel, Carsten and J. Peter Neary, “Multi-Product Firms and Flexible Manufacturing in the Global Economy,” *Review of Economic Studies*, January 2010, *77* (1), 188–217.

Feenstra, Robert C. and Hong Ma, “Optimal Choice of Product Scope for Multiproduct Firms under Monopolistic Competition,” in Elhanan Helpman, Dalia Marin, and Thierry Verdier, eds., *The Organization of Firms in a Global Economy*, Cambridge, Mass.: Harvard University Press, November 2008, chapter 6, pp. 173–199.

__ , Robert E. Lipsey, Haiyan Deng, Alyson C. Ma, and Hengyong Mo, “World Trade Flows: 1962-2000,” *NBER Working Paper*, January 2005, *11040.*

Helpman, Elhanan, Marc J. Melitz, and Stephen R. Yeaple, “Export versus FDI with Heterogeneous Firms,” *American Economic Review*, March 2004, *94* (1), 300–316.

Krugman, Paul R., “Scale Economies, Product Differentiation, and the Pattern of Trade,” *American Economic Review*, 1980, *70* (5), 950–59.

Mayer, Thierry, Marc J. Melitz, and Gianmarco I.P. Ottaviano, "Market Size, Competition, and the Product Mix of Exporters," *NBER Working Paper*, April 2011, *16959.*

Melitz, Marc J., "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, November 2003, *71* (6), 1695–1725.

Nocke, Volker and Stephen R. Yeaple, "Globalization and Endogenous Firm Scope," *NBER Working Paper*, June 2006, *12322.*