

The Extensive Margin of Exporting Products: A Firm-level Analysis*

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Abstract

We examine multi-product exporters and use firm-product-destination data to quantify export entry barriers. Our general-equilibrium model of multi-product firms generalizes earlier models. To match main facts about multi-product exporters, we estimate our model with rich demand and access cost shocks for Brazilian firms. The estimates document that additional products farther from a firm's core competency incur higher unit costs, but face lower market access costs. We find that these market access costs differ across destinations and evaluate a scenario that standardizes market access between countries. The resulting welfare gains are similar to eliminating all current tariffs.

Keywords: International trade; heterogeneous firms; multi-product firms; firm and product panel data; Brazil

JEL Classification: F12, L11, F14

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1 Introduction

Multi-product firms dominate the domestic market and international trade. Their preponderance has informed recent advances in the theory of firm and exporter growth.¹ While market entry with additional products is a significant margin of firm and exporter expansion, our understanding of the associated market access costs and the welfare benefits of product expansion is still limited. This shortcoming confines our insight into determinants of export growth and inhibits the application of the theory to policy issues. Meanwhile, international trade policy has shifted interest to facilitating market access by dismantling non-tariff measures (NTMs), which are now more relevant as a remaining avenue for liberalizing trade than are import tariffs.² Despite the apparent relevance of NTMs our limited ability to measure them has impeded research to quantify their importance.

In this paper we build a framework of multi-product exporters that generalize earlier multi-product models and offers a flexible setup to rigorously quantify the relevance of market access costs for exporter expansion. We use Brazilian data to empirically recover these policy-dependent costs in a multi-product model and relate them to welfare. Our framework extends the monopolistic competition model of Melitz (2003) by embedding a multi-product setup into a conventional constant elasticity of substitution (CES) demand system.

We model within-firm product heterogeneity with two key mechanisms. First, we assume as in Eckel and Neary (2010, henceforth EN) that a firm faces declining efficiency in supplying additional products that are farther from its core competency. Second, we

¹See, for example, Eckel and Neary (2010); Goldberg, Khandelwal, Pavcnik and Topalova (2010); Bernard, Redding and Schott (2010, 2011). Bernard, Jensen and Schott (2009) document for U.S. trade in 2000 that firms that export more than five products at the HS 10-digit level make up 30 percent of exporters but account for 97 percent of all exports. In our Brazilian exporter data for 2000, 25 percent of all manufacturing exporters ship more than ten products at the internationally comparable HS 6-digit level and account for 75 percent of total exports. Similar findings are reported by Iacovone and Javorcik (2008) for Mexico and Álvarez, Faruq and López (2013) for Chile.

²See, for example, OECD (2005); UNCTAD (2010); WTO (2012). Kee, Nicita and Olarreaga (2009) estimate that, for a majority of tariff lines in 78 countries, the ad valorem equivalent of the NTMs today exceeds the import tariff. Rounds of trade negotiations have converted conventional quantity restrictions such as quotas into tariffs (“tariffication”) and then brought tariffs to unprecedentedly low levels. Tariffs on industrial products in developed countries, for instance, have come down to an average of just 3.8 percent (www.wto.org accessed 11/29/2015). Recent surveys of exporting firms in numerous countries document that “*technical measures and customs rules and procedures* . . . are [consistently] among the five most reported categories of [trade] barriers” (OECD 2005, p. 24). Similarly, the recently concluded Trans-Pacific Partnership (TPP) agreement among 12 Asia-Pacific economies targets NTMs by streamlining customs rules and procedures (chapter 5), sanitary and phytosanitary regulations (ch. 7), and technical barriers to trade (ch. 8) as well as implementing regulatory coherence (ch. 25). Our specific definition of market access costs will not include tariffs, in contrast to an occasionally broader use of the term in the trade policy discourse.

introduce local product appeal shocks (similar to Eaton, Kortum and Kramarz 2011) and thus nest a version of the Bernard et al. (2011, henceforth BRS) model that attributes within-firm product heterogeneity to local demand shocks. In our framework, the firm faces two extensive margins and one intensive margin: it chooses its presence at export destinations, its *exporter scope* (the number of products) at each destination, and the quantities (prices) for each individual product at each destination.

We consider three types of costs. First, as mentioned above, there are product-specific production costs at the firm level similar to EN (core competence). Second, there are shipping costs (iceberg trade costs), which vary with sales but do not depend on scope. Both production costs and shipping costs deter trade at all margins. Third, to capture the specificities of non-tariff barriers for market access, we consider a flexible schedule of fixed exporting costs by firm, product, and destination market, generalizing the firm-destination level exporting costs in Chaney (2008). This *market access cost* schedule can vary by firm-product and accommodates the possible cases of economies and diseconomies of scope.³ It affects only the two extensive margins: a firm’s entry into a destination with the first product and its exporter scope there.

The micro-foundation of market access costs allows us to use data on multi-product exporters to estimate these costs and to relate them to measures of non-tariff barriers. Our approach thus differs from firm-level research, including Arkolakis (2010), in that we give substance to market access costs and directly estimate those costs from product entry and product sales.⁴ Most importantly, while differences in market penetration costs in that paper affect the exporter sales distribution by destination, that distribution is largely invariant across destinations in the data, therefore leaving no room for policy related to market penetration costs. In contrast, we explicitly exploit the variation of the relationship between exporter scale and scope by destination to identify policy relevant differences in market access costs.

To inform theory we document individual product sales and exporter scope by destination. We elicit three main facts. First, within firms and destinations, we look at the sales distribution by product. Wide-scope exporters sell large amounts of their top-selling

³Seminal references on economies of scope are Panzar and Willig (1977) and (1981). Formally, there are economies of scope for sales x and y of two products if the cost function satisfies $C(x+y) < C(x) + C(y)$, that is if the cost function is *subadditive*.

⁴The parametrization of our estimation model fully nests the Arkolakis (2010) market penetration costs for an exporter’s product composite. In an Online Supplement, we present a generalization of our model to nest market penetration costs as in Arkolakis (2010). We demonstrate that the stochastic components in our simulated method of moments estimator fully absorb the market penetration costs that firms choose to incur for their product lines at a destination, rendering our estimation consistent with Arkolakis (2010).

products. Moreover, they sell considerably smaller amounts of their lowest-selling products than do narrow-scope exporters. Second, within destinations and across firms, we look at exporter scope: there are few dominant exporters with wide scope but many narrow-scope firms. The median exporter only ships one or two products per destination. We also find that the average exporter scope is larger at geographically closer destinations, indicating varying incremental market access costs. Finally, within destinations and across firms, firm average sales per product and exporter scope exhibit a strong positive covariation in distant destinations.

These facts have a number of implications for the theory. For a wide-scope firm to profitably sell minor amounts of its lowest-selling products, incremental market access costs must be low at wide scope. The finding is at odds with models of multi-product firms where market access costs are fixed or constant for additional products and underlies our flexible market access cost schedule that allows for potential economies of scope. For example, fixed market access costs are constant in BRS, EN and Mayer, Melitz and Ottaviano (2014). Our combination of scope-dependent production costs and market access costs delivers variation in average exporter scope on the one hand and generates the correlation of average sales and exporter scope across destinations on the other hand, consistent with our second and third facts.

For our quantification we adopt a simulated method of moments estimator in order to handle the three stochastic elements of the model. These elements—Pareto distributed firm-level productivity, a stochastic firm-level market access cost component, and local product appeal shocks—are needed to match the empirical regularities in the Brazilian exporting transaction data. We also document, for practical purposes, that results from ordinary least squares under only one stochastic element (firm-level productivity) provide a useful approximation to the full simulated method of moments estimation. In the main estimation, we target our first two facts, which the estimated model closely matches. We also illustrate the success of this estimation by showing that the estimated model fits the third fact (on the destination-specific correlation of average product sales and exporter scope), which we deliberately do not target in the estimation. A decomposition of the variance in product sales shows that product- and firm-level heterogeneity accounts for two-fifths of the variation in product sales, while idiosyncratic product appeal shocks abroad account for three-fifths. This finding highlights both the relevance of our extended framework of multi-product exporting and the important interplay of a firm’s core competency with local demand conditions abroad.

The estimation reveals that additional products farther from a firm’s core competency

incur higher unit costs but also reveal differences in the economies of scope in market access costs between destinations. In addition, these estimated differences are poorly explained by geographic and other invariable gravity predictors, but appear susceptible to economic conditions or policy. We simulate a reduction in market access costs for additional products and its effect on global trade. To capture only components of market access costs that appear amenable to policy, we hypothetically reduce market access costs worldwide to the schedules observed in nearby destinations with low incremental market access costs. This counterfactual harmonization of incremental market access costs across destinations highlights the potential importance of reducing NTMs on exporters' additional products. Our simulation generates welfare gains similar to eliminating today's remaining observable tariffs.

Our approach to countries' market access costs asks how their protection affects a typical country's exports, and global welfare, and is therefore closely related to trade restrictiveness measurement by Kee et al. (2009) in partial equilibrium.⁵ We adopt a general-equilibrium framework and allow for rich micro-foundations for the incidence of market access costs on firm and product entry. However, our complementary approach foregoes NTM survey information by source country and tariff line. Examples of incremental market access costs among NTMs are product-level health regulations, safety standards, certifications and licenses.⁶

Over the past few years, research into multi-product firms has expanded markedly (see for example, BRS, EN, Mayer et al. (2014), Eckel, Iacovone, Javorcik and Neary (2015), among others).⁷ This work stresses the significance of multi-product firms either from an empirical perspective or from a theoretical. Instead, our work aims to make contact of these two large parts of the literature by bringing together theory and data: we use facts

⁵Earlier indexes of trade restrictiveness ask how harmful protection is to a country itself (for surveys see Feenstra 1995; Anderson and Neary 2005). An index of a country's trade restrictiveness is akin to a single hypothetical ad-valorem tariff that would be equivalent either in terms of welfare (Anderson and Neary 1996) or import volumes (Anderson and Neary 2003) to the country's overall set of protectionist measures.

⁶Some NTMs are arguably market access costs that an exporter incurs prior to the shipment of the first unit of a product and not again (UNCTAD 2010), while other NTMs such as customs procedures may also act like shipping costs in that they lengthen the duration of export financing. As the empirical literature on NTMs starts to make available more precise NTM variables, they can be embedded into our framework's shipping cost and market access cost functions. For now, our market access cost estimates do not discern individual NTMs from other so-called "natural" trade barriers at the border, such as language. Our counterfactual simulations, however, are designed to capture policy relevant market entry and behind-the-border costs.

⁷Nocke and Yeaple (2014) and Dhingra (2013) study multi-product exporters but do not generate a within-firm sales distribution, which lies at the heart of our analysis. Other empirical work includes Thomas (2011); Amador and Oromolla (2013); and Álvarez et al. (2013).

about multi-product firms to understand the costs and benefits of expanding product lines. In turn, we use the general equilibrium structure of our model to assess the implications of policies related to removing product expansion costs.⁸

Arkolakis, Costinot and Rodriguez-Clare (2012) show for a wide family of models, which includes ours, that conditional on identical observed trade flows these models predict identical ex-post welfare gains irrespective of firm turnover and product-market reallocation. Their findings also imply, however, that models in that family differ in their predictions for trade flows and welfare with respect to ex-ante changes in market access costs. Our framework provides market-specific micro-foundations for such market access costs, and we use it to compute the impact of the elimination of these costs on trade flows and welfare.

The paper is organized in five more sections. In Section 2 we describe the model. Section 3 presents the dataset and observed empirical patterns. Section 4 presents the simulated method of moments (SMM) estimator that we use to uncover the model's parameters. Counterfactuals involving variations in market access costs are in Section 5. We conclude with Section 6.

2 Model

Our model rests on firm productivity as a key source of heterogeneity. This variability generates dispersion in total sales and in the number of products sold. There are two main additional ingredients. First, we introduce firm-product-destination specific preferences that affect individual product sales through a stochastic demand component under a constant elasticity of substitution. Second, we specify market access costs, which depend deterministically on the number of products that a firm sells in a destination market but depend stochastically on a firm-destination specific entry cost draw. These elements allow us both to closely match the data and to incorporate key features of recent multi-product exporter models.⁹

⁸Timoshenko (2015) empirically analyzes multi-product firm dynamics. Qiu and Zhou (2013) document the importance of variety-specific introduction fees, which we term incremental market access costs. Morales, Sheu and Zahler (2014) structurally study the path-dependent sequential entry of multi-product firms into additional export markets.

⁹For example, we can nest a version of the BRS model by using a simplification of our market access costs, where access costs are a linear function of exporter scope. We model core competency following EN and similar to Mayer et al. (2014), but use a different consumer utility function, so we can capture a variant of their predictions under a constant elasticity of substitution by removing individual preference shocks and keeping market access costs constant in our framework. For an appropriately defined market access cost schedule that depends on the choice of consumers reached through marketing, we can nest the

2.1 Setup

There are N countries. The export source country is denoted with s and the export destination with d . There is a measure of L_d consumers at destination d . Consumers have symmetric preferences with a constant elasticity of substitution σ over a continuum of varieties. In this multi-product setting, a “variety” offered by a firm ω from source country s to destination d is the product composite

$$X_{sd}(\omega) \equiv \left(\sum_{g=1}^{G_{sd}(\omega)} \xi_{sdg}(\omega)^{\frac{1}{\sigma}} x_{sdg}(\omega)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where $G_{sd}(\omega)$ is the exporter scope (the number of products) that firm ω sells in country d , g is the running index of a firm’s product at destination d , $\xi_{sdg}(\omega)$ is an i.i.d. shock to firm ω ’s g -th product’s appeal (with mean $\mathbb{E}[\xi_{sdg}(\omega)] = 1$, positive support and known realization at the time of consumer choice), and $x_{sdg}(\omega)$ is the quantity of product g that consumers consume. In marketing terminology, the product composite is often called a firm’s product line or product mix. We assume that every product line is uniquely offered by a single firm, but a firm may ship different product lines to different destinations.

2.2 Consumers

The consumer’s utility at destination d is

$$\left(\sum_{k=1}^N \int_{\omega \in \Omega_{kd}} X_{kd}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \quad \text{for } \sigma > 1, \quad (1)$$

where Ω_{kd} is the set of firms that ship from source country k to destination d . For simplicity we assume that the elasticity of substitution across a firm’s products is the same as the elasticity of substitution between varieties of different firms.¹⁰ It is straightforward to generalize the model to consumer preferences with two nests (see Appendix C for an outline and the Online Supplement to this paper for a derivation). If the firm’s products in the inner nest were closer substitutes to each other than product lines are substitutable across firms, then a firm’s additional products would cannibalize the sales of its infra-marginal products. As we show in the Online Supplement, the cannibalization effect

Arkolakis (2010) model with our (stochastic) market entry components (see our Online Supplement).

¹⁰Allanson and Montagna (2005) adopt a similar nested CES form to study the product life-cycle and market structure, and Atkeson and Burstein (2008) use a similar nested CES form in a heterogeneous-firms model of trade but do not consider multi-product firms.

would not substantively alter our main theoretical results and estimation equations.¹¹

The representative consumer earns a wage w_d from inelastically supplying her unit of labor endowment to producers in country d and receives a per-capita dividend distribution π_d equal to her share $1/L_d$ in total profits at national firms. We denote total income with $Y_d = (w_d + \pi_d)L_d$. The consumer observes the product appeal shocks $\xi_{sdg}(\omega)$ prior to consumption choice so that the first-order conditions of utility maximization imply a deterministic product demand

$$x_{sdg}(\omega) = \left(\frac{p_{sdg}(\omega)}{P_d} \right)^{-\sigma} \xi_{sdg}(\omega) \frac{T_d}{P_d}, \quad (2)$$

where p_{sdg} is the price of product g in destination d and we denote by T_d the total expenditure of consumers in country d . In the calibration, we will allow for the possibility that total consumption expenditure T_d is different from country output Y_d (allowing for trade imbalances), so we use different notation for the two terms. We define the corresponding ideal price index P_d as

$$P_d \equiv \left[\sum_{k=1}^N \int_{\omega \in \Omega_{kd}} \sum_{g=1}^{G_{kd}(\omega)} \xi_{kdg}(\omega) p_{kdg}(\omega)^{-(\sigma-1)} d\omega \right]^{-\frac{1}{\sigma-1}}. \quad (3)$$

2.3 Firms

Firms face three types of costs: variable production costs (which are constant for a given product but higher for products farther away from a firm's core competency), variable shipping costs (iceberg trade costs), and market access costs (which depend on a firm's local exporter scope but do not vary with sales). Each firm draws a productivity parameter ϕ and a destination specific market access cost shock $c_d \in (0, \infty)$. The firm chooses how many products to ship to a given destination and what price to charge for each product at a destination. Following the firms' choices, consumers learn the product specific taste shocks $\xi_{sdg}(\omega)$ for each firm-product. Then production and sales are realized. Firms from country s with identical productivity ϕ and identical market access cost shock c_d face an identical optimization problem in every destination d at the time of their market access and exporter scope decision. A firm produces each product g with a linear production

¹¹The main estimation relationships in this paper would remain unaltered under that generalization. Some estimated coefficients would reflect elasticities in the inner nest and other coefficients the elasticities of the outer nest. Hottman, Redding and Weinstein (2014) follow up on the cannibalization effect by using data on overall expenditure shares and prices to calculate both intra-firm and inter-firm elasticities of substitution separately.

technology, employing source-country labor given a firm-product specific efficiency ϕ_g .

Following Chaney (2008), we assume that there is a continuum of potential producers of measure J_s in each source country s . When exported, products incur standard iceberg trade costs so that $\tau_{sd} > 1$ units must be shipped from s for one unit to arrive at destination d . We normalize $\tau_{ss} = 1$ for domestic sales. This iceberg trade cost is common to all firms and to all firm-products shipping from s to d .

Without loss of generality we order each firm's products in terms of their efficiency, from most efficient to least efficient, so that $\phi_1 \geq \phi_2 \geq \dots \geq \phi_{G_{sd}}$. Under this convention we write the efficiency of the g -th product of a firm ϕ as

$$\phi_g \equiv \phi/h(g) \quad \text{with} \quad h'(g) > 0. \quad (4)$$

Related to the marginal-cost schedule $h(g)$ we define the average product efficiency index in destination d when the firm sells G_{sd} products there as

$$\bar{H}(G_{sd}) \equiv \left(\sum_{g=1}^{G_{sd}} h(g)^{-(\sigma-1)} \right)^{-\frac{1}{\sigma-1}}. \quad (5)$$

This efficiency index decreases with exporter scope, because firms add less efficient products as they widen scope, and will play an important role in the firm's optimality condition for scope choice.

2.3.1 Firm market access costs

The firm faces a product-destination specific *incremental market access cost* $c_d f_{sd}(g)$. A firm that adopts an exporter scope of G_{sd} therefore incurs a total market access cost of

$$F_{sd}(G_{sd}, c_d) = c_d \sum_{g=1}^{G_{sd}} f_{sd}(g) \quad (6)$$

if its idiosyncratic market access cost is c_d . The firm's market access cost is zero at zero scope and strictly positive otherwise:

$$f_{sd}(0) = 0 \quad \text{and} \quad f_{sd}(g) > 0 \quad \text{for all } g = 1, 2, \dots, G_{sd}$$

where $f_{sd}(g)$ is a continuous function in $[1, +\infty)$.¹² Similar to Eaton et al. (2011), we assume the access cost shock c_d to be i.i.d. across firms and destinations.

The incremental market access cost $c_d f_{sd}(g)$ accommodates fixed costs of production (e.g. with $0 < f_{ss}(g) < f_{sd}(g)$). In a given destination market, the incremental market access costs $c_d f_{sd}(g)$ may increase or decrease with exporter scope. But a firm's total market access costs $F_{sd}(G_{sd}, c_d)$ necessarily increase with exporter scope G_{sd} in country d because $f_{sd}(g) > 0$.¹³ We assume that the incremental market access costs $c_d f_{sd}(g)$ require labor from the destination country d so that $F_{sd}(G_{sd}, c_d)$ is homogeneous of degree one in w_d . Combined with the varying firm-product efficiencies ϕ_g , this market access cost structure allows us to endogenize the exporter scope choice at each destination. Whereas the incremental market access cost is meant to capture the barriers to access that may differ for different exporters depending on the number of products sold, the idiosyncratic access cost shock implies that there is no strict hierarchy of destinations across exporters. Some exporters may sell to less popular destinations but not to the most popular ones.

In summary, there are two scope-dependent cost components: the marginal cost schedule $h(g)$ and the incremental market access cost $f_{sd}(g)$. Suppose for a moment that the incremental market access cost is constant in destination d and independent of g with $f_{sd}(g) = f_{sd}$. Then a firm in our model faces diseconomies of scope in destination d because the marginal-cost schedule $h(g)$ strictly increases with the product index g . But, if incremental market access costs decrease sufficiently strongly with g , our functional forms would allow for overall economies of scope in destination market d .

Before we proceed to firm optimization, we introduce a parameterized example for these functions that will later allow us to quantitatively match the patterns observed in the Brazilian data. For quantification, we will specify

$$\begin{aligned} f_{sd}(g) &= f_{sd} \cdot g^{\delta_{sd}} \quad \text{for } \delta_{sd} \in (-\infty, +\infty) & \text{and} \\ h(g) &= g^\alpha \quad \text{for } \alpha \in [0, +\infty). \end{aligned} \tag{7}$$

The choice of these two functions is guided by the log-linear relationships that we will present in Section 3. Introducing the example at this stage helps us provide intuition for

¹²Brambilla (2009) adopts a related specification but its implications are not explored in an equilibrium firm-product model.

¹³This specification accommodates a potentially separate firm-level access cost (sometimes referred to as a one-time beachhead cost), which can be subsumed in the first product's market access cost. The only requirement is that our later assumptions on the shape of the market access cost schedule are satisfied. In continuous product space with nested CES utility, in contrast, market access costs must be non-zero at zero scope because a firm would otherwise export to all destinations worldwide (Bernard et al. 2011; Arkolakis and Muendler 2010).

the role that the parameters δ_{sd} and α will play in later estimation. The parameter δ_{sd} is the scope elasticity of market access cost. The product $\alpha(\sigma-1)$ is the scope elasticity of product efficiency and its estimated value will determine how fast sales drop for additional firm products. We allow δ_{sd} to vary across destinations, unlike α . While α governs production of a product within a single source country, market access costs are paid repeatedly at every destination. We show in the Online Supplement that the market access cost specification (7) is readily reformulated to accommodate the functional form of Arkolakis (2010) market penetration costs for a firm's product composite, where f_{sd} may depend on the optimal share of consumers reached. Market penetration costs do not affect our final estimation model because the relevant marketing cost parameters get subsumed in the (stochastic) market access cost component $c_d f_{sd}$.

2.3.2 Firm optimization

Conditional on destination market access, the firm chooses individual product prices given consumer demand under monopolistic competition. The resulting first-order conditions from the profit maximizing equation produce identical markups over marginal cost $\tilde{\sigma} \equiv \sigma/(\sigma-1) > 1$ for $\sigma > 1$.¹⁴

Firms with the same productivity ϕ and the same access cost shock for a given destination c_d , make identical product entry decisions in equilibrium. It is therefore convenient to name firms selling to a given destination d by their common characteristic (ϕ, c_d) . We will suppress the ω notation whenever there is no risk of confusion. A type (ϕ, c_d) firm chooses

¹⁴After a firm observes each product g 's appeal shock at a destination $\xi_{sdg}(\omega)$, its total profit from selling an optimal number of products G_{sd} to destination market d is

$$\pi_{sd}(\phi, c_d) = \max_{G_{sd}} \sum_{g=1}^{G_{sd}} \left[\max_{\{p_{sdg}\}_{g=1}^{G_{sd}}} \left(p_{sdg} - \tau_{sd} \frac{w_s}{\phi/h(g)} \right) \left(\frac{p_{sdg}}{P_d} \right)^{-\sigma} \xi_{sdg} \frac{T_d}{P_d} \right] - F_{sd}(G_{sd}, c_d).$$

Suppose the firm sets every individual price p_{sdg} after it observes the appeal shocks. Its first-order conditions with respect to every individual price p_{sdg} imply an optimal product price

$$p_{sdg}(\phi) = \tilde{\sigma} \tau_{sd} w_s h(g) / \phi$$

with an identical markup over marginal cost $\tilde{\sigma} \equiv \sigma/(\sigma-1) > 1$ for $\sigma > 1$. Importantly, product price does not depend on the appeal shock realization because the shock enters profits multiplicatively; it is therefore not relevant for the firm's choice problem whether prices are set before or after the firm observes the product appeal shocks. In other words, maximizing total *expected* profit would result in the same first-order conditions for individual price. We adopt the convention that a firm commits to its price prior to the realization of product appeal shocks, and then ships the demanded quantities given price. The price commitment is credible and renegotiation proof because price choice remains optimal ex post. Firms may face a loss in the market if the demand shock realization implies that sales fail to cover the market entry costs, as market entry costs are sunk prior to the demand shock realization.

an exporter scope $G_{sd}(\phi, c_d)$. Plugging the optimal pricing decision into the firm's profit function we obtain expected profits at a destination d for a firm ϕ selling G_{sd} products,

$$\pi_{sd}(\phi, c_d) = \max_{G_{sd}} D_{sd} \phi^{\sigma-1} \bar{H}(G_{sd})^{-(\sigma-1)} - c_d \sum_{g=1}^{G_{sd}} f_{sd}(g),$$

with the revenue shifter

$$D_{sd} \equiv \left(\frac{P_d}{\tilde{\sigma} \tau_{sd} w_s} \right)^{\sigma-1} \frac{T_d}{\sigma}. \quad (8)$$

For profit maximization with respect to exporter scope to be well defined, we make the following assumption.

Assumption 1 (Strictly increasing combined incremental scope costs). *Combined incremental scope costs $z_{sd}(G, c_d) \equiv c_d f_{sd}(G) h(G)^{\sigma-1}$ strictly increase in exporter scope G .*

Under this assumption, and given the pricing decision, the optimal product choice is the largest $G \in \{0, 1, \dots\}$ such that operating profits from that product G equal (or exceed) the incremental market access costs:

$$\pi_{sd}^{g=1}(\phi) \equiv D_{sd} \phi^{\sigma-1} \geq c_d f_{sd}(G) h(G)^{\sigma-1} \equiv z_{sd}(G, c_d), \quad (9)$$

where $\pi_{sd}^{g=1}(\phi)$ are the operating profits from the core product. In our parameterized example, Assumption 1 requires that the sum $\delta_{sd} + \alpha(\sigma - 1)$ is larger than zero since $z_{sd}(G, c_d) = c_d f_{sd}(1) G^{\delta_{sd} + \alpha(\sigma-1)}$.

We can express the condition for optimal scope more intuitively and evaluate optimal exporter scope of different firms. A given firm ϕ with access cost shock c_d exports from s to d if and only if $\pi_{sd}(\phi, c_d) \geq 0$. At the break-even point $\pi_{sd}(\phi, c_d) = 0$, the firm is indifferent between selling its first product in destination market d or not selling at all. Equivalently, reformulating the break-even condition and using the above expression for minimum profitable scope, the productivity threshold $\phi_{sd}^*(c_d)$ for exporting at all from s to d is given by

$$\phi_{sd}^*(c_d)^{\sigma-1} \equiv c_d f_{sd}(1) / D_{sd}. \quad (10)$$

In general, using the above definition, we can define the productivity threshold $\phi_{sd}^{*,G}(c_d)$ such that firms with $\phi \geq \phi_{sd}^{*,G}(c_d)$ sell at least G_{sd} products at destination d with

$$\phi_{sd}^{*,G}(c_d)^{\sigma-1} \equiv \frac{z_{sd}(G, c_d)}{c_d f_{sd}(1)} \phi_{sd}^*(c_d)^{\sigma-1} = \frac{z_{sd}(G, c_d)}{D_{sd}} \quad \text{with} \quad z_{sd}(G, c_d) \equiv c_d f_{sd}(G) h(G)^{\sigma-1}, \quad (11)$$

adopting the notational simplification $\phi_{sd}^*(c_d) \equiv \phi_{sd}^{*,1}(c_d)$. Note that if Assumption 1 holds then $\phi_{sd}^*(c_d) < \phi_{sd}^{*,2}(c_d) < \phi_{sd}^{*,3}(c_d) < \dots$ so that more productive firms introduce more products in a given destination. As a result, $G_{sd}(\phi, c_d)$ is a step-function that weakly increases in ϕ for any given c_d .

The firm's optimal price choice for each product precedes the realization of the appeal shock ξ_{sdg} . Once the vector $\boldsymbol{\xi}$ of appeal shocks for a firm ω is realized, the firm supplies the market-clearing quantity of each product under the product's constant marginal cost. Using consumer demand (2) and the above definitions, we can express each individual product's sales by a firm of type (ϕ, c_d) in equilibrium as¹⁵

$$y_{sdg}(\phi, c_d, \xi_{sdg}) = \sigma z_{sd}(G_{sd}(\phi, c_d), c_d) \left(\frac{\phi}{\phi_{sd}^{*,G}(c_d)} \right)^{\sigma-1} h(g)^{-(\sigma-1)} \xi_{sdg}. \quad (12)$$

Summing over g , the firm's total sales at a destination become

$$t_{sd}(\phi, c_d, \boldsymbol{\xi}) = \sigma c_d f_{sd}(1) \left(\frac{\phi}{\phi_{sd}^*(c_d)} \right)^{\sigma-1} H(G_{sd}(\phi, c_d), \boldsymbol{\xi})^{-(\sigma-1)} \quad (13)$$

in equilibrium, where

$$H(G_{sd}(\phi, c_d), \boldsymbol{\xi}) \equiv \left(\sum_{g=1}^{G_{sd}(\phi, c_d)} h(g)^{-(\sigma-1)} \xi_{sdg} \right)^{-\frac{1}{\sigma-1}}.$$

The firm's realization of total sales $t_{sd}(\phi, c_d, \boldsymbol{\xi})$ in equilibrium and optimal exporter scope $G_{sd}(\phi, c_d)$ determine its *exporter scale*

$$a_{sd}(\phi, c_d, \boldsymbol{\xi}) \equiv t_{sd}(\phi, c_d, \boldsymbol{\xi}) / G_{sd}(\phi, c_d)$$

at destination d , the average sales per product, conditional on exporting from s to d .

Proposition 1 *If Assumption 1 holds, then for all $s, d \in \{1, \dots, N\}$*

- *exporter scope $G_{sd}(\phi, c_d)$ is positive and weakly increases in ϕ for $\phi \geq \phi_{sd}^*(c_d)$, and*
- *total firm exports $t_{sd}(\phi, c_d, \boldsymbol{\xi})$ are positive and strictly increase in ϕ for $\phi \geq \phi_{sd}^*(c_d)$.*

Proof. The first statement follows immediately from the discussion above. The second statement follows because $H(G_{sd}(\phi, c_d), \boldsymbol{\xi})$ strictly increases in $G_{sd}(\phi, c_d)$ a.s., given the

¹⁵The shocks ξ_{sdg} and $\boldsymbol{\xi}$ could be written as $\xi_{sdg}(\omega)$ and $\boldsymbol{\xi}(\omega)$ to emphasize that they are firm specific.

positive support of ξ_{sdg} , but $G_{sd}(\phi, c_d)$ weakly increases in ϕ , so $H(G_{sd}(\phi, c_d), \xi)$ weakly increases in ϕ . By (13), $t_{sd}(\phi, c_d, \xi)$ also monotonically depends on ϕ itself, so $t_{sd}(\phi, c_d)$ strictly increases in ϕ . ■

2.4 Model aggregation and equilibrium

To aggregate the model we specify a Pareto distribution of firm productivity following Helpman, Melitz and Yeaple (2004) and Chaney (2008). This assumption yields convenient functional forms. We specify the cumulative distribution function $\Pr = 1 - (b_s)^\theta / \phi^\theta$ over the support $[b_s, +\infty)$, where θ is the Pareto shape parameter, common across all source countries, and more advanced countries are thought to have a higher location parameter b_s . We also define $\tilde{\theta} \equiv \theta / (\sigma - 1)$ to simplify notation.

The resulting conditional probability density function of the distribution of entrants is then

$$\mu(\phi | \phi_{sd}^*, \theta) = \begin{cases} \theta(\phi_{sd}^*)^\theta / \phi^{\theta+1} & \text{if } \phi \geq \phi_{sd}^*, \\ 0 & \text{otherwise.} \end{cases} \quad (14)$$

We use the shorthand ϕ_{sd}^* for the productivity cutoff but note that $\phi_{sd}^*(c_d)$ depends on a firm's access cost realization by (10). Integrating over the density of the market access cost distribution, we obtain M_{sd} , the measure of firms that sell to destination d from source country s

$$M_{sd} = \kappa \frac{J_s b_s^\theta}{[f_{sd}(1) / D_{sd}]^{\tilde{\theta}}} \quad (15)$$

by (10). The parameter

$$\kappa \equiv \int_{c_d} c_d^{-\tilde{\theta}} dF(c_d)$$

reflects the expected access deterring effect of the firm-destination specific market access cost component c_d on the mass of active exporters at a destination.

We denote aggregate bilateral sales from country s to d with T_{sd} . The corresponding average expected sales per firm are defined as \bar{T}_{sd} , so that $T_{sd} = M_{sd} \bar{T}_{sd}$ and

$$\bar{T}_{sd} \equiv \int_{c_d} \bar{T}_{sd}(c_d) dF(c_d), \quad (16)$$

where $\bar{T}_{sd}(c_d)$ is the mean expected sales per firm for a given market access cost draw c_d . Similarly, we define average market access costs as

$$\bar{F}_{sd} \equiv \int_{c_d} \bar{F}_{sd}(c_d) dF(c_d), \quad (17)$$

where $\bar{F}_{sd}(c_d)$ is the mean market access cost for a given draw c_d .¹⁶

For aggregation we also require the following two assumptions to hold to guarantee that average sales per firm are positive and finite.

Assumption 2 (Pareto probability mass in low tail). *The Pareto shape parameter is such that $\tilde{\theta} > 1$.*

Assumption 3 (Bounded market access costs and product efficiency). *Incremental market access costs and product efficiency satisfy $\sum_{G=1}^{\infty} f_{sd}(G)^{-(\tilde{\theta}-1)} h(G)^{-\theta} \in (0, +\infty)$.*

Lemma 1 *Suppose Assumptions 1, 2 and 3 hold. Then for all $s, d \in \{1, \dots, N\}$, average sales per firm are a constant multiple of average market access costs:*

$$\bar{T}_{sd} = \frac{\tilde{\theta} \sigma}{\tilde{\theta} - 1} \bar{F}_{sd}. \quad (18)$$

Proof. See Appendix A.1. ■

Despite our rich micro-foundations at the firm-product level and idiosyncratic shocks by destination, in the aggregate the share of market access costs in bilateral exports $\bar{F}_{sd}/\bar{T}_{sd}$ only depends on parameters θ and σ , while mean market access costs \bar{F}_{sd} vary by source and destination country. Bilateral average sales can be summarized with a function only of the parameters θ and σ and the properties of mean market access costs \bar{F}_{sd} .

Finally, we can use the measure of exporters M_{sd} from equation (15), expression (18) for average sales and the definition of the revenue shifter D_{sd} in (8) to derive the share of products from country s in country d 's expenditure:

$$\lambda_{sd} = \frac{M_{sd} \bar{T}_{sd}}{\sum_k M_{kd} \bar{T}_{kd}} = \frac{J_s(b_s)^\theta (w_s \tau_{sd})^{-\theta} f_{sd}(1)^{-\tilde{\theta}} \bar{F}_{sd}}{\sum_k J_k(b_k)^\theta (w_k \tau_{kd})^{-\theta} f_{kd}(1)^{-\tilde{\theta}} \bar{F}_{kd}}, \quad (19)$$

where $f_{sd}(1)^{-\tilde{\theta}} \bar{F}_{sd} = \sum_{G=1}^{\infty} f_{sd}(G)^{-(\tilde{\theta}-1)} h(G)^{-\theta}$ by Lemma 1 (see equation (A.3) in Appendix A.1). Our framework generates a bilateral gravity equation. As in Eaton and Kortum (2002) and Chaney (2008), the elasticity of trade with respect to variable trade costs is $-\theta$.¹⁷ The difference between our model, in terms of bilateral trade flows, and the framework of Eaton and Kortum (2002) is that market access costs influence bilateral trade similar to Chaney (2008) in the aggregate. At the firm-product level, however, our framework provides rigorously quantifiable foundations for the relevant market access costs. The

¹⁶ $\bar{T}_{sd}(c_d)$ and $\bar{F}_{sd}(c_d)$ follow from integrating over firm productivity conditional on exporting.

¹⁷In our model, the elasticity of trade with respect to trade costs is the negative Pareto shape parameter, whereas it is the negative Fréchet shape parameter in Eaton and Kortum (2002).

gravity relationship (19) clarifies how those micro-founded market access cost components relate to aggregate bilateral trade through the weighted sum $\sum_{G=1}^{\infty} f_{sd}(G)^{-(\tilde{\theta}-1)} h(G)^{-\theta}$. We thus offer a tool to evaluate the responsiveness of overall trade to changes in individual market access cost components.

The partial elasticity $\eta_{\lambda, f(g)}$ of trade with respect to a product g 's access cost component is $-(\tilde{\theta}-1)$ times the product's share $h(g)^{-\theta}$ in the weighted sum. To assess the relative importance of the extensive margin of product entry, relative to firm entry with the core product, we can compare elasticities using the ratio

$$\frac{\eta_{\lambda, f(g)}}{\eta_{\lambda, f(1)}} = \frac{f_{sd}(g)^{-(\tilde{\theta}-1)} h(g)^{-\theta}}{f_{sd}(1)^{-(\tilde{\theta}-1)}}. \quad (20)$$

This ratio simplifies to the function $g^{-\delta_{sd}(\tilde{\theta}-1)-\alpha\theta}$ in our parametrization. The power is strictly negative if and only if $\delta_{sd} + \alpha(\sigma-1) > \delta_{sd}/\tilde{\theta}$. It therefore depends on the sign and magnitude of δ_{sd} whether the elasticity of trade with respect to an additional product's incremental market access cost is higher or lower than the elasticity of firm entry.

We can also compute mean exporter scope in a destination. For the average number of products to be finite we will need the following necessary assumption.

Assumption 4 (Strongly increasing combined incremental scope costs). *Combined incremental scope costs satisfy $\sum_{G=1}^{\infty} z_{sd}(G, c_d)^{-\tilde{\theta}} \in (0, +\infty)$.*

This assumption is in general more restrictive than Assumption 1. It requires that combined incremental scope costs $Z(G)$ increase in G at a rate asymptotically faster than $1/\tilde{\theta}$ (a result that follows from the ratio rule, see Rudin 1976, ch. 3). Mean exporter scope in a destination is¹⁸

$$\bar{G}_{sd} = \kappa f_{sd}(1)^{\tilde{\theta}} \sum_{G=1}^{\infty} z_{sd}(G)^{-\tilde{\theta}}. \quad (21)$$

For our parameterized example, the expression implies that mean exporter scope is invariant to destination market size.¹⁹

¹⁸The expression is derived (omitting firm access cost $f_{sd}(1)$ and integration over c_d for brevity) using

$$\bar{G}_{sd} = \int_{\phi_{sd}^*} G_{sd}(\phi) \theta \frac{(\phi_{sd}^*)^{\theta}}{(\phi)^{\theta+1}} d\phi = (\phi_{sd}^*)^{\theta} \theta \left[\int_{\phi_{sd}^*}^{\phi_{sd}^{*,2}} \phi^{-(\theta+1)} d\phi + \int_{\phi_{sd}^{*,2}}^{\phi_{sd}^{*,3}} 2\phi^{-(\theta+1)} d\phi + \dots \right].$$

Completing the integration, rearranging terms and using equation (11), we obtain (21), where we use the shorthand $z_{sd}(G) \equiv z_{sd}(G, c_d)/c_d = f_{sd}(G) h(G)^{\sigma-1}$.

¹⁹To directly test that mean exporter scope is largely unresponsive to destination market size we present

We turn to the model's equilibrium. Total sales of a country s equal its total sales across all destinations (including domestic sales):

$$Y_s = \sum_{k=1}^N \lambda_{sk} T_k, \quad (22)$$

where T_k is consumer expenditure at destination k . Additionally, Lemma 1 implies that a country's total expense for market access costs is a constant (source country invariant) share of bilateral exports. This result implies that the share of wages and profits in total income is constant (source country invariant) and given by

$$w_s L_s = \frac{\tilde{\theta}\sigma - 1}{\tilde{\theta}\sigma} Y_s \quad \text{and} \quad \pi_s L_s = \frac{1}{\tilde{\theta}\sigma} Y_s. \quad (23)$$

See Appendix A.2 for a derivation.

This concludes the presentation of equilibrium conditions when trade is balanced ($Y_d = T_d$). We will relax the assumption of balanced trade in our calibration and defer the discussion of the full solution.

2.5 Closed-form structural equations

To conclude the presentation of our framework, we derive quantitative predictions. We relate these predictions to empirical regularities in Section 3 and to the structural equations for estimation in Section 4. To simplify notation, we define $\tilde{\alpha} \equiv \alpha(\sigma - 1)$ and $\tilde{\theta} \equiv \theta/(\sigma - 1)$. Assumptions 1 through 4 guarantee that the quantitative predictions are well defined. Table 1 reports the equivalent parameter restrictions of those necessary assumptions under our functional forms (7).²⁰

The optimal exporter scope for firms with $\phi \geq \phi_{sd}^*(c_d)$ is given by (9) and can be

this relationship in Figure B.2 (Appendix B.1). This implication as well as the robust scope and sales distributions are related to the Pareto assumption. Another implication of the Pareto assumption is that the relationship between the number of exporters shipping to a destination and the destination market size becomes linear in logs—a salient feature of both our Brazilian and French exporter data (Eaton, Kortum and Kramarz 2004).

²⁰Assumption 4 implies Assumption 1 but it depends on the sign of δ_{sd} whether Assumption 3 implies Assumption 1 (or Assumption 4). The necessary conditions for equilibrium existence can be summarized compactly with

$$\min \left\{ \delta_{sd}(\tilde{\theta} - 1), \delta_{sd}\tilde{\theta} \right\} + \alpha\theta > 1 \quad \text{and} \quad \tilde{\theta} > 1.$$

By parametrization (7), the combined market access cost function $f_{sd}(1)^{-\tilde{\theta}} \bar{F}_{sd}(\nu) \equiv f_{sd}(1)^{\tilde{\theta}-1} \sum_{G=1}^{\infty} G^{-\nu}$ contains a Riemann zeta function $\zeta(\nu) \equiv \sum_{G=1}^{\infty} G^{-\nu}$ for a real parameter $\nu \equiv \tilde{\theta}[\delta_{sd} + \tilde{\alpha}] + \delta_{sd}$.

Table 1: **Parametric Functional Forms**

Assumption	Parameter values
1. Strictly increasing combined incremental scope costs	$\delta_{sd} + \tilde{\alpha} > 0$
2. Pareto probability mass in low tail	$\tilde{\theta} > 1$
3. Bounded market access costs	$\delta_{sd} + \tilde{\alpha} > (\delta_{sd} + 1)/\tilde{\theta}$
4. Strongly increasing combined incremental scope costs	$\delta_{sd} + \tilde{\alpha} > 1/\tilde{\theta}$

Note: Functional forms $f_{sd}(g) = f_{sd} \cdot g^{\delta_{sd}}$ and $h(g) = g^\alpha$ by (7); definitions $\tilde{\alpha} \equiv \alpha(\sigma-1)$ and $\tilde{\theta} \equiv \theta/(\sigma-1)$.

written as

$$G_{sd}(\phi, c_d) = \text{integer} \left\{ \left[\phi / \phi_{sd}^*(c_d) \right]^{\frac{\sigma-1}{\delta_{sd} + \tilde{\alpha}}} \right\}. \quad (24)$$

Using this relationship and equation (12) we can express optimal sales of the g -th product in destination d for a firm (ϕ, c_d) as a function of the total number of products that the firm sells in d :²¹

$$y_{sdg}(\phi, c_d, \xi_{sdg}) = \sigma c_d f_{sd}(1) G_{sd}(\phi, c_d)^{\delta_{sd} + \tilde{\alpha}} g^{-\tilde{\alpha}} \left(\phi / \phi_{sd}^{*,G}(c_d) \right)^{\sigma-1} \xi_{sdg}. \quad (25)$$

Summing over a firm's products g , we arrive at the firm's total sales $t_{sd}(\phi, c_d, \xi) = \sum_g y_{sdg}(\phi, c_d, \xi_{sdg})$ and, dividing total sales by exporter scope, we obtain average sales per product, or average *exporter scale*. Given (25), exporter scale takes the form

$$a_{sd}(\phi, c_d, \xi) = \sigma c_d f_{sd}(1) G_{sd}(\phi, c_d)^{\delta_{sd} + \tilde{\alpha} - 1} \left(\phi / \phi_{sd}^{*,G}(c_d) \right)^{\sigma-1} H(G_{sd}(\phi, c_d), \xi)^{-(\sigma-1)}, \quad (26)$$

where $H(G_{sd}, \xi)^{-(\sigma-1)} \equiv \sum_{g=1}^{G_{sd}} g^{-\tilde{\alpha}} \xi_{sdg}$.

3 Data and Regularities

Our Brazilian exporter data originate from all merchandise exports declarations for 2000. From these customs records we construct a three-dimensional panel of exporters, their destination countries, and their export products at the Harmonized System (HS) 6-digit level. In this section, we document new regularities concerning multi-product firms and

²¹Under our parametrization $f_{sd}(G) = f_{sd}(1)G^{\delta_{sd}}$, average sales per firm become $\bar{T}_{sd} = [\tilde{\theta}\sigma/(\tilde{\theta}-1)]f_{sd}(1)\sum_{G=1}^{\infty} G^{-\delta_{sd}(\tilde{\theta}-1)}h(G)^{-\theta}$ and the access costs $f_{sd}(1)$ can be recovered recursively from

$$\frac{\bar{T}_{s\ell}}{\bar{T}_{sk}} = \frac{f_{s\ell}(1)}{f_{sk}(1)}$$

for any pair of destinations ℓ and k , after normalizing $f_{sd}(1)$ for one destination.

elicit novel aspects of known facts that are documented elsewhere (Eaton et al. 2004; Bernard et al. 2011; Arkolakis and Muendler 2013). These regularities form a body of facts that any theory of multi-product firms with heterogeneous productivity should match. We pay particular attention to differences between nearby and far-away destination markets.

3.1 Data sources and sample characteristics

Products in the original SECEX (*Secretaria de Comercio Exterior*) exports data for 2000 are reported using 8-digit codes (under the common Mercosur nomenclature), of which the first six digits coincide with 6-digit Harmonized System (HS) codes. We aggregate the data to the HS 6-digit product and firm level so that the resulting dataset is comparable to data for other countries.²²

We restrict our sample to manufacturing firms and their exports of manufactured products, removing intermediaries and their commercial resales of manufactures. The restriction to manufacturing firms and their manufactured products makes our findings closely comparable to BRS and Eaton et al. (2011). Manufacturing firms ship 86 percent of Brazil’s manufactured product exports. The resulting manufacturing firm sample has 10,215 exporters selling 3,717 manufactured products at the 6-digit HS level to 170 foreign destinations, and a total of 162,570 exporter-destination-product observations. Appendix B describes our data with additional detail.

3.2 Regularities

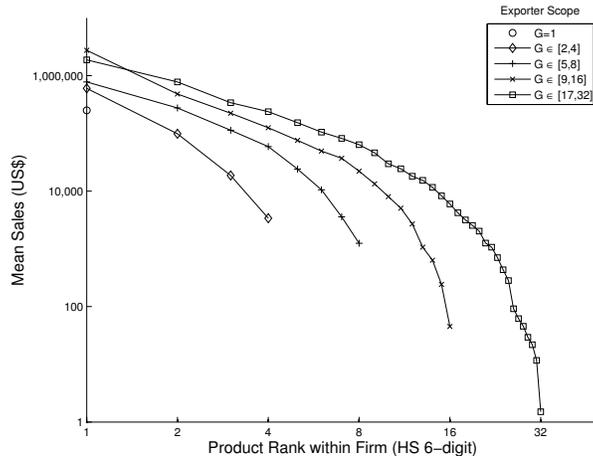
To characterize firms, we decompose a firm ω ’s total exports to destination d , $t_d(\omega)$, into the number of products $G_d(\omega)$ sold at d (the *exporter scope* in d) and the average sales per export product $a_d(\omega) \equiv t_d(\omega) / G_d(\omega)$ in d (the *exporter scale* in d). We elicit three major stylized facts from the data at three levels of aggregation, ranging from the individual product level within firms to the exporter scope and exporter scale distributions across firms.

Fact 1 *Within firms and destinations,*

1. *wide-scope exporters sell large amounts of their top-selling products, with exports concentrated in a few products, and*
2. *wide-scope exporters sell small amounts of their lowest-selling products.*

²²Our Online Supplement documents that our findings are similar at the common Mercosur nomenclature 8-digit level, which is closely related to the HS 8-digit level for other countries.

Figure 1: Firm-product Sales Distributions by Exporter Scope



Source: SECEX 2000, manufacturing firms and their manufactured products.

Note: Products at the HS 6-digit level, shipments to Argentina. We group firms by their exporter scope G in Argentina (Argentina is the most common export destination). The product rank \hat{g} refers to the sales rank of an exporter’s product in Argentina. Mean product sales is the average of individual firm-product sales $\sum_{\omega \in \{\omega: G_{\text{ARG}}(\omega)=G\}} y_{\omega \text{ ARG } g}^G / N_G$, computed for all firm-products with individual rank \hat{g} at the M_{ARG}^G firms exporting to Argentina with scope $G_{\text{ARG}} = G$.

Figure 1 documents this fact. For the figure, we limit our sample to exporters at a single destination and show only firms that ship at least one product to Argentina (the most common export destination) and group the exporters by their exporter scope G in Argentina. Results at other export destinations are similar.²³ For each scope group G and for each product rank g , we then take the average of the log of product sales $\log y_{\omega \text{ ARG } g}^G$ for those firm-products in Argentina. The graph plots the average log product sales against the log product rank by exporter scope group. The figure shows that a firm’s sales within a destination are concentrated in a few core products consistent with the core competency view of EN. In the model, the degree of concentration is regulated by how fast $f_d(g)$ and $h(g)$ change with g (the elasticities δ_d and $\tilde{\alpha} \equiv \alpha(\sigma - 1)$). Figure 1 also documents that wide-scope exporters sell more of their top-selling products than firms with few products. The model’s equation (25) matches this aspect under Assumption 1.

The product ranking of sales within firms need not be globally deterministic, as $f_d(g)$ and $h(g)$ would suggest, but the local product rankings can differ across destinations in reality, which we model with product-specific taste shocks similar to BRS. Comparing ranks across destinations, we can assess the relative importance of core competency versus product-specific taste shocks: for each given HS 6-digit product that a Brazilian firm sells

²³We present plots for the United States and Uruguay in Appendix B (Figure B.1). Argentina, the United States and Uruguay are the top three destinations in terms of presence of Brazilian manufacturing exporters in 2000.

in Argentina, we can correlate the firm-product’s rank elsewhere with the firm-product’s Argentinean rank. We find a correlation coefficient of .785 and a Spearman’s rank correlation coefficient of .860, indicating an important role for core competency.

To assess the first statement of Fact 1 for all export destinations, we regress the logarithm of the revenues of the best-selling products $y_{\omega d1}$ for firm ω to destination d on log exporter scope $G_{\omega d}$, discerning effects separately for Latin American and Caribbean (LAC) and non-LAC destinations and conditional on a firm fixed effect $\chi_{\omega}\mathbb{I}_{\omega}$:

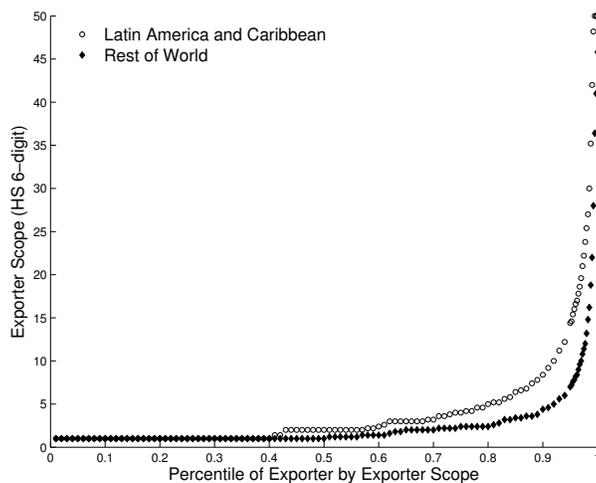
$$\log y_{\omega d1} = \underset{(.04)}{-0.16} \mathbb{I}_{d \in \text{LAC}} + \underset{(.04)}{1.30} \log G_{\omega d} - \underset{(.05)}{0.18} \mathbb{I}_{d \in \text{LAC}} \times \log G_{\omega d} + \chi_{\omega} \mathbb{I}_{\omega} + \epsilon_{\omega d}.$$

This regression is a version of the model’s equation (25) for a firm’s core product $g = 1$. The goodness of fit R^2 is .54 (standard error in parentheses clustered at firm level) for 170 destinations and 7,096 firms (46,208 observations). The coefficient estimate on $\log G_{\omega d}$ shows that sales of the best-selling product increase with an elasticity of 1.3 as exporter scope in a market widens. However, for LAC destinations, the elasticity is only 1.1 (1.30-0.18). In light of the model’s equation (25) for $g = 1$, this coefficient can be interpreted as an estimate of the sum $\delta_{\text{LAC}} + \tilde{\alpha}$. This variation by destination is closely related to our later estimation finding that there are destination-specific elasticities of incremental market access costs with respect to exporter scope. In subsection 3.3 below, we will assess the first statement of Fact 1 yet more rigorously and estimate the model’s equation (25) at the individual product g level (not just for the first product).

The second statement in Fact 1 that wide-scope exporters sell their lowest-ranked products for small amounts is also consistent with our model’s equation (25). The equation implies for a firm’s least-selling product $g = G_{\omega d}$ that its sales fall with a firm’s scope if and only if market access costs decline with additional products (δ_d is negative). The finding is at odds with models of multi-product firms where product access costs are fixed or absent, such as BRS or Mayer et al. (2014), and underlies our choice of product-specific market access costs. The second statement in Fact 1 closely relates to our later simulation result that falling access costs induce more trade mostly through the entry of new exporters with their first product, whereas falling barriers to product entry raise trade by less than similar relative declines in variable trade costs.

To assess the second statement in Fact 1 quantitatively, we regress the lowest-ranked product’s log sales $y_{\omega dG}$ on a firm’s log exporter scope $G_{\omega d}$ in a destination, conditioning on fixed effects for firm ω and destination d , and obtain an elasticity of -2.07 under an R^2 of .39 (standard error of .02 clustered at firm level) for the same number of observations as

Figure 2: **Exporter Scope Distribution**



Source: SECEX 2000, manufacturing firms and their manufactured products.

Note: Products at the HS 6-digit level. The percentile of exporters by scope is calculated within a given destination. Exporter scope is the corresponding scope for a given percentile averaged across the five most common destinations within each of the two regions (LAC or non-LAC).

above. The coefficient estimate on $\log G_{\omega d}$ shows that sales of the lowest-selling product fall with an elasticity of 2.1 as exporter scope at a destination widens. In light of the model's equation (25) for $g = G_{\omega d}$, this coefficient can be interpreted as an estimate of δ_{LAC} .

Fact 2 *At each destination, there are a few wide-scope and many narrow-scope exporters.*

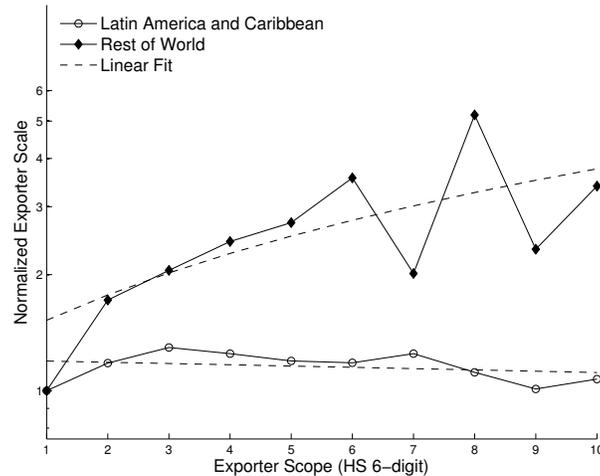
Figure 2 plots average exporter scope in the top five destinations of a region (LAC or non-LAC) against the percentile of an exporter in terms of scope at the destination. The median firm, conditional on exporting, only ships one or two products to any given destination. Within a destination, the exporter scope distribution exhibits a concentration in the upper tail reminiscent of a Pareto distribution.

The exporter scope distribution varies between destinations. Plotted in open dots is the average exporter scope at top LAC destinations, and with solid dots the exporter scope at non-LAC destinations. Brazilian exporters have a wider exporter scope at LAC destinations than at non-LAC destinations. To quantify the difference in exporter scope across destinations, we run a simple regression of log exporter scope $G_{\omega d}$ on an indicator for LAC destinations and condition on firm fixed effects $\chi_{\omega} \mathbb{I}_{\omega}$:

$$\log G_{\omega d} = \underset{(.02)}{.35} \mathbb{I}_{d \in LAC} + \chi_{\omega} \mathbb{I}_{\omega} + \epsilon_{\omega d}.$$

The R^2 is .55 (standard error in parentheses clustered at firm level) for 170 destinations

Figure 3: **Exporter Scope and Exporter Scale**



Source: SECEX 2000, manufacturing firms and their manufactured products.

Note: Products at the HS 6-digit level. Exporter scope is the number of products exported to a given destination. Exporter scale is a firm’s total sales at a destination divided by its exporter scope within the destination. We normalize exporter scale by the average total sales of single-product exporters at the destination, so that the normalized exporter scale for single-product exporters is one. We report mean exporter scope and mean exporter scale over the five most common destinations within a region (LAC or non-LAC). The dashed lines depict the ordinary least-squares fit.

and 7,096 firms (46,208 observations). In light of the model’s equation (24), a wider exporter scope in nearby LAC countries, conditional on the common firm effects across destinations, is consistent with a lower sum $\delta_{LAC} + \tilde{\alpha}$ than in the rest of the world, similar to evidence on the first statement in Fact 1.

Fact 3 *Average sales per product (exporter scale) and exporter scope exhibit varying destination-specific degrees of correlation, with the correlation positive and highest in distant destinations.*

On average across destinations, exporter scale is increasing in the number of exported products. When comparing across destinations, an exporter’s average product sales exhibit a stronger positive correlation with exporter scope in more distant destinations. For Brazilian exporters to LAC destinations, for example, the estimated elasticity of average product sales with respect to exporter scope is just .02 in a regression of log exporter scope on log exporter scale, conditional on industry fixed effects (and the elasticity is not statistically significantly different from zero).²⁴ However, among exporters to non-LAC destinations, the elasticity of exporter scale with respect to exporter scope is markedly

²⁴The absence of a strong correlation between exporter scale and exporter scope among Brazilian firms exporting to close-by LAC countries is reminiscent of the finding by BRS that scale and scope hardly correlate among U.S. exporters to Canada.

higher, reaching .15 (which is statistically significantly different from zero). Figure 3 illustrates these results. The logarithm of average exporter scale $a_{\omega d}$ at the top-five destinations in a region is plotted against average exporter scope $G_{\omega d}$ at the top-five destinations in the region. In light of our model’s equation (26), a consistent explanation is again that δ_d is negative and in absolute magnitude larger in nearby countries, similar to evidence from the previous two facts. Exporters to a nearby destination therefore experience a rapid decline in market access costs for additional products, permitting low-selling products into a nearby market.²⁵

3.3 Scale-scope-rank regression

We conclude our descriptive exploration of the data with an empirical assessment of Fact 1 (Figure 1) at the product level. For this purpose, we simplify the model and set both the market access cost and the local product appeal to unity across all firms and destinations: $c_d = \xi_{dg} = 1$. Using equation (25), we can express firm ω ’s log sales $y_{\omega dg}$ of the g -th product in destination d as a function of the firm’s log exporter scope $G_{\omega d}$ and the log local rank of the firm’s product g :

$$\ln y_{\omega dg} = (\delta_d + \tilde{\alpha}) \ln G_{\omega d} - \tilde{\alpha} \ln g - (1/\tilde{\theta}) \ln(1 - \Pr_{\omega d}^G) + \ln \sigma [f_d(1)/f(1)] \mathbb{I}_{d \in \text{LAC}} + \chi_{\omega} \mathbb{I}_{\omega} + \epsilon_{\omega dg}, \quad (27)$$

using the fact that our model implies $(\sigma - 1) \ln(\phi_{\omega}/\phi_d^{*,G}) = -(1/\tilde{\theta}) \ln(1 - \Pr_{\omega d}^G)$ for $c_d = 1$. To measure $1 - \Pr_{\omega d}^G$, we compute a Brazilian firm’s local sales percentile among the Brazilian exporters with minimum exporter scope G and include the log percentile as a regressor. We augment the estimation equation with a combined disturbance $\chi_{\omega} \mathbb{I}_{\omega} + \epsilon_{\omega dg}$, simply recognizing that the equation will only hold with some empirical error, and condition out a firm’s worldwide fixed effect χ_{ω} . The (exhaustive) set of firm effects absorbs the worldwide average log fixed cost $\ln \sigma f(1)$.

There are concerns using estimation equation (27). The equation is misspecified if local sales shocks ξ_{dg} permute the global rank order of a firm’s products and turn the order into different location-specific rankings. This misspecification makes the equation “memoryless” in that estimation does not register a firm-product’s identity across locations and therefore loses account of the firm-product’s ranking outside a given location d . Moreover, the estimation equation suffers an omitted variable bias because unobserved

²⁵We find the positive scale and scope association at more distant destinations also confirmed in regressions conditional on a firm fixed effect: Brazilian firms exporting to non-LAC destinations have an elasticity of exporter scale with respect to exporter scope nearly 50 percent higher than at LAC destinations.

Table 2: **Fit of Individual Product Sales**

	δ_{LAC}	δ_{ROW}	$\tilde{\alpha}$	$\tilde{\theta}$	$\delta_{\text{LAC}} - \delta_{\text{ROW}}$
Baseline: $g = 1; G = 2$	-1.82 (0.09)	-1.61 (0.11)	3.04 (0.08)	2.35 (0.31)	-0.21 (0.10)
Variant 1: $g = 2; G = 2$	-1.23 (0.10)	-1.13 (0.12)	3.04 (0.08)	2.10 (0.29)	-0.10 (0.14)
Variant 2: $g = 1; G = 16$	-1.41 (0.11)	-1.19 (0.12)	2.62 (0.10)	2.35 (0.31)	-0.22 (0.10)

Source: SECEX 2000, manufacturing firms and their manufactured products.

Note: Products at the HS 6-digit level. OLS-FE firm fixed effects estimation of equation (27) for firm ω 's individual product g sales at destination d in two parts, (i) under the baseline restriction $g = 1$,

$$\ln y_{\omega dg} = \underset{(0.04)}{1.22} \ln G_{\omega, d \in \text{LAC}} + \underset{(0.07)}{1.43} \ln G_{\omega, d \in \text{ROW}} - \underset{(0.06)}{0.43} \ln(1 - \text{Pr}_{\omega d}^G) - \underset{(0.05)}{0.32} \mathbb{I}_{d \in \text{LAC}} + \chi_{\omega} \mathbb{I}_{\omega} + \epsilon_{\omega dg},$$

and (ii) under the baseline restriction $G_{\omega d} = 2$

$$(\ln y_{\omega dg} - 0.43 \ln(1 - \text{Pr}_{\omega d}^G) - 0.32 \mathbb{I}_{d \in \text{LAC}}) = \underset{(0.08)}{-3.04} \ln g_{\omega d} + \chi_{\omega} \mathbb{I}_{\omega} + \epsilon_{\omega dg}.$$

Robust standard errors from the delta method, clustered at the industry level, in parentheses. Estimates of δ_{LAC} measure the scope elasticity of market access costs for Brazilian firms shipping to other LAC destinations, δ_{ROW} for Brazilian firms shipping to destinations outside LAC.

positive firm-destination product appeal shocks will both tend to raise exporter scope and to systematically permute the local rank order of firm products; this omitted variable bias would expectedly distort the estimates of δ_d . To mitigate the concerns, we estimate equation (27) in two parts by restricting the estimation sample: (i) we isolate the intercept of the graphs in Figure 1 by restricting the sample just the best selling (or second-best selling) product, $g = 1$ (or $g = 2$), and estimate how the intercept varies with exporter scope for two location groups $G_{\omega, d \in \text{LAC}}$ (LAC) and $G_{\omega, d \in \text{ROW}}$ (non-LAC destinations); (ii) we measure the slope of the graphs in Figure 1 by restricting the sample to $G_{\omega, d \in \text{LAC}} = G_{\omega, d \in \text{ROW}} = 2$ (or $G_{\omega, d} = 16$). To obtain mutually consistent results from this two-part estimation, we use the estimated coefficients on $\mathbb{I}_{d \in \text{LAC}}$ and $\ln(1 - \text{Pr}_{\omega d}^G)$ from the first part (i) as constraints on the second part (ii). Given the potential misspecification under any pair of restrictions, the regressions merely offer a descriptive exploration of the data.

Table 2 reports results from the two-part regression exercise under three combinations of restrictions. The baseline specification uses the restrictions $g = 1$ and $G_{\omega d} = 2$ for a pair of regressions under firm fixed effects (standard errors clustered at the level of 259 industries). The first variation uses the restrictions $g = 2$ and $G_{\omega d} = 2$ for a separate

pair of firm fixed effects regressions and the second variation combines the restrictions $g = 1$ and $G_{\omega d} = 16$ for a final pair of firm fixed effects regressions.²⁶ As expected from the different relationships between exporter scope and scale outside LAC and within LAC (Figure 3), δ_{LAC} exceeds δ_{ROW} in absolute magnitude. Overall δ_d falls in the range between -1.13 and -1.82 across specifications and regions, while $\tilde{\alpha}$ lies in the range from 2.62 to 3.04 and $\tilde{\theta}$ between 2.10 and 2.35 . In the baseline specification, the magnitudes of the δ_d estimates imply that incremental local entry costs drop at an elasticity of -1.61 when manufacturers introduce additional products in markets outside LAC, and with -1.82 within LAC. But firm-product efficiency drops off even faster with an elasticity of around 3.04 in the baseline. Adding the two fixed scope cost coefficients suggests that there are net overall diseconomies of scope with a scope elasticity of 1.22 in LAC and 1.43 in non-LAC destinations. The coefficient estimates suggest that Assumptions 1 and 2 in Table 1 are satisfied in our data.

Based on these initial descriptive explorations, the power in the partial elasticity ratio (20) is strictly negative across all specifications because we find $\delta_d + \tilde{\alpha} > 0 > \delta_d/\tilde{\theta}$. The partial elasticity of trade with respect to an additional product's fixed cost is therefore lower than the elasticity with respect to the core product. In other words, these initial descriptive estimates imply that product entry at multi-product exporters should matter less than firm entry with the core product. We now turn from descriptive explorations to an internally consistent estimator and will use the measured parameter magnitudes to assess the importance of each margin for overall trade.

4 Estimation

We adopt a method of simulated moments for parameter estimation.²⁷ We specify the product appeal shocks ξ_{dg} and the market access costs shocks c_d to be distributed log-normally with mean zero and variance σ_c and σ_ξ , respectively.

²⁶We also explored industry and destination fixed effects regressions in pairs and found results to be broadly similar.

²⁷The potential importance of market access cost and product appeal shocks can render conventional estimators problematic. A firm ω 's market access cost shock $c_{\omega d}$ is unobserved and can affect the firm's local choice of exporter scope beyond the firm's productivity ϕ_ω , especially if the market access cost shock c_d is widely dispersed. A firm-product's rank g_ω in production is unobserved and can differ from the firm-product's observed local rank in sales ($\hat{g}_{\omega d} \equiv 1 + \sum_{k=1}^G \mathbf{1}_{[y_{\omega dk}(\xi_{dk}) > y_{\omega dg}(\xi_{dg})]}$), especially if the product appeal shock ξ_{dg} is widely dispersed. The possible stochastic permutations of exporter scopes and product ranks introduce an exacting dimensionality that is hard to handle with a maximum likelihood estimator and the need for numerical computation of higher moments makes a general method of moments difficult to implement.

We need to identify five parameters $\Theta = \{\delta, \tilde{\alpha}, \tilde{\theta}, \sigma_\xi, \sigma_c\}$, where $\tilde{\alpha} \equiv \alpha(\sigma - 1)$ and $\tilde{\theta} \equiv \theta/(\sigma - 1)$. These five parameters fully characterize the relevant shapes of our functional forms and the dispersion of the three stochastic elements—Pareto distributed firm-level productivity, the random firm-level market access cost component, and local product appeal shocks. Our moments are standardized relative to the median firm or top firm-product at a destination. This produces a simulation estimator invariant to two deterministic shifters in the firms’ cost and revenue functions: a destination-specific market access cost shifter $\sigma f_d(1)$ and a destination-specific revenue shifter D_d , which are both common across exporters at a destination. Moreover, we specify the domestic access cost components ξ_{BRA_g} and c_{BRA} to be deterministic so that every exporter sells in the home market with certainty. In our ultimate implementation of the simulated moments estimator, we adopt an extension to destination-specific scope elasticities of market access costs with δ_d varying between LAC and non-LAC countries.

4.1 Moments

At any iteration of the simulation, we use the candidate parameters $\hat{\Theta}$ to compute a simulated vector of moments $\mathbf{m}^{\text{sim}}(\hat{\Theta})$, analogous to moments in the data \mathbf{m}^{data} . We use five sets of simulated moments. Each set is designed to characterize select parameters and to capture a salient fact from Section 3 or from the literature. However, we exclude moments related to Fact 3 from our set of targeted moments. Instead, we will use Fact 3 to assess the fit of our estimates to that regularity after estimation. We now summarize the simulated moments and discuss how they contribute to parameter identification. Additional details on the moment definitions as well as the simulation algorithm can be found in Appendix D.

1. *Sales of the top-selling product across firms within destinations.* Based on the first statement of Fact 1, we characterize the top-selling products’ sales across firms with the same exporter scope. Among firms exporting three or four products to Argentina, for example, we take the ratio of the top-selling product at the 95th percentile across firms and the top-selling product of the median firm. Our restriction to the top product and our standardization by the median firm with the same scope isolate the stochastic components by equation (25) and therefore help identify the dispersion of product appeal shocks (and partly the dispersion of the market access cost shock).
2. *Within-destination and within-firm product sales concentration.* We then use the second statement in Fact 1 and the ratios between the sales of given lower-ranked

products and the sales of the top product to characterize the concentration of sales within firms. The comparison of sales within firms neutralizes a firm’s global productivity ranking and eliminates the role of exporter scope as well as destination-specific determinants by equation (25). The within-firm within-destination sales ratios therefore help pin down the scope elasticity of product efficiency $\tilde{\alpha}$ and help identify the dispersion of product appeal shocks.

3. *Within-destination exporter scope distribution.* We then turn to Fact 2 and compute, within destinations, the shares of exporters with certain exporter scopes. For example, we calculate the proportion of exporters to Argentina, shipping three or four products. The frequencies of firms with a given exporter scope help identify the shape parameter $\tilde{\theta}$ of the Pareto firm size distribution (and partly the dispersion of the market access cost shock) and help pin down the scope elasticity $\delta + \tilde{\alpha}$, which translates productivity into exporter scope by equation (24).
4. *Market presence combinations.* Mirroring similar regularities documented in Eaton et al. (2011), we use the frequency of firms shipping to any permutation of Brazil’s top five export destinations in LAC and the top five destinations outside of LAC. For example, we target the number of exporters that ship to Argentina and Chile, but not to Bolivia, Paraguay and Uruguay. Matching these market presence patterns helps us identify the dispersion of market access cost shocks.
5. *Within-firm export proportions between destination pairs.* It is a widely documented fact that a firm’s sales are positively correlated across destinations. For each firm, we pair its total sales to a given destination with its sales to Brazil’s respective top destination in LAC or outside LAC. The ratio of a firm’s total sales to two destinations depends on the firm’s respective exporter scopes by equation (26) and therefore helps pin down the scope elasticity of sales $\delta + \tilde{\alpha}$. The pairwise sales ratios also help identify the dispersion of product appeal shocks and market access shocks.

4.2 Inference

Inference proceeds as follows. To find an estimate of Θ , we first stack the differences between observed and simulated moments $\Delta \mathbf{m}(\Theta) = \mathbf{m}^{\text{data}} - \mathbf{m}^{\text{sim}}(\hat{\Theta})$.

The true parameter Θ_0 satisfies $\mathbb{E}[\Delta \mathbf{m}(\Theta_0)] = 0$, so we search for the $\hat{\Theta}$ that minimizes the weighted sum of squares, $\Delta \mathbf{m}(\Theta)' \mathbf{W} \Delta \mathbf{m}(\Theta)$, where \mathbf{W} is a positive semi-definite weighting matrix. Ideally we would obtain $\mathbf{W} = \mathbf{V}^{-1}$ where \mathbf{V} is the variance-covariance

Table 3: **Estimation Results**

	δ_{LAC}	δ_{ROW}	$\tilde{\alpha}$	$\tilde{\theta}$	σ_{ξ}	σ_c	$\delta_{\text{LAC}} - \delta_{\text{ROW}}$
Baseline	-1.16 (0.04)	-0.86 (0.06)	1.76 (0.04)	1.72 (0.08)	1.82 (0.04)	0.58 (0.02)	-0.30 (0.06)
No product appeal shocks ($\sigma_{\xi} = 0$)	-1.40 (0.04)	-1.19 (0.06)	2.42 (0.04)	1.00 (0.004)		0.99 (0.01)	-0.22 (0.05)
No market access, cost shocks ($\sigma_c = 0$)	-1.20 (0.06)	-0.91 (0.09)	1.78 (0.04)	1.76 (0.12)	2.00 (0.03)		-0.28 (0.05)

Source: SECEX 2000, manufacturing firms and their manufactured products.

Note: Products at the HS 6-digit level. Standard errors from 30 bootstraps in parentheses. Estimates of δ_{LAC} measure the scope elasticity of market access costs for Brazilian firms shipping to other LAC destinations, δ_{ROW} for Brazilian firms shipping to destinations outside LAC.

matrix of the moments. However, the true matrix is unknown, so we use the empirical analogue:

$$\hat{\mathbf{V}} = \frac{1}{N^{\text{sample}}} \sum_{n=1}^{N^{\text{sample}}} (\mathbf{m}^{\text{data}} - \mathbf{m}_n^{\text{sample}}) (\mathbf{m}^{\text{data}} - \mathbf{m}_n^{\text{sample}})',$$

where $\mathbf{m}_n^{\text{sample}}$ are the moments from a random sample drawn with replacement of the original firms in the dataset and N^{sample} is the number of those draws.²⁸ To search for $\hat{\Theta}$ we use a derivative-free Nelder-Mead downhill simplex search. We compute standard errors using a bootstrap method that allows for sampling and simulation error.²⁹

4.3 Results

We simulate one million firms so that we obtain approximately thirty-thousand exporters. The number of simulated firms is roughly three times as large as the number of 331,528 actual Brazilian manufacturing firms and 10,215 exporters. We use an excess number of simulated firms to reduce the noise in our simulation draws.

To allow for some cross-destination variation, we estimate separate scope elasticities of market access costs for LAC destinations (δ_{LAC}) and the rest of the world (δ_{ROW}).³⁰ Table 3

²⁸Currently, we use $N^{\text{sample}} = 1,000$. Due to adding up constraints, we cannot invert this matrix $\hat{\mathbf{V}}$. Instead, we take a Moore–Penrose pseudo-inverse.

²⁹For the bootstrap we repeat the estimation process 30 times, replacing \mathbf{m}^{data} with $\mathbf{m}^{\text{bootstrap sample}}$ to generate standard errors. The bootstrapped standard errors are not centered.

³⁰We observe a concentration of exporter presence at specific pairs of destinations within regions. For example, exporters to Paraguay frequently also export to Argentina; exporters to the United Kingdom frequently also ship to the United States. However, there is no clear association between exporting to the United Kingdom and Paraguay. In reality, there is a complex set of factors that might connect market

presents our baseline estimates in the first row. The baseline estimates for δ_{LAC} and δ_{ROW} are both negative, significantly different from zero, and also significantly different from each other. The negative sign implies that exporting an additional product to a destination is less costly in terms of market access costs than any previous product. The difference in the estimated scope elasticities between LAC and non-LAC destinations means that incremental market access costs to LAC destinations fall almost 30 percent faster than incremental market access costs to the rest of the world. The scope elasticity of production efficiency $\tilde{\alpha}$ is positive and significantly different from zero. The estimate of $\tilde{\alpha} \approx 1.7$ implies that an additional product has more than proportionally higher unit production costs than any infra-marginal product. In both the LAC region and the rest of the world, the combined scope elasticity $\delta_d + \tilde{\alpha}$ is strictly positive and implies strictly increasing incremental scope costs (Assumption 1 is therefore empirically satisfied). Overall, the estimates from the simulated method of moments are similar in broad terms to those from our baseline descriptive data exploration in the preceding section (Table 2) but all coefficients are smaller in absolute magnitude in the current baseline specification.

Our baseline estimate for $\tilde{\theta}$ is statistically significantly above 1 and significantly less than 2 (Assumption 2 is therefore also empirically satisfied, consistent with a Pareto distribution of firm productivity). The baseline estimate of the variance of firm-product appeal shocks σ_ξ is approximately 2 and implies that, conditional on market access cost shocks and firm productivity, the ratio of the 75th firm sales percentile to the 25th firm sales percentile is over 10. This large disparity stands in contrast to the implications of our baseline estimates for σ_c , which imply that the ratio of the 75th firm sales percentile to the 25th firm sales percentile is only about 2, far less than 10.³¹

To explore the implications of our baseline estimates for the sources of variation in

access costs between destinations. For example, customs unions, common markets, shared destination languages, and unified distribution systems could link market access costs between countries. Our model does not explicitly take those potential connections into account. Instead, we implement a simplification and jointly simulate firms to identify separate moments for Latin American and Caribbean (LAC) export destinations as well as the rest of the world (ROW).

³¹Eaton et al. (2011), in a similar model but without multi-product firms and under slightly different sources of heterogeneity, find $\tilde{\theta} \approx 2.5$, which is larger than our estimate of 1.72. Their estimate of $\tilde{\theta}$ captures the elasticity of substitution between firms, whereas ours reflects the elasticity of substitution between firm-product varieties. Our estimate for σ_ξ is inline with Eaton et al. (2011) who find that their firm-specific appeal shock has a variance of 1.69. However, our appeal shock is firm-product specific (not just firm specific), so the estimates are not directly comparable.

firm-product sales more systematically, we apply a log decomposition to equation (25)³²

$$\log y_{\omega dg}^G = \underbrace{-\tilde{\alpha} \log g_{\omega}}_A + \underbrace{(\delta_d + \tilde{\alpha}) \log G_{\omega d}}_B + \underbrace{\log c_{\omega d} + (\sigma - 1) \log[\phi_{\omega} / \phi_d^{*G}(c_{\omega d})]}_C + \underbrace{\log \xi_{\omega dg}}_D.$$

Our estimates for LAC destinations imply that components A and B, which account for both a firm-product’s global production rank g_{ω} as well as the firm’s local exporter scope $G_{\omega d}$, explain 34 percent of sales variation. Component C, which reflects the combined market access costs shock and productivity for firms with identical exporter product scope, accounts for 17 percent of sales variation at LAC destinations. Individual firm-product appeal shocks in component D account for 50 percent of sales variations in LAC countries. The breakdown is slightly different non-LAC destinations. Components A and B explain only 21 percent of firm-product sales, component C accounts for 20 percent, and component D accounts for the remaining 60 percent. This difference between LAC and non-LAC destinations is entirely due to the difference between δ_{LAC} and δ_{ROW} , which for LAC destinations augments the importance of exporter scope and reduces the dependence on individual product appeal shocks.

An interpretation of components A and B is that they show firm-level competency (core competency in a particular firm-product and overall firm capability with regards to exporter scope), whereas component C reflects idiosyncratic firm heterogeneity, and component D the randomness of individual product appeal. Product appeal shocks play a dominant role in firm sales. However, the combination of firm-level competency and firm heterogeneity plays nearly as important a part. Our estimates highlight that a reduction in the scope elasticity of incremental market access costs from their magnitude in non-LAC countries to the magnitude in LAC countries raises the importance of firm-level competency considerably.

In two departures from our baseline specification, we re-estimate the model dropping one source of heterogeneity at a time: we first omit product appeal shocks and then drop market access cost shocks. We report the resulting estimates in the second and third row of Table 3. When we omit product appeal shocks (setting $\sigma_{\xi} = 0$), the estimated dispersion of market access costs expectedly increases. In addition, the estimated magnitudes of the scope elasticities δ_d and $\tilde{\alpha}$ markedly increase. The estimate of the shape parameter of the Pareto firm size distribution $\hat{\theta}$ hits a corner solution (and barely satisfies Assumption 2). Those salient changes in parameter estimates underscore the importance of specifying product appeal shocks. Interestingly, however, the regional difference in the

³²We standardize firm-product sales by $\sigma f_d(1)$ in estimation.

scope elasticity of market access costs $\hat{\delta}_{LAC} - \hat{\delta}_{ROW}$ remains similar to that under our baseline estimates. Dropping market access cost shocks has only minor effects on the remaining estimates.³³

Relating our results back to the findings from Table 2, which were based on simple log linear estimators dropping both sources of heterogeneity (product appeal shocks and market access cost shocks), we find qualitatively similar results. This broad similarity across estimators suggests that both simulated method of moments and its simpler counterparts identify comparable principal variation in the data but the quantitative differences indicate the importance of heterogeneity in the product appeal and market access costs.

To assess the sensitivity of our results to potential heterogeneity in product types and heterogeneity in destinations, we repeat estimation for numerous alternative specifications: we demean firm-product sales at the HS 6-digit level by average Brazilian exports at the HS 2-digit level, we restrict the sample to firms in high-tech manufacturing industries, we separate Mercosur member countries from other LAC destinations, and we drop both Argentina and the United States from the sample. We find our estimates broadly confirmed and report the details of the sensitivity exercises in an Online Supplement. To document the properties of our method of simulated moments, we also report results from Monte Carlo simulations of our estimator in the Online Supplement.

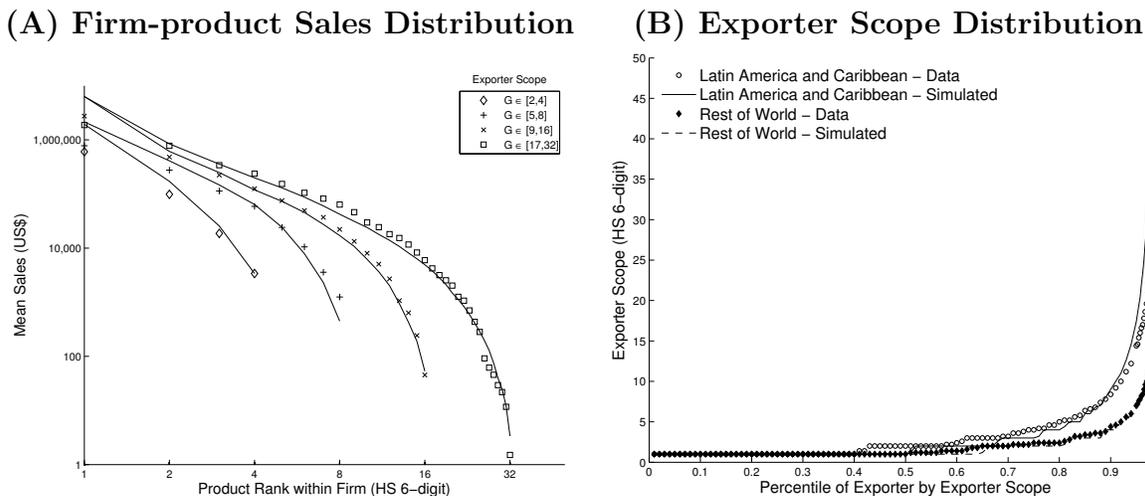
4.4 Model fit

To gauge the fit of our estimates, we plot simulated data using the baseline parameter estimates (from the first row of Table 3) alongside the actual data. We first assess how well we capture features of the data that our simulated moments target directly. Figure 4 shows our targeted moments and illustrate the close fit of our simulated data. The simulated data, depicted with lines in Figures 4A and 4B, match our Facts 1 and 2 closely, as shown with individual dots. Figure 4A presents the within-firm distribution of product sales in Argentina for firms with different exporter scopes. Figure 4B shows the exporter scope distributions, averaging over the five most common destinations in the LAC and non-LAC regions.

We now turn to regularities in the data that our simulated moments in the estimation routine do not target. We deliberately exclude from our estimation any moments that relate to Fact 3. However, as Figure 5A documents, our simulated firms line up closely

³³We cannot compare the goodness of fit in meaningful ways across specifications because the moments used under the restrictions differ from the baseline estimation. For $\sigma_{\xi} = 0$, we have to limit the set of moments 2 to the median because there is no variation by percentile in the simulation. For $\sigma_c = 0$, we have to exclude the set of moments 4 and 5.

Figure 4: **Fit of Targeted Moments**



Source: SECEX 2000, manufacturing firms and their manufactured products.

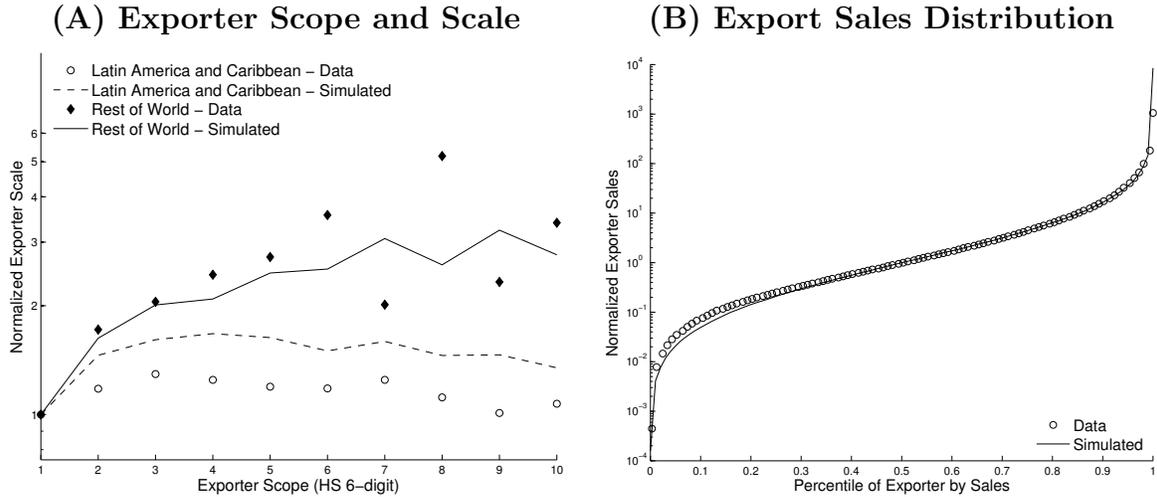
Note: Products at the HS 6-digit level. Data plots replicate those in Figures 1 and 2. Predicted curves based on simulations in Section D.1, using the baseline parameter estimates in Table 3. Panel A shows shipments to Argentina, grouping firms by their local exporter scope and firm-products by their local sales rank. Panel B shows the exporter scope by percentile, averaged across the five most common destinations within each of the two regions (LAC and non-LAC).

with the observed data. Our estimates detect clearly different scale-scope correlations in LAC destinations and non-LAC destinations. Figure 5A depicts the distribution of total sales by percentile within destinations. Our estimation routine includes simulated moments that relate to the distribution of sales across firm-products (within firms), to the distribution of exporter scope (within destinations), and to the proportion of total sales between pairs of destinations (within firms). None of those moments fully captures the distribution of total sales across firms (within destinations) because sales depend on all three sources of stochastic variation in the model: firm productivity, market access cost draws and product appeal shocks. Even though we do not directly target the total sales distribution with our simulated moments, Figure 5B documents that we find a close fit between our simulated firms and the data.

4.5 Policy implications

In our model, fixed costs of exporting G products to destination d take the form of equation (6), which depends on both the fixed cost of introducing the first product at an export market $f_d(1)$ and the elasticity of additional products' fixed costs with respect to exporter scope δ_g . We take our estimates for σ , θ , and α and minimize deviations between the model and data to find the set of δ_d that best match our empirical moments for each

Figure 5: Fit of Non-Targeted Moments



Source: SECEX 2000, manufacturing firms and their manufactured products.

Note: Products at the HS 6-digit level. Data plot in Panel A replicates that in Figure 3. Predicted curves based on simulations in Section D.1, using the baseline parameter estimates in Table 3. Panel A shows exporter scale (a firm’s total sales at a destination divided by its exporter scope at the destination) against exporter scope, averaging each variable over the five most common destinations within a region (LAC and non-LAC) and normalizing scale by the average total sales of single-product exporters at the destination. Panel B shows total firm exports by percentile, averaging a firm’s total exports over the five most common destinations within each of the two regions (LAC and non-LAC) and normalizing total sales by the median firm’s total at the destination.

destination country reached by 60 or more Brazilian exporters. This procedure yields 74 δ_d estimates.

To evaluate the policy relevance of the $f_d(1)$ and δ_d estimates, we study the extent to which they are correlated with policy variables or exogenous geographic and economic factors. We conduct an analysis of variance (ANOVA) for both $f_d(1)$ and δ_d with respect to five potential explanatory variables: the logarithms of distance from Brazil, destination population, destination gross domestic product, and destination area, as well as an indicator for countries in a regional trade agreement with Brazil.³⁴ The first four predictors are policy invariant variables, while the fifth predictor is policy dependent.

We report the ANOVA results in Table 4. The predictors explain the bulk of the variance in the fixed cost of exporting the first good $f_d(1)$, with an R^2 of .81. The test statistics on individual regressors fail to reject relevance only in the case of geographic distance. However, when it comes to the elasticity of market access costs with respect to exporter scope δ_d , the candidate gravity predictors fail to explain the variance. The R^2 is only .09. No single gravity variable has statistically significant explanatory power

³⁴All predictors are from the CEPII gravity database for 2000.

Table 4: ANOVA for $f_d(1)$ and δ_d

Factor	Initial fixed export cost: $f_d(1)$		Elasticity to exporter scope: δ_d	
	F-Statistic	p -value	F-statistic	p -value
$\log(DIST_d)$	0.55	.46	1.37	.25
$\log(POP_d)$	5.89	.01	0.06	.80
$\log(GDP_p)$	155.05	<.00	2.31	.13
$\log(AREA_d)$	4.93	.03	0.67	.42
RTA_d	3.43	.07	2.58	.11
Overall Model	123.93	<.00	1.42	.23
Observations	151		74	
R^2	.81		.09	

Source: SECEX 2000, manufacturing firms and their manufactured products. CEPII 2000 gravity database

Notes: Products at the HS 6-digit level. Analysis weighted by the number of exporting companies to a particular destination. Method of obtaining $\log(f_d(1))$ from footnote 21, δ_d described in the text.

at conventional significance levels. Out of all the candidate predictors, the single policy variable—the indicator for a regional trade agreement with Brazil RTA_d —comes closest to conventional statistical significance.

Taken together these findings provide evidence consistent with the hypothesis that exogenous geography related, and therefore policy invariant, factors play no statistically relevant role for the determination of the elasticity of incremental market access costs δ_d , in contrast with the usual specification of fixed market access costs $f_d(1)$. We therefore maintain the tenet that market related economic determinants, amenable to policy, plausibly shape δ_d and proceed to study the impact of reducing related market access barriers.

5 General Equilibrium Counterfactual

We conduct a counterfactual simulation to quantify the implied impact of our baseline estimates (first row of Table 3) for changes to bilateral trade when destination-specific market access costs are brought down. Brazil is close to the median country in exports per capita, so we consider our baseline parameter estimates informative for global trade.³⁵

³⁵By the WTF and WDI data for all industries and countries, Brazil ranks at the 48th percentile (top 100th country out of 192) in terms of exports per capita in 2000. Brazil's total exports in 2000 are at the 88th percentile worldwide (top 27th country out of 205).

Table 5: Percentage Change in Simulated Welfare ($\theta = 2.59$)

Country	Counterfactual				Country	Counterfactual			
	(1)	(2)	(3)	(4)		(1)	(2)	(3)	(4)
	$\Delta f(g)$	$\Delta f(g)_C$	$\Delta \delta$	$\Delta \delta_C$		$\Delta f(g)$	$\Delta f(g)_C$	$\Delta \delta$	$\Delta \delta_C$
Armenia	0.75	0.51	0.84	0.57	Kuwait	2.68	1.52	2.99	1.69
Australia	1.29	0.76	1.44	0.85	Kyrgyzstan	1.46	0.88	1.63	0.98
Austria	2.72	0.71	3.03	0.80	Latvia	2.14	0.43	2.38	0.49
Azerbaijan	1.31	0.69	1.47	0.77	Lithuania	2.12	0.46	2.37	0.52
BelgmLuxNthl	3.82	1.37	4.26	1.53	Malta	5.30	2.27	5.92	2.54
Bolivia	1.09	0.58	1.22	0.64	Mexico	2.62	0.54	2.91	0.60
Brazil	0.68	0.64	0.76	0.71	Morocco	1.61	1.50	1.80	1.67
Bulgaria	1.95	0.68	2.17	0.76	Norway	1.88	0.59	2.10	0.66
Canada	2.83	0.49	3.16	0.55	Oman	2.74	2.80	3.06	3.12
Chile	1.52	1.39	1.70	1.56	Poland	1.91	0.57	2.13	0.64
ChinaHKG	1.78	1.06	1.99	1.19	Portugal	2.13	0.50	2.38	0.56
Colombia	0.88	0.80	0.98	0.89	Romania	2.15	0.64	2.40	0.71
CostaRica	0.66	0.43	0.74	0.48	RussianFed	2.20	1.15	2.45	1.28
Cyprus	2.95	1.38	3.29	1.54	Senegal	1.52	1.16	1.70	1.29
Ethiopia	0.81	0.60	0.90	0.67	Slovenia	3.50	0.93	3.90	1.04
Finland	2.21	0.76	2.46	0.85	SouthKorea	1.89	1.02	2.11	1.14
FranceMonaco	1.77	0.61	1.97	0.68	Spain	1.73	0.50	1.93	0.56
Germany	1.93	0.79	2.15	0.88	SriLanka	0.76	0.26	0.85	0.30
Hungary	4.68	1.32	5.22	1.47	Sweden	2.02	0.71	2.26	0.80
IndMalSgThai	5.10	2.98	5.68	3.32	TrinidadTbg	0.67	0.32	0.75	0.36
India	2.09	1.35	2.34	1.51	Tunisia	2.73	2.60	3.05	2.90
Iran	0.15	0.06	0.17	0.07	Turkey	1.23	1.02	1.37	1.14
Ireland	5.03	1.99	5.60	2.23	UK	1.62	0.76	1.81	0.85
Israel	1.64	1.35	1.83	1.51	USA	0.89	0.59	0.99	0.66
Italy	1.49	0.52	1.66	0.58	Ukraine	1.68	0.46	1.87	0.51
Japan	0.50	0.32	0.56	0.35	Uruguay	0.97	0.68	1.09	0.75
Jordan	1.80	1.17	2.01	1.30	VietNam	2.58	0.79	2.88	0.88
Kazakhstan	2.04	1.46	2.27	1.63	Yemen	1.05	0.57	1.17	0.64
Kenya	1.31	1.02	1.46	1.13	RestOfWorld	4.73	4.94	5.28	5.51
					Mean	2.02	1.02	2.26	1.13

Note: Counterfactual experiments (1) and (3) reduce market access costs everywhere, experiments (2) and (4) reduce market access costs only at destinations outside a source country's own continent, with ROW treated as a different continent. Experiments use baseline parameter estimates of $\Theta = \{\delta, \bar{\alpha}, \bar{\theta}\} = \{-1.16, 1.76, 1.72\}$ (see Table 3). Pareto shape parameter $\theta = 2.59$ imputed from Crozet and Koenig (2010) and estimates in Table 3. See Appendix B.2 for data construction. Following Dekle, Eaton and Kortum (2007), we collapse (i) Hong Kong, Macao and mainland China, (ii) Belgium, Luxembourg and the Netherlands, (iii) Indonesia, Malaysia, Singapore, and Thailand and (iv) France and Monaco into single markets.

Our main counterfactual exercise harmonizes market access cost schedules across destinations. We reduce the market access cost for an additional product (not counting a firm’s first product) at distant destinations to the level at nearby destinations. In a broad sense, this exercise helps apprise the importance of multi-product exporters when it comes to the reduction of market access costs for additional products. Examples of relevant market access costs for additional products are health regulations and safety standards, certifications and licenses.

To perform the counterfactual experiments, we add three elements to the model following Eaton et al. (2011). (i) We introduce intermediate inputs as in Eaton and Kortum (2002). In particular, we assume that the production of each product uses a Cobb-Douglas aggregate of labor and a composite of all other manufactured products with cost P_d . The labor share in manufacturing production is β , and the share of intermediate inputs $1 - \beta$. The total input cost is therefore $w_d = W_d^\beta P_d^{1-\beta}$, where we now think of w_d as the input cost and W_d as the wage. (ii) There is a non-manufacturing sector in each country as in Alvarez and Lucas (2007) that combines manufactures with labor, in a Cobb-Douglas production function, where manufactures have a share γ in GDP. The price of final output in country d is proportional to $P_d^\gamma W_d^{1-\gamma}$. We state the resulting equations in Appendix A. (iii) We allow for a manufacturing trade deficit B_d , and for an overall trade deficit B_d^T in goods and services. Both deficits are set to their observed levels in 2000.

We compute the share of manufacturing in GDP for each country using data on GDP, manufacturing production and trade (as described in Appendix B.2). We set the labor share in manufacturing production to $\beta = .330$, the sample average for countries with available information (Appendix B.2). To compute the impact of a counterfactual change in market access costs, we use the Dekle et al. (2007) methodology (details in Appendix E). The merit of this method is that it requires no information on the initial level of technology, iceberg trade costs, and market access costs. Instead, we can compute the changes in all equilibrium variables using the percentage change in the underlying parameter of interest (market access cost parameters in our case).

For our primary results we consider a baseline value of $\theta = 2.59$ from Crozet and Koenig (2010), which we obtain by averaging across 34 industries.³⁶ For all other starting parameter values, we use our baseline estimates from row 1 in Table 3.

³⁶Crozet and Koenig (2010) obtain results for σ , we obtain θ using our estimated value for $\tilde{\theta}$. Eaton et al. (2011) find an estimate of $\theta = 4.87$. In a related set of models, Eaton and Kortum (2002), Bernard, Eaton, Jensen and Kortum (2003) and Simonovska and Waugh (2014) find estimates of θ between 3.60 and 8.28.

5.1 Changes in total market access costs

We initially decrease all market access costs for all products (f_a) by 15 percent to all international destinations. In this and all following experiments, we do not change any domestic trade costs and set the change in total domestic access costs to $\hat{F}_{ss} = 1$. Table 5 shows the results of the counterfactual exercise in terms of changes in welfare (see Appendix E for derivations). The results of the first experiment are labeled as counterfactual (1) in Table 5.

In a second experiment, we reduce market access costs only to countries not on the same continent by 15 percent. The results are shown as counterfactual (2) in Table 5. This experiment, while crude, highlights changes in market access costs to distant locations.³⁷ In both exercises, we see significant increases in welfare. Considering a simple average across all 58 countries in our sample, welfare increases by 2.0 percent in the first counterfactual experiment and by 1.0 percent in the second counterfactual experiment.

5.2 Changes in incremental market access costs

In our third and fourth counterfactual experiments we evaluate scenarios under which market access costs only for incremental products are brought down. This counterfactual stands in for eliminating various non-tariff barriers and directly utilizes our baseline results from Table 3. Those baseline results show that the incremental market access costs of shipping additional products to LAC destination drop nearly 30 percent faster with exporter scope than the incremental market access costs elsewhere. We conduct a counterfactual experiment with a 30-percent drop in the scope elasticity of market access costs. Since δ is negative, the experiment amounts to a 30-percent increase in the absolute value of δ . Note that we do not alter the cost of a firm's initial market entry with its first product. This 30-percent increase in the absolute value of δ is applied to all destinations in counterfactual (3) but only to destinations in other continents, which proxy for distant countries, in counterfactual (4).

In both counterfactual experiments we see results broadly in line with those from dropping overall market access costs. Dropping the incremental export costs to all foreign destinations increases average welfare by 2.3 percent and to destinations on different continents by 1.1 percent. While these increases in welfare may seem small, they operate only through multi-product firms and are unrelated to the entry costs of exporting the first

³⁷While lumping countries by continents is an admittedly imprecise way of classifying nearby and distant locations, preferential trade agreements and trade partnerships to date typically do link countries within continents (think of the European Union, NAFTA or Mercosur).

product. A 30-percent drop in the scope elasticity of incremental market access costs for multi-product firms has an effect that is broadly similar to reducing market access costs for all firms by 15 percent.

5.3 Changes in tariffs

Finally, to compare changes in market access costs to changes in conventional variable trade costs, we evaluate the welfare gains from the elimination of all tariff barriers. Under the assumption that remaining tariffs today represent around 4 percent of the value of exports, we experiment with a counterfactual decline of 4 percentage points in variable trade costs to mimic the elimination of tariffs.³⁸ Using our parameter estimates, we find an average welfare gain across markets of approximately 1.8 percent; this is broadly comparable to the gains from reductions in incremental market access costs.

Our estimate of $\theta = 2.59$ comes from French firm level data used by Crozet and Koenig (2010). Alternatively, we can use Simonovska and Waugh (2014), who aggregate trade flows from 123 countries. Their various estimates of θ range from 2.79 to 4.46, with their preferred specification producing $\theta = 4.41$. Using that latter estimate, our counterfactual experiment (4), in which we reduce only trade costs to distant destinations, results in an average welfare increase of 0.8 percent across destinations. Similarly, eliminating all tariffs increases welfare by 1.7 percent. In summary, our counterfactual experiments with plausible reductions of market access costs result in welfare gains of a similar magnitude as the elimination of remaining tariffs.

6 Conclusion

We develop a model that accounts for a number of pertinent facts on multi-product exporters, which we uncover using Brazilian exporting micro-data. The model allows us to estimate market access costs that regulate the entry of exporters and their products, and are important elements in trade theories with heterogeneous firms. Our estimates indicate that additional products farther from a firm's core competency incur higher unit costs but also that the elasticity of market access costs with respect to additional products declines at an almost one-third faster rate in nearby destinations. We conduct counterfactual exercises that accordingly reduce the scope elasticity of market access costs by one-third and

³⁸Novy (2013) finds that the average total variable trade costs for a set of OECD countries in terms of tariff equivalents is 94 percent in 2000; the same countries have ad-valorem tariff rates of approximately 4 percent (Anderson and Neary 2005).

find welfare gains similar in magnitude to a complete elimination of currently remaining tariffs. Results of these counterfactual exercises are reminiscent of surveys for numerous countries (OECD 2005, Ch. 1) and evidence on product trade (Reyes 2011), which suggest that non-tariff measures deter the market access of small and narrow-scope firms more heavily.

While we have incorporated many available dimensions of the trade data, more can be done. Our approach leaves unexplored recently available information on unit prices and time series trends, for example. Such additional information may prove valuable in understanding more precisely the patterns of product market access and exporter expansions. Similarly, we leave specific mechanisms that may shape market access cost determinants open for further investigation.

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Appendix

A Proofs

A.1 Proof of Lemma 1

We will show that, conditional on a market access cost draw c_d , average sales are proportional to average market access costs:

$$\bar{F}_{sd}(c_d) = \frac{\tilde{\theta}-1}{\tilde{\theta}\sigma} \bar{T}_{sd}(c_d). \quad (\text{A.1})$$

We can then integrate (A.1) over the market access cost distribution to establish Lemma 1 after aggregation across firms:

$$\bar{F}_{sd} = \int \bar{F}_{sd}(c_d) dF(c_d) = \int \frac{\tilde{\theta}-1}{\tilde{\theta}\sigma} \bar{T}_{sd}(c_d) dF(c_d) = \frac{\tilde{\theta}-1}{\tilde{\theta}\sigma} \bar{T}_{sd}.$$

We now prove (A.1). We drop the argument c_d for brevity. Expected total sales per firm in s shipping to d are

$$\begin{aligned} \bar{T}_{sd} &= \int_{\phi_{sd}^*} \mathbb{E}[t_{sd}(\phi, \boldsymbol{\xi})] \mu(\phi | \phi_{sd}^*, \theta) d\phi \\ &= \int_{\phi_{sd}^*} \sigma f_{sd}(1) \left(\frac{\phi}{\phi_{sd}^*} \right)^{\sigma-1} \sum_{g=1}^{G_{sd}(\phi)} h(g)^{-(\sigma-1)} \mathbb{E}[\xi_{sdg}] \cdot \theta \frac{(\phi_{sd}^*)^\theta}{(\phi)^{\theta+1}} d\phi \\ &= \int_{\phi_{sd}^*} \sigma f_{sd}(1) \left(\frac{\phi}{\phi_{sd}^*} \right)^{\sigma-1} \sum_{g=1}^{G_{sd}(\phi)} h(g)^{-(\sigma-1)} \cdot \theta \frac{(\phi_{sd}^*)^\theta}{(\phi)^{\theta+1}} d\phi \end{aligned}$$

by optimal total exports (13) and the independence of product appeal ξ_{sdg} and firm productivity ϕ . Consider the term $\int_{\phi_{sd}^*} \phi^{\sigma-1-(\theta+1)} \sum_{g=1}^{G_{sd}(\phi)} h(g)^{-(\sigma-1)} d\phi$. Rewrite the term as a piecewise integral

$$\begin{aligned} \int_{\phi_{sd}^*} \sum_{g=1}^{G_{sd}(\phi)} \frac{\phi^{\sigma-1-(\theta+1)}}{h(g)^{\sigma-1}} d\phi &= \int_{\phi_{sd}^*} \sum_{g=1}^{\phi_{sd}^{*,2}} \frac{\phi^{\sigma-1-(\theta+1)}}{h(g)^{\sigma-1}} d\phi + \int_{\phi_{sd}^{*,2}}^{\phi_{sd}^{*,3}} \sum_{g=1}^2 \frac{\phi^{\sigma-1-(\theta+1)}}{h(g)^{\sigma-1}} d\phi + \dots \\ &= \frac{1}{h(1)^{\sigma-1}} \int_{\phi_{sd}^*}^{\infty} \phi^{\sigma-1-(\theta+1)} d\phi + \frac{1}{h(2)^{\sigma-1}} \int_{\phi_{sd}^{*,2}}^{\infty} \phi^{\sigma-1-(\theta+1)} d\phi + \dots \\ &= \frac{1}{h(1)^{\sigma-1}} \frac{(\phi_{sd}^*)^{(\sigma-1)-\theta}}{\theta - (\sigma-1)} + \frac{1}{h(2)^{\sigma-1}} \frac{(\phi_{sd}^{*,2})^{(\sigma-1)-\theta}}{\theta - (\sigma-1)} + \dots \end{aligned}$$

for $\theta > \sigma - 1$. Using the definitions of ϕ_{sd}^* , $\phi_{sd}^{*,2}$, etc. from (11), we have

$$\int_{\phi_{sd}^*}^{G_{sd}(\phi)} \sum_{g=1} \frac{\phi^{\sigma-1-(\theta+1)}}{h(g)^{\sigma-1}} d\phi = \frac{1}{\theta - (\sigma-1)} \left(\frac{f_{sd}(1)}{(\phi_{sd}^*)^{\sigma-1}} \right)^{\tilde{\theta}-1} \sum_{G=1}^{\infty} \frac{f_{sd}(G)^{-(\tilde{\theta}-1)}}{h(G)^\theta} \quad (\text{A.2})$$

with $\tilde{\theta} \equiv \theta/(\sigma-1)$. Therefore

$$\bar{T}_{sd} = \frac{\tilde{\theta} \sigma}{\tilde{\theta} - 1} f_{sd}(1)^{\tilde{\theta}} \sum_{G=1}^{\infty} f_{sd}(G)^{-(\tilde{\theta}-1)} h(G)^{-\theta},$$

proving the first equality in (18). The expression is finite by Assumption 3.

Average market access costs paid by firms in s selling to d are

$$\begin{aligned} \bar{F}_{sd} &= \int_{\phi_{sd}^*}^{\phi_{sd}^{*,2}} F_{sd}(1) \theta \frac{(\phi_{sd}^*)^\theta}{\phi^{\theta+1}} d\phi + \int_{\phi_{sd}^{*,2}}^{\phi_{sd}^{*,3}} F_{sd}(2) \theta \frac{(\phi_{sd}^*)^\theta}{\phi^{\theta+1}} d\phi + \\ &= F_{sd}(1) (\phi_{sd}^*)^\theta \left[(\phi_{sd}^*)^{-\theta} - (\phi_{sd}^{*,2})^{-\theta} \right] + F_{sd}(2) (\phi_{sd}^*)^\theta \left[(\phi_{sd}^{*,2})^{-\theta} - (\phi_{sd}^{*,3})^{-\theta} \right] + \dots \end{aligned}$$

Using the definition $F_{sd}(G_{sd}) = \sum_{g=1}^{G_{sd}} f_{sd}(g)$ and collecting terms with a common $\phi_{sd}^{*,G}$ we can rewrite the above expression as

$$\bar{F}_{sd} = f_{sd}(1) + (\phi_{sd}^{*,2})^{-\theta} (\phi_{sd}^*)^\theta f_{sd}(2) + (\phi_{sd}^{*,3})^{-\theta} (\phi_{sd}^*)^\theta f_{sd}(3) + \dots$$

Using the definition of $\phi_{sd}^{*,G}$ from equation (11) in the above equation we get

$$\begin{aligned} \bar{F}_{sd} &= f_{sd}(1) + \left(\frac{f_{sd}(2)^{1/(\sigma-1)} h(2)}{f_{sd}(1)^{1/(\sigma-1)} h(1)} \right)^{-\theta} f_{sd}(2) + \dots \\ &= \left[f_{sd}(1) + f_{sd}(1)^{\tilde{\theta}} (f_{sd}(2)^{1/(\sigma-1)} h(2))^{-\theta} f_{sd}(2) + \dots \right] \\ &= f_{sd}(1)^{\tilde{\theta}} \left[f_{sd}(1)^{-(\tilde{\theta}-1)} + f_{sd}(2)^{-(\tilde{\theta}-1)} h(2)^{-\theta} + \dots \right] \\ &= f_{sd}(1)^{\tilde{\theta}} \sum_{G=1}^{\infty} f_{sd}(G)^{-(\tilde{\theta}-1)} h(G)^{-\theta} \\ &= \frac{\tilde{\theta} - 1}{\tilde{\theta} \sigma} \bar{T}_{sd}. \end{aligned} \quad (\text{A.3})$$

This proves the second equality in (18). The ratio $\bar{F}_{sd}/\bar{T}_{sd}$ is therefore a destination invariant constant.

A.2 Share of wages and profits

We show here that the share of wages and profits in total income is constant (source country invariant). Note that the share of net profits from bilateral sales is the share of gross variable profits in total sales $1/\sigma$, less the market access costs paid, divided by total sales

$(\tilde{\theta}-1)/\tilde{\theta}\sigma$. Thus, using the result of Lemma 1, $\pi_{sd}L_d/T_{sd} = 1/\sigma - (\tilde{\theta}-1)/(\tilde{\theta}\sigma) = 1/(\tilde{\theta}\sigma)$. Total profits for country s are $\pi_s L_s = \sum_k \lambda_{sk} T_k / (\tilde{\theta}\sigma)$, where $\sum_k \lambda_{sk} T_k$ is the country's total income by (22) and T_k is consumer expenditure at destination k . So profit income and wage income can be expressed as constant shares of total income as in the main text, equation (23).

B Data

B.1 Brazilian exporter-product-destination data

We identify an exporter's sector from the firm's reported *CNAE* four-digit industry (for 654 industries across all sectors of the economy) in the administrative RAIS records (*Relação Anual de Informações Sociais*) at the Brazilian labor ministry. The level of detail in *CNAE* is comparable to the *NAICS 2007* five-digit level. To map from the HS 6-digit codes to ISIC revision 2 at the two-digit level we use an extended SITC-to-ISIC concordance, augmenting an OECD concordance for select manufacturing industries to all industries.³⁹

As Table B.1 shows in columns 5 and 6, our Brazilian manufacturer sample includes 10,215 firms with shipments of 3,717 manufactured products at the 6-digit Harmonized System level to 170 destinations, and a total of 162,570 exporter-destination-product observations.⁴⁰ Exporters shipping multiple products dominate. They ship more than 90 percent of all exports from Brazil, and their global top-selling product accounts for 60 percent of Brazilian exports worldwide. We report the top exporting products of Brazilian firms in our Online Supplement.⁴¹

To calculate summary medians and means of these variables for regional aggregates and the world as a whole in Table B.1 (columns 3 to 6), we treat each aggregate as if it were a single destination and collapse all product shipments to different countries within the aggregate into a single product shipment. In most data treatments in the text, in contrast, we analyze these variables country by country, consistent with our main hypothesis that market-access determinants of trade matter repeatedly destination by destination.

The median exporter is a relatively small exporter, with sales to the rest of the world totaling around US\$ 89,000. The mean exporter, in contrast, sells around US\$ 3.7 million abroad, more than 40 times as much as the median exporter. Exporter scope and exporter scale exhibit similarly stark differences between mean and median. The median Brazilian manufacturer sells two products worldwide, but the mean scope per firm is 5.3 products. The median Brazilian exporter has a product scale of around US\$ 37,000 per product, but the exporter scale per exporter is US\$ 705,000, or around 20 times as high as that for the

³⁹Our SITC-to-ISIC concordance is available at URL econ.ucsd.edu/muendler/resource.

⁴⁰We remove export records with zero value from the Brazilian data, which include shipments of commercial samples but also potential reporting errors, and lose 408 of initially 162,978 exporter-destination-product observations. Our results on exporter scope do not materially change when including or excluding zero-shipment products from the product count.

⁴¹The top-5 selling products of Brazilian exporters at the 6-digit level are: 1. Airplanes heavier than 2 tons, 2. Chemical woodpulp, 3. Soybean oilcake, 4. Passenger vehicles with engines above 1,500 cc, 5. Transmissions.

Table B.1: **Sample Characteristics by Destination**

to destination d	From Brazil				
	Argentina (1)	USA (2)	non-OECD (3)	OECD (4)	World (5)
# of Firms (M)	4,590	3,083	8,664	5,041	10,215
# of Destinations (N)	1	1	147	23	170
# of HS-6 products (G)	2,814	2,144	3,537	2,772	3,717
# of Observations	21,623	10,775	126,211	36,359	162,570
Destination share in Tot. exp.	.144	.257	.441	.559	1
Firm shares in Total exports					
Single-prod. firms	.086	.123	.069	.142	.090
Multi-prod. firms' top prod.	.555	.662	.573	.625	.597
Multi-prod. firms' other prod.	.359	.215	.359	.233	.313
Median Total exp. ($T_d(m)$)	.068	.120	.066	.137	.089
Median Exp. scope ($G_d(m)$)	2	1	2	2	2
Median Exp. scale ($a_d(m)$)	.031	.068	.028	.070	.037
Mean Total exports (\bar{t}_d)	1.192	3.170	1.932	4.217	3.720
Mean Exp. scope (\bar{G}_d)	4.711	3.495	5.176	3.933	5.278
Mean Exp. scale (\bar{a}_d)	.253	.907	.373	1.072	.705

Source: SECEX 2000 manufacturing firms and their manufactured products at the HS 6-digit level, destinations linked to WTF (Feenstra, Lipsey, Deng, Ma and Mo 2005) and Unido Industrial Statistics (UNIDO 2005).

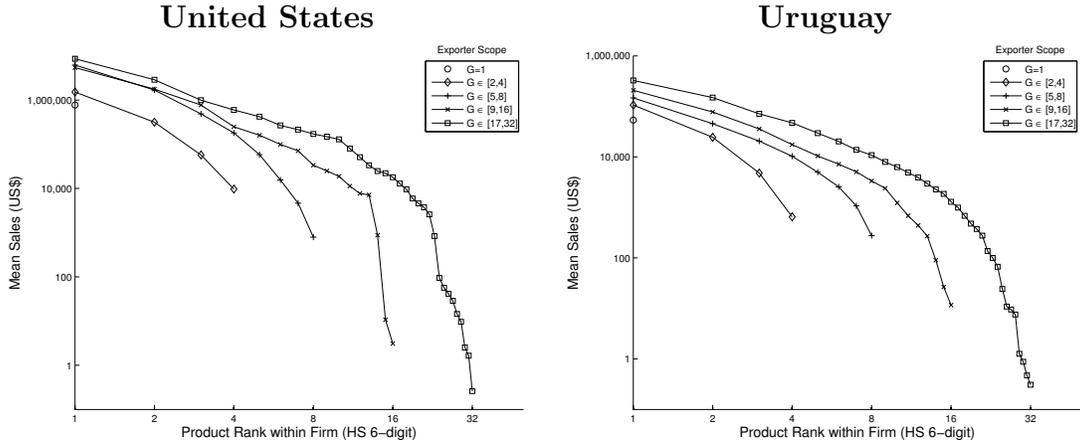
Note: Each aggregate region (world, OECD, non-OECD) treated as a single destination, collapsing product shipments to different countries into single product shipment. Products at the HS 6-digit level. Exports in US\$ million fob. Firms' exporter scale (ad in US\$ million fob) is the scope-weighted arithmetic mean of exporter scales. OECD includes all OECD members in 1990. Argentina is Brazil's top export destination in terms of presence of Brazilian manufacturing exporters in 2000, the United States second to top.

median firm.

The importance of the top-selling product at multi-product exporters and the mean-median ratios are similar across destinations. To investigate the robustness across countries, we select Brazil's top two export destinations in terms of presence of Brazilian manufacturing exporters (Argentina and United States), as well as the non-OECD and OECD aggregates. Our theory emphasizes the importance of exporting behavior within destinations. Within single countries, the mean manufacturer's exports exceed the median manufacturer's exports by similarly large factors as in the aggregate, between 14 (in Argentina, column 1) and 26 (in the United States, column 2). In the non-OECD aggregate (column 3), exports of the mean firm exceed the exports of the median firm by a factor of about 30. The same mean-median ratio of about 30 prevails in the OECD aggregate.

Figure B.1 documents Fact 1 for the United States and Uruguay, complementing Figure 1 for Argentina in the text. In each plot, we limit our sample to exporters and their

Figure B.1: U.S. and Uruguayan Within-firm Sales Distributions



Source: SECEX 2000, manufacturing firms and their manufactured products.

Note: Products at the HS 6-digit level, shipments to the United States and Uruguay. We group firms by their exporter scope G at a destination d (United States or Uruguay). The product rank \hat{g} refers to the sales rank of an exporter's product in that destination. Mean product sales is the average of individual firm-product sales $\sum_{\omega \in \{\omega: G_d(\omega)=G\}} y_{\omega dg}^G / N_G$, computed for all firm-products with individual rank \hat{g} at the M_d^G firms exporting to the destination with scope $G_d = G$.

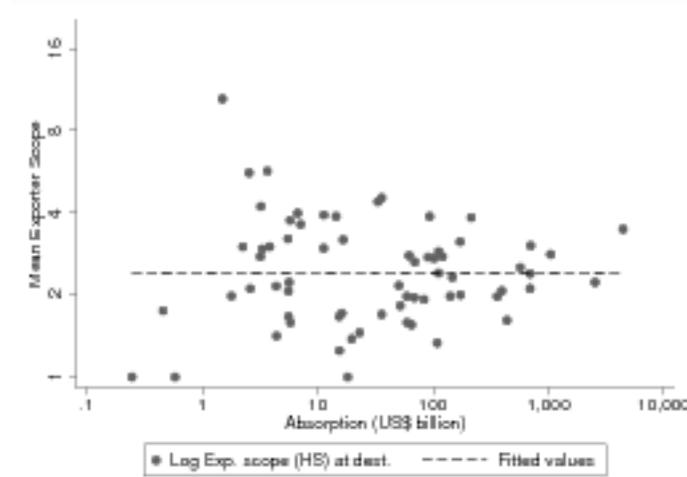
shipments to the respective destination (Argentina, the United States and Uruguay are the top three destinations in terms of presence of Brazilian manufacturing exporters in 2000) and group the exporters by their local exporter scope G . For each scope group G and for each product rank g , we then take the average of the log of product sales $\log y_{\omega dg}^G$ for those firm-products over all destinations. The graphs for the United States and Uruguay confirm Fact 1 that a few core products dominate local sales and that the least-selling products sell for smaller amounts the wider the firm's exporter scope.

We further investigate the striking similarity of firm scope choices across destinations by relating the mean number of products to destination market size. Figure B.2 shows a scatter plot of the log mean exporter scope \bar{G}_{sd} against the log of total absorption at the destination T_d . The depicted fitted line, from an ordinary least squares regression, has a slope that is not significantly different from zero at conventional levels. In other words, most of the variation in firms' exports to destinations of different size is due to variation in the firms' mean scale per product. At the firm level, the Brazilian data exhibit destination-presence patterns that resemble those in the French and U.S. firm-destination data. Similar to Eaton et al. (2011), for instance, the elasticity of the number of firms with respect to the number of export destinations is about -2.5, just as for French exporters.

B.2 Data for counterfactual analysis

For bilateral trade and trade balances in manufactured products, we use World Trade Flow (WTF) data in U.S. dollars for the year 2000 (Feenstra et al. 2005). To mitigate the effect of entrepot trade, we follow Dekle et al. (2007) and collapse (i) Hong Kong, Macao and mainland China, (ii) Belgium, Luxembourg and the Netherlands, and (iii) Indonesia,

Figure B.2: Mean Exporter Scope and Absorption by Destination



Source: SECEX 2000 manufacturing firms and their manufactured products at the HS 6-digit level, destinations linked to WTF and Unido Industrial Statistics

Malaysia, Singapore, and Thailand into single entities. In 2000, import information for India is missing from WTF. We obtain information for India in 2000 from UN Comtrade. We keep only manufactured products from the WTF data, using a concordance from the OECD at the SITC revision-2 4-digit level to determine manufactured products, and exclude agricultural and mining merchandise. By our construction, the world's trade balance is zero.

For information on GDP, manufacturing value added and the overall trade balances in goods and services in 2000 we use the World Bank's World Development Indicators 2009 (WDI). India included, our initial WTF sample has 132 countries that can be matched to the WDI data, and we collapse bilateral trade for the rest of the world by trade partner into a 133rd observation. We compute GDP and manufacturing value added for the rest of the world as the WDI reported world total less the sample total of our 132 matched countries. We set the overall trade balances in goods and services for the rest of the world so that the world total is zero.

We obtain β from the UNIDO ISIC level (UNIDO 2005 revision 2), which offers both manufacturing value added and manufacturing gross production for 51 of our sample countries and the rest of the world. Averaging the ratio of manufacturing value added to manufacturing output in 2000 over these countries yields $\beta = .330$. This worldwide β estimate enters our computation of γ_d by (E.8).

We need information on manufacturing absorption. Following Eaton et al. (2011), we infer manufacturing absorption as manufacturing output (from UNIDO 2005) plus the trade deficit (from WTF). The UNIDO data for manufacturing output are considerably less complete than either WTF or WDI. We obtain manufacturing output for Brazil from the Brazilian statistical agency IBGE (2010). Our final country sample for which we have manufacturing absorption contains 57 countries. By the model in Appendix E, γ_d

is given by (E.8). We use our WTF-WDI-UNIDO data to calculate γ_d for 57 countries. For the rest of the world, we set γ_d to the average of our sample ($\gamma = .244$) and back out manufacturing absorption from (E.8).

C Nested Utility

We can generalize the model to consumer preferences

$$\left(\sum_{s=1}^N \int_{\omega \in \Omega_{sd}} \left[\sum_{g=1}^{G_{sd}(\omega)} \xi_{sdg}(\omega)^{\frac{1}{\varepsilon}} x_{sdg}(\omega)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1} \frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \quad \text{where } \varepsilon > 1, \sigma > 1, \varepsilon \neq \sigma.$$

In this case we redefine the product efficiency index as:

$$H(G_{sd}) \equiv \left(\sum_{g=1}^{G_{sd}(\phi)} h(g)^{-(\varepsilon-1)} \right)^{-\frac{1}{\varepsilon-1}}. \quad (\text{C.4})$$

With this new definition, the expressions for firm product sales (12) and for aggregate bilateral trade (18) in Lemma 1 remain unaltered. For remaining details on the generalized model see our Online Supplement.

Under this generalization, a firm's individual products can be less substitutable among themselves than with outside products (if $\varepsilon < \sigma$) or more substitutable ($\varepsilon > \sigma$). In the latter case, a firm's additional products cannibalize sales of its infra-marginal products. The cannibalization effect is symmetric for all products, so relative sales of a firm's existing products are not affected by the introduction of additional products. This constancy of relative sales in our model does not carry over to models with CES-preferences and a countable number of firms such as Feenstra and Ma (2008) or to models with non-CES preferences such as Mayer et al. (2014) and Dhingra (2013).

D Simulation Algorithm and Moments

D.1 Simulation algorithm

Given a candidate estimate Θ , we simulate the export behavior for $J^{\text{sim}} = 1,000,000$ hypothetical Brazilian firms $\omega = 1, \dots, J^{\text{sim}}$ shipping to destinations $d = 1, \dots, N$ using our model (N is the observed number of destinations). In order to maintain the stochastic components unchanged as we search over Θ , prior to the simulation routine we draw (i) J^{sim} independent realizations of the firm's productivity percentile (ϕ_ω/ϕ^*) from the standard uniform distribution, (ii) $J^{\text{sim}} \times N$ independent realizations of the firm-specific market access costs $c_{\omega d}$ from the standard log normal distribution, and (iii) $J^{\text{sim}} \times N \times \bar{G}$ independent realizations of individual product appeal shocks $\xi_{\omega dg}$ from the standard log

normal distribution (where \bar{G} is the maximum observed exporter scope of any firm at any destination).

A given iteration of the model simulation requires a set of candidate parameters Θ and the number of Brazilian firms selling to each destination M_d . An iteration of the simulation proceeds in the following steps.

- (i). Scale the $J^{\text{sim}} \times N$ standard log normal market access cost draws by the current candidate dispersion parameter σ_c . Then, for each Brazilian firm ω and any destination d , compute the entry-relevant adjusted firm productivity parameter

$$\phi_{\omega d} \equiv c_{\omega d} \cdot (\phi_{\omega} / \phi^*)^{-1/\tilde{\theta}},$$

using the standard uniform firm productivity percentile (ϕ_{ω} / ϕ^*) .

- (ii). Back out the local entry threshold ϕ_d^* at destination d using the observed number M_d of Brazilian exporters at the destination and the known number of Brazilian firms M_{BRA} ,

$$\frac{M_d}{M_{\text{BRA}}} = \frac{1}{J^{\text{sim}}} \sum_{\omega=1}^{J^{\text{sim}}} \mathbb{I}\{\phi_{\omega d} > \phi_d^*\}.$$

The local entry cutoff ϕ_d^* depends on the mean of the $c_{\omega d}$ realizations. The cutoff is lower when the market access cost draws are lower on average.

To obtain M_{BRA} we merge the RAIS database of the formal-sector universe of Brazilian firms in 2000 with the SECEX export database. We find that 3.1 percent of Brazilian manufacturing firms export a manufactured product.⁴²

- (iii). Generate a firm-product-destination indicator $\mathbf{1}_{\omega dg}$ for each firm ω that exports its g -th product to destination d . For this purpose, compute the local product-level entry cutoffs

$$\phi_d^{*,G} \equiv G^{\delta + \tilde{\alpha}} \phi_d^*.$$

Given the cutoffs, the firm-product-destination indicators are $\mathbf{1}_{\omega dg} = \mathbb{I}\{\phi_{\omega d} > \phi_d^{*,g}\}$. Compute the exporter scope for each firm ω at a destination d ,

$$G_{\omega d} = \sum_{g=1}^{\bar{G}} \mathbf{1}_{\omega dg}.$$

- (iv). Scale the $J^{\text{sim}} \times N \times \bar{G}$ standard log normal product appeal draws by the current candidate dispersion parameter σ_{ξ} . Then generate the sales of a firm ω 's g -th ranked

⁴²The exporter share of 3.1 percent may seem low, but the Brazilian RAIS database includes all formal-sector firms and establishments with at least one employee. In contrast, censuses and surveys in most developing and some industrialized countries truncate their target population of firms from below with thresholds up to 20 employees. Truncation of the Brazilian manufacturing firm sample at a threshold of at least 10 employees would raise the exporter share to 10.7 percent. Truncation at a 20-employee threshold would raise the exporter share to 17.9 percent. The estimates in Table 3 are not sensitive to this convention. Using the alternative assumption that 10 percent of Brazilian firms export does not alter the reported results appreciably.

product at destination d , where the firm has an exporter scope $G_{\omega d}$:

$$y_{\omega dg}^G = \mathbf{1}_{\omega dg} \cdot G_{\omega d}^{\delta+\tilde{\alpha}} \cdot g_{\omega}^{-\tilde{\alpha}} \cdot \frac{\phi_{\omega d}}{\phi_d^{*,G}} \cdot \xi_{\omega dg} = \mathbf{1}_{\omega dg} \cdot g_{\omega}^{-\tilde{\alpha}} \cdot \frac{(\phi_{\omega}/\phi^*)^{-1/\tilde{\theta}}}{\phi_d^*/c_{\omega d}} \cdot \xi_{\omega dg}.$$

This expression for product revenue $y_{\omega dg}^G$ omits the destination-specific market access cost shifter $\sigma f_d(1)$ and the destination-specific revenue shifter D_d (which does not enter ϕ_d^* in the simulation). Both shifters are common across exporters at a destination and firm invariant in our simulation, because we normalize relevant moments by the according destination-specific median or extremum. See the following subsection for the definition of moments.

- (v). At each destination d and for every firm ω , rank order the firm's products by their local sales $y_{\omega dg}^G$ and compute the local rank for each firm-product g as

$$\hat{g}_{\omega d} \equiv 1 + \sum_{k=1}^{G_{\omega d}} \mathbb{I}\{y_{\omega dk}(\xi_{dk}) > y_{\omega dg}(\xi_{dg})\}.$$

In general, the local rank will differ from the firm-level rank in production $\hat{g}_{\omega d} \neq g_{\omega}$ due to the product appeal shock $\xi_{\omega dg}$.

D.2 Moments

We now define and discuss the moments used in the simulated method of moments algorithm. To isolate the parameters that are relevant for the shapes of our functional forms and the dispersion of the stochastic components, we adopt moments that are comparable across destinations by neutralizing destination-specific shifters with adequate factors of proportionality, based on the destination median or a destination extremum. To separately identify δ_{LAC} and δ_{ROW} , we use sets of moments for both LAC and non-LAC destinations.

D.2.1 Within-destination sales of top-selling products across firms

Our first set of moments compares the sales $y_{\omega d1}^G$ of the firms' top-selling products $\hat{g}_{\omega d} = 1$ across firms within a destination d . We compute these moments for groups of firms that share the same exporter scope $G_{\omega d} \in G$. Within each destination, we start with single-product firms (firms with an exporter scope $G_{\omega d} = 1$) and rank order the firms by their single product's sales from largest to smallest within the destination d . From the rank order of product sales we pick firms at select percentiles $P(\omega) = p$, overusing higher percentiles to match mostly upper-tail behavior. Then we repeat the computations for the group of firms with an exporter scope of two or three products sold ($G_{\omega d} \in \{2, 3\}$), and again rank only their top-selling products by sales across firms within destination, and so forth. Normalizing with the sales of the top-product at the median firm $P(\omega) = .5$

within an exporter-scope group $G_{\omega d} = G$, we obtain a first set of moments

$$M1_{pd}^G \equiv \log \left(\frac{y_{P(\omega)=p,d1}^G}{y_{P(\omega)=.5,d1}^G} \right), \quad p \in \{.95, .90, .85, .80, .70, .60, .25\},$$

$$G \in \{\{1\} \{2, 3\} \{4, 5, 6\} \{7, \dots\}\}.$$

This procedure would provide us with $7 \times 4 \times N$ moments for N destinations. For simplicity, we use the weighted geometric average across LAC and non-LAC destinations and obtain just $7 \times 4 \times 2$ moments $M1_{pd}^G$.

The sales dispersion across the firms' top-selling products is driven by the product appeal realization and partly by a firm's market access cost draw because product sales are larger on average in markets with higher access costs (see step (iv) of the algorithm).

D.2.2 Within-destination and within-firm product sales concentration

The second set of moments compares the sales $y_{\omega d1}^G$ of a firm's top-selling product and the sales $y_{\omega d\hat{g}}^G$ of the same firm's \hat{g} -th ranked product within a destination d . We compute these moments for groups of firms that share the same global scope $\max_d \{G_{\omega d}\} \in G$ across all destinations. For all firms that have a global scope of $\max_d \{G_{\omega d}\} \in G$, within each destination we compute the firm ω 's sales ratio $y_{\omega d\hat{g}}^{\max G} / y_{\omega d1}^{\max G}$ for the following three groups of lower-ranked products $\hat{g} \in \{\{2, 3\} \{4, 5, 6\} \{7, \dots\}\}$. For each group of lower-ranked products, we then pool over all destinations within a region and pool over all scope groups the sales ratios $y_{\omega d\hat{g}}^{\max G} / y_{\omega d1}^{\max G}$, rank order the sales ratios $y_{\omega d\hat{g}}^{\max G} / y_{\omega d1}^{\max G}$ from highest to lowest and pick firm observations at select percentiles $P(\omega) = p$. We obtain the second set of moments

$$M2_{pd}^{\hat{g}} \equiv \log \left(\frac{y_{P(\omega)=p,d\hat{g}}^G}{y_{P(\omega)=p,d1}^G} \right), \quad p \in \{.90, .75, .50, .25, .10\},$$

$$\hat{g} \in \{\{2, 3\} \{4, 5, 6\} \{7, \dots\}\}.$$

We compute the moments separately for LAC and non-LAC destinations, so this procedure generates $5 \times 3 \times 2$ moments.

The comparison of sales within firms of a given global scope implicitly conditions on the firm's global productivity percentile (ϕ_ω / ϕ^*), and the comparison within destinations removes destination specific variation including a firm's market access shock at a destination (see step (iv) of the algorithm). The within-firm and within-destination sales ratio $y_{\omega d\hat{g}}^{\max G} / y_{\omega d1}^{\max G}$ therefore varies with $\tilde{\alpha}$ and captures the product appeal shock dispersion.

D.2.3 Within-destination exporter scope distribution

The third set of moments characterizes the exporter scope distribution by destination. We count the exporters with an exporter scope of at least $G_{\omega d} \geq G$ at every destination and compute their share in the total number of exporters at the destination. We obtain a

Table D.2: **Firm Counts of Destination Strings**

Latin America and Caribbean (LAC)		Rest of World (non-LAC)	
Destination String	# Firms	Destination String	# Firms
ARG	1,647	USA	1,651
ARG-URY	507	USA-DEU	236
ARG-URY-CHL	296	USA-DEU-ITA	52
ARG-URY-CHL-PRY	225	USA-DEU-ITA-GBR	87
ARG-URY-CHL-PRY-BOL	600	USA-DEU-ITA-GBR-ESP	138
Other	4,799	Other	4,777
Total	8,074	Total	5,471

Source: SECEX 2000, manufacturing firms and their manufactured products.

Note: Strings denote Argentina (ARG), Uruguay (URY), Chile (CHL), Paraguay (PRY) and Bolivia (BOL); United States (USA), Germany (DEU), Italy (ITA), United Kingdom (GBR) and Spain (ESP). Those are the top five destinations within LAC and within non-LAC in terms of Brazilian manufacturing firm presence with manufactured product exports.

third set of moments

$$M3_d^G \equiv \frac{\sum_{\omega \in \Omega_d} \mathbb{I}\{G_{\omega d} \geq G\}}{\sum_{\omega \in \Omega_d} \mathbb{I}\{G_{\omega d} \geq 1\}}, \quad G \in \{2, 4, 6, 8, 12, 16\}.$$

This procedure would provide us with $6 \times N$ moments for N destinations. For simplicity, we use the weighted geometric average separately across LAC and non-LAC destinations and obtain just 6×2 moments $M3_d^G$.

The within-destination share of firms with a given exporter scope addresses the parameter $\tilde{\theta}$ of the firm productivity distribution and also the scope elasticity $\delta + \tilde{\alpha}$, which translates productivity into exporter scope (see steps (ii) and (iii) of the algorithm). The share of firms with a given exporter scope captures the dispersion of market access cost draws in addition because exporter scope is larger on average in markets with lower access costs.

D.2.4 Market presence combinations

For the fourth set of moments, we take the top five export destinations within LAC and within non-LAC in terms of the presence of Brazilian manufacturing exporters. We calculate the shares of exporters that sell to any of the permutations of those five destinations. The top five most common destinations within LAC are Argentina (ARG), Uruguay (URY), Chile (CHL), Paraguay (PRY) and Bolivia (BOL), within non-LAC they are the United States (USA), Germany (DEU), Italy (ITA), United Kingdom (GBR) and Spain (ESP). We summarize the possible permutations with strings of up to five destinations. For example, the single-destination string ARG means selling to Argentina but to no other among the top five destinations in LAC; the string ARG-URY means selling to Argentina

and Uruguay but not to Chile, Paraguay or Bolivia. See Table D.2 for frequencies of select permutations. This collection of destination combinations produces a total of $2 \times 2^5 = 64$ moments, denoted $M4_{\{d\}\text{-COMB}}$.

These moments reflect every firm ω 's exact destination combination and therefore help assess the dispersion of market access cost draws.

D.2.5 Within-firm export proportions between destination pairs

The fifth set of moments compares a firm ω 's total exports $t_{\omega d}$ to a destination d and its total exports to Argentina for LAC ($t_{\omega \text{ARG}}$) or the United States for non-LAC ($t_{\omega \text{USA}}$). We compute the total export ratios $t_{\omega d}/t_{\omega \text{ARG}}$ and $t_{\omega d}/t_{\omega \text{USA}}$ by destination d and firm ω for the four destinations Uruguay, Chile, Paraguay and Bolivia in LAC (which together with Argentina are the top five LAC destinations in terms of presence of Brazilian manufacturing exporters) and for the four destinations Germany, Italy, United Kingdom and Spain in non-LAC (which together with the United are the five most common non-LAC destinations). Within each region LAC and non-LAC we then rank order the firms by their export ratios $t_{\omega d}/t_{\omega \text{ARG}}$ and $t_{\omega d}/t_{\omega \text{USA}}$ from largest to smallest for each of the four close-to-top destinations. From the rank order of product sales we pick firms at select percentiles $P(\omega) = p$. Normalizing with the exports ratio at the median firm $P(\omega) = .5$, we obtain the fifth set of moments

$$M5_{pd\text{LAC}} \equiv \log \left(\frac{t_{P(\omega)=p,d}/t_{P(\omega)=p,\text{ARG}}}{t_{P(\omega)=.5,d}/t_{P(\omega)=.5,\text{ARG}}} \right), \quad p \in \{.95, .90, .85, .70, .60, .25\},$$

$$d_{\text{LAC}} \in \{\text{URY, CHL, PRY, BOL}\}.$$

and

$$M5_{pd\text{non-LAC}} \equiv \log \left(\frac{t_{P(\omega)=p,d}/t_{P(\omega)=p,\text{USA}}}{t_{P(\omega)=.5,d}/t_{P(\omega)=.5,\text{USA}}} \right), \quad p \in \{.95, .90, .85, .70, .60, .25\},$$

$$d_{\text{non-LAC}} \in \{\text{DEU, ITA, GBR, ESP}\}.$$

This procedure generates $6 \times 4 \times 2$ moments.

The exports ratio between destination pairs captures the dispersion of market access cost draws, which alter exporter scope and therefore total sales, and the ratio captures the dispersion of product appeal shocks, which change product sales directly. A firm's total sales ratio depends on the firm's respective exporter scopes with an elasticity of $\delta + \tilde{\alpha}$ by equation (26).

E Counterfactuals and Calibration

We follow a procedure similar to Alvarez and Lucas (2007), Dekle et al. (2007) and Eaton et al. (2011), and extend our framework to a setting with:

- Immobile labor between countries, but mobile labor between sectors, so there is a single wage W_s in country s ;

- An input bundle that consists of labor and intermediate goods, so such an input costs

$$w_s = W_s^\beta P_s^{1-\beta};$$

- A non manufacturing, non-traded final-product sector that only requires labor input and produces with a Cobb-Douglas combination of the non-manufacturing and manufacturing sectors, so final good prices are

$$P_s^f = P_i^\gamma W_i^{1-\gamma};$$

- Market access costs that require labor at the export destination and are homogeneous of degree $1 - \tilde{\theta}$ in foreign wages, so we can rewrite

$$f_{sd}(1)\bar{F}_{sd} = W_d^{1-\tilde{\theta}}\tilde{F}_{sd},$$

where \tilde{F}_{sd} denotes mean market access cost costs in terms of labor units;

- Unchanging trade deficits in manufacturing and non-manufacturing sectors;
- Technological parameters and labor endowments that are time invariant.

Using equation (19) for current trade shares λ_{sd} , we can express counterfactual trade shares as

$$\lambda'_{sd} = \frac{\lambda_{sd} \left(\hat{W}_s^\beta \hat{P}_s^{1-\beta} \right)^{-\theta} \hat{\tau}_{sd}^{-\theta} \widehat{F}_{sd}}{\sum_k \lambda_{kd} \left(\hat{W}_k^\beta \hat{P}_k^{1-\beta} \right)^{-\theta} \hat{\tau}_{kd}^{-\theta} \widehat{F}_{kd}}. \quad (\text{E.5})$$

The price index (3) can be derived as

$$\begin{aligned} P_d^{1-\sigma} &= \sum_k \int_{c_d} \left[\int_{\phi_{kd}^*(c_d)} \left[\sum_{g=1}^{G_{kd}(\phi)} \left(\tilde{\sigma} \frac{w_k}{\phi/h(g)} \tau_{kd} \right)^{1-\sigma} \right] \frac{\theta (\phi_{kd}^*(c_d))^\theta}{\phi^{\theta+1}} d\phi \right] dF(c_d) \\ &= \sum_k (\tilde{\sigma} w_k \tau_{kd})^{1-\sigma} J_k b_k^\theta \int_{c_d} \left[\int_{\phi_{kd}^*(c_d)} \left[\sum_{g=1}^{G_{kd}(\phi)} \frac{h(g)^{1-\sigma}}{\phi^{2-\sigma+\theta}} \right] d\phi \right] dF(c_d) \\ &= \sum_k (\tilde{\sigma} w_k \tau_{kd})^{1-\sigma} J_s b_s^\theta \left[\frac{1}{\theta - (\sigma - 1)} \left(\frac{f_{sd}(1)}{(\phi_{sd}^*)^{\sigma-1}} \right)^{\tilde{\theta}-1} \sum_{G=1}^{\infty} \frac{f_{sd}(G)^{-(\tilde{\theta}-1)}}{h(G)^\theta} \right] \int_{c_d} c_d^{-\tilde{\theta}} dF(c_d) \\ &= \sum_k (\tilde{\sigma} w_k \tau_{kd})^{1-\sigma} J_s b_s^\theta \left[\frac{1}{\theta - (\sigma - 1)} \left(\left(\frac{P_d}{\tilde{\sigma} \tau_{sd} w_s} \right)^{\sigma-1} \frac{T_d}{\sigma} \right)^{\tilde{\theta}-1} \bar{F}_{kd} f_{kd}(1)^{-\tilde{\theta}} \right] \kappa. \end{aligned}$$

The second step uses equation (15). The third step uses Lemma 1 to replace the integral term. The fourth step uses the log-normal distribution of c_d as well as equations (10)

and (15). Finally, collecting terms and solving for $P_d^{-\theta}$ yields

$$P_d^{-\theta} = \kappa (T_d)^{\tilde{\theta}-1} \frac{\sigma^{-(\tilde{\theta}-1)} \tilde{\sigma}^{-\theta}}{1 - 1/\tilde{\theta}} \sum_k J_k b_k^\theta \left(W_k^\beta P_k^{1-\beta} \right)^{-\theta} \tau_{kd}^{-\theta} W_d^{-(\tilde{\theta}-1)} \tilde{F}_{kd}, \quad (\text{E.6})$$

which can be restated in terms of relative changes as⁴³

$$\hat{P}_d = \left[\sum_k \lambda_{kd} \left(\hat{W}_k^\beta \hat{P}_k^{1-\beta} \right)^{-\theta} \hat{\tau}_{kd}^{-\theta} \hat{F}_{kd} \right]^{-1/\theta} \left(\frac{\hat{T}_d}{\hat{W}_d} \right)^{1/\theta - 1/(\sigma-1)}. \quad (\text{E.7})$$

As regards notation, \hat{x} denotes a gross relative change: $\hat{x} \equiv x'/x$, where x' is the new value. The above result is a system of equations that determines relative changes of prices as a function of relative changes in wages. To complete the procedure, we follow Eaton et al. (2011, Appendix E). Total manufacturing absorption is

$$T_d = \underbrace{\gamma_d \cdot (Y_d^T + B_d^T)}_{\text{final demand (labor income + profits)}} + \underbrace{(1-\beta) \cdot \frac{\sigma-1}{\sigma} Y_d}_{\text{demand for intermediates by manufacturing sector}}$$

where Y_d^T is total GDP of country d , including labor income and profits, B_d^T is the current account deficit and Y_d output of the manufacturing sector. We allow the share γ_d of manufacturing value added in GDP to be country specific. Manufacturing expenditure equals $T_d = Y_d + B_d$, where B_d is the trade deficit in the manufacturing sector. We can therefore solve for T_d and Y_d and obtain

$$\begin{aligned} T_d &= \frac{\gamma_d (Y_d^T + B_d^T) - (1-\beta)(1-1/\sigma) B_d}{1/\sigma + \beta(1-1/\sigma)}, \\ Y_d &= \frac{\gamma_d (Y_d^T + B_d^T) - B_d}{1/\sigma + \beta(1-1/\sigma)}. \end{aligned} \quad (\text{E.8})$$

⁴³We can use expression (E.6) together with equation (19) to obtain

$$P_d^{-\theta} = (T_d)^{\tilde{\theta}-1} \frac{(\sigma)^{-(\tilde{\theta}-1)} (\tilde{\sigma})^{-\theta} J_d b_d^\theta \left(W_d^\beta P_d^{1-\beta} \right)^{-\theta} W_d^{-(\tilde{\theta}-1)} \tilde{F}_{dd}}{1 - 1/\tilde{\theta} \lambda_{dd}}.$$

Thus changes in real wage are

$$\left(\frac{\widehat{W}_d}{\widehat{P}_d} \right) = \left(\widehat{\lambda}_{dd} \right)^{-1/\theta} \frac{\left(\widehat{T}_d / \widehat{W}_d \right)^{\frac{1-\tilde{\theta}}{\theta}}}{\left(\widehat{F}_{dd} \right)^{-1/\theta}}.$$

We consider $\widehat{F}_{dd} = 1$ in our counterfactual exercise, so this expression differs for domestic access costs from a similar one in Arkolakis et al. (2012) inasmuch as changes in the ratio $\widehat{T}_d / \widehat{W}_d$ reflect changes in the ratio of total absorption to wages (which is not one due to non-zero deficits).

We assume γ_d is time invariant, so we solve equation (E.8) for γ_d using 2000 baseline data.

To summarize, using the Dekle et al. (2007) algorithm, we can compute how given relative changes in market access costs \widehat{F}_{kd} lead to $\widehat{\lambda}_{sd}$, \widehat{P}_d , \widehat{W}_d . Denoting future variables with a prime, we find T'_d , Y'_d by inspecting equations (E.5), (E.6) and imposing the market clearing condition

$$Y'_s L_s = \sum_{k=1}^N \lambda'_{sk} T'_k. \quad (\text{E.9})$$

Online Supplement to

The Extensive Margin of Exporting Products: A Firm-level Analysis

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This Online Supplement is organized in five sections. Section S1 generalizes the Arkolakis, Ganapati and Muendler (2015, henceforth AGM) model to nested consumer preferences (as previewed in Appendix C to AGM). Each inner nest holds the products in a firm's product line with an elasticity of substitution that differs from that of the outer nest over product lines of different firms. In Section S2, we accommodate market penetration costs as in Arkolakis (2010) and show that our simulated methods of moments estimator is invariant to marketing costs at the level of product lines. Section S3 presents tabulations of the underlying Brazilian export data for 2000. In Section S4, we report Monte Carlo simulations that assess identification under our estimation routine. Finally, Section S5 offers a variety of robustness checks for our main estimates, using alternative assumptions and restricted samples.

S1 Nested Preferences with Different Elasticities

We analyze a generalized version of the AGM model of multi-product firms that allows for within-firm cannibalization effects. The main result is that the qualitative properties of the AGM model are retained: the size distribution of firm sales and the distribution of the firms' numbers of products is consistent with regularities in Brazilian exporter data as well as other data sets. More importantly, the general equilibrium properties of the model do not depend on the inner nests' elasticity (the elasticity across the products of a given firm's product composite) so that the general equilibrium of the model can be easily characterized using the tools of Dekle et al. (2007).

While the model is highly tractable, the introduction of one more demand elasticity adds a further degree of freedom. This degree of freedom can be disciplined using independent estimates for the outer and inner nests' elasticities, such as those of Broda and Weinstein (2006). Under an according parametrization, the model can be used for counterfactual exercises that simulate the impact of changes in trade costs on the firm size distribution and the distribution of the firms' numbers of products.

In the following subsection we present and solve the generalized model. We derive its aggregate properties in subsection S1.2. Subsection S1.3 concludes the presentation of the model with nested preferences.

S1.1 Model

There is a countable number of countries. We label the source country of an export shipment with s and the export destination with d .

We adopt a two-tier nested CES utility function for consumer preferences.⁴⁴ Each inner nest of consumer preferences aggregates a firm's products with a CES utility function and an elasticity of substitution ε . Using marketing terminology, the product composite of the inner nest can be called a firm's product line or product mix. The product lines of different firms are then aggregated using an outer CES utility nest with an elasticity σ . Each firm offers a countable number of products but there is a continuum of firms in the world. We assume that every product line is uniquely offered by a single firm, but a firm may ship different product lines to different destinations. Formally, the representative consumer's utility function at destination d is given by

$$U_d = \left(\sum_s \int_{\Omega_{sd}} \left(\sum_{g=1}^{G_{sd}(\omega)} q_{sdg}(\omega)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\sigma-1}{\sigma} \frac{\varepsilon}{\varepsilon-1}} d\omega \right)^{\frac{\sigma}{\sigma-1}}$$

where $q_{sdg}(\omega)$ is the quantity consumed of the g -th product of firm ω , producing in country s . Ω_{sd} is the set of firms from source country s selling to country d .

The representative consumer's first-order conditions imply that demand for the g -th product of firm ω in market d is

$$q_{sdg}(\omega) = p_{sdg}(\omega)^{-\varepsilon} P_{sd}(\omega; G_{sd})^{\varepsilon-\sigma} P_d^{\sigma-1} T_d,$$

where $p_{sdg}(\omega)$ is the price of that product,

$$P_{sd}(\omega; G_{sd}) \equiv \left[\sum_{g=1}^{G_{sd}(\omega)} p_{sdg}(\omega)^{-(\varepsilon-1)} \right]^{-1/(\varepsilon-1)}$$

is the ideal price index for the product line of firm ω selling $G_{sd}(\omega)$ products in market d , and

$$P_d \equiv \left[\sum_s \int_{\Omega_{sd}} P_{sd}(\omega; G_{sd})^{-(\sigma-1)} d\omega \right]^{-1/(\sigma-1)}$$

is the ideal consumer price index in market d . T_d is total consumption expenditure.

⁴⁴Atkeson and Burstein (2008) use a similar nested CES form in a heterogeneous-firms model of trade but their outer nest refers to different industries and the inner nests to different firms within the industry. Eaton and Kortum (2010) present a stochastic model with nested CES preferences to characterize the firm size distribution and their products under Cournot competition. In our model, firms do not strategically interact with other firms. This property of the model allows us to characterize general equilibrium beyond the behavior of individual firms.

S1.1.1 Firm optimization

We assume that the firm has a linear production function for each product. A firm with overall productivity ϕ faces an efficiency $\phi/h(g)$ in producing its g 'th product, where $h(g)$ is an increasing function with $h(1) = 1$. We call the firm's total number of products G_{sd} at destination d its *exporter scope* at d . Productivity is the only source of firm heterogeneity so that, under the model assumptions below, firms of the same type ϕ from country s face an identical optimization problem in every destination d . Since all firms with productivity ϕ will make identical decisions in equilibrium, it is convenient to name them by their common characteristic ϕ from now on.⁴⁵

The firm also incurs local entry costs to sell its g -th product in market d : $f_{sd}(g) > 0$ for $g > 1$, with $f_{sd}(0) = 0$. These incremental product-specific fixed costs may increase or decrease with exporter scope. The overall entry cost for market d is denoted by $F_{sd}(G) \equiv \sum_{g=1}^G f_{sd}(g)$ and strictly increases in exporter scope by definition.

Profits of a firm with productivity ϕ from country s that sells products $g = 1, \dots, G_{sd}$ in d at prices p_{sdg} are

$$\pi_{sd}(\phi) = \sum_{g=1}^{G_{sd}} \left(p_{sdg} - \frac{w_s}{\phi/h(g)} \tau_{sd} \right) P_{sdg}^{-\varepsilon} \cdot P_{sd}(\phi; G_{sd})^{\varepsilon-\sigma} P_d^{\sigma-1} T_d - F_{sd}(G_{sd}). \quad (\text{S.1})$$

We consider the first-order conditions with respect to the prices p_{sdg} of each product g , consistent with an optimal product-line price $P_{sd}(\phi; G_{sd})$, and also with respect to exporter scope G_{sd} . As shown in Appendix S-A to this Online Supplement, the first-order conditions with respect to prices imply a constant markup over marginal cost for all products equal to $\tilde{\sigma} \equiv \sigma/(\sigma-1)$.

Using the constant markup rule in demand for the g -th product of a firm with exporter scope G_{sd} yields optimal sales of the product

$$p_{sdg}(\phi) q_{sdg}(\phi) = \left(\tilde{\sigma} \frac{w_s \tau_{sd}}{\phi/h(g)} \right)^{-(\varepsilon-1)} P_{sd}(\phi; G_{sd})^{\varepsilon-\sigma} P_d^{\sigma-1} T_d. \quad (\text{S.2})$$

Using this result and the definition of $P_{sd}(\phi; G_{sd})$, we can rewrite profits that a firm generates at destination d selling G_{sd} products as

$$\begin{aligned} \pi_{sd}(\phi; G_{sd}) &= P_{sd}(\phi; G_{sd})^{-(\sigma-1)} \frac{P_d^{\sigma-1} T_d}{\sigma} - F_{sd}(G_{sd}) \\ &= H(G_{sd})^{-(\sigma-1)} (\tilde{\sigma} w_s \tau_{sd})^{-(\sigma-1)} \frac{\phi^{\sigma-1} P_d^{\sigma-1} T_d}{\sigma} - F_{sd}(G_{sd}), \end{aligned} \quad (\text{S.3})$$

⁴⁵To simplify the exposition, we assume here that firms face no other idiosyncratic cost components, whereas the AGM model also allows for a destination specific market access cost shock c_d so that a firm in that model is characterized by a pair of shocks (ϕ, c_d) . The derivations here can be readily generalized to such idiosyncratic market access costs.

where

$$H(G_{sd}) \equiv \left[\sum_{g=1}^{G_{sd}} h(g)^{-(\varepsilon-1)} \right]^{-1/(\varepsilon-1)}$$

is the firm's product efficiency index.

Similar to Assumption 1 in AGM, we impose the following assumption, which is necessary for optimal exporter scope to be well defined.

Assumption S.1 *Parameters are such that $Z_{sd}(G) = f_{sd}(G)/[H(G)^{-(\sigma-1)} - H(G-1)^{-(\sigma-1)}]$ strictly increases in G .*

The expression for $Z_{sd}(G)$ reduces to $Z_{sd}(G) = f_{sd}(G)h(G)^{\sigma-1}$ when $\varepsilon = \sigma$. In that case, Assumption S.1 is identical to the one considered in AGM.

For a firm to enter a destination market, its productivity has to exceed a threshold ϕ_{sd}^* , where ϕ_{sd}^* is implicitly defined by zero profits for the first product:

$$P_d^{\sigma-1} T_d [P_{sd}(\phi_{sd}^*; 1)]^{-(\sigma-1)} = \sigma f_{sd}(1).$$

Using the convention $h(1) = 1$ for $G = 1$ in (S.3) yields

$$(\phi_{sd}^*)^{\sigma-1} = \sigma f_{sd}(1) \frac{(\tilde{\sigma} w_s \tau_{sd})^{\sigma-1}}{P_d^{\sigma-1} T_d}. \quad (\text{S.4})$$

Similarly, we can define the threshold productivity of selling G products in market d . The firm is indifferent between introducing a G -th product or stopping with an exporter scope of $G - 1$ at the product-entry threshold $\phi_{sd}^{*,G}$ if

$$\pi_{sd}(\phi_{sd}^{*,G}; G) - \pi_{sd}(\phi_{sd}^{*,G}; G - 1) = 0. \quad (\text{S.5})$$

Using equations (S.3) and (S.4) in this profit equivalence condition, we can solve out for the implicitly defined product-entry threshold $\phi_{sd}^{*,G}$, at which the firm sells G_{sd} or more products,

$$\left(\phi_{sd}^{*,G}\right)^{\sigma-1} = \frac{(\phi_{sd}^*)^{\sigma-1}}{H(G_{sd})^{-(\sigma-1)} - H(G_{sd} - 1)^{-(\sigma-1)}} \frac{f_{sd}(G_{sd})}{f_{sd}(1)} = \frac{(\phi_{sd}^*)^{\sigma-1}}{f_{sd}(1)} Z_{sd}(G_{sd}), \quad (\text{S.6})$$

where we define $\phi_{sd}^{*,1} \equiv \phi_{sd}^*$. So, under Assumption S.1, the profit equivalence condition (S.5) implies that the product-entry thresholds $\phi_{sd}^{*,G}$ strictly increase with G and more productive firms will weakly raise exporter scope compared to less productive firms.

Export sales can be written succinctly as

$$\begin{aligned} t_{sd}(\phi) &= \left(\tilde{\sigma} \frac{w_s \tau_{sd}}{\phi} \right)^{1-\varepsilon} P_d^{\sigma-1} T_d \sum_{g=1}^{G_{sd}} h(g)^{1-\varepsilon} [P_d(G_{sd}(\phi))]^{\varepsilon-\sigma} \\ &= \sigma f_{sd}(1) \left(\frac{\phi}{\phi_{sd}^*} \right)^{\sigma-1} H(G_{sd}(\phi))^{-(\sigma-1)} \end{aligned} \quad (\text{S.7})$$

using equation (S.4). This sales relationship is similar in both models with $\varepsilon \neq \sigma$ and models with $\varepsilon = \sigma$. The only difference between the two types of models is that $H(G_{sd})$ depends on ε by (S.3). If the term $H(G_{sd})$ converges to a constant for $G_{sd} \rightarrow \infty$, then export sales are Pareto distributed in the upper tail if ϕ is Pareto distributed. Similar to Proposition 1 in the main text, we can state

Proposition S.1 *Suppose Assumption S.1 holds. Then for all s, d :*

- *exporter scope $G_{sd}(\phi)$ is positive and weakly increases in ϕ for $\phi \geq \phi_{sd}^*$;*
- *total firm exports $t_{sd}(\phi)$ are positive and strictly increase in ϕ for $\phi \geq \phi_{sd}^*$.*

Proof. The first statement follows directly from the discussion above. The second statement follows because $H(G_{sd}(\phi))^{-(\sigma-1)}$ strictly increases in $G_{sd}(\phi)$ and $G_{sd}(\phi)$ weakly increases in ϕ so that $t_{sd}(\phi)$ strictly increases in ϕ by (S.7). ■

Similar to AGM, we define *exporter scale* (an exporter's mean sales) in market d as

$$a_{sd}(\phi) = \sigma f_{sd}(1) \left(\frac{\phi}{\phi_{sd}^*} \right)^{\sigma-1} \frac{H(G_{sd}(\phi))^{1-\sigma}}{G_{sd}(\phi)}$$

Under a mild condition, exporter scale $a_{sd}(\phi)$ increases with ϕ and thus with a firm's total sales $t_{sd}(\phi)$. The following sufficient condition ensures that exporter scale increases with total sales.

Case C1 *The function $Z_{sd}(g)$ strictly increases in g with an elasticity*

$$\frac{\partial \ln Z_{sd}(g)}{\partial \ln g} > 1.$$

Case S.1 is more restrictive than Assumption S.1 in that the condition not only requires Z_{sd} to increase with g but that the increase be more than proportional. We can formally state the following result.

Proposition S.2 *If $Z_{sd}(g)$ satisfies Case C1, then sales per export product $a_{sd}(\phi)$ strictly increase at the discrete points $\phi = \phi_{sd}^*, \phi_{sd}^{*,2}, \phi_{sd}^{*,3}, \dots$*

Proof. Compared to AGM, $Z_{sd}(g)$ is defined in more general terms, but it enters the relevant relationships in the same way as in AGM before. Case C1 therefore also suffices in the nested-utility model, and the proposition holds (see the Appendix in AGM for details of the proof for non-nested utility). ■

S1.1.2 Within-firm sales distribution

We revisit optimal sales per product and their relationship to exporter scope and the product's rank in a firm's sales distribution. The relationship lends itself to estimation in

micro data. Using the productivity thresholds for firm entry (S.4) and product entry (S.6) in optimal sales (S.2) and simplifying yields

$$\begin{aligned}
p_{sdg}(\phi)x_{sdg}(\phi) &= \sigma Z_{sd}(G_{sd})H(G_{sd})^{\varepsilon-\sigma} \left(\frac{\phi}{\phi_{sd}^{*,G}} \right)^{\sigma-1} h(g)^{-(\varepsilon-1)} \\
&= \sigma \frac{f_{sd}(G_{sd})H(G_{sd})^{\varepsilon-1}}{1 - [1 - h(G_{sd})^{-(\varepsilon-1)}/H(G_{sd})^{-(\varepsilon-1)}]^{\frac{\sigma-1}{\varepsilon-1}}} \left(\frac{\phi}{\phi_{sd}^{*,G}} \right)^{\sigma-1} h(g)^{-(\varepsilon-1)}.
\end{aligned} \tag{S.8}$$

Note that $H(G)^{\varepsilon-\sigma}$ strictly falls in G if $\varepsilon > \sigma$. Under Case C1, the term $Z_{sd}(G_{sd})H(G_{sd})^{\varepsilon-\sigma}$ must strictly increase in G , however, because individual product sales strictly drop as the product index g increases and $h(g)^{-(\varepsilon-1)}$ falls. So, if $Z_{sd}(G_{sd})H(G_{sd})^{\varepsilon-\sigma}$ did not strictly increase in G , average sales per product would not strictly increase, contrary to Proposition S.2.

Compared to AGM, the relationship (S.8) is not log-linear if $\varepsilon \neq \sigma$ and requires a non-linear estimator, similar to the general case in continuous product space (Arkolakis and Muendler 2010).

S1.2 Aggregation

To derive clear predictions for equilibrium we specify a Pareto distribution of firm productivity following Helpman et al. (2004) and Chaney (2008). A firm's productivity ϕ is drawn from a Pareto distribution with a source-country dependent location parameter b_s and a shape parameter θ over the support $[b_s, +\infty)$ for all destinations s . The cumulative distribution function of ϕ is $\Pr = 1 - (b_s)^\theta/\phi^\theta$ and the probability density function is $\theta(b_s)^\theta/\phi^{\theta+1}$, where more advanced countries are thought to have a higher location parameter b_s . Therefore the measure of firms selling to country d , that is the measure of firms with productivity above the threshold ϕ_{sd}^* , is

$$M_{sd} = J_s \frac{b_s^\theta}{(\phi_{sd}^*)^\theta}. \tag{S.9}$$

As a result, the probability density function of the conditional productivity distribution for entrants is given by

$$\mu_{sd}(\phi) = \begin{cases} \theta(\phi_{sd}^*)^\theta/\phi^{\theta+1} & \text{if } \phi \geq \phi_{sd}^* \\ 0 & \text{otherwise.} \end{cases} \tag{S.10}$$

We define the resulting Pareto shape parameter of the total sales distribution as $\tilde{\theta} \equiv \theta/(\sigma-1)$.

With these distributional assumptions we can compute a number of aggregate statistics from the model. We denote aggregate bilateral sales of firms from s to country d as T_{sd} .

The corresponding average sales are defined as \bar{T}_{sd} , so that $T_{sd} = M_{sd}\bar{T}_{sd}$ and

$$\bar{T}_{sd} \equiv \int_{\phi_{sd}^*} t_{sd}(\phi) \mu_{sd}(\phi) d\phi. \quad (\text{S.11})$$

Similarly, we define average local entry costs as

$$\bar{F}_{sd} \equiv \int_{\phi_{sd}^*} F_{sd}(G_{sd}(\phi)) \mu_{sd}(\phi) d\phi.$$

To compute \bar{T}_{sd} , we impose two additional assumptions mirroring Assumptions 2 and 3 in AGM.

Assumption S.2 *Parameters are such that $\theta > \sigma - 1$.*

Assumption S.3 *Parameters are such that the mean market access cost*

$$\tilde{F}_{sd} \equiv \sum_{G=1}^{\infty} f_{sd}(G)^{1-\tilde{\theta}} [H(G)^{1-\sigma} - H(G-1)^{1-\sigma}]^{\tilde{\theta}}$$

is strictly positive and finite.

Then we can make the following statement.

Proposition S.3 *Suppose Assumptions S.1, S.2 and S.3 hold. Then average sales \bar{T}_{sd} per firm are a constant multiple of average local entry costs \bar{F}_{sd}*

$$\bar{T}_{sd} = \frac{\tilde{\theta}\sigma}{\tilde{\theta} - 1} \bar{F}_{sd} = f_{sd}(1)^{\tilde{\theta}} \tilde{F}_{sd}.$$

Proof. See Appendix S-C to this Online Supplement. ■

As a result, bilateral expenditure trade shares can be expressed as

$$\lambda_{sd} = \frac{M_{sd}\bar{T}_{sd}}{\sum_k M_{kd}\bar{T}_{kd}} = \frac{J_s(b_s)^\theta (w_s \tau_{sd})^{-\theta} f_{sd}(1)^{-\tilde{\theta}} \bar{F}_{sd}}{\sum_k J_k(b_k)^\theta (w_k \tau_{kd})^{-\theta} f_{kd}(1)^{-\tilde{\theta}} \bar{F}_{kd}}, \quad (\text{S.12})$$

an expression that depends on the values of ε and σ only insofar as these parameters affect \bar{F}_{sd} through $H(G)$.

We can also compute mean exporter scope at a destination:

$$\begin{aligned} \bar{G}_{sd} &= \int_{\phi_{sd}^*} G_{sd}(\phi) \mu_{sd}(\phi) d\phi \\ &= (\phi_{sd}^*)^\theta \theta \left[\int_{\phi_{sd}^*}^{\phi_{sd}^{*,2}} \phi^{-\theta-1} d\phi + \int_{\phi_{sd}^{*,2}}^{\phi_{sd}^{*,3}} 2\phi^{-\theta-1} d\phi + \dots \right] \\ &= \frac{(\phi_{sd}^{*,2})^{-\theta} - (\phi_{sd}^*)^{-\theta}}{(\phi_{sd}^*)^{-\theta}} + \frac{(\phi_{sd}^{*,3})^{-\theta} - (\phi_{sd}^{*,2})^{-\theta}}{(\phi_{sd}^*)^{-\theta}} + \dots \end{aligned}$$

Completing the integration, rearranging terms and using equation (S.6), we obtain

$$\bar{G}_{sd} = f_{sd}(1)^{\tilde{\theta}} \sum_{g=1}^{\infty} Z_{sd}(g)^{-\tilde{\theta}}. \quad (\text{S.13})$$

For the average number of products to be well defined and finite we require one more assumption:

Assumption S.4 *Parameters are such that $\sum_{g=1}^{\infty} Z_{sd}(g)^{-\tilde{\theta}}$ is strictly positive and finite.*

In Appendix S-C to this Online Supplement we show that the firms' fixed cost expense is a constant share of their total sales (where we denote means using a bar), as summarized in Proposition S.3:

$$\frac{\bar{F}_{sd}}{\bar{T}_{sd}} = \frac{\tilde{\theta} - 1}{\tilde{\theta}\sigma}.$$

We derive aggregate welfare in Appendix S-D to this Online Supplement and demonstrate in Appendix S-E to this Supplement that wage income and profit income can be expressed as a constant share of total output y_s per capita:

$$\pi_s = \eta y_s, \quad w_s = (1 - \eta) y_s,$$

where $\eta \equiv 1/(\tilde{\theta}\sigma)$. Since aggregates of the model do not depend on ε , the equilibrium definition is the same as in AGM.

S1.3 Summary

We have characterized an extension of the AGM model, in which the elasticity of substitution between a firm's individual products does not equal the elasticity of substitution across product lines of different firms. The extended model retains the main qualitative implications of the baseline AGM model, in which the two elasticities are the same. Future work using the structure of the generalized model to obtain estimates of the two elasticities may lead to a better understanding of the substitution effects within and across firms.

S2 Combination of Market Access and Market Penetration Cost Definitions

We turn to a generalization of AGM to nest both market access costs (as in AGM) and market penetration costs (as in Arkolakis 2010) as special cases.

S2.1 Restatement of AGM market access costs

We retain from AGM the specification that a firm draws not only a productivity parameter ϕ but also a destination specific market access cost shock c_d with well defined moments (and possibly a non-unitary mean). Suppose any two firms from source country s happen

to draw identical productivity ϕ and happen to draw an identical market access cost parameter $c_d \in (0, \infty)$; those two firms face an identical optimization problem in every destination d at the time of their product access decision. The pair of shocks (ϕ, c_d) therefore completely characterizes a firm's market access decision.

To accommodate market penetration costs as in Arkolakis (2010), we extend the market access cost definition (from AGM) and postulate that a firm's incremental market access cost also depends on its optimal choice of market penetration: a firm from country s decides the fraction n_{sd} of the L_d consumers who the firms wants to reach with its product composite (product line or mix) shipped to destination d . Consistent with the treatment of a firm in Arkolakis (2010) as the seller of a single product line (brand), we adopt the convention that a firm picks a common penetration rate for all its $g = 1, \dots, G_{sd}$ products shipped to a destination ($n_{sdg} = n_{sd}$ for all g).⁴⁶

As in AGM, a firm (ϕ, c_d) faces a product-destination specific *incremental market access cost* $c_d \bar{f}_{sd}(g; n_{sd})$, where $c_d \in (0, \infty)$ is a stochastic firm-specific market access cost shock. A firm that adopts an exporter scope of G_{sd} at destination d therefore incurs a total market access cost of

$$F_{sd}(G_{sd}, c_d; n_{sd}) = \sum_{g=1}^{G_{sd}} c_d \bar{f}_{sd}(g; n_{sd}). \quad (\text{S.14})$$

For any positive market penetration choice $n_{sd} > 0$, the firm's market access cost is zero at zero scope and strictly positive otherwise:

$$\bar{f}_{sd}(0; n_{sd}) = 0 \quad \text{and} \quad \bar{f}_{sd}(g; n_{sd}) > 0 \quad \text{for all } g = 1, 2, \dots, G_{sd},$$

where $\bar{f}_{sd}(g; n_{sd})$ is a continuous function in $[1, +\infty) \times [0, +\infty)$.

Arkolakis (2010) uses specific functional forms for market penetration costs, derived from primitives on consumer demand and product marketing. We discuss the generalized market access and market penetration cost definition also in terms of those specific functional forms. Extending (7) in AGM, we specify

$$\begin{aligned} \bar{f}_{sd}(g; n_{sd}) &= f_{sd}(n_{sd}) \cdot g^{\delta_{sd}} & \text{for } \delta_{sd} \in (-\infty, +\infty) & \quad \text{and} \\ h(g) &= g^\alpha & \text{for } \rho \in [0, +\infty). \end{aligned} \quad (\text{S.15})$$

The market access cost parameter $f_{sd}(n_{sd})$ is zero at zero penetration and strictly positive otherwise:

$$f_{sd}(0) = 0 \quad \text{and} \quad f_{sd}(n_{sd}) > 0 \quad \text{for all } n_{sd} > 0$$

where $f_{sd}(n_{sd})$ is a continuous function in $[0, +\infty)$.

Recall from AGM that a firm also faces a multiplicative i.i.d. shock ξ_{sdg} to its g -th product's appeal at a destination d (with mean $\mathbb{E}[\xi_{sdg}(\omega)] = 1$, positive support and known realization at the time of consumer choice). Under CES consumer demand, it

⁴⁶A further generalization that allows for firm-product specific optimal choices of n_{sdg} would result in interesting novel relations between core competency in production and market penetration choices: a firm's efficiency schedule $\phi_g \equiv \phi/h(g)$ would interact with product-specific market penetration costs in the product adoption decisions. We leave this generalization for future work.

is irrelevant whether the firm sets optimal price before or after the firm observes the product's appeal realization (see footnote 14 in AGM); price is a proportional markup over the firm-product's marginal production cost irrespective of the size of demand.

However, consistent with AGM and the deterministic setup of Arkolakis (2010), the firm has to take both the product entry (exporter scope G_{sd}) and the market penetration decision (consumer fraction n_{sd}) prior to observing any product appeal shock. It follows that, for a firm with an optimal and strictly positive market penetration at a destination d ($n_{sd} > 0$), the first-order conditions for a firm (ϕ, c_d) and therefore its optimal exporter scope G_{sd} and individual product sales are identical to those presented in AGM—with only two differences in interpretation:⁴⁷ we replace the product-invariant part of incremental market access costs with $f_{sd} \equiv f_{sd}(n_{sd})$ and we replace the revenue shifter (equation (8) in AGM) with $D_{sd} = D_{sd}(n_{sd})$ using

$$D_{sd}(n_{sd}) \equiv n_{sd} \cdot \bar{D}_{sd} \quad \text{for } \bar{D}_{sd} \equiv \left(\frac{P_d}{\tilde{\sigma} \tau_{sd} w_s} \right)^{\sigma-1} \frac{T_d}{\sigma} \quad (\text{S.16})$$

under a given (optimal) market penetration rate $n_{sd} \in (0, 1]$. Our original AGM model is the special case with $n_{sd} = 1$.

S2.2 Generalization of market penetration costs from Arkolakis (2010)

We now show that a specific functional form for $f_{sd}(n_{sd})$ accommodates Arkolakis (2010) market penetration costs as a special case and preserves a firm's relevant optimality conditions from Arkolakis (2010). The wage bill required to reach n_{sd} consumers in a market of size L_d is $F_{sd}(\cdot, \cdot; n_{sd})$, where L_d is a parameter for the firm and n_{sd} is a decision variable.⁴⁸ For a firm with given optimal exporter scope G_{sd} and market access cost draw c_d , define the firm's market penetration cost function $F_{sd}(\cdot, \cdot; n_{sd})$ to be equal to its *total market access cost* from (S.14) with

$$F_{sd}(\cdot, \cdot; n_{sd}) \equiv \sum_{g=1}^{G_{sd}} c_d \bar{f}_{sd}(g; n_{sd}) = \frac{(L_d)^\rho}{\psi_{sd}(\cdot, \cdot)} \frac{1 - (1 - n_{sd})^{1-\beta}}{1 - \beta} \quad \text{for } \beta \in (0, 1) \cup (1, +\infty), \quad (\text{S.17})$$

where

$$\psi_{sd}(G_{sd}, c_d) \equiv \frac{\bar{\psi}}{(w_s)^\gamma (w_d)^{1-\gamma}} \frac{1}{c_d \cdot \sum_{g=1}^{G_{sd}} g^{\delta_{sd}}}$$

⁴⁷In AGM, f_{sd} is sometimes also stated as $f_{sd}(1)$ for the first product.

⁴⁸Arkolakis (2010) includes market size L_d as an argument in the market penetration cost function and formally states a functional form for $F_{sd}(\cdot, \cdot; n_{sd}) \equiv f(n_{sd}; L_d)$ in equation (2). Arkolakis (2010) treats $f(n_{sd}; L_d)$ as the labor requirement needed to reach $n_{sd}L_d$ consumers and uses a factor of proportionality ψ to standardize the requirement given the composite wage payment $(w_s)^\gamma (w_d)^{1-\gamma}$. For comparability to AGM, we think of $F_{sd}(\cdot, \cdot; n_{sd})$ in wage bill equivalents and standardize with an accordingly scaled factor of proportionality ψ_{sd} .

and $\bar{\psi}$ is a positive scalar (similar to the original ψ from Arkolakis 2010). Importantly, the generalization of the market penetration cost from Arkolakis (2010) to $F_{sd}(\cdot, \cdot; n_{sd})$ in equation (S.17) preserves the four relevant properties of the market penetration cost function: (i) the market penetration cost vanishes at zero penetration since $F_{sd}(\cdot, \cdot; 0) = 0$, (ii) it strictly increases in n since $\partial F_{sd}(\cdot, \cdot; n)/\partial n > 0$ for $n \in [0, 1]$, (iii) it is convex in n since $\partial^2 F_{sd}(\cdot, \cdot; n)/(\partial n)^2 > 0$ for $n \in [0, 1]$, and (iv) it is unbounded since $\lim_{n \rightarrow \infty} F_{sd}(\cdot, \cdot; n) = +\infty$.

Equivalently, for consistency with AGM and $c_d \bar{f}_{sd}(g; n_{sd}) = c_d f_{sd}(n_{sd}) \cdot g^{\delta_{sd}}$ by equation (S.15), we define

$$f_{sd}(n_{sd}) \equiv \frac{(w_s)^\gamma (w_d)^{1-\gamma} (L_d)^\rho}{\bar{\psi}} \frac{1 - (1 - n_{sd})^{1-\beta}}{1 - \beta} \quad \text{for } \beta \in (0, 1) \cup (1, +\infty). \quad (\text{S.18})$$

These mutually consistent but alternative fixed cost definitions allow us to now switch perspective between a firm's optimality conditions for the market penetration rate n_{sd} (given the optimal exporter scope G_{sd} and the market access cost draw c_d) on the one hand side, and a firm's optimality conditions for exporter scope G_{sd} (given the optimal market penetration rate n_{sd} and the market access cost draw c_d) on the other hand side. Both the optimal exporter scope G_{sd} and the market penetration rate n_{sd} decisions need to be made prior to observing the products' appeal shocks (ξ_{sdg}), so a firm takes the two decisions simultaneously.

We already pointed out above that the optimality conditions on exporter scope G_{sd} are the same as those in AGM, merely replacing $f_{sd} \equiv f_{sd}(n_{sd})$ and $D_{sd} \equiv D_{sd}(n_{sd})$ for given optimal n_{sd} . We now turn to showing that the new optimality conditions for the market penetration rate n_{sd} , given optimal exporter scope G_{sd} and the market access cost draw c_d , are a straightforward restatement of the related optimality conditions from Arkolakis (2010), simply generalizing the cost scalar to $\psi_{sd}(G_{sd}, c_d)$. The original Arkolakis (2010) model is the special case with $G_{sd} = c_d = 1$. The original Melitz (2003) model is a special case with both $G_{sd} = c_d = 1$ and $\beta = 0$.

S2.3 Optimal market penetration costs given optimal exporter scope

Given the simultaneous choice of exporter scope and the market penetration rate, we can solve without loss of generality for the market penetration rate n_{sd} presuming that exporter scope G_{sd} is optimal. Suppose, conditional on destination market access, a type (ϕ, c_d) firm is setting optimal individual product prices (facing a fraction n_{sd} of consumer demand under monopolistic competition) and optimal exporter scope (given market penetration n_{sd}). The resulting first-order conditions from the profit maximizing equation imply identical markups over marginal cost $\tilde{\sigma} \equiv \sigma/(\sigma-1) > 1$ for each firm-product under

$\sigma > 1$.⁴⁹ Given optimal exporter scope $G_{sd}(\phi, c_d)$, and using the optimal pricing decision in the firm's profit function, we obtain the firm's expected profits (prior to product appeal shock realizations) at destination d :

$$\pi_{sd}(\phi, c_d) = \max_{n_{sd}} D_{sd}(n_{sd}) \phi^{\sigma-1} \bar{H}(G_{sd})^{-(\sigma-1)} - F_{sd}(G_{sd}, c_d; n_{sd}),$$

with $F_{sd}(G_{sd}, c_d; n_{sd})$ given by (S.17), the penetration dependent revenue shifter given by

$$D_{sd}(n_{sd}) \equiv n_{sd} \cdot \bar{D}_{sd} \quad \text{for } \bar{D}_{sd} \equiv \left(\frac{P_d}{\tilde{\sigma} \tau_{sd} w_s} \right)^{\sigma-1} \frac{T_d}{\sigma}$$

and the average product efficiency index in destination d for a firm with exporter scope G_{sd} given by $\bar{H}(G_{sd})^{-(\sigma-1)} \equiv \sum_{g=1}^{G_{sd}} h(g)^{-(\sigma-1)}$.

The first-order condition for maximizing profit $\pi_{sd}(\phi, c_d)$ with respect to the market penetration rate n_{sd} is equivalent to

$$\frac{\bar{D}_{sd}}{L_d} \phi^{\sigma-1} \bar{H}(G_{sd})^{-(\sigma-1)} = \frac{1}{\psi_{sd}(G_{sd}, c_d) (L_d)^{1-\rho}} \frac{1}{(1 - n_{sd})^\beta}, \quad (\text{S.19})$$

given $\beta \in (0, 1) \cup (1, +\infty)$ for a type (ϕ, c_d) firm with optimal exporter scope $G_{sd}(\phi, c_d)$. Similar to the first-order condition in Arkolakis (2010, equation (8)), the left-hand side of the condition shows the marginal revenue of a firm's product line (net of labor production cost) per consumer and the right-hand side the marginal cost per consumer of bringing the product line to destination d .

The zero-consumer threshold of minimal productivity for a firm to start penetrating a market can be found by setting $n_{sd} = 0$ in (S.19) and solving out for productivity. The zero-consumer threshold for productivity is

$$\phi_{sd}^{*, n=0}(c_d)^{\sigma-1} \equiv \frac{(L_d)^{\rho-1}}{\psi_{sd}(G_{sd}, c_d)} \frac{\bar{H}(G_{sd})^{\sigma-1}}{\bar{D}_{sd}/L_d}. \quad (\text{S.20})$$

A firm compares the marginal per-consumer revenue from reaching an infinitesimally small mass of consumers (the left-hand side of (S.19)) to the marginal per-consumer cost of reaching that infinitesimally small mass (the right-hand side of (S.19)). Given elastic CES

⁴⁹A firm selling an optimal number of products G_{sd} to destination market d has an expected profit of

$$\pi_{sd}(\phi, c_d) = \max_{G_{sd}} \sum_{g=1}^{G_{sd}} \mathbb{E} \left[\max_{\{p_{sdg}\}_{g=1}^{G_{sd}}} \left(p_{sdg} - \tau_{sd} \frac{w_s}{\phi/h(g)} \right) \left(\frac{p_{sdg}}{P_d} \right)^{-\sigma} \xi_{sdg} \frac{T_d}{P_d} \right] - F_{sd}(G_{sd}, c_d; n_{sd}).$$

The firm's first-order conditions with respect to every individual price p_{sdg} imply an optimal product price

$$p_{sdg}(\phi) = \tilde{\sigma} \tau_{sd} w_s h(g) / \phi$$

with an identical markup over marginal cost $\tilde{\sigma} \equiv \sigma/(\sigma-1) > 1$ for $\sigma > 1$. Product price does not depend on the appeal shock realization because the shock enters profits multiplicatively, so it is irrelevant whether a firm is setting price before or after the appeal shock is observed.

demand, more productive firms extract higher marginal per-consumer revenue, so they choose higher rates of market penetration. Similar to Arkolakis (2010, Proposition 1), for $\beta > 0$, a type (ϕ, c_d) firm will choose to stay out of destination d and set $n_{sd}(\phi, c_d) = 0$ if $\phi < \phi_{sd}^{*,n=0}(c_d)$. Conversely, two firms of types (ϕ_1, c_d) and (ϕ_2, c_d) will enter and set $n_{sd}(\phi_2, c_d) > n_{sd}(\phi_1, c_d) \geq 0$ if $\phi_2 > \phi_1 \geq \phi_{sd}^{*,n=0}(c_d)$.

Inverting the first-order condition (S.19) to solve for the optimal $n_{sd}(\phi, c_d)$, and using (S.20), yields the optimal market penetration rate for a firm's product line

$$n_{sd}(\phi, c_d) = 1 - \left(\frac{\phi_{sd}^{*,n=0}(c_d)}{\phi} \right)^{(\sigma-1)/\beta} \quad \text{if } \phi \geq \phi_{sd}^{*,n=0}(c_d). \quad (\text{S.21})$$

S2.4 Equilibrium properties

In the combined model with both AGM market access costs and Arkolakis (2010) market penetration costs, the optimal exporter scope choice implies that the productivity threshold $\phi_{sd}^{*,1}(c_d)$ for exporting at all from s to d is $\phi_{sd}^{*,1}(c_d)$, while the zero-consumer threshold is $\phi_{sd}^{*,n=0}$. Both need to be satisfied, so the *effective entry threshold* is $\phi_{sd}^*(c_d) = \max[\phi_{sd}^{*,1}(c_d), \phi_{sd}^{*,n=0}]$.⁵⁰

Note that $\psi_{sd}(G_{sd}, c_d)$ strictly monotonically decreases in c_d by (S.17), so the zero-consumer threshold $\phi_{sd}^{*,n=0}(c_d)$ strictly monotonically increases in c_d by (S.20).⁵¹ Similarly, by AGM's equation (10) the productivity threshold for exporting at all ($G_{sd} \geq 1$) strictly increases in c_d . We conclude that, also in the combined model with both AGM market access costs and Arkolakis (2010) market penetration costs, a higher market access cost draw c_d strictly raises the effective entry threshold $\phi_{sd}^*(c_d)$.

However, the effect of a higher market access cost draw c_d on realized total market access cost $F_{sd}(G_{sd}, c_d; n_{sd})$ is ambiguous by (S.17). The reason is that $\psi_{sd}(G_{sd}, c_d)$ strictly monotonically decreases in c_d with a unitary elasticity, thus raising F_{sd} with a unitary

⁵⁰By AGM's equation (10), the productivity threshold for exporting at all ($G_{sd} = 1$) is implicitly given by

$$\begin{aligned} \phi_{sd}^{*,1}(c_d)^{\sigma-1} &\equiv \frac{c_d f_{sd}(n_{sd}(\phi_{sd}^{*,1}, c_d))}{D_{sd}(n_{sd}(\phi_{sd}^{*,1}, c_d))} \\ &= \frac{(L_d)^\rho}{\psi_{sd}(1, c_d) \bar{D}_{sd}} \frac{1 - \left(\frac{\phi_{sd}^{*,n=0}(c_d)}{\phi_{sd}^{*,1}(c_d)} \right)^{(\sigma-1)(1-\beta)/\beta}}{1 - \beta} \frac{1}{1 - \left(\frac{\phi_{sd}^{*,n=0}(c_d)}{\phi_{sd}^{*,1}(c_d)} \right)^{(\sigma-1)/\beta}}, \end{aligned} \quad (\text{S.22})$$

where the latter equality follows from (S.17), (S.18) and (S.21) under the condition that $\phi_{sd}^{*,1}(c_d) \geq \phi_{sd}^{*,n=0}(c_d)$. Restating (S.20), the zero-consumer threshold for productivity is

$$\phi_{sd}^{*,n=0}(c_d)^{\sigma-1} \equiv \frac{(L_d)^\rho}{\psi_{sd}(1, c_d) \bar{D}_{sd}} \bar{H}(1)^{\sigma-1}$$

for a firm's first product ($G_{sd} = 1$), where $\bar{H}(1) = 1$. Using the latter condition in (S.22), it follows that $\phi_{sd}^{*,1}(c_d) \geq \phi_{sd}^{*,n=0}(c_d)$ need not hold for $\beta < 1$ or $\beta > 1$.

⁵¹The optimal market penetration rate $n_{sd}(\phi, c_d)$ therefore strictly decreases in c_d by (S.21) for $\sigma > 1$ and $\beta > 0$.

elasticity, while $n_{sd}(\phi, c_d)$ strictly decreases in c_d with a non-unitary elasticity by (S.21), thus lowering F_{sd} with a non-unitary elasticity. The net effect of a c_d shock on a firm's realized $F_{sd}(G_{sd}, c_d; n_{sd})$ is therefore ambiguous.

S2.5 Implications for estimation

A firm's optimal market penetration rate $n_{sd}(\phi, c_d)$ for its product line shifts the product-invariant part of incremental market access costs $c_d f_{sd}(n_{sd})$. Our estimator flexibly allows for a firm-destination specific market access cost shock c_d , which also shifts the product-invariant part of incremental market access costs $c_d f_{sd}(n_{sd})$. Our estimator therefore subsumes within the stochastic market access cost parameter c_d any firm-destination specific variation in the market penetration rate, and fully accounts for the possibility that firms optimally set their market penetration rate n_{sd} .

Similarly, our estimator flexibly allows for a non-unitary firm-destination specific average product appeal shock $\bar{\xi}_{sd} = \sum_{g=1}^{G_{sd}} \xi_{sdg} / G_{sd}$. Our estimator therefore subsumes within the average product appeal shock ξ_{sd} any firm-destination specific variation in the revenue shifter $D_{sd}(n_{sd}) = n_{sd} \bar{D}_{sd}$, and fully accounts for the possibility that firms optimally set their market penetration rate n_{sd} .

In summary, our existing estimation framework in AGM flexibly allows for the consequences of market penetration costs as in Arkolakis (2010).

S3 Export Products and Export Destinations

Tables S.1 and S.2 show Brazil's top ten export destinations by number of exporters and the top ten exported HS 6-digit product codes by total value in 2000. In Table S.1, Argentina is the most common export destination and the United States receives most Brazilian exports in value. In Table S.2, medium-sized aircraft is the leading export product in value, followed by wood pulp and biofuel products.

Table S.1: Top Brazilian Export Destinations

Destination	# Exporters	Export Value (USD)
Argentina	4,590	5,472,333,618
Uruguay	3,251	504,642,201
USA	3,083	9,772,577,557
Chile	2,342	1,145,161,210
Paraguay	2,319	561,065,104
Bolivia	1,799	282,543,791
Mexico	1,336	1,554,452,204
Venezuela	1,333	658,281,591
Germany	1,217	1,364,610,059
Peru	1,191	329,896,577

Source: SECEX 2000, manufacturing firms and their manufactured products.

Table S.2: **Top Brazilian Exported Items**

HS Code	Description	Value (USD)
880230	Airplanes between 2 and 15 tons	2,618,856,983
470329	Bleached non-coniferous chemical wood pulp	1,523,403,942
230400	Soybean oil-cake and other solid residues	1,245,752,048
870323	Passenger vehicles between 1,500 and 3,000 cc	1,197,222,859
852520	Transmission apparatus incorporating reception apparatus	926,618,451
640399	Footwear, with outer soles	854,950,667
720712	Semifinished products of iron or nonalloy steel	802,801,270
760110	Unwrought aluminum, not alloyed	765,195,563
200911	Orange juice, frozen	561,103,666
170111	Raw solid cane sugar	520,544,094

Source: SECEX 2000, manufacturing firms and their manufactured products.

S4 Monte Carlo Simulations

To document identification under our simulated method of moments estimator, we run Monte Carlo tests with generated data. We generate 333,000 Brazilian firms under the initial parameters Θ , where

$$\Theta = \left\{ \delta_1, \delta_2, \tilde{\alpha}, \tilde{\theta}, \sigma_\xi, \sigma_c \right\} = \{-1.17, -0.90, 1.73, 1.84, 1.89, 0.53\}$$

reflects the baseline estimates from Table 2 in the main text. The generated data have approximately 10,000 exporters. We then apply our simulated method of moments routine to the generated data and find the optimum, recovering an estimate of the parameter vector $\hat{\Theta}$. We repeat the data generation and estimation procedure 30 times and report in Table S.3 the mean and standard deviation of the elements of $\hat{\Theta}$.

The Monte Carlo results in Table S.3 document that our procedure accurately pinpoints all parameters of interest. In particular $(\hat{\alpha}, \hat{\sigma}_\xi, \hat{\sigma}_c)$ are precisely estimated, with point estimates close to the initial parameters behind the generated data and with standard errors less than 2 percent of the true value. Similarly, $\hat{\delta}$ and $\hat{\theta}$ are estimated close to their true values, their standard errors are under 4 percent of their true values. The proximity of our parameter estimates to the initial parameters underlying the data generation, and their precision, substantiate the hypothesis that our simulated method of moments estimator identifies the AGM model's parameters of interest.

S5 Sensitivity Analysis

To assess the robustness of our baseline estimates in AGM, we perform a number of modifications to our main specification. Overall, we find that our baseline results are

Table S.3: Monte Carlo Results

Θ	δ_1	δ_2	$\tilde{\alpha}$	$\tilde{\theta}$	σ_ξ	σ_c	$\delta_1 - \delta_2$
Parameter of generated data	-1.17	-0.90	1.73	1.84	1.89	0.53	-0.27
Estimate (mean) (s.e.)	-1.21 (0.03)	-0.93 (0.03)	1.71 (0.02)	1.91 (0.05)	1.88 (0.03)	0.51 (0.01)	-0.28 (0.01)

remarkably robust to sample restrictions and alternative variable definitions.

S5.1 Adjusted sales

Our baseline estimates imply a large and statistically significant difference between δ_{LAC} and δ_{ROW} . In a first robustness check, we strive to rule out that this difference could be driven by different typical sales across sets of products that Brazilian firms ship to LAC and non-LAC countries. We therefore correct sales and control for the mean sales of product groups at the HS 2-digit level. Concretely, we take the upward or downward deviation of a firm's HS 6-digit product sales to a destination $\log y_{\omega d}^p$ from the worldwide product-group sales mean of Brazilian exporters:

$$\tilde{y}_{\omega dg} = \exp \left\{ \log y_{\omega dg} - \frac{1}{M} \sum_{\omega} \frac{1}{N} \sum_d \sum_{g \in \text{HS } 2} \log y_{\omega dg} \right\}.$$

This adjustment does not reduce the sample size. We report the results in the row *1. Adjusted sales* of Table S.4. The estimates are broadly consistent with the baseline, but the estimated scope elasticities of market access costs δ and of product efficiency $\tilde{\alpha}$ are lower in absolute magnitude, and so is the estimated Pareto shape parameter $\tilde{\theta}$. These estimates imply that both the within-firm product distribution is more concentrated in the top product and the between-firm sales distribution has more firms in the upper tail with extremely high sales. An intuitive explanation is that demeaning sales by the average exporter's typical sale in a product group exacerbates sales deviations of specific products, thus making distributions appear more extreme. However, signs of our estimates stay the same and broad magnitudes remain qualitatively similar to the baseline estimates.

S5.2 Advanced manufacturing

A related robustness concern is that estimation could be driven by different feasible exporter scopes across product groups that Brazilian firms ship to LAC and non-LAC destination. For example, more differentiated industries, or more technology driven industries, might allow for the export of more potential varieties, or the HS classification system might simply provide more individual HS 6-digit products within more refined HS 2-digit product groups. In a second robustness exercise we therefore split the sample into firms

that are active in relatively “advanced manufacturing” industries. We present results from our definition of advanced manufacturing as three top-level NACE sectors “Manufacture of machinery and equipment”, “Manufacture of electrical and optical equipment” and “Manufacture of transport equipment” (codes DK, DL and DM).

Under this sectoral restriction, a markedly reduced sample size of only 2,539 Brazilian manufacturing exporters remains. Despite the considerable drop in sample size, however, results in the row labeled *2. Advanced manufacturing* in Table S.4 are broadly consistent with the baseline. In advanced manufacturing industries, the difference in scope elasticities of market access costs δ is larger between LAC and non-LAC countries. While market access costs drop off even faster with scope in LAC countries in advanced manufacturing industries, in non-LAC destination the converse is the case: market access costs remain more elevated at higher scopes than in the average manufacturing industry. The gap between the LAC and non-LAC scope elasticities of market access costs is almost double as wide in advanced manufacturing industries as it is in the average industry. For our counterfactual exercise, this wider difference of market access cost elasticities between LAC and non-LAC countries implies even more pronounced benefits of harmonizing market access costs across the world. Other parameter estimates are similar to the baseline, and specially the scope elasticity of product efficiency $\tilde{\alpha}$ is not statistically significantly different in advanced industries compared to the average industry. Overall, every sign remains the same and estimates that are statistically different from the baseline remain comparable in their qualitative economic implications.

S5.3 Eight-digit NCM product categories

To make our results closely comparable to evidence from other countries, in our main text we define a product as a Harmonized System (HS) 6-digit code, which is internationally comparable by requirement of the World Customs Organization (WCO) across its 200 member countries. To query the potential sensitivity of our results to a refined product classification, we use the Mercosur 8-digit level (*Nomenclatura Comum do Mercosul* NCM8), which roughly corresponds to the 8-digit HS level by the World Customs Organization. As the row *3. NCM 8-digit manufacturing* in Table S.4 shows, our results are hardly sensitive at all to the change in level of disaggregation. No single estimate is statistically different from our baseline estimates.

S5.4 Sensitivity to destinations Argentina and United States

Two destination markets dominate Brazilian manufacturing exports: Argentina (the top destination in terms of exporter counts) and the United States (the top destination in terms of export value). To assure ourselves that the estimates are not driven by potential outlier behavior of export flows to those two destinations, we remove them from the sample. As Table S.4 in the row labeled *4. Dropping ARG, USA* shows, only the estimate of the firm size distribution’s shape parameter θ becomes statistically significantly different from the baseline estimate. All other estimates are statistically indistinguishable from the baseline estimates. When we omit Argentina and the United States, the higher estimate for the

Table S.4: **Robustness for Select Subsamples**

estimate (s.e.)	δ_{LAC}	δ_{ROW}	$\tilde{\alpha}$	$\tilde{\theta}$	σ_{ξ}	σ_c	$\delta_{\text{LAC}} - \delta_{\text{ROW}}$
Baseline (all manufacturing)	-1.17 (0.09)	-0.90 (0.11)	1.73 (0.09)	1.84 (0.09)	1.89 (0.04)	0.53 (0.03)	-0.27 (0.05)
1. Adjusted sales	-0.89 (0.06)	-0.70 (0.06)	1.53 (0.06)	1.63 (0.07)	1.92 (0.03)	0.60 (0.02)	-0.18 (0.03)
2. Advanced manufacturing	-1.27 (0.14)	-0.78 (0.17)	1.77 (0.12)	1.64 (0.23)	2.01 (0.08)	0.58 (0.06)	-0.50 (0.13)
3. NCM 8-digit manufacturing	-1.16 (0.06)	-0.84 (0.08)	1.72 (0.05)	1.76 (0.10)	1.82 (0.04)	0.55 (0.03)	-0.32 (0.05)
4. Dropping ARG, USA (all manufacturing)	-1.22 (0.08)	-0.86 (0.07)	1.75 (0.07)	2.04 (0.09)	1.73 (0.04)	0.55 (0.02)	-0.36 (0.07)
5. Exporter share 10 percent	-1.43 (0.11)	-1.19 (0.14)	1.98 (0.10)	1.86 (0.11)	1.81 (0.04)	0.54 (0.03)	-0.24 (0.05)

Source: SECEX 2000, manufacturing firms and their manufactured products.

Note: Products at the HS 6-digit level. Estimates of δ_{LAC} indicates the scope elasticity for incremental product access costs for Brazilian firms shipping to other LAC destinations. Similarly δ_{ROW} perform the same role for exports to non-LAC destinations. See text for full description of various specifications.

Pareto tail index $\tilde{\theta}$ implies a lower probability mass in the upper tail of firms with extremely high sales. Even though Argentina and the United States attract a large number of export entrants from Brazil, these markets also exhibit a stronger concentration of exports among just a few top-selling firms than the average Brazilian export destination.

S5.5 Exporter share

An arguably important moment for our simulated method of moments is the share of formally established Brazilian manufacturing firms that export. Among the universe of Brazilian firms with at least one employee, only three percent of firms are exporters in 2000. This share is similar to that observed in other countries, for which data on the universe of firms with at least one employee is available. However, censuses and surveys in most developing and some industrialized countries truncate their target population of firms from below with thresholds up to 20 employees. To query sensitivity of our estimates to the share of exporters, we hypothetically consider an alternative share of 10 percent of Brazilian firms exporting. This exercise serves two purposes. First, comparisons of our findings to future results in other countries may depend on using a hypothetically truncated target population of firms from below. Second, we can check how our results might depend on a hypothetically more export oriented manufacturing sector such as, for instance, the U.S. manufacturing sector.

Table S.4 reports the results in the row 5. *Exporter share 10 percent.* Compared to the baseline, the scope elasticities of market access costs δ and of product efficiency $\tilde{\alpha}$ increase

Table S.5: **Alternative Regional Aggregates**

δ_1	δ_2	$\delta_1 - \delta_2$
Baseline	Baseline	-0.27
Latin America and Caribbean (LAC)	Rest of World	(0.05)
1. Mercosur	Rest of LAC (Non-Mercosur)	-0.03 (0.03)
2. Mercosur	Rest of World (Non-Mercosur)	-0.18 (0.03)

Source: SECEX 2000, manufacturing firms and their manufactured products.

Note: Products at the HS 6-digit level.

in absolute magnitude. Intuitively, the estimator tries to “explain” the hypothetically higher share of exporters with relatively faster declines in market access costs as exporter scope increases but to offset those access cost reductions with relatively steeper declines in product efficiency away from core competency, so as to keep matching the overall pattern of exporter scopes across destinations. The other three parameter estimates remain similar to the baseline estimates. This final robustness exercise therefore clarifies how the firm entry margin influences identification: if firm entry with the first product were hypothetically more prevalent, then for a given common market access cost component $f_{sd}(1)$ the access cost schedule would need to decline faster with scope, leading to wider exporters scopes everywhere, unless production efficiency also declines faster with scope.

S5.6 Sensitivity to Mercosur

In a final set of robustness exercises, we alternate the pairings of regional aggregates. In the baseline, we split the world into LAC (Latin American and the Caribbean) and the Rest of the World (non-LAC). In a first alteration, we drop destination countries outside of LAC from our sample and split LAC into Mercosur destinations in 2000 (Argentina, Paraguay, Uruguay) and non-Mercosur destinations. In the row labelled *1. Mercosur–Rest of LAC*, Table S.5 reports the results for the difference in the scope elasticities of market access costs between the two sub-regions within LAC, and the difference is negative as in the baseline but small (and not statistically different from zero). This finding justifies our treatment of LAC in the baseline as a relatively homogeneous region for Brazilian exporters. In a second alteration of the regional split, we discern between Mercosur destinations in 2000 and the Rest of the World, where the Rest of the World includes LAC countries outside Mercosur as well as non-LAC destinations. Expectedly, given the earlier results in the baseline and in the first alteration, the difference is negative but not quite as pronounced in magnitude as the difference between LAC and the Rest of the World. We therefore conclude that LAC countries outside Mercosur are more similar to Mercosur than to the Rest of the World and consider our baseline split of destinations into LAC and non-LAC an adequate country grouping.

Appendix to the Online Supplement

S-A Optimal Product Prices

We characterize the first-order conditions for the firm's optimal pricing rules at every destination d . There are $G_{sd}(\phi)$ first-order conditions with respect to p_{sdg} . For any $G_{sd}(\phi)$, taking the first derivative of profits $\pi_{sd}(\phi)$ from (S.1) with respect to p_{sdg} and dividing by $p_{sdg}^{-\varepsilon} P_{sd}(\phi; G_{sd})^{\varepsilon-\sigma} P_d^{\sigma-1} T_d$ yields

$$\begin{aligned} \frac{\partial \pi_{sd}(\phi)}{\partial p_{sdg}} &= P_d^{\sigma-1} T_d \cdot P_{sd}(\phi; G_{sd})^{\varepsilon-\sigma} p_{sdg}^{-\varepsilon} \left\{ 1 - \varepsilon \left(1 - \frac{w_s}{\phi/h(g)} \tau_{sd} p_{sdg}^{-1} \right) \right. \\ &\quad \left. + (\varepsilon - \sigma) P_{sd}(\phi; G_{sd})^{\varepsilon-1} \sum_{k=1}^{G_{sd}(\phi)} \left(p_{sdk} - \frac{w_s}{\phi/h(k)} \tau_{sd} \right) p_{sdk}^{-\varepsilon} \right\}. \end{aligned} \quad (\text{S-A.1})$$

The first-order conditions require that (S-A.1) is equal to zero for all products $g = 1, \dots, G_{sd}(\phi)$. Use the first-order conditions for any two products g and g' and reformulate to find

$$p_{sdg}/p_{sdg'} = h(g)/h(g').$$

So the firm must optimally charge an identical markup over the marginal costs for all products. Define this optimal markup as \bar{m} . To solve out for \bar{m} in terms of primitives, use $p_{sdg} = \bar{m} w_s \tau_{sd} / [\phi/h(g)]$ in the first-order condition above and simplify:

$$1 - \varepsilon \frac{1}{\bar{m}} + (\varepsilon - \sigma) P_{sd}(\phi; G_{sd})^{\varepsilon-1} \frac{\bar{m} - 1}{\bar{m}} \sum_{k=1}^{G_{sd}(\phi)} p_{sdk}^{-(\varepsilon-1)} = 0.$$

Note that $\sum_{k=1}^{G_{sd}(\phi)} p_{sdk}^{-(\varepsilon-1)} = P_{sd}(\phi; G_{sd})^{-(\varepsilon-1)}$. Solving the first-order condition for \bar{m} , we find the optimal markup over each product g 's marginal cost

$$\bar{m} = \tilde{\sigma} \equiv \sigma / (\sigma - 1).$$

A firm with productivity ϕ optimally charges a price

$$p_{sdg}(\phi) = \tilde{\sigma} w_s \tau_{sd} / [\phi/h(g)] \quad (\text{S-A.2})$$

for its products $g = 1, \dots, G_{sd}(\phi)$.

S-B Second-order Conditions

We now turn to the second-order conditions for price choice. To find the entries along the diagonal of the Hessian matrix, take the first derivative of condition (S-A.1) with respect to the own price p_{sdg} and then replace $w_s \tau_{sd} / [\phi/h(g)] = p_{sdg}(\phi) / \tilde{\sigma}$ by the first-order condition

to find

$$\begin{aligned}
\frac{\partial^2 \pi_{sd}(\phi)}{(\partial p_{sdg})^2} &= P_d^{\sigma-1} T_d \cdot P_{sd}(\phi; G_{sd})^{\varepsilon-\sigma} p_{sdg}^{-\varepsilon} \left\{ -\frac{\varepsilon}{\tilde{\sigma}} p_{sdg}^{-1} \right. \\
&\quad + (\varepsilon - \sigma) P_{sd}(\phi; G_{sd})^{\varepsilon-1} [-(\varepsilon - 1) + \varepsilon/\tilde{\sigma}] p_{sdg}^{-\varepsilon} \\
&\quad \left. + (\varepsilon - \sigma)(\varepsilon - 1) P_{sd}(\phi; G_{sd})^{2(\varepsilon-1)} p_{sdg}^{-\varepsilon} \cdot \sum_{k=1}^{G_{sd}} (1 - 1/\tilde{\sigma}) p_{sdk}^{-(\varepsilon-1)} \right\} \\
&= P_{sd}(\phi; G_{sd})^{\varepsilon-\sigma} P_d^{\sigma-1} T_d \cdot \left\{ -\varepsilon p_{sdg}^{-\varepsilon-1} + (\varepsilon - \sigma) P_{sd}(\phi; G_{sd})^{\varepsilon-1} p_{sdg}^{-2\varepsilon} \right\} / \tilde{\sigma}.
\end{aligned} \tag{S-B.3}$$

This term is strictly negative if and only if

$$(\varepsilon - \sigma) P_{sd}(\phi; G_{sd})^{\varepsilon-1} p_{sdg}^{-(\varepsilon-1)} < \varepsilon.$$

If $\varepsilon \leq \sigma$, this last condition is satisfied because the left-hand side is weakly negative and $\varepsilon > 0$. If $\varepsilon > \sigma$, then we can rewrite the condition as $p_{sdg}^{-(\varepsilon-1)} / [\sum_{k=1}^{G_{sd}} p_{sdk}^{-(\varepsilon-1)}] < 1 < \varepsilon / (\varepsilon - \sigma)$ so that the condition is satisfied. The diagonal entries of the Hessian matrix are therefore strictly negative for any demand elasticity configuration across nests.

To derive the entries off the diagonal of the Hessian matrix, we take the derivative of condition (S-A.1) for product g with respect to any other price $p_{sdg'}$ and then replace $w_s \tau_{sd} / [\phi / h(g')] = p_{sdg'}(\phi) / \tilde{\sigma}$ by the first-order condition to find

$$\begin{aligned}
\frac{\partial^2 \pi_{sd}(\phi)}{\partial p_{sdg} \partial p_{sdg'}} &= P_d^{\sigma-1} T_d \cdot P_{sd}(\phi; G_{sd})^{\varepsilon-\sigma} p_{sdg}^{-\varepsilon} \left\{ (\varepsilon - \sigma) P_{sd}(\phi; G_{sd})^{\varepsilon-1} [-(\varepsilon - 1) + \varepsilon/\tilde{\sigma}] p_{sdg'}^{-\varepsilon} \right. \\
&\quad \left. + (\varepsilon - \sigma)(\varepsilon - 1) P_{sd}(\phi; G_{sd})^{2(\varepsilon-1)} p_{sdg'}^{-\varepsilon} \sum_{k=1}^{G_{sd}} (1 - 1/\tilde{\sigma}) p_{sdk}^{-(\varepsilon-1)} \right\} \\
&= P_d^{\sigma-1} T_d \cdot (\varepsilon - \sigma) P_{sd}(\phi; G_{sd})^{\varepsilon-\sigma+\varepsilon-1} p_{sdg}^{-\varepsilon} p_{sdg'}^{-\varepsilon} / \tilde{\sigma}.
\end{aligned} \tag{S-B.4}$$

This term is strictly positive if and only if $\varepsilon > \sigma$.

Having derived the entries of the Hessian matrix, it remains to establish the conditions under which the Hessian is negative definite. We discern two cases. First the case of $\varepsilon \leq \sigma$ and then the case $\varepsilon > \sigma$.

S-B.1 Negative definiteness of Hessian if $\varepsilon \leq \sigma$

By (S-B.3) and (S-B.4), the Hessian matrix can be written as

$$\mathbf{H} = P_{sd}(\phi; G_{sd})^{\varepsilon-\sigma} P_d^{\sigma-1} T_d [\mathbf{H}_A + (\varepsilon - \sigma) P_{sd}(\phi; G_{sd})^{\varepsilon-1} \mathbf{H}_B],$$

where

$$\mathbf{H}_A \equiv \begin{pmatrix} -\varepsilon p_{sd1}^{-\varepsilon-1} & & & \\ 0 & -\varepsilon p_{sd2}^{-\varepsilon-1} & & \\ 0 & 0 & -\varepsilon p_{sd3}^{-\varepsilon-1} & \\ \dots & & & \dots \end{pmatrix} \text{ and } \mathbf{H}_B \equiv \begin{pmatrix} p_{sd1}^{-\varepsilon} p_{sd1}^{-\varepsilon} & & & \\ p_{sd2}^{-\varepsilon} p_{sd1}^{-\varepsilon} & p_{sd2}^{-\varepsilon} p_{sd2}^{-\varepsilon} & & \\ p_{sd3}^{-\varepsilon} p_{sd1}^{-\varepsilon} & p_{sd3}^{-\varepsilon} p_{sd2}^{-\varepsilon} & p_{sd3}^{-\varepsilon} p_{sd3}^{-\varepsilon} & \\ \dots & & & \dots \end{pmatrix}.$$

The Hessian matrix \mathbf{H} is negative definite if and only if the negative Hessian

$$-\mathbf{H} = P_{sd}(\phi; G_{sd})^{\varepsilon-\sigma} P_d^{\sigma-1} T_d [-\mathbf{H}_A + (\sigma-\varepsilon) P_{sd}(\phi; G_{sd})^{\varepsilon-1} \mathbf{H}_B]$$

is positive definite. Note that the sum of one positive definite matrix and any number of positive semidefinite matrices is positive definite. Hence if $-\mathbf{H}_A$ and \mathbf{H}_B are positive semidefinite and at least one of the two matrices is positive definite (given $\varepsilon \leq \sigma$), then the Hessian is negative definite.

A necessary and sufficient condition for a matrix to be positive definite is that the leading principal minors of the matrix are positive. The leading principal minors of $-\mathbf{H}_A$ are positive, so $-\mathbf{H}_A$ is positive definite. For \mathbf{H}_B , the first leading principal minor is positive, and all remaining principal minors are equal to zero. So \mathbf{H}_B is positive semidefinite. Therefore the Hessian matrix \mathbf{H} is negative definite.

S-B.2 Negative definiteness of Hessian if $\varepsilon > \sigma$

Another necessary and sufficient condition for the Hessian matrix \mathbf{H} to be negative definite is that the leading principal minors alternate sign, with the first principal minor being negative. The first diagonal entry is strictly negative as is any diagonal entry by (S-B.3). An application of the leading principal minor test in our case requires a recursive computation of the determinants of $G_{sd}(\phi)$ submatrices (a solution of polynomials with order up to $G_{sd}(\phi)$). We choose to check for negative definiteness of the Hessian in two separate ways when $\varepsilon > \sigma$. First, we derive a sufficient (but not necessary) condition for negative definiteness of the Hessian and query its empirical validity. Second, we present a necessary (but not sufficient) condition for negative definiteness of the Hessian for any pair of two products.

Sufficiency. A sufficient condition for the Hessian to be negative definite is due to McKenzie (1960): a symmetric diagonally dominant matrix with strictly negative diagonal entries is negative definite. A matrix is diagonally dominant if, in every row, the absolute value of the diagonal entry strictly exceeds the sum of the absolute values of all off-diagonal entries. By our derivations above, all diagonal entries of the Hessian are strictly negative.

For $\varepsilon > \sigma$, the condition for the Hessian to be diagonally dominant is

$$\sum_{k \neq g}^{G_{sd}} (\varepsilon - \sigma) P_{sd}(\phi; G_{sd})^{\varepsilon-1} p_{sdk}^{-\varepsilon} p_{sdg}^{-\varepsilon} < \varepsilon p_{sdg}^{-\varepsilon-1} - (\varepsilon - \sigma) P_{sd}(\phi; G_{sd})^{\varepsilon-1} p_{sdg}^{-2\varepsilon}$$

for all of a firm ϕ 's products (rows of its Hessian), where we cancelled the strictly positive terms $P_d^{\sigma-1} T_d P_{sd}(\phi; G_{sd})^{\varepsilon-\sigma} / \tilde{\sigma}$ from the inequality.

Using the optimal price (S-A.2) of product g from the first-order condition and rearranging terms yields the following condition

$$\frac{\sum_{k=1}^{G_{sd}} h(k)^{-\varepsilon}}{\sum_{k=1}^{G_{sd}} h(k)^{-(\varepsilon-1)}} < \frac{\varepsilon}{\varepsilon-\sigma} h(g)^{-1} \quad (\text{S-B.5})$$

for the Hessian to be a diagonally dominant matrix at the optimum.

By convention and without loss of generality $h(1) = 1$ for a firm with productivity ϕ . So the product efficiency schedule $h(g)$ strictly exceeds unity for the second product and subsequent products. As a result, the left-hand side of the inequality is bounded above for an exporter with a scope of at least two products at a destination:

$$\frac{\sum_{k=1}^{G_{sd}} h(k)^{-\varepsilon}}{\sum_{k=1}^{G_{sd}} h(k)^{-(\varepsilon-1)}} < \frac{\sum_{k=1}^{G_{sd}} h(k)^{-(\varepsilon-1)}}{\sum_{k=1}^{G_{sd}} h(k)^{-(\varepsilon-1)}} = 1.$$

A sufficient (but not necessary) condition for the Hessian to be negative definite is therefore

$$1 \leq h(g) < \frac{\varepsilon}{\varepsilon-\sigma}$$

for all of the firm's products. However, the Hessian can still be negative definite even if this condition fails. Clearly, the Hessian becomes negative definite the closer is ε to σ because then the off-diagonal entries approach zero and the Hessian is trivially diagonally dominant. Moreover, the Hessian can be negative definite even if it is not a diagonally dominant matrix.

To query the empirical validity of the sufficient condition $h(g) < \varepsilon/(\varepsilon-\sigma)$, consider evidence on products and brands in Broda and Weinstein (2006). Their preferred estimates for ε and σ within and across domestic U.S. brand modules are 11.5 and 7.5. Estimates in AGM suggest that $\alpha(\varepsilon-1)$ is around 1.84 under the specification that $h(g) = g^\alpha$. These parameters imply that the condition $h(g) < \varepsilon/(\varepsilon-\sigma)$ is satisfied for Hessians with up to 414 products. In the AGM data, no firm-country observations involve 415 or more products in a market (with a median of one product and a mean of 3.52). Even if additional products individually violate the sufficient condition, Hessians with more products may still be negative definite.

Necessity. Consider any two products g and g' . Negative definiteness of the Hessian must be independent of the ordering of products, so these two products can be assigned the first and second row in the Hessian without loss of generality. As stated before, a necessary and sufficient condition for the Hessian to be negative definite is that the leading principal minors of the Hessian alternate sign, with the first principal minor being negative. A necessary condition for the Hessian to be negative definite is therefore that the principal minors of any two products (first and second in the Hessian) alternate sign, with the first principal minor negative and the second positive.

The first principle minor is strictly negative because all diagonal entries are strictly negative by (S-B.3). The second principal minor is strictly positive if and only if the determinant satisfies

$$2 \varepsilon^2 P_{sd}(\phi; G_{sd})^{-(\varepsilon-1)} - \varepsilon(\varepsilon-\sigma) \left(p_{sdg}^{-(\varepsilon-1)} + p_{sdg'}^{-(\varepsilon-1)} \right) - (\varepsilon-\sigma)^2 (p_{sdg} p_{sdg'})^{-(\varepsilon-1)} P_{sd}(\phi; G_{sd})^{\varepsilon-1} > 0, \quad (\text{S-B.6})$$

where we cancelled the strictly positive terms $P_d^{2(\sigma-1)} T_d P_{sd}(\phi; G_{sd})^{2(\varepsilon-\sigma)} / \tilde{\sigma}^2$ from the inequality and multiplied both sides by $p_{sdg}^{\varepsilon-1} p_{sdg'}^{\varepsilon-1} P_{sd}(\phi; G_{sd})^{-(\varepsilon-1)}$.

To build intuition, consider the dual-product case with $G_{sd}(\phi) = 2$. Then condition (S-B.6) simplifies to

$$\frac{h(g)^{-(\varepsilon-1)}}{\sum_{k=1}^{G_{sd}} h(k)^{-(\varepsilon-1)}} \cdot \frac{h(g')^{-(\varepsilon-1)}}{\sum_{k=1}^{G_{sd}} h(k)^{-(\varepsilon-1)}} < \frac{\varepsilon}{\varepsilon-\sigma} \frac{\varepsilon+\sigma}{\varepsilon-\sigma}.$$

For $\varepsilon > \sigma$, both terms in the product on the right-hand side strictly exceed unity while the terms in the product on the left-hand side are strictly less than one, and the condition is satisfied.

In the multi-product case with $G_{sd}(\phi) > 2$, replace $p_{sdg}^{-(\varepsilon-1)} + p_{sdg'}^{-(\varepsilon-1)} = P_{sd}(\phi; G_{sd})^{-(\varepsilon-1)} - \sum_{k \neq g, g'} p_{sdk}^{-(\varepsilon-1)}$ in condition (S-B.6) and simplify to find

$$\frac{h(g)^{-(\varepsilon-1)}}{\sum_{k=1}^{G_{sd}} h(k)^{-(\varepsilon-1)}} \cdot \frac{h(g')^{-(\varepsilon-1)}}{\sum_{k=1}^{G_{sd}} h(k)^{-(\varepsilon-1)}} < \frac{\varepsilon}{\varepsilon-\sigma} \frac{\varepsilon+\sigma}{\varepsilon-\sigma} + \frac{\varepsilon}{\varepsilon-\sigma} \frac{\sum_{k \neq g, g'} h(k)^{-(\varepsilon-1)}}{\sum_{k=1}^{G_{sd}} h(k)^{-(\varepsilon-1)}}.$$

For $\varepsilon > \sigma$, the necessary condition on any two products of a multi-product firm is trivially satisfied by the above derivations because the additional additive term on the right-hand side is strictly positive.

In summary, parameters of our model are such that, for any two products of a multi-product firm, the second-order condition is satisfied.

S-C Proof of Proposition S.3

Average sales from s to d are

$$\bar{T}_{sd} = \int_{\phi_{sd}^*} y_{sd}(G_{sd}) \frac{\theta (\phi_{sd}^*)^\theta}{\phi^{\theta+1}} d\phi = \sigma f_{sd}(1) \theta \int_{\phi_{sd}^*} \frac{\phi^{\sigma-2-\theta} / (\phi_{sd}^*)^{\sigma-1-\theta}}{H(G_{sd}(\phi))^{\sigma-1}} d\phi.$$

The proof of the proposition follows from the following Lemma.

Lemma 2 *Suppose Assumptions S.1, S.2 and S.3 hold. Then*

$$\int_{\phi_{sd}^*} \frac{\phi^{\sigma-2-\theta} / (\phi_{sd}^*)^{\sigma-1-\theta}}{H(G_{sd}(\phi))^{\sigma-1}} d\phi = \frac{f_{sd}(1)^{\tilde{\theta}-1}}{\theta - (\sigma-1)} \tilde{F}_{sd},$$

where

$$\tilde{F}_{sd} \equiv \sum_{v=1}^{\infty} \frac{[f_{sd}(v)]^{1-\tilde{\theta}}}{[H(v)^{1-\sigma} - H(v-1)^{1-\sigma}]^{-\tilde{\theta}}}.$$

Proof. Note that

$$\begin{aligned} \int_{\phi_{sd}^*} \frac{\phi^{\sigma-2-\theta} / (\phi_{sd}^*)^{\sigma-1-\theta}}{H(G_{sd}(\phi))^{\sigma-1}} d\phi &= H(1)^{1-\sigma} \int_{\phi_{sd}^*}^{\phi_{sd}^{*,2}} \phi^{\sigma-2-\theta} d\phi + H(2)^{1-\sigma} \int_{\phi_{sd}^{*,2}}^{\phi_{sd}^{*,3}} \phi^{\sigma-2-\theta} d\phi + \dots \\ &= H(1)^{1-\sigma} \left[\frac{(\phi_{sd}^{*,2})^{\sigma-1-\theta} - (\phi_{sd}^*)^{\sigma-1-\theta}}{[\theta - (\sigma-1)] (\phi_{sd}^*)^{\sigma-1-\theta}} \right] \\ &\quad + H(2)^{1-\sigma} \left[\frac{(\phi_{sd}^{*,3})^{\sigma-1-\theta} - (\phi_{sd}^{*,2})^{\sigma-1-\theta}}{[\theta - (\sigma-1)] (\phi_{sd}^*)^{\sigma-1-\theta}} \right] + \dots \end{aligned}$$

Also note that, using equations (S.4) and (S.6), the ratio

$$\left[(\phi_{sd}^{*,2})^{\sigma-1-\theta} - (\phi_{sd}^*)^{\sigma-1-\theta} \right] / (\phi_{sd}^*)^{\sigma-1-\theta}$$

can be rewritten as

$$\begin{aligned} &\frac{(\phi_{sd}^{*,G})^{\sigma-1-\theta} - (\phi_{sd}^{*,G-1})^{\sigma-1-\theta}}{(\phi_{sd}^*)^{\sigma-1-\theta}} = \\ &= \frac{\left[\frac{(\phi_{sd}^*)^{\sigma-1}}{H(g)^{1-\sigma} - H(g-1)^{1-\sigma}} \frac{f_{sd}(g)}{f_{sd}(1)} \right]^{\frac{\sigma-1-\theta}{\sigma-1}} - \left[\frac{(\phi_{sd}^*)^{\sigma-1}}{H(g-1)^{1-\sigma} - H(g-2)^{1-\sigma}} \frac{f_{sd}(g-1)}{f_{sd}(1)} \right]^{\frac{\sigma-1-\theta}{\sigma-1}}}{\left[(\phi_{sd}^*)^{\sigma-1} \right]^{\frac{\sigma-1-\theta}{\sigma-1}}} \\ &= f_{sd}(1)^{\tilde{\theta}-1} \left\{ \frac{f_{sd}(g)^{1-\tilde{\theta}}}{[H(g)^{1-\sigma} - H(g-1)^{1-\sigma}]^{1-\tilde{\theta}}} - \frac{f_{sd}(g-1)^{1-\tilde{\theta}}}{[H(g-1)^{1-\sigma} - H(g-2)^{1-\sigma}]^{1-\tilde{\theta}}} \right\}. \end{aligned}$$

We define⁵²

$$\begin{aligned}
\tilde{F}_{sd} &\equiv \sum_{v=1} H(v)^{1-\sigma} \left[\frac{[f_{sd}(v+1)]^{1-\tilde{\theta}}}{[H(v+1)^{1-\sigma} - H(v)^{1-\sigma}]^{1-\tilde{\theta}}} - \frac{[f_{sd}(v)]^{1-\tilde{\theta}}}{[H(v)^{1-\sigma} - H(v-1)^{1-\sigma}]^{1-\tilde{\theta}}} \right] \\
&= \sum_{v=1} \left[[H(v)^{1-\sigma} - H(v-1)^{1-\sigma}] \frac{[f_{sd}(v)]^{1-\tilde{\theta}}}{[H(v)^{1-\sigma} - H(v-1)^{1-\sigma}]^{1-\tilde{\theta}}} \right] \\
&= \sum_{v=1} \frac{[f_{sd}(v)]^{1-\tilde{\theta}}}{[H(v)^{1-\sigma} - H(v-1)^{1-\sigma}]^{-\tilde{\theta}}}.
\end{aligned}$$

With this definition we obtain

$$\int_{\phi_{sd}^*} \frac{\phi^{\sigma-2-\theta} / (\phi_{sd}^*)^{\sigma-1-\theta}}{H(G_{sd}(\phi))^{\sigma-1}} d\phi = \frac{f_{sd}(1)^{\tilde{\theta}-1}}{\theta - (\sigma-1)} \tilde{F}_{sd}.$$

S-D Welfare

We have that

$$\begin{aligned}
P_d^{1-\sigma} &= \sum_s \int_{\phi_{sd}^*} [P_{sd}(\phi)]^{1-\sigma} \mu(\phi) d\phi \\
&= \sum_s \int_{\phi_{sd}^*} M_{sd} \left[\sum_{v=1}^{G_{sd}(\phi)} \left(\tilde{\sigma} \frac{w_s}{\phi/h(g)} \tau_{sd} \right)^{1-\varepsilon} \right]^{\frac{1-\sigma}{1-\varepsilon}} \frac{\theta (\phi_{sd}^*)^\theta}{\phi^{\theta+1}} d\phi \\
&= \sum_s (\tilde{\sigma} w_s \tau_{sd})^{1-\sigma} b_s^\theta \theta \left[H(1)^{1-\sigma} \left(\frac{(\phi_{sd}^{*,2})^{\sigma-1-\theta} - (\phi_{sd}^{*,1})^{\sigma-1-\theta}}{\theta - (\sigma-1)} \right) + \dots \right] \\
&= \sum_s (\tilde{\sigma} w_s \tau_{sd})^{-\theta} b_s^\theta \theta \left(\frac{f_{sd}(1)}{\frac{1}{\sigma} T_d} \right)^{1-\tilde{\theta}} \left[H(1)^{1-\sigma} \left(\frac{(\phi_{sd}^{*,2})^{\sigma-1-\theta} - (\phi_{sd}^{*,1})^{\sigma-1-\theta}}{(\phi_{sd}^{*,1})^{\sigma-1-\theta}} \right) + \dots \right],
\end{aligned}$$

⁵²In the special case with $\varepsilon = \sigma$, we can rearrange the terms and find

$$\tilde{F}_{sd} = \sum_{v=1}^{\infty} \frac{[f_{sd}(v)]^{1-\tilde{\theta}}}{[h(v)^{\sigma-1}]^{-\tilde{\theta}}} = \sum_{v=1}^{\infty} \frac{[f_{sd}(v)]^{1-\tilde{\theta}}}{h(v)^{-\theta}}.$$

where we use the definition of $\phi_{sd}^{*,1}$ for the last step. The final term in parentheses equals \tilde{F}_{sd} so

$$P_d^{-\theta} = \frac{\theta(\tilde{\sigma})^{-\theta}}{\left(\frac{1}{\sigma}\right)^{1-\theta/(\sigma-1)} T_d^{1-\tilde{\theta}}} \sum_s b_s^\theta (w_s \tau_{sd})^{-\theta} \tilde{F}_{sd}.$$

Using this relationship in equation (S.12), we obtain

$$\left(\frac{T_d}{P_d}\right)^\theta = \left(\frac{T_d}{w_d}\right)^\theta \frac{\theta(\tilde{\sigma})^{-\theta}}{(\sigma)^{\tilde{\theta}-1}} \frac{b_d^\theta}{\lambda_{dd}^{-\theta}} \frac{\tilde{F}_{dd}(1)}{T_d^{1-\tilde{\theta}}}.$$

If trade is balanced then $T_d = Y_d$, where T_d is consumption expenditure and Y_d is output. By the definition of $\tilde{F}_{dd}(1)$, this variable is homogeneous of degree $1 - \tilde{\theta}$ in wages, and the wage bill share in output $w_d L_d / Y_d$ is constant in all equilibria (see proof below). We therefore arrive at the same welfare expression as in Arkolakis et al. (2012): the share of domestic sales in consumption expenditure λ_{dd} and the coefficient of the Pareto distribution are sufficient statistics to characterize aggregate welfare in the case of balanced trade.

The final step is to verify that the wage w_d is a constant fraction of per-capita output y_d so that the first ratio on the right-hand side is constant. We demonstrate this next.

S-E Constant Wage Share in Output per Capita

We show that the ratio w_d / y_d is a constant number. We first look at the share of fixed costs in bilateral sales. Average fixed costs incurred by firms from s selling to d are

$$\begin{aligned} \bar{F}_{sd} &= \int_{\phi_{sd}^*}^{\phi_{sd}^{*,2}} F_{sd}(1) \theta \frac{(\phi_{sd}^*)^\theta}{\phi^{\theta+1}} d\phi + \int_{\phi_{sd}^{*,2}}^{\phi_{sd}^{*,3}} F_{sd}(2) \theta \frac{(\phi_{sd}^*)^\theta}{\phi^\theta} d\phi + \\ &= -F_{sd}(1) (\phi_{sd}^*)^\theta \left[(\phi_{sd}^{*,2})^{-\theta} - (\phi_{sd}^*)^{-\theta} \right] - F_{sd}(2) (\phi_{sd}^*)^\theta \left[(\phi_{sd}^{*,3})^{-\theta} - (\phi_{sd}^{*,2})^{-\theta} \right] - \dots \end{aligned}$$

Using the definition $F_{sd}(G_{sd}) \equiv \sum_{g=1}^{G_{sd}} f_{sd}(g)$ and collecting terms with respect to $\phi_{sd}^{*,G}$ we can write the above expression as

$$\bar{F}_{sd} = f_{sd}(1) + (\phi_{sd}^{*,2})^{-\theta} (\phi_{sd}^*)^\theta f_{sd}(2) + (\phi_{sd}^{*,3})^{-\theta} (\phi_{sd}^*)^\theta f_{sd}(3) + \dots$$

Using the definition of $\phi_{sd}^{*,G}$ from equation (S.6) to replace terms in the above equation, we obtain

$$\left(\phi_{sd}^{*,G}\right)^{\sigma-1} = \frac{(\phi_{sd}^*)^{\sigma-1}}{H(G_{sd})^{-(\sigma-1)} - H(G_{sd}-1)^{-(\sigma-1)}} \frac{f_{sd}(G_{sd})}{f_{sd}(1)}.$$

Therefore

$$\begin{aligned}
\bar{F}_{sd} &= f_{sd}(1) + \left(\frac{f_{sd}(2)^{1/(\sigma-1)} [H(2)^{-(\sigma-1)} - H(1)^{-(\sigma-1)}]^{-1/(\sigma-1)}}{f_{sd}(1)^{1/(\sigma-1)} [H(1)^{-(\sigma-1)}]^{-1/(\sigma-1)}} \right)^{-\theta} f_{sd}(2) + \dots \\
&= \left[f_{sd}(1) + f_{sd}(1)^{\tilde{\theta}} \left(f_{sd}(2)^{1/(\sigma-1)} [H(2)^{-(\sigma-1)} - H(1)^{-(\sigma-1)}]^{-1/(\sigma-1)} \right)^{-\theta} f_{sd}(2) + \dots \right] \\
&= [f_{sd}(1)]^{\tilde{\theta}} \left[f_{sd}(1)^{1-\tilde{\theta}} + \frac{f_{sd}(2)^{1-\tilde{\theta}}}{[H(2)^{-(\sigma-1)} - H(1)^{-(\sigma-1)}]^{-\tilde{\theta}}} + \frac{f_{sd}(3)^{1-\tilde{\theta}}}{[H(3)^{-(\sigma-1)} - H(2)^{-(\sigma-1)}]^{-\tilde{\theta}}} \dots \right] \\
&= [f_{sd}(1)]^{\tilde{\theta}} \tilde{F}_{sd}
\end{aligned}$$

and hence

$$\frac{\bar{F}_{sd}}{\bar{T}_{sd}} = \frac{f_{sd}(1)^{\tilde{\theta}} \left[f_{sd}(1)^{1-\tilde{\theta}} + \frac{f_{sd}(2)^{1-\tilde{\theta}}}{h(2)^\theta} + \dots \right]}{\frac{f_{sd}(1)^{\tilde{\theta}} \theta \sigma}{\theta - (\sigma - 1)} \sum_{g=1}^{\infty} \frac{f_{sd}(g)^{1-\tilde{\theta}}}{h(g)^\theta}} = \frac{\theta - (\sigma - 1)}{\theta \sigma} \frac{\sum_{g=1}^{\infty} \frac{f_{sd}(g)^{1-\tilde{\theta}}}{h(g)^\theta}}{\sum_{g=1}^{\infty} \frac{f_{sd}(g)^{1-\tilde{\theta}}}{h(g)^\theta}} < 1.$$

Finally, the share of profits generated by the corresponding bilateral sales is the share of variable profits in total sales ($1/\sigma$) minus the average fixed costs paid, as derived above. So

$$\frac{\bar{\pi}_{sd}}{\bar{T}_{sd}} = \frac{1}{\sigma} - \frac{\theta - (\sigma - 1)}{\theta \sigma} = \frac{\sigma - 1}{\theta \sigma} = \frac{1}{\tilde{\theta} \sigma} \equiv \eta.$$

This finding implies that the wage is a constant fraction of per capita income. The reason is that total profits for country s are $\pi_s L_s = \sum_k \lambda_{sk} T_k / (\tilde{\theta} \sigma)$, where $\sum_k \lambda_{sk} T_k$ is the country's total income because total manufacturing sales of a country s equal its total sales across all destinations. So profit income and wage income can be expressed as constant shares of total income:

$$\pi_s L_s = \frac{1}{\tilde{\theta} \sigma} Y_s \quad \text{and} \quad w_s L_s = \frac{\tilde{\theta} \sigma - 1}{\tilde{\theta} \sigma} Y_s.$$