

The Extensive Margin of Exporting Products: A Firm-level Analysis *Online Supplement**

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Abstract

This Online Supplement contains four parts. First, we generalize the Arkolakis, Ganapati and Muendler (2014) model to nested consumer preferences. Each inner nest holds the products in a firm's product line with an elasticity of substitution that differs from that of the outer nest over product lines of different firms. Second, we present tabulations of the underlying Brazilian export data for 2000. Third, we conduct Monte Carlo simulations to assess identification under our estimation routine. Finally, we offer a variety of robustness checks for our main estimates, using alternative assumptions and restricted samples.

Keywords: International trade; heterogeneous firms; multi-product firms; firm and product panel data; Brazil

JEL Classification: F12, L11, F14

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1 Nested Preferences with Different Elasticities

We analyze a generalized version of the (Arkolakis et al. 2014, henceforth AGM) model of multi-product firms that allows for within-firm cannibalization effects. The main result is that the qualitative properties of the AGM model are retained: the size distribution of firm sales and the distribution of the firms' numbers of products is consistent with regularities in Brazilian exporter data as well as other data sets. More importantly, the general equilibrium properties of the model do not depend on the inner nests' elasticity (the elasticity across the products of a given firm) so that the general equilibrium of the model can be easily characterized using the tools of Dekle, Eaton and Kortum (2007).

While the model is highly tractable, the introduction of one more demand elasticity adds a further degree of freedom. This degree of freedom can be disciplined using independent estimates for the outer and inner nests' elasticities, such as those of Broda and Weinstein (2006). Under an according parametrization, the model can be used for counterfactual exercises that simulate the impact of changes in trade costs on the firm size distribution and the distribution of the firms' numbers of products.

In the following subsection we present and solve the generalized model. We derive its aggregate properties in subsection 1.2. Subsection 1.3 concludes the presentation of the model with nested preferences.

1.1 Model

There is a countable number of countries. We label the source country of an export shipment with s and the export destination with d .

We adopt a two-tier nested CES utility function for consumer preferences.¹ Each inner nest of consumer preferences aggregates a firm's products with a CES utility function and an elasticity of substitution ε . Using marketing terminology, the product bundle of the inner nest can be called a firm's product line (or product mix). The product lines of different firms are then aggregated using an outer CES utility nest with an elasticity σ . Each firm offers a countable number of products but there is a continuum of firms in the world. We assume that every product line is uniquely offered by a single firm, but a firm may ship different product lines to different destinations. Formally, the representative consumer's utility function at destination d is given by

$$U_d = \left(\sum_s \int_{\Omega_{sd}} \left(\sum_{g=1}^{G_{sd}(\omega)} q_{sdg}(\omega)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\sigma-1}{\sigma} \frac{\varepsilon}{\varepsilon-1}} d\omega \right)^{\frac{\sigma}{\sigma-1}}$$

¹Atkeson and Burstein (2008) use a similar nested CES form in a heterogeneous-firms model of trade but their outer nest refers to different industries and the inner nests to different firms within the industry. Eaton and Kortum (2010) present a stochastic model with nested CES preferences to characterize the firm size distribution and their products under Cournot competition. In our model, firms do not strategically interact with other firms. This property of the model allows us to characterize general equilibrium beyond the behavior of individual firms.

where $q_{sdg}(\omega)$ is the quantity consumed of the g -th product of firm ω , producing in country s . Ω_{sd} is the set of firms from source country s selling to country d .

The representative consumer's first order conditions imply that demand for the g -th product of firm ω in market d is

$$q_{sdg}(\omega) = p_{sdg}(\omega)^{-\varepsilon} P_{sd}(\omega; G_{sd})^{\varepsilon-\sigma} P_d^{\sigma-1} T_d,$$

where $p_{sdg}(\omega)$ is the price of that product,

$$P_{sd}(\omega; G_{sd}) \equiv \left[\sum_{g=1}^{G_{sd}(\omega)} p_{sdg}(\omega)^{-(\varepsilon-1)} \right]^{-1/(\varepsilon-1)}$$

is the ideal price index for the product line of firm ω selling $G_{sd}(\omega)$ products in market d , and

$$P_d \equiv \left[\sum_s \int_{\Omega_{sd}} P_{sd}(\omega; G_{sd})^{-(\sigma-1)} d\omega \right]^{-1/(\sigma-1)}$$

is the ideal consumer price index in market d . T_d is total consumption expenditure.

1.1.1 Firm optimization

We assume that the firm has a linear production function for each product. A firm with overall productivity ϕ faces an efficiency $\phi/h(g)$ in producing its g 'th product, where $h(g)$ is an increasing function with $h(1) = 1$. We call the firm's total number of products G_{sd} at destination d its *exporter scope* at d . Productivity is the only source of firm heterogeneity so that, under the model assumptions below, firms of the same type ϕ from country s face an identical optimization problem in every destination d . Since all firms with productivity ϕ will make identical decisions in equilibrium, it is convenient to name them by their common characteristic ϕ from now on.²

The firm also incurs local entry costs to sell its g -th product in market d : $f_{sd}(g) > 0$ for $g > 1$, with $f_{sd}(0) = 0$. These incremental product-specific fixed costs may increase or decrease with exporter scope. The overall entry cost for market d is denoted by $F_{sd}(G) \equiv \sum_{g=1}^G f_{sd}(g)$ and strictly increases in exporter scope by definition.

Profits of a firm with productivity ϕ from country s that sells products $g = 1, \dots, G_{sd}$ in d at prices p_{sdg} are

$$\pi_{sd}(\phi) = \sum_{g=1}^{G_{sd}} \left(p_{sdg} - \frac{w_s}{\phi/h(g)} \tau_{sd} \right) p_{sdg}^{-\varepsilon} \cdot P_{sd}(\phi; G_{sd})^{\varepsilon-\sigma} P_d^{\sigma-1} T_d - F_{sd}(G_{sd}). \quad (1)$$

We consider the first-order conditions with respect to the prices p_{sdg} of each product g , consistent with an optimal product-line price $P_{sd}(\phi; G_{sd})$, and also with respect to exporter

²To simplify the exposition, we assume here that firms face no other idiosyncratic cost components, whereas the Arkolakis et al. (2014) model also allows for a destination specific market access cost shock c_d so that a firm in that model is characterized by a pair of shocks (ϕ, c_d) .

scope G_{sd} . As shown in Appendix A, the first-order conditions with respect to prices imply a constant markup over marginal cost for all products equal to $\tilde{\sigma} \equiv \sigma/(\sigma-1)$.

Using the constant markup rule in demand for the g -th product of a firm with exporter scope G_{sd} yields optimal sales of the product

$$p_{sdg}(\phi)q_{sdg}(\phi) = \left(\tilde{\sigma} \frac{w_s \tau_{sd}}{\phi/h(g)} \right)^{-(\varepsilon-1)} P_{sd}(\phi; G_{sd})^{\varepsilon-\sigma} P_d^{\sigma-1} T_d. \quad (2)$$

Using this result and the definition of $P_{sd}(\phi; G_{sd})$, we can rewrite profits that a firm generates at destination d selling G_{sd} products as

$$\begin{aligned} \pi_{sd}(\phi; G_{sd}) &= P_{sd}(\phi; G_{sd})^{-(\sigma-1)} \frac{P_d^{\sigma-1} T_d}{\sigma} - F_{sd}(G_{sd}) \\ &= H(G_{sd})^{-(\sigma-1)} (\tilde{\sigma} w_s \tau_{sd})^{-(\sigma-1)} \frac{\phi^{\sigma-1} P_d^{\sigma-1} T_d}{\sigma} - F_{sd}(G_{sd}), \end{aligned} \quad (3)$$

where

$$H(G_{sd}) \equiv \left[\sum_{g=1}^{G_{sd}} h(g)^{-(\varepsilon-1)} \right]^{-1/(\varepsilon-1)}$$

is the firm's product efficiency index.

We impose the following assumption, which is necessary for optimal exporter scope to be well defined.

Assumption 1 *Parameters are such that $Z_{sd}(G) = f_{sd}(G)/[H(G)^{-(\sigma-1)} - H(G-1)^{-(\sigma-1)}]$ strictly increases in G .*

The expression for $Z_{sd}(G)$ reduces to $Z_{sd}(G) = f_{sd}(G)h(G)^{\sigma-1}$ when $\varepsilon = \sigma$. So, in that case, Assumption 1 is identical to the one considered in AGM.

For a firm to enter a destination market, its productivity has to exceed a threshold ϕ_{sd}^* , where ϕ_{sd}^* is implicitly defined by zero profits for the first product:

$$P_d^{\sigma-1} T_d [P_{sd}(\phi_{sd}^*; 1)]^{-(\sigma-1)} = \sigma f_{sd}(1).$$

Using the convention $h(1) = 1$ for $G = 1$ in (3) yields

$$(\phi_{sd}^*)^{\sigma-1} = \sigma f_{sd}(1) \frac{(\tilde{\sigma} w_s \tau_{sd})^{\sigma-1}}{P_d^{\sigma-1} T_d}. \quad (4)$$

Similarly, we can define the threshold productivity of selling G products in market d . The firm is indifferent between introducing a G -th product or stopping with an exporter scope of $G-1$ at the product-entry threshold $\phi_{sd}^{*,G}$ if

$$\pi_{sd}(\phi_{sd}^{*,G}; G) - \pi_{sd}(\phi_{sd}^{*,G}; G-1) = 0. \quad (5)$$

Using equations (3) and (4) in this profit equivalence condition, we can solve out for the implicitly defined product-entry threshold $\phi_{sd}^{*,G}$, at which the firm sells G_{sd} or more products,

$$\left(\phi_{sd}^{*,G}\right)^{\sigma-1} = \frac{(\phi_{sd}^*)^{\sigma-1}}{H(G_{sd})^{-(\sigma-1)} - H(G_{sd}-1)^{-(\sigma-1)}} \frac{f_{sd}(G_{sd})}{f_{sd}(1)} = \frac{(\phi_{sd}^*)^{\sigma-1}}{f_{sd}(1)} Z_{sd}(G_{sd}), \quad (6)$$

where we define $\phi_{sd}^{*,1} \equiv \phi_{sd}^*$. So, under Assumption 1, the profit equivalence condition (5) implies that the product-entry thresholds $\phi_{sd}^{*,G}$ strictly increase with G and more productive firms will weakly raise exporter scope compared to less productive firms. Based on these relationships, we can make the following statement

Export sales can be written succinctly as

$$\begin{aligned} t_{sd}(\phi) &= \left(\tilde{\sigma} \frac{w_s \tau_{sd}}{\phi}\right)^{1-\varepsilon} P_d^{\sigma-1} T_d \sum_{g=1}^{G_{sd}} h(g)^{1-\varepsilon} [P_d(G_{sd}(\phi))]^{\varepsilon-\sigma} \\ &= \sigma f_{sd}(1) \left(\frac{\phi}{\phi_{sd}^*}\right)^{\sigma-1} H(G_{sd}(\phi))^{-(\sigma-1)} \end{aligned} \quad (7)$$

using equation (4). This sales relationship is similar in both models with $\varepsilon \neq \sigma$ and models with $\varepsilon = \sigma$. The only difference between the two types of models is that $H(G_{sd})$ depends on ε by (3). If the term $H(G_{sd})$ converges to a constant for $G_{sd} \rightarrow \infty$, then export sales are Pareto distributed in the upper tail if ϕ is Pareto distributed.

Proposition 1 *Suppose Assumption 1 holds. Then for all s, d :*

- *exporter scope $G_{sd}(\phi)$ is positive and weakly increases in ϕ for $\phi \geq \phi_{sd}^*$;*
- *total firm exports $t_{sd}(\phi)$ are positive and strictly increase in ϕ for $\phi \geq \phi_{sd}^*$.*

Proof. The first statement follows directly from the discussion above. The second statement follows because $H(G_{sd}(\phi))^{-(\sigma-1)}$ strictly increases in $G_{sd}(\phi)$ and $G_{sd}(\phi)$ weakly increases in ϕ so that $t_{sd}(\phi)$ strictly increases in ϕ by (7). ■

Similar to AGM, we define *exporter scale* (an exporter's mean sales) in market d as

$$a_{sd}(\phi) = \sigma f_{sd}(1) \left(\frac{\phi}{\phi_{sd}^*}\right)^{\sigma-1} \frac{H(G_{sd}(\phi))^{1-\sigma}}{G_{sd}(\phi)}$$

Under a mild condition, exporter scale $a_{sd}(\phi)$ increases with ϕ and thus with a firm's total sales $t_{sd}(\phi)$. The following sufficient condition ensures that exporter scale increases with total sales.

Case 1 *The function $Z_{sd}(g)$ strictly increases in g with an elasticity*

$$\frac{\partial \ln Z_{sd}(g)}{\partial \ln g} > 1.$$

Case 1 is more restrictive than Assumption 1 in that the condition not only requires Z_{sd} to increase with g but that the increase be more than proportional. We can formally state the following result.

Proposition 2 *If $Z_{sd}(g)$ satisfies Case 1, then sales per export product $a_{sd}(\phi)$ strictly increase at the discrete points $\phi = \phi_{sd}^*, \phi_{sd}^{*,2}, \phi_{sd}^{*,3}, \dots$*

Proof. Compared to AGM, $Z_{sd}(g)$ is defined in more general terms, but it enters the relevant relationships in the same way as in AGM before. Case 1 therefore also suffices in the nested-utility model, and the proposition holds (see the Appendix in AGM for details of the proof for non-nested utility). ■

1.1.2 Within-firm sales distribution

We revisit optimal sales per product and their relationship to exporter scope and the product's rank in a firm's sales distribution. The relationship lends itself to estimation in micro data. Using the productivity thresholds for firm entry (4) and product entry (6) in optimal sales (2) and simplifying yields

$$\begin{aligned} p_{sdg}(\phi)x_{sdg}(\phi) &= \sigma Z_{sd}(G_{sd})H(G_{sd})^{\varepsilon-\sigma} \left(\frac{\phi}{\phi_{sd}^{*,G}} \right)^{\sigma-1} h(g)^{-(\varepsilon-1)} \\ &= \sigma \frac{f_{sd}(G_{sd})H(G_{sd})^{\varepsilon-1}}{1 - [1 - h(G_{sd})^{-(\varepsilon-1)}/H(G_{sd})^{-(\varepsilon-1)}]^{\frac{\sigma-1}{\varepsilon-1}}} \left(\frac{\phi}{\phi_{sd}^{*,G}} \right)^{\sigma-1} h(g)^{-(\varepsilon-1)}. \end{aligned} \quad (8)$$

Note that $H(G)^{\varepsilon-\sigma}$ strictly falls in G if $\varepsilon > \sigma$. Under Case 1, the term $Z_{sd}(G_{sd})H(G_{sd})^{\varepsilon-\sigma}$ must strictly increase in G , however, because individual product sales strictly drop as the product index g increases and $h(g)^{-(\varepsilon-1)}$ falls. So, if $Z_{sd}(G_{sd})H(G_{sd})^{\varepsilon-\sigma}$ did not strictly increase in G , average sales per product would not strictly increase, contrary to Proposition 2.

Compared to AGM, the relationship (8) is not log-linear if $\varepsilon \neq \sigma$ and requires a non-linear estimator, similar to the general case in continuous product space (Arkolakis and Muendler 2010).

1.2 Aggregation

To derive clear predictions for the model equilibrium we specify a Pareto distribution of firm productivity following Helpman, Melitz and Yeaple (2004) and Chaney (2008). A firm's productivity ϕ is drawn from a Pareto distribution with a source-country dependent location parameter b_s and a shape parameter θ over the support $[b_s, +\infty)$ for all destinations s . The cumulative distribution function of ϕ is $\Pr = 1 - (b_s)^\theta/\phi^\theta$ and the probability density function is $\theta(b_s)^\theta/\phi^{\theta+1}$, where more advanced countries are thought to have a higher location parameter b_s . Therefore the measure of firms selling to country d , that is the measure of firms with productivity above the threshold ϕ_{sd}^* , is

$$M_{sd} = J_s \frac{b_s^\theta}{(\phi_{sd}^*)^\theta}. \quad (9)$$

As a result, the probability density function of the conditional productivity distribution for entrants is given by

$$\mu_{sd}(\phi) = \begin{cases} \theta(\phi_{sd}^*)^\theta / \phi^{\theta+1} & \text{if } \phi \geq \phi_{sd}^* \\ 0 & \text{otherwise.} \end{cases} \quad (10)$$

We define the resulting Pareto shape parameter of the total sales distribution as $\tilde{\theta} \equiv \theta/(\sigma-1)$.

With these distributional assumptions we can compute a number of aggregate statistics from the model. We denote aggregate bilateral sales of firms from s to country d as T_{sd} . The corresponding average sales are defined as \bar{T}_{sd} , so that $T_{sd} = M_{sd}\bar{T}_{sd}$ and

$$\bar{T}_{sd} = \int_{\phi_{sd}^*} t_{sd}(\phi) \mu_{sd}(\phi) d\phi. \quad (11)$$

Similarly, we define average local entry costs as

$$\bar{F}_{sd} = \int_{\phi_{sd}^*} F_{sd}(G_{sd}(\phi)) \mu_{sd}(\phi) d\phi.$$

To compute \bar{T}_{sd} , we impose two additional assumptions.

Assumption 2 *Parameters are such that $\theta > \sigma - 1$.*

Assumption 3 *Parameters are such that $\tilde{F}_{sd} \equiv \sum_{G=1}^{\infty} f_{sd}(G)^{1-\tilde{\theta}} [H(G)^{1-\sigma} - H(G-1)^{1-\sigma}]^{\tilde{\theta}}$ is strictly positive and finite.*

Then we can make the following statement.

Proposition 3 *Suppose Assumptions 1, 2 and 3 hold. Then average sales \bar{T}_{sd} per firm are a constant multiple of average local entry costs \bar{F}_{sd}*

$$\bar{T}_{sd} = \frac{\tilde{\theta}\sigma}{\tilde{\theta}-1} \bar{F}_{sd} = f_{sd}(1)^{\tilde{\theta}} \tilde{F}_{sd}.$$

Proof. See Appendix C. ■

As a result, bilateral expenditure trade shares can be expressed as

$$\lambda_{sd} = \frac{M_{sd}\bar{T}_{sd}}{\sum_k M_{kd}\bar{T}_{kd}} = \frac{J_s(b_s)^\theta (w_s \tau_{sd})^{-\theta} f_{sd}(1)^{-\tilde{\theta}} \bar{F}_{sd}}{\sum_k J_k(b_k)^\theta (w_k \tau_{kd})^{-\theta} f_{kd}(1)^{-\tilde{\theta}} \bar{F}_{kd}}, \quad (12)$$

an expression that depends on the values of ε and σ only insofar as these parameters affect \bar{F}_{sd} through $H(G)$.

We can also compute mean exporter scope at a destination:

$$\begin{aligned}
\bar{G}_{sd} &= \int_{\phi_{sd}^*} G_{sd}(\phi) \mu_{sd}(\phi) d\phi \\
&= (\phi_{sd}^*)^\theta \theta \left[\int_{\phi_{sd}^*}^{\phi_{sd}^{*,2}} \phi^{-\theta-1} d\phi + \int_{\phi_{sd}^{*,2}}^{\phi_{sd}^{*,3}} 2\phi^{-\theta-1} d\phi + \dots \right] \\
&= \frac{(\phi_{sd}^{*,2})^{-\theta} - (\phi_{sd}^*)^{-\theta}}{(\phi_{sd}^*)^{-\theta}} + \frac{(\phi_{sd}^{*,3})^{-\theta} - (\phi_{sd}^{*,2})^{-\theta}}{(\phi_{sd}^*)^{-\theta}} + \dots
\end{aligned}$$

Completing the integration, rearranging terms and using equation (6), we obtain

$$\bar{G}_{sd} = f_{sd}(1)^{\tilde{\theta}} \sum_{g=1}^{\infty} Z_{sd}(g)^{-\tilde{\theta}}. \tag{13}$$

For the average number of products to be a well defined and finite number we require

Assumption 4 *Parameters are such that $\sum_{g=1}^{\infty} Z_{sd}(g)^{-\tilde{\theta}}$ is strictly positive and finite.*

In Appendix C we show that fixed costs expenditure is a constant share of firm sales (where we denote means using a bar), as summarized in Proposition 3:

$$\frac{\bar{F}_{sd}}{\bar{T}_{sd}} = \frac{\tilde{\theta} - 1}{\tilde{\theta}\sigma}.$$

We derive aggregate welfare in Appendix D and demonstrate in Appendix E that wage income and profit income can be expressed as a constant share of total output y_s per capita:

$$\pi_s = \eta y_s, \quad w_s = (1 - \eta) y_s,$$

where $\eta \equiv 1/(\tilde{\theta}\sigma)$. Since aggregates of the model do not depend on ε , the equilibrium definition is the same as in AGM.

1.3 Conclusion

We have characterized an extension of the AGM model when the elasticity of substitution between a firm's individual products does not equal the elasticity of substitution across products lines of different firms. The extended model retains the main qualitative implications of the baseline AGM model, in which the two elasticities are the same. Future work using the structure of the generalized model to obtain estimates of the two elasticities may lead to a better understanding of the substitution effects within and across firms.

2 Data: Exported Products and Export Destinations

Tables 1 and 2 show Brazil's top ten export destinations by number of exporters and the top ten exported HS 6-digit product codes by total value in 2000. In Table 1, Argentina is the most common export destination and the United States receives most Brazilian exports in value. In Table 2, medium-sized aircraft is the leading export product in value, followed by wood pulp and biofuel products.

Table 1: **Top Brazilian Export Destinations**

| Destination | # Exporters | Export Value (USD) |
|-------------|-------------|--------------------|
| Argentina | 4,590 | 5,472,333,618 |
| Uruguay | 3,251 | 504,642,201 |
| USA | 3,083 | 9,772,577,557 |
| Chile | 2,342 | 1,145,161,210 |
| Paraguay | 2,319 | 561,065,104 |
| Bolivia | 1,799 | 282,543,791 |
| Mexico | 1,336 | 1,554,452,204 |
| Venezuela | 1,333 | 658,281,591 |
| Germany | 1,217 | 1,364,610,059 |
| Peru | 1,191 | 329,896,577 |

Source: SECEX 2000, manufacturing firms and their manufactured products.

Table 2: **Top Brazilian Exported Items**

| HS Code | Description | Value (USD) |
|---------|--|---------------|
| 880230 | Airplanes between 2 and 15 tons | 2,618,856,983 |
| 470329 | Bleached non-coniferous chemical wood pulp | 1,523,403,942 |
| 230400 | Soybean oil-cake and other solid residues | 1,245,752,048 |
| 870323 | Passenger vehicles between 1,500 and 3,000 cc | 1,197,222,859 |
| 852520 | Transmission apparatus incorporating reception apparatus | 926,618,451 |
| 640399 | Footwear, with outer soles | 854,950,667 |
| 720712 | Semifinished products of iron or nonalloy steel | 802,801,270 |
| 760110 | Unwrought aluminum, not alloyed | 765,195,563 |
| 200911 | Orange juice, frozen | 561,103,666 |
| 170111 | Raw solid cane sugar | 520,544,094 |

Source: SECEX 2000, manufacturing firms and their manufactured products.

3 Sensitivity of Monte Carlo Estimates

To document identification under our simulated method of moments estimator, we run Monte Carlo tests with generated data. We generate 333,000 Brazilian firms under the initial parameters Θ , where

$$\Theta = \{\delta_1, \delta_2, \tilde{\alpha}, \tilde{\theta}, \sigma_\xi, \sigma_c\} = \{-1.17, -0.90, 1.73, 1.84, 1.89, 0.53\}$$

reflects the baseline estimates from Table 2 in the main text. The generated data have approximately 10,000 exporters. We then apply our simulated method of moments routine to the generated data and find the optimum, recovering an estimate of the parameter vector $\hat{\Theta}$. We repeat the data generation and estimation procedure 30 times and report in Table 3 the mean and standard deviation of the elements of $\hat{\Theta}$.

The Monte Carlo results in Table 3 document that our procedure accurately pinpoints all parameters of interest. In particular $(\hat{\alpha}, \hat{\sigma}_\xi, \hat{\sigma}_c)$ are precisely estimated, with point estimates close to the initial parameters behind the generated data and with standard errors less than 2 percent of the true value. Similarly, $\hat{\delta}$ and $\hat{\theta}$ are estimated close to their true values, their standard errors are under 4 percent of their true values. The proximity of our parameter estimates to the initial parameters underlying the data generation, and their precision, substantiate that our simulated method of moments estimator identifies the AGM model’s parameters of interest.

Table 3: Monte Carlo Results

| Θ | δ_1 | δ_2 | $\tilde{\alpha}$ | $\tilde{\theta}$ | σ_ξ | σ_c | $\delta_1 - \delta_2$ |
|-----------------------------|------------|------------|------------------|------------------|--------------|------------|-----------------------|
| Parameter of generated data | -1.17 | -0.90 | 1.73 | 1.84 | 1.89 | 0.53 | -0.27 |
| Estimate (mean) | -1.21 | -0.93 | 1.71 | 1.91 | 1.88 | 0.51 | -0.28 |
| (s.e.) | (0.03) | (0.03) | (0.02) | (0.05) | (0.03) | (0.01) | (0.01) |

4 Sensitivity Analysis

To assess the robustness of our baseline estimates in AGM, we perform a number of modifications to our main specification. Overall, we find that our baseline results are remarkably robust to sample restrictions and alternative variable definitions.

4.1 Adjusted sales

Our baseline estimates imply a large and statistically significant difference between δ_{LAC} and δ_{ROW} . In a first robustness check, we strive to rule out that this difference could be driven by different typical sales across sets of products that Brazilian firms ship to LAC and non-LAC countries. We therefore correct sales and control for the mean sales of product groups at the HS 2-digit level. Concretely, we take the upward or downward deviation of a firm’s HS 6-digit product sales to a destination $\log y_{\omega d}^p$ from the worldwide product-group sales mean of Brazilian exporters:

$$\tilde{y}_{\omega dg} = \exp \left\{ \log y_{\omega dg} - \frac{1}{M} \sum_{\omega} \frac{1}{N} \sum_d \sum_{g \in \text{HS } 2} \log y_{\omega dg} \right\}.$$

This adjustment does not reduce the sample size. We report the results in the row labeled *1. Adjusted sales* in Table 4. The estimates are broadly consistent with the baseline, but the estimated scope elasticities of market access costs δ and of product efficiency $\tilde{\alpha}$ are lower in absolute magnitude, and so is the estimated Pareto shape parameter $\tilde{\theta}$. These estimates imply that both the within-firm product distribution is more concentrated in the top product and the between-firm sales distribution has more firms in the upper tail with extremely high sales. An intuitive explanation is that demeaning sales by the average exporter’s typical sale in a product group exacerbates sales deviations of specific products, thus making distributions appear more extreme. However, signs of our estimates stay the same and broad magnitudes remain qualitatively similar to the baseline estimates.

4.2 Advanced manufacturing

A related robustness concern is that estimation could be driven by different feasible exporter scopes across product groups that Brazilian firms ship to LAC and non-LAC destination. For example, more differentiated industries, or more technology driven industries, might allow for the export of more potential varieties, or the HS classification system might simply provide more individual HS 6-digit products within more refined HS 2-digit product groups. In a second robustness exercise we therefore split the sample into firms that are active in relatively “advanced manufacturing” industries. We present results from our definition of advanced manufacturing as three top-level NACE sectors “Manufacture of machinery and equipment n.e.c”, “Manufacture of electrical and optical equipment” and “Manufacture of transport equipment” (codes DK, DL and DM).

Under this sectoral restriction, a markedly reduced sample size of only 2,539 Brazilian manufacturing exporters remains. Despite the considerable drop in sample size, however,

results in the row labeled *2. Advanced manufacturing* in Table 4 are broadly consistent with the baseline. In advanced manufacturing industries, the difference in scope elasticities of market access costs δ is larger between LAC and non-LAC countries. While market access costs drop off even faster with scope in LAC countries in advanced manufacturing industries, in non-LAC destination the converse is the case: market access costs remain more elevated at higher scopes than in the average manufacturing industry. The gap between the LAC and non-LAC scope elasticities of market access costs is almost double as wide in advanced manufacturing industries as it is in the average industry. For our counterfactual exercise, this wider difference of market access cost elasticities between LAC and non-LAC countries implies even more pronounced benefits of harmonizing market access costs across the world. Other parameter estimates are similar to the baseline, and specially the scope elasticity of product efficiency $\tilde{\alpha}$ is not statistically significantly different in advanced industries compared to the average industry. Overall, every sign remains the same and estimates that are statistically different from the baseline remain comparable in their qualitative economic implications.

4.3 Eight-digit NCM product categories

To make our results closely comparable to evidence from other countries, in our main text we define a product as a Harmonized System (HS) 6-digit code, which is internationally comparable by requirement of the World Customs Organization (WCO) across its 200 member countries. To query the potential sensitivity of our results to a refined product classification, we use the Mercosur 8-digit level (*Nomenclatura Comum do Mercosul* NCM8), which roughly corresponds to the 8-digit HS level by the World Customs Organization. As the row *3. NCM 8-digit manufacturing* in Table 4 shows, our results are hardly sensitive at all to the change in level of disaggregation. No single estimate is statistically different from our baseline estimates.

4.4 Sensitivity to destinations Argentina and United States

Two destination markets dominate Brazilian manufacturing exports: Argentina (the top destination in terms of exporter counts) and the United States (the top destination in terms of export value). To assure ourselves that the estimates are not driven by potential outlier behavior of export flows to those two destinations, we remove them from the sample. As Table 4 in the row labeled *4. Dropping ARG, USA* shows, only the estimate of the firm size distribution's shape parameter $\tilde{\theta}$ becomes statistically significantly different from the baseline estimate. All other estimates are statistically indistinguishable from the baseline estimates. When we omit Argentina and the United States, the higher estimate for the Pareto tail index $\tilde{\theta}$ implies a lower probability mass in the upper tail of firms with extremely high sales. Even though Argentina and the United States attract a large number of export entrants from Brazil, these markets also exhibit a stronger concentration of exports among just a few top-selling firms than the average Brazilian export destination.

Table 4: **Robustness for Select Subsamples**

| estimate (s.e.) | δ_{LAC} | δ_{ROW} | $\tilde{\alpha}$ | $\tilde{\theta}$ | σ_{ξ} | σ_c | $\delta_{\text{LAC}} - \delta_{\text{ROW}}$ |
|---|-----------------------|-----------------------|------------------|------------------|----------------|----------------|---|
| Baseline (all manufacturing) | -1.17 (0.09) | -0.90 (0.11) | 1.73 (0.09) | 1.84 (0.09) | 1.89 (0.04) | 0.53 (0.03) | -0.27 (0.05) |
| 1. Adjusted sales | -0.89 (0.06) | -0.70 (0.06) | 1.53 (0.06) | 1.63 (0.07) | 1.92 (0.03) | 0.60 (0.02) | -0.18 (0.03) |
| 2. Advanced manufacturing | -1.27 (0.14) | -0.78 (0.17) | 1.77 (0.12) | 1.64 (0.23) | 2.01 (0.08) | 0.58 (0.06) | -0.50 (0.13) |
| 3. NCM 8-digit manufacturing | -1.16 (0.06) | -0.84 (0.08) | 1.72 (0.05) | 1.76 (0.10) | 1.82 (0.04) | 0.55 (0.03) | -0.32 (0.05) |
| 4. Dropping ARG, USA (all manufacturing) | -1.22 (0.08) | -0.86 (0.07) | 1.75 (0.07) | 2.04 (0.09) | 1.73 (0.04) | 0.55 (0.02) | -0.36 (0.07) |
| 5. Exporter share 10 percent | -1.43 (0.11) | -1.19 (0.14) | 1.98 (0.10) | 1.86 (0.11) | 1.81 (0.04) | 0.54 (0.03) | -0.24 (0.05) |

Source: SECEX 2000, manufacturing firms and their manufactured products.

Note: Products at the HS 6-digit level. Estimates of δ_{LAC} indicates the scope elasticity for incremental product access costs for Brazilian firms shipping to other LAC destinations. Similarly δ_{ROW} perform the same role for exports to non-LAC destinations. See text for full description of various specifications.

4.5 Exporter share

An arguably important moment for our simulated method of moments is the share of formally established Brazilian manufacturing firms that export. Among the universe of Brazilian firms with at least one employee, only three percent of firms are exporters in 2000. This share is similar to that observed in other countries, for which data on the universe of firms with at least one employee is available. However, censuses and surveys in most developing and some industrialized countries truncate their target population of firms from below with thresholds up to 20 employees. To query sensitivity of our estimates to the share of exporters, we hypothetically consider an alternative share of 10 percent of Brazilian firms exporting. This exercise serves two purposes. First, comparisons of our findings to future results in other countries may depend on using a hypothetically truncated target population of firms from below. Second, we can check how our results might depend on a hypothetically more export oriented manufacturing sector such as, for instance, the U.S. manufacturing sector.

Table 4 reports the results in the row labeled 5. *Exporter share 10 percent*. Compared to the baseline, the scope elasticities of market access costs δ and of product efficiency $\tilde{\alpha}$ increase in absolute magnitude. Intuitively, the estimator tries to “explain” the hypothetically higher share of exporters with relatively faster declines in market access costs as exporter scope increases but to offset those access cost reductions with relatively steeper declines in product efficiency away from core competency, so as to keep matching the overall pattern of exporter scopes across destinations. The other three parameter esti-

Table 5: **Alternative Regional Aggregates**

| δ_1 | δ_2 | $\delta_1 - \delta_2$ |
|-----------------------------------|------------------------------|-----------------------|
| Baseline | Baseline | -0.27 |
| Latin America and Caribbean (LAC) | Rest of World | (0.05) |
| 1. Mercosur | Rest of LAC (Non-Mercosur) | -0.03 (0.03) |
| 2. Mercosur | Rest of World (Non-Mercosur) | -0.18 (0.03) |

Source: SECEX 2000, manufacturing firms and their manufactured products.

Note: Products at the HS 6-digit level.

mates remain similar to the baseline estimates. This final robustness exercise therefore clarifies how the firm entry margin influences identification: if firm entry with the first product were hypothetically more prevalent, then for a given common market access cost component $f_{sd}(1)$ the access cost schedule would need to decline faster with scope, leading to wider exporter scopes everywhere, unless production efficiency also declines faster with scope.

4.6 Sensitivity to Mercosur

In a final set of robustness exercises, we alternate the pairings of regional aggregates. In the baseline, we split the world into LAC (Latin American and the Caribbean) and the Rest of the World (non-LAC). In a first alteration, we drop destination countries outside of LAC from our sample and split LAC into Mercosur destinations in 2000 (Argentina, Paraguay, Uruguay) and non-Mercosur destinations. In the row labeled *1. Mercosur–Rest of LAC*, Table 5 reports the results for the difference in the scope elasticities of market access costs between the two sub-regions within LAC, and the difference is negative as in the baseline but small (and not statistically different from zero). This finding justifies our treatment of LAC in the baseline as a relatively homogeneous region for Brazilian exporters. In a second alteration of the regional split, we discern between Mercosur destinations in 2000 (Argentina, Paraguay, Uruguay) and the Rest of the World, where the Rest of the World includes LAC countries outside Mercosur as well as non-LAC destinations. Expectedly, given the earlier results in the baseline and in the first alteration, the difference is negative but not quite as pronounced in magnitude as the difference between LAC and the Rest of the World. We therefore conclude that LAC countries outside Mercosur are more similar to Mercosur than to the Rest of the World and consider our baseline split of destinations into LAC and non-LAC an adequate country grouping.

Appendix

A Optimal product prices

We characterize the first-order conditions for the firm's optimal pricing rules at every destination d . There are $G_{sd}(\phi)$ first-order conditions with respect to p_{sdg} . For any $G_{sd}(\phi)$, taking the first derivative of profits $\pi_{sd}(\phi)$ from (1) with respect to p_{sdg} and dividing by $p_{sdg}^{-\varepsilon} P_{sd}(\phi; G_{sd})^{\varepsilon-\sigma} P_d^{\sigma-1} T_d$ yields

$$\begin{aligned} \frac{\partial \pi_{sd}(\phi)}{\partial p_{sdg}} = & P_d^{\sigma-1} T_d \cdot P_{sd}(\phi; G_{sd})^{\varepsilon-\sigma} p_{sdg}^{-\varepsilon} \left\{ 1 - \varepsilon \left(1 - \frac{w_s}{\phi/h(g)} \tau_{sd} p_{sdg}^{-1} \right) \right. \\ & \left. + (\varepsilon - \sigma) P_{sd}(\phi; G_{sd})^{\varepsilon-1} \sum_{k=1}^{G_{sd}(\phi)} \left(p_{sdk} - \frac{w_s}{\phi/h(k)} \tau_{sd} \right) p_{sdk}^{-\varepsilon} \right\}. \end{aligned} \quad (\text{A.1})$$

The first-order conditions require that (A.1) equals zero for all products $g = 1, \dots, G_{sd}(\phi)$. Use the first-order conditions for any two products g and g' and reformulate to find

$$p_{sdg}/p_{sdg'} = h(g)/h(g').$$

So the firm must optimally charge an identical markup over the marginal costs for all products. Define this optimal markup as \bar{m} . To solve out for \bar{m} in terms of primitives, use $p_{sdg} = \bar{m} w_s \tau_{sd} / [\phi/h(g)]$ in the first-order condition above and simplify:

$$1 - \varepsilon \frac{1}{\bar{m}} + (\varepsilon - \sigma) P_{sd}(\phi; G_{sd})^{\varepsilon-1} \frac{\bar{m} - 1}{\bar{m}} \sum_{k=1}^{G_{sd}(\phi)} p_{sdk}^{-(\varepsilon-1)} = 0.$$

Note that $\sum_{k=1}^{G_{sd}(\phi)} p_{sdk}^{-(\varepsilon-1)} = P_{sd}(\phi; G_{sd})^{-(\varepsilon-1)}$. Solving the first-order condition for \bar{m} , we find the optimal markup over each product g 's marginal cost

$$\bar{m} = \tilde{\sigma} \equiv \sigma / (\sigma - 1).$$

A firm with productivity ϕ optimally charges a price

$$p_{sdg}(\phi) = \tilde{\sigma} w_s \tau_{sd} / [\phi/h(g)] \quad (\text{A.2})$$

for its products $g = 1, \dots, G_{sd}(\phi)$.

B Second-order conditions

We now turn to the second-order conditions for price choice. To find the entries along the diagonal of the Hessian matrix, take the first derivative of condition (A.1) with respect to

the own price p_{sdg} and then replace $w_s \tau_{sd} / [\phi / h(g)] = p_{sdg}(\phi) / \tilde{\sigma}$ by the first-order condition to find

$$\begin{aligned} \frac{\partial^2 \pi_{sd}(\phi)}{(\partial p_{sdg})^2} &= P_d^{\sigma-1} T_d \cdot P_{sd}(\phi; G_{sd})^{\varepsilon-\sigma} p_{sdg}^{-\varepsilon} \left\{ -\frac{\varepsilon}{\tilde{\sigma}} p_{sdg}^{-1} \right. \\ &\quad + (\varepsilon - \sigma) P_{sd}(\phi; G_{sd})^{\varepsilon-1} [-(\varepsilon - 1) + \varepsilon / \tilde{\sigma}] p_{sdg}^{-\varepsilon} \\ &\quad \left. + (\varepsilon - \sigma)(\varepsilon - 1) P_{sd}(\phi; G_{sd})^{2(\varepsilon-1)} p_{sdg}^{-\varepsilon} \cdot \sum_{k=1}^{G_{sd}} (1 - 1/\tilde{\sigma}) p_{sdk}^{-(\varepsilon-1)} \right\} \\ &= P_{sd}(\phi; G_{sd})^{\varepsilon-\sigma} P_d^{\sigma-1} T_d \cdot \left\{ -\varepsilon p_{sdg}^{-\varepsilon-1} + (\varepsilon - \sigma) P_{sd}(\phi; G_{sd})^{\varepsilon-1} p_{sdg}^{-2\varepsilon} \right\} / \tilde{\sigma}. \quad (\text{B.3}) \end{aligned}$$

This term is strictly negative if and only if

$$(\varepsilon - \sigma) P_{sd}(\phi; G_{sd})^{\varepsilon-1} p_{sdg}^{-(\varepsilon-1)} < \varepsilon.$$

If $\varepsilon \leq \sigma$, this last condition is satisfied because the left-hand side is weakly negative and $\varepsilon > 0$. If $\varepsilon > \sigma$, then we can rewrite the condition as $p_{sdg}^{-(\varepsilon-1)} / [\sum_{k=1}^{G_{sd}} p_{sdk}^{-(\varepsilon-1)}] < 1 < \varepsilon / (\varepsilon - \sigma)$ so that the condition is satisfied. So, the diagonal entries of the Hessian matrix are strictly negative.

To derive the entries off the diagonal of the Hessian matrix, we take the derivative of condition (A.1) for product g with respect to any other price $p_{sdg'}$ and then replace $w_s \tau_{sd} / [\phi / h(g')] = p_{sdg'}(\phi) / \tilde{\sigma}$ by the first-order condition to find

$$\begin{aligned} \frac{\partial^2 \pi_{sd}(\phi)}{\partial p_{sdg} \partial p_{sdg'}} &= P_d^{\sigma-1} T_d \cdot P_{sd}(\phi; G_{sd})^{\varepsilon-\sigma} p_{sdg}^{-\varepsilon} \left\{ (\varepsilon - \sigma) P_{sd}(\phi; G_{sd})^{\varepsilon-1} [-(\varepsilon - 1) + \varepsilon / \tilde{\sigma}] p_{sdg'}^{-\varepsilon} \right. \\ &\quad \left. + (\varepsilon - \sigma)(\varepsilon - 1) P_{sd}(\phi; G_{sd})^{2(\varepsilon-1)} p_{sdg'}^{-\varepsilon} \sum_{k=1}^{G_{sd}} (1 - 1/\tilde{\sigma}) p_{sdk}^{-(\varepsilon-1)} \right\} \\ &= P_d^{\sigma-1} T_d \cdot (\varepsilon - \sigma) P_{sd}(\phi; G_{sd})^{\varepsilon-\sigma+\varepsilon-1} p_{sdg}^{-\varepsilon} p_{sdg'}^{-\varepsilon} / \tilde{\sigma}. \quad (\text{B.4}) \end{aligned}$$

This term is strictly positive if and only if $\varepsilon > \sigma$.

Having derived the entries of the Hessian matrix, it remains to establish the conditions under which the Hessian is negative definite. We discern two cases. First the case of $\varepsilon \leq \sigma$ and then the case $\varepsilon > \sigma$.

B.1 Negative definiteness of Hessian if $\varepsilon \leq \sigma$

By (B.3) and (B.4), the Hessian matrix can be written as

$$\mathbf{H} = P_{sd}(\phi; G_{sd})^{\varepsilon-\sigma} P_d^{\sigma-1} T_d [\mathbf{H}_A + (\varepsilon - \sigma) P_{sd}(\phi; G_{sd})^{\varepsilon-1} \mathbf{H}_B],$$

where

$$\mathbf{H}_A \equiv \begin{pmatrix} -\varepsilon p_{sd1}^{-\varepsilon-1} & & & \\ 0 & -\varepsilon p_{sd2}^{-\varepsilon-1} & & \\ 0 & 0 & -\varepsilon p_{sd3}^{-\varepsilon-1} & \\ \dots & & & \dots \end{pmatrix} \text{ and } \mathbf{H}_B \equiv \begin{pmatrix} p_{sd1}^{-\varepsilon} p_{sd1}^{-\varepsilon} & & & \\ p_{sd2}^{-\varepsilon} p_{sd1}^{-\varepsilon} & p_{sd2}^{-\varepsilon} p_{sd2}^{-\varepsilon} & & \\ p_{sd3}^{-\varepsilon} p_{sd1}^{-\varepsilon} & p_{sd3}^{-\varepsilon} p_{sd2}^{-\varepsilon} & p_{sd3}^{-\varepsilon} p_{sd3}^{-\varepsilon} & \\ \dots & & & \dots \end{pmatrix}.$$

The Hessian matrix \mathbf{H} is negative definite if and only if the negative Hessian

$$-\mathbf{H} = P_{sd}(\phi; G_{sd})^{\varepsilon-\sigma} P_d^{\sigma-1} T_d [-\mathbf{H}_A + (\sigma-\varepsilon)P_{sd}(\phi; G_{sd})^{\varepsilon-1} \mathbf{H}_B]$$

is positive definite. Note that the sum of one positive definite matrix and any number of positive semidefinite matrices is positive definite. So, if $-\mathbf{H}_A$ and \mathbf{H}_B are positive semidefinite and at least one of the two matrices is positive definite (given $\varepsilon \leq \sigma$), then the Hessian is negative definite.

A necessary and sufficient condition for a matrix to be positive definite is that the leading principal minors of the matrix are positive. The leading principal minors of $-\mathbf{H}_A$ are positive, so $-\mathbf{H}_A$ is positive definite. For \mathbf{H}_B , the first leading principal minor is positive, and all remaining principal minors are equal to zero. So \mathbf{H}_B is positive semidefinite. Therefore the Hessian matrix \mathbf{H} is negative definite.

B.2 Negative definiteness of Hessian if $\varepsilon > \sigma$

Another necessary and sufficient condition for the Hessian matrix \mathbf{H} to be negative definite is that the leading principal minors alternate sign, with the first principal minor being negative. The first principal minor the first diagonal entry is strictly negative as is any diagonal entry by (B.3). An application of the leading principal minor test in our case requires a recursive computation of the determinants of $G_{sd}(\phi)$ submatrices (a solution of polynomials with order up to $G_{sd}(\phi)$). We choose to check for negative definiteness of the Hessian in two separate ways when $\varepsilon > \sigma$. First, we derive a sufficient (but not necessary) condition for negative definiteness of the Hessian and query its empirical validity. Second, we present a necessary (but not sufficient) condition for negative definiteness of the Hessian for any pair of two products.

Sufficiency. A sufficient condition for the Hessian to be negative definite is due to McKenzie (1960): a symmetric diagonally dominant matrix with strictly negative diagonal entries is negative definite. A matrix is diagonally dominant if, in every row, the absolute value of the diagonal entry strictly exceeds the sum of the absolute values of all off-diagonal entries. By our derivations above, all diagonal entries of the Hessian are strictly negative.

For $\varepsilon > \sigma$, the condition for the Hessian to be diagonally dominant is

$$\sum_{k \neq g}^{G_{sd}} (\varepsilon - \sigma) P_{sd}(\phi; G_{sd})^{\varepsilon-1} p_{sdk}^{-\varepsilon} p_{sdg}^{-\varepsilon} < \varepsilon p_{sdg}^{-\varepsilon-1} - (\varepsilon - \sigma) P_{sd}(\phi; G_{sd})^{\varepsilon-1} p_{sdg}^{-2\varepsilon}$$

for all of a firm ϕ 's products (rows of its Hessian), where we cancelled the strictly positive terms $P_d^{\sigma-1} T_d P_{sd}(\phi; G_{sd})^{\varepsilon-\sigma} / \tilde{\sigma}$ from the inequality. So, for $\varepsilon = \sigma$ the Hessian is diagonally dominant.

Using the optimal price (A.2) of product g from the first-order condition and rearranging terms yields the following condition

$$\frac{\sum_{k=1}^{G_{sd}} h(k)^{-\varepsilon}}{\sum_{k=1}^{G_{sd}} h(k)^{-(\varepsilon-1)}} < \frac{\varepsilon}{\varepsilon - \sigma} h(g)^{-1} \quad (\text{B.5})$$

for the Hessian to be a diagonally dominant matrix at the optimum.

By convention and without loss of generality $h(1) = 1$ for a firm with productivity ϕ . So the product efficiency schedule $h(g)$ strictly exceeds unity for the second product and subsequent products. As a result, the left-hand side of the inequality is bounded above for an exporter with a scope of at least two products at a destination:

$$\frac{\sum_{k=1}^{G_{sd}} h(k)^{-\varepsilon}}{\sum_{k=1}^{G_{sd}} h(k)^{-(\varepsilon-1)}} < \frac{\sum_{k=1}^{G_{sd}} h(k)^{-(\varepsilon-1)}}{\sum_{k=1}^{G_{sd}} h(k)^{-(\varepsilon-1)}} = 1.$$

A sufficient (but not necessary) condition for the Hessian to be negative definite is therefore

$$1 \leq h(g) < \frac{\varepsilon}{\varepsilon - \sigma}$$

for all of the firm's products. However, the Hessian can still be negative definite even if this condition fails. Clearly, the Hessian becomes negative definite the closer is ε to σ because then the off-diagonal entries approach zero and the Hessian is trivially diagonally dominant. Moreover, the Hessian can be negative definite even if it is not a diagonally dominant matrix.

To query the empirical validity of the sufficient condition $h(g) < \varepsilon/(\varepsilon - \sigma)$, consider evidence on products and brands in Broda and Weinstein (2006). Their preferred estimates for ε and σ within and across domestic U.S. brand modules are 11.5 and 7.5. Estimates in Arkolakis et al. (2014) suggest that $\alpha(\varepsilon - 1)$ is around 1.84 under the specification that $h(g) = g^\alpha$. These parameters imply that the condition $h(g) < \varepsilon/(\varepsilon - \sigma)$ is satisfied for Hessians with up to 414 products. In the Arkolakis et al. (2014) data, no firm-country observations involve 415 or more products in a market (with a median of one product and a mean of 3.52). Even if additional products individually violate the sufficient condition, Hessians with more products may still be negative definite.

Necessity. Consider any two products g and g' . Negative definiteness of the Hessian must be independent of the ordering of products, so these two products can be assigned the first and second row in the Hessian without loss of generality. As stated before, a necessary and sufficient condition for the Hessian to be negative definite is that the leading principal minors of the Hessian alternate sign, with the first principal minor being negative. So a necessary condition for the Hessian to be negative definite is that the principal minors of any two products (first and second in the Hessian) alternate sign, with the first principal minor negative and the second positive.

The first principle minor is strictly negative because all diagonal entries are strictly negative by (B.3). The second principal minor is strictly positive if and only if the determinant satisfies

$$2\varepsilon^2 P_{sd}(\phi; G_{sd})^{-(\varepsilon-1)} - \varepsilon(\varepsilon - \sigma) \left(p_{sdg}^{-(\varepsilon-1)} + p_{sdg'}^{-(\varepsilon-1)} \right) - (\varepsilon - \sigma)^2 (p_{sdg} p_{sdg'})^{-(\varepsilon-1)} P_{sd}(\phi; G_{sd})^{\varepsilon-1} > 0, \quad (\text{B.6})$$

where we cancelled the strictly positive terms $P_d^{2(\sigma-1)} T_d P_{sd}(\phi; G_{sd})^{2(\varepsilon-\sigma)} / \tilde{\sigma}^2$ from the inequality and multiplied both sides by $p_{sdg}^{\varepsilon-1} p_{sdg'}^{\varepsilon-1} P_{sd}(\phi; G_{sd})^{-(\varepsilon-1)}$.

To build intuition, consider the dual-product case with $G_{sd}(\phi) = 2$. Then condition (B.6) simplifies to

$$\frac{h(g)^{-(\varepsilon-1)}}{\sum_{k=1}^{G_{sd}} h(k)^{-(\varepsilon-1)}} \cdot \frac{h(g')^{-(\varepsilon-1)}}{\sum_{k=1}^{G_{sd}} h(k)^{-(\varepsilon-1)}} < \frac{\varepsilon}{\varepsilon-\sigma} \frac{\varepsilon+\sigma}{\varepsilon-\sigma}.$$

For $\varepsilon > \sigma$, both terms in the product on the right-hand side strictly exceed unity while the terms in the product on the left-hand side are strictly less than one, and the condition is satisfied.

In the multi-product case with $G_{sd}(\phi) > 2$, replace $p_{sdg}^{-(\varepsilon-1)} + p_{sdg'}^{-(\varepsilon-1)} = P_{sd}(\phi; G_{sd})^{-(\varepsilon-1)} - \sum_{k \neq g, g'} p_{sdk}^{-(\varepsilon-1)}$ in condition (B.6) and simplify to find

$$\frac{h(g)^{-(\varepsilon-1)}}{\sum_{k=1}^{G_{sd}} h(k)^{-(\varepsilon-1)}} \cdot \frac{h(g')^{-(\varepsilon-1)}}{\sum_{k=1}^{G_{sd}} h(k)^{-(\varepsilon-1)}} < \frac{\varepsilon}{\varepsilon-\sigma} \frac{\varepsilon+\sigma}{\varepsilon-\sigma} + \frac{\varepsilon}{\varepsilon-\sigma} \frac{\sum_{k \neq g, g'} h(k)^{-(\varepsilon-1)}}{\sum_{k=1}^{G_{sd}} h(k)^{-(\varepsilon-1)}}.$$

For $\varepsilon > \sigma$, the necessary condition on any two products of a multi-product firm is trivially satisfied by the above derivations because the additional additive term on the right-hand side is strictly positive.

In summary, parameters of our model are such that, for any two products of a multi-product firm, the second-order condition is satisfied.

C Proof of Proposition 3

Average sales from s to d are

$$\bar{T}_{sd} = \int_{\phi_{sd}^*} y_{sd}(G_{sd}) \frac{\theta (\phi_{sd}^*)^\theta}{\phi^{\theta+1}} d\phi = \sigma f_{sd}(1) \theta \int_{\phi_{sd}^*} \frac{\phi^{\sigma-2-\theta} / (\phi_{sd}^*)^{\sigma-1-\theta}}{H(G_{sd}(\phi))^{\sigma-1}} d\phi.$$

The proof of the proposition follows from the following Lemma.

Lemma 1 *Suppose Assumptions 1, 2 and 3 hold. Then*

$$\int_{\phi_{sd}^*} \frac{\phi^{\sigma-2-\theta} / (\phi_{sd}^*)^{\sigma-1-\theta}}{H(G_{sd}(\phi))^{\sigma-1}} d\phi = \frac{f_{sd}(1)^{\tilde{\theta}-1}}{\theta - (\sigma-1)} \tilde{F}_{sd},$$

where

$$\tilde{F}_{sd} \equiv \sum_{v=1}^{\infty} \frac{[f_{sd}(v)]^{1-\tilde{\theta}}}{[H(v)^{1-\sigma} - H(v-1)^{1-\sigma}]^{-\tilde{\theta}}}.$$

Proof. Note that

$$\begin{aligned} \int_{\phi_{sd}^*} \frac{\phi^{\sigma-2-\theta} / (\phi_{sd}^*)^{\sigma-1-\theta}}{H(G_{sd}(\phi))^{\sigma-1}} d\phi &= H(1)^{1-\sigma} \int_{\phi_{sd}^*}^{\phi_{sd}^{*,2}} \phi^{\sigma-2-\theta} d\phi + H(2)^{1-\sigma} \int_{\phi_{sd}^{*,2}}^{\phi_{sd}^{*,3}} \phi^{\sigma-2-\theta} d\phi + \dots \\ &= H(1)^{1-\sigma} \left[\frac{(\phi_{sd}^{*,2})^{\sigma-1-\theta} - (\phi_{sd}^*)^{\sigma-1-\theta}}{[\theta - (\sigma-1)] (\phi_{sd}^*)^{\sigma-1-\theta}} \right] \\ &\quad + H(2)^{1-\sigma} \left[\frac{(\phi_{sd}^{*,3})^{\sigma-1-\theta} - (\phi_{sd}^{*,2})^{\sigma-1-\theta}}{[\theta - (\sigma-1)] (\phi_{sd}^*)^{\sigma-1-\theta}} \right] + \dots \end{aligned}$$

Also note that, using equations (4) and (6), the ratio $\left[(\phi_{sd}^{*,2})^{\sigma-1-\theta} - (\phi_{sd}^*)^{\sigma-1-\theta} \right] / (\phi_{sd}^*)^{\sigma-1-\theta}$ can be rewritten as

$$\begin{aligned} & \frac{\left(\phi_{sd}^{*,G} \right)^{\sigma-1-\theta} - \left(\phi_{sd}^{*,G-1} \right)^{\sigma-1-\theta}}{\left(\phi_{sd}^* \right)^{\sigma-1-\theta}} = \\ & = \frac{\left[\frac{\left(\phi_{sd}^* \right)^{\sigma-1} f_{sd}(g)}{H(g)^{1-\sigma} - H(g-1)^{1-\sigma}} \right]^{\frac{\sigma-1-\theta}{\sigma-1}} - \left[\frac{\left(\phi_{sd}^* \right)^{\sigma-1} f_{sd}(g-1)}{H(g-1)^{1-\sigma} - H(g-2)^{1-\sigma}} \right]^{\frac{\sigma-1-\theta}{\sigma-1}}}{\left[\left(\phi_{sd}^* \right)^{\sigma-1} \right]^{\frac{\sigma-1-\theta}{\sigma-1}}} \\ & = f_{sd}(1)^{\tilde{\theta}-1} \left\{ \frac{f_{sd}(g)^{1-\tilde{\theta}}}{\left[H(g)^{1-\sigma} - H(g-1)^{1-\sigma} \right]^{1-\tilde{\theta}}} - \frac{f_{sd}(g-1)^{1-\tilde{\theta}}}{\left[H(g-1)^{1-\sigma} - H(g-2)^{1-\sigma} \right]^{1-\tilde{\theta}}} \right\}. \end{aligned}$$

We define³

$$\begin{aligned} \tilde{F}_{sd} & \equiv \sum_{v=1} H(v)^{1-\sigma} \left[\frac{[f_{sd}(v+1)]^{1-\tilde{\theta}}}{\left[H(v+1)^{1-\sigma} - H(v)^{1-\sigma} \right]^{1-\tilde{\theta}}} - \frac{[f_{sd}(v)]^{1-\tilde{\theta}}}{\left[H(v)^{1-\sigma} - H(v-1)^{1-\sigma} \right]^{1-\tilde{\theta}}} \right] \\ & = \sum_{v=1} \left[\left[H(v)^{1-\sigma} - H(v-1)^{1-\sigma} \right] \frac{[f_{sd}(v)]^{1-\tilde{\theta}}}{\left[H(v)^{1-\sigma} - H(v-1)^{1-\sigma} \right]^{1-\tilde{\theta}}} \right] \\ & = \sum_{v=1} \frac{[f_{sd}(v)]^{1-\tilde{\theta}}}{\left[H(v)^{1-\sigma} - H(v-1)^{1-\sigma} \right]^{-\tilde{\theta}}}. \end{aligned}$$

With this definition we obtain

$$\int_{\phi_{sd}^*} \frac{\phi^{\sigma-2-\theta} / (\phi_{sd}^*)^{\sigma-1-\theta}}{H(G_{sd}(\phi))^{\sigma-1}} d\phi = \frac{f_{sd}(1)^{\tilde{\theta}-1}}{\theta - (\sigma-1)} \tilde{F}_{sd}.$$

■

³In the special case with $\varepsilon = \sigma$, we can rearrange the terms and find

$$\tilde{F}_{sd} = \sum_{v=1}^{\infty} \frac{[f_{sd}(v)]^{1-\tilde{\theta}}}{\left[h(v)^{\sigma-1} \right]^{-\tilde{\theta}}} = \sum_{v=1}^{\infty} \frac{[f_{sd}(v)]^{1-\tilde{\theta}}}{h(v)^{-\tilde{\theta}}}.$$

D Welfare

We have that

$$\begin{aligned}
P_d^{1-\sigma} &= \sum_s \int_{\phi_{sd}^*} [P_{sd}(\phi)]^{1-\sigma} \mu(\phi) d\phi \\
&= \sum_s \int_{\phi_{sd}^*} M_{sd} \left[\sum_{v=1}^{G_{sd}(\phi)} \left(\tilde{\sigma} \frac{w_s}{\phi/h(g)} \tau_{sd} \right)^{1-\varepsilon} \right]^{\frac{1-\sigma}{1-\varepsilon}} \frac{\theta (\phi_{sd}^*)^\theta}{\phi^{\theta+1}} d\phi \\
&= \sum_s (\tilde{\sigma} w_s \tau_{sd})^{1-\sigma} b_s^\theta \theta \left[H(1)^{1-\sigma} \left(\frac{(\phi_{sd}^{*,2})^{\sigma-1-\theta} - (\phi_{sd}^{*,1})^{\sigma-1-\theta}}{\theta - (\sigma-1)} \right) + \dots \right] \\
&= \sum_s (\tilde{\sigma} w_s \tau_{sd})^{-\theta} b_s^\theta \theta \left(\frac{f_{sd}(1)}{\frac{1}{\sigma} T_d} \right)^{1-\tilde{\theta}} \left[H(1)^{1-\sigma} \left(\frac{(\phi_{sd}^{*,2})^{\sigma-1-\theta} - (\phi_{sd}^{*,1})^{\sigma-1-\theta}}{(\phi_{sd}^{*,1})^{\sigma-1-\theta}} \right) + \dots \right],
\end{aligned}$$

where we use the definition of $\phi_{sd}^{*,1}$ for the last step. The final term in parentheses equals \tilde{F}_{sd} so

$$P_d^{-\theta} = \frac{\theta (\tilde{\sigma})^{-\theta}}{\left(\frac{1}{\sigma}\right)^{1-\theta/(\sigma-1)} T_d^{1-\tilde{\theta}}} \sum_s b_s^\theta (w_s \tau_{sd})^{-\theta} \tilde{F}_{sd}.$$

Using this relationship in equation (12), we obtain

$$\left(\frac{T_d}{P_d} \right)^\theta = \left(\frac{T_d}{w_d} \right)^\theta \frac{\theta (\tilde{\sigma})^{-\theta}}{(\sigma)^{\tilde{\theta}-1}} \frac{b_d^\theta}{\lambda_{dd}^{-\theta}} \frac{\tilde{F}_{dd}(1)}{T_d^{1-\tilde{\theta}}}.$$

If trade is balanced then $T_d = y_d$, where y_d is output per capita. Since, by the definition of $\tilde{F}_{dd}(1)$, this variable is homogeneous of degree $1 - \tilde{\theta}$ in wages and w_d/y_d is constant in all equilibria (see proof below) we arrive at the same welfare expression as in Arkolakis, Costinot and Rodriguez-Clare (2012): the share of domestic sales in expenditure λ_{dd} and the coefficient of the Pareto distribution are sufficient statistics to characterize aggregate welfare in the case of balanced trade.

The final step is to verify that the wage w_d is a constant fraction of per-capita output y_d so that the first ratio on the right-hand side is constant. We demonstrate this next.

E Constant wage share in output per capita

We show that the ratio w_d/y_d is a constant number. We first look at the share of fixed costs in bilateral sales. Average fixed costs incurred by firms from s selling to d are

$$\begin{aligned}
\bar{F}_{sd} &= \int_{\phi_{sd}^*}^{\phi_{sd}^{*,2}} F_{sd}(1) \theta \frac{(\phi_{sd}^*)^\theta}{\phi^{\theta+1}} d\phi + \int_{\phi_{sd}^{*,2}}^{\phi_{sd}^{*,3}} F_{sd}(2) \theta \frac{(\phi_{sd}^*)^\theta}{\phi^\theta} d\phi + \\
&= -F_{sd}(1) (\phi_{sd}^*)^\theta \left[(\phi_{sd}^{*,2})^{-\theta} - (\phi_{sd}^*)^{-\theta} \right] - F_{sd}(2) (\phi_{sd}^*)^\theta \left[(\phi_{sd}^{*,3})^{-\theta} - (\phi_{sd}^{*,2})^{-\theta} \right] - \dots
\end{aligned}$$

Using the definition $F_{sd}(G_{sd}) \equiv \sum_{g=1}^{G_{sd}} f_{sd}(g)$ and collecting terms with respect to $\phi_{sd}^{*,G}$ we can write the above expression as

$$\bar{F}_{sd} = f_{sd}(1) + (\phi_{sd}^{*,2})^{-\theta} (\phi_{sd}^*)^\theta f_{sd}(2) + (\phi_{sd}^{*,3})^{-\theta} (\phi_{sd}^*)^\theta f_{sd}(3) + \dots$$

Using the definition of $\phi_{sd}^{*,G}$ from equation (6) to replace terms in the above equation, we obtain

$$\left(\phi_{sd}^{*,G}\right)^{\sigma-1} = \frac{(\phi_{sd}^*)^{\sigma-1} f_{sd}(G_{sd})}{H(G_{sd})^{-(\sigma-1)} - H(G_{sd}-1)^{-(\sigma-1)}} \frac{f_{sd}(G_{sd})}{f_{sd}(1)}.$$

Therefore

$$\begin{aligned} \bar{F}_{sd} &= f_{sd}(1) + \left(\frac{f_{sd}(2)^{1/(\sigma-1)} \left[H(2)^{-(\sigma-1)} - H(1)^{-(\sigma-1)} \right]^{-1/(\sigma-1)}}{f_{sd}(1)^{1/(\sigma-1)} \left[H(1)^{-(\sigma-1)} \right]^{-1/(\sigma-1)}} \right)^{-\theta} f_{sd}(2) + \dots \\ &= \left[f_{sd}(1) + f_{sd}(1)^{\tilde{\theta}} \left(f_{sd}(2)^{1/(\sigma-1)} \left[H(2)^{-(\sigma-1)} - H(1)^{-(\sigma-1)} \right]^{-1/(\sigma-1)} \right)^{-\theta} f_{sd}(2) + \dots \right] \\ &= [f_{sd}(1)]^{\tilde{\theta}} \left[f_{sd}(1)^{1-\tilde{\theta}} + \frac{f_{sd}(2)^{1-\tilde{\theta}}}{\left[H(2)^{-(\sigma-1)} - H(1)^{-(\sigma-1)} \right]^{-\tilde{\theta}}} + \frac{f_{sd}(3)^{1-\tilde{\theta}}}{\left[H(3)^{-(\sigma-1)} - H(2)^{-(\sigma-1)} \right]^{-\tilde{\theta}}} \dots \right] \\ &= [f_{sd}(1)]^{\tilde{\theta}} \tilde{F}_{sd} \end{aligned}$$

and hence

$$\frac{\bar{F}_{sd}}{\bar{T}_{sd}} = \frac{f_{sd}(1)^{\tilde{\theta}} \left[f_{sd}(1)^{1-\tilde{\theta}} + \frac{f_{sd}(2)^{1-\tilde{\theta}}}{h(2)^\theta} + \dots \right]}{\frac{f_{sd}(1)^{\tilde{\theta}} \theta \sigma}{\theta - (\sigma - 1)} \sum_{g=1}^{\infty} \frac{f_{sd}(g)^{1-\tilde{\theta}}}{h(g)^\theta}} = \frac{\theta - (\sigma - 1)}{\theta \sigma} \frac{\sum_{g=1}^{\infty} \frac{f_{sd}(g)^{1-\tilde{\theta}}}{h(g)^\theta}}{\sum_{g=1}^{\infty} \frac{f_{sd}(g)^{1-\tilde{\theta}}}{h(g)^\theta}} < 1.$$

Finally, the share of profits generated by the corresponding bilateral sales is the share of variable profits in total sales ($1/\sigma$) minus the average fixed costs paid, as derived above. So

$$\frac{\bar{\pi}_{sd}}{\bar{T}_{sd}} = \frac{1}{\sigma} - \frac{\theta - (\sigma - 1)}{\theta \sigma} = \frac{\sigma - 1}{\theta \sigma} = \frac{1}{\tilde{\theta} \sigma} \equiv \eta.$$

This finding implies that the wage is a constant fraction of per capita income. The reason is that total profits for country s are $\pi_s L_s = \sum_k \lambda_{sk} T_k / (\tilde{\theta} \sigma)$, where $\sum_k \lambda_{sk} T_k$ is the country's total income because total manufacturing sales of a country s equal its total sales across all destinations. So profit income and wage income can be expressed as constant shares of total income:

$$\pi_s L_s = \frac{1}{\tilde{\theta} \sigma} Y_s \quad \text{and} \quad w_s L_s = \frac{\tilde{\theta} \sigma - 1}{\tilde{\theta} \sigma} Y_s.$$

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