B Online Appendix (not for publication)

This Online Appendix provides some additional results referenced in the paper.

B.1 Quasi-symmetric trade costs

In this subsection, we show that when trade costs are quasi-symmetric, then balanced trade implies that the origin and destination fixed effects of the gravity trade flow expression are equal up to scale.

We first formally define "quasi-symmetry." We say that the set of trade frictions $\{\tau_{ij}\}_{i,j\in S\times S}$ are *quasi-symmetric* if there exists a set of origin scalars $\left\{\tau_i^A\right\}_{i\in S}$, destination scalars $\left\{\tau_i^B\right\}_{i\in S}$, and a symmetric matrix $\{\tilde{\tau}_{ij}\}_{i,j\in S\times S}$ where $\tilde{\tau}_{ij}=\tilde{\tau}_{ji}$ for all $i,j\in S\times S$ such that we can write:

$$\tau_{ij}=\tau_i^A\tau_i^B\tilde{\tau}_{ij}\ \forall i,j\in S\times S.$$

Loosely speaking, quasi-symmetric trade frictions are those that are reducible to a symmetric component and exporter- and importer-specific components. While restrictive, it is important to note that the vast majority of papers which estimate gravity equations assume that trade frictions are quasi-symmetric; for example Eaton and Kortum (2002) and Waugh (2010) assume that trade costs are composed by a symmetric component that depends on bilateral distance and on a destination or origin fixed effect.

Combining the universal gravity conditions C.1 and C.2 allows us to write the value of bilateral trade flows from i to j as:

$$X_{ij}=\tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j,$$

which we now re-write as:

$$X_{ij}=\tau_{ij}^{-\phi}\gamma_i\delta_j, \tag{22}$$

where we call $\gamma_i\equiv p_i^{-\phi}$ the *origin fixed effect* and $\delta_j\equiv P_j^{\phi}E_j$ the *destination fixed effect.*

Proposition 1. *If trade costs are quasi-symmetric, then in any model within the universal gravity framework, the product of the equilibrium origin fixed effect and the origin scalar will be equal to the product of the equilibrium destination fixed effects and the destination fixed effect up to scale, i.e.:*

$$\left(\tau_i^A\right)^{-\phi}\gamma_i\propto\left(\tau_i^B\right)^{-\phi}\delta_i\ \forall i\in S.$$

Proof. We first note that market clearing condition C.4 and balanced trade condition C.5 together imply that: $\sum_{j\in S}X_{ij}=\sum_{j\in S}X_{ji}\ \forall i\in S$. Combining this with the gravity expression (22) and quasi-symmetry implies:

$$\frac{\left(\tau_i^A\right)^{-\phi}\gamma_i}{\left(\tau_i^B\right)^{-\phi}\delta_i}=\frac{\sum_{j\in S}\tilde{\tau}_{ij}^{-\phi}\left(\tau_j^A\right)^{-\phi}\gamma_j}{\sum_{j\in S}\tilde{\tau}_{ij}^{-\phi}\left(\tau_j^B\right)^{-\phi}\delta_j}=\sum_{j\in S}\frac{\tilde{\tau}_{ij}^{-\phi}\left(\tau_j^B\right)^{-\phi}\delta_j}{\sum_{k\in S}\tilde{\tau}_{ik}^{-\phi}\left(\tau_k^B\right)^{-\phi}\delta_k}\times\frac{\left(\tau_j^A\right)^{-\phi}\gamma_j}{\left(\tau_j^B\right)^{-\phi}\delta_j}.$$

It is easy to show that $\frac{\left(\tau_i^A\right)^{-\phi}\gamma_i}{\left(\tau_i^B\right)^{-\phi}\delta_i}=1$ is a solution to this problem. From the Perron-Frobenius theorem, this solution is unique up to scale. Therefore we have:

$$\left(\tau_i^A\right)^{-\phi}\gamma_i \propto \left(\tau_i^B\right)^{-\phi}\delta_i \;\forall i \in S,$$

as required. □

Proposition 1 has a number of important implications. First, Proposition 1 allows one to simplify the equilibrium system of equations 1 and 2 into a single non-linear equation:

$$\left(p_i^{-\phi}\right)^{\frac{1+\psi+\phi}{\psi-\phi}} = \lambda \sum_{j\in S} \tau_{ji}^{-\phi}\left(\frac{\tau_i^A}{\tau_i^B}\right)^{\frac{\phi^2}{\psi-\phi}} C_i^{\frac{\phi}{\psi-\phi}} p_j^{-\phi}, \tag{23}$$

which simplifies the characterization of the theoretical and empirical properties of the equilibrium.

Second, by showing that the origin and destination fixed effects are equal up to scale, Proposition 1 provides offers an analytical characterization of the equilibrium. For example, given the definition of the origin and destination fixed effects, Proposition 1 can equivalently be expressed as:

$$\frac{p_i}{P_i} \propto \frac{\tau_i^B}{\tau_i^A} E_i^{-\frac{1}{\phi}},$$

i.e. real factor returns in a location are inversely related to total expenditure in a location.

Third, it is straightforward to show that quasi-symmetry implies that equilibrium trade flows will be bilaterally symmetric, i.e. $X_{ij} = X_{ji} \;\forall i, j \in S \times S$, allowing one to test whether trade costs are quasi-symmetric directly from observed trade flow data.

Finally, we should note that the results of Proposition 1 have already been used in the literature for particular models, albeit implicitly. The most prominent example is Anderson and Van Wincoop (2003), who use the result to show the bilateral resistance is equal to the price index.[31] To our knowledge, Head and Mayer (2013) are the first to recognize the importance of balanced trade and market clearing in generating the result for the Armington model; however, Proposition 1 shows that the result applies more generally to any model with quasi-symmetrical trade costs in the universal gravity framework.

B.2 Proofs of the lemmas used in Theorem (1)

There are 4 lemmas which are not proven in the paper. In this section, we discuss them carefully. Before proving these lemmas, we discuss how we use them in the proof. In the proof, we show a fixed point for the "scaled" system, not the actual system. Therefore it needs to be shown that there exists a fixed point for the actual system, which is shown in Lemma 1. Then we argue that the solution we obtain is strictly positive, which is guaranteed by Assumption 1. We emphasize the connectivity assumption is crucial here. These two lemmas are used in **Part i)** Theorem 1.

[31] The result is also used in economic geography by Allen and Arkolakis (2014) to simplify a set on non-linear integral equations into a single integral equation.

Part ii) shows that there exists an unique solution. During the proof, we argue that 18 should hold with strict inequality. Again the connectivity allows us to show this result (Lemma 3). After establishing this strict inequality, we follow the argument by Allen, Arkolakis, and Li (2014), which requires that the largest absolute eigenvalues for $|A|$ are less than 1. Since A is a 2-by-2 matrix, we can compute the eigenvalues by hand and show that one of them is exactly 1, and the other is less than 1 if the conditions in **Part ii)** are satisfied.

The last lemma (Lemma 5) shows that if trade costs are quasi-symmetric, then we have a simpler system N equations instead of $2N$, and the parameter values ensuring uniqueness expand.

Lemma 1. *Suppose that z solves (15). Then there exists $\widehat{z}$ solving (14).*

Proof. First it is easy to show[32]

$$\sum_{i,j\in S} K_{ij}C_i^{-1}C_j x_j^{a_{11}} y_j^{a_{12}} = \sum_{i,j\in S} K_{ji} x_j^{a_{21}} y_j^{a_{22}}. \tag{26}$$

Guess a solution

$$\widehat{z} = \begin{pmatrix} (\widehat{x}_i)_i \\ (\widehat{y}_i)_i \end{pmatrix} = \begin{pmatrix} t^{-1}(x_i)_i \\ t^{-1}(y_i)_i \end{pmatrix}, \tag{27}$$

where $t = \left(\sum_{i,j\in S} K_{ij}C_i^{-1}C_j x_j^{a_{21}} y_j^{a_{22}}\right)^{\frac{1}{1-a_{11}-a_{12}}} = \left(\sum_{i,j\in S} K_{ji} x_j^{a_{21}} y_j^{a_{22}}\right)^{\frac{1}{1-a_{21}-a_{22}}}$.[33] Then it is easy to verify that (27) solves (14); in particular, note that

$$\begin{aligned}\widehat{x}_i &= t^{-1}\frac{\sum_{j\in S} K_{ij}C_i^{-1}C_j x_j^{a_{11}} y_j^{a_{12}}}{\sum_{i,j\in S} K_{ij}C_i^{-1}C_j x_j^{a_{11}} y_j^{a_{12}}} = t^{1-a_{11}-a_{12}}\frac{\sum_{j\in S} K_{ij}C_i^{-1}C_j (\widehat{x}_j)^{a_{11}} \widehat{y}_j^{a_{12}}}{\sum_{i,j\in S} K_{ij}C_i^{-1}C_j x_j^{a_{11}} y_j^{a_{12}}} \\ &= \sum_{j\in S} K_{ij}C_i^{-1}C_j \widehat{x}_j^{a_{11}} \widehat{y}_j^{a_{12}}.\end{aligned}$$

[32]To see this, multiply $C_i x_i^{a_{21}} y_i^{a_{22}} = C_i p_i^{-\phi}$, to the first equations of (15) and sum over i;

$$\sum_i C_i p_i^{1+\psi} P_i^{-\psi} = \frac{\sum_i \sum_j K_{ij} C_j x_i^{a_{21}} y_i^{a_{22}} x_j^{a_{11}} y_j^{a_{12}}}{\sum_{i,j} K_{ij} C_i^{-1} C_j x_j^{a_{11}} y_j^{a_{12}}}. \tag{24}$$

Also multiply $C_i x_i^{a_{11}} y_i^{a_{12}} = C_i P_i^{\phi-\psi} p_i^{1+\psi}$ to the second equations (15) and sum over i;

$$\sum_{i\in S} C_i p_i^{1+\psi} P_i^{-\psi} = \frac{\sum_{i\in S} \sum_{j\in S} K_{ij} C_j x_i^{a_{21}} y_i^{a_{22}} x_j^{a_{11}} y_j^{a_{12}}}{\sum_{i\in S, j\in S} K_{ji} x_j^{a_{21}} y_j^{a_{22}}}. \tag{25}$$

Notice that the LHS is the same as one in (24). Also the numerator of the RHS in (24) is the same as one in (25). Therefore the following double sum terms should be the same:

$$\sum_{i,j} K_{ij} C_i^{-1} C_j x_j^{a_{11}} y_j^{a_{12}} = \sum_{i\in S, j\in S} K_{ji} x_j^{a_{21}} y_j^{a_{22}}.$$

[33]Notice that $a_{11} + a_{12} = a_{21} + a_{22}$.

We can also show that the second equations in 14 are also solved in the same vein:

$$\begin{aligned}\widehat{y}_i &= t\frac{\sum_{j\in S} K_{ji} x_j^{a_{21}} y_j^{a_{22}}}{\sum_{i,j\in S} K_{ji} x_j^{a_{21}} y_j^{a_{22}}} = t^{1-a_{21}-a_{22}} \frac{\sum_{j\in S} K_{ji} \widehat{x}_j^{a_{21}} \widehat{y}_j^{a_{22}}}{\sum_{i,j\in S} K_{ji} x_j^{a_{21}} y_j^{a_{22}}} \\ &= \sum_{j\in S} K_{ji} \widehat{x}_j^{a_{21}} \widehat{y}_j^{a_{22}}.\end{aligned}$$

The above two equations confirm that $\widehat{x}_i$ and $\widehat{y}_i$ is a solution to (14) □

Lemma 2. *If τ_{ij} satisfies the strong connectivity, then the fixed point for (15) z is strictly positive.*

Proof. We need to consider four different cases for the combinations of α_{11}, α_{12} satisfying different inequalities. We will consider the case $\alpha_{11}, \alpha_{12} > 0$ since the logic in the other cases is the same. Suppose that $x_i = 0$. Consider an arbitrary location $n \neq i$ and consider a connected path, $K_{in}^c \equiv K_{i\pi(i')} \times ... \times K_{\pi(i')n} > 0$. Then, from the first of equations in (14) notice that

$$x_i = \sum_{j\in S} K_{ij} x_j^{\alpha_{11}} y_j^{\alpha_{12}} \geq K_{in} x_{\pi(k)}^{\alpha_{11}} y_{\pi(i)}^{\alpha_{12}}$$

then either x_n or y_n or both are zero. If $x_n = 0$ this argument holds for any n so this is a contradiction with the non-zero equilibrium proved above. Else if $y_n = 0$ we can repeat the argument the second of the equations in (14) to establish another contradiction. Notice that if either of $\alpha_{11}, \alpha_{12} = 0$ a contradiction is also easy to establish. □

Lemma 3. *Equation 18 holds with strict inequality.*

To that end, define the set of directly connected countries to each location i as $S_i^c \equiv \{j : K_{ij} > 0\}$. Then notice that equation (16) combined with our equality assumption on (18) yields

$$\frac{x_i}{\widehat{x}_i} = \frac{1}{\widehat{x}_i} \sum_{j\in S_i^c} K_{ij} C_i^{-1} C_j \left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{11}} \left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{12}} (\widehat{x}_j)^{\alpha_{11}} (\widehat{y}_j)^{\alpha_{12}} = \max_{j\in S} \left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{11}} \max_{j\in S} \left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{12}}.$$

Notice that given that $\hat{x}_i$ is a solution, this implies that the following has to be true for all $j \in S_i^c$

$$\left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{11}} = \max_{j\in S} \left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{11}} \quad \left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{12}} = \max_{j\in S} \left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{12}}.$$

Now notice that if $\alpha_{11} \neq 0$ then for all the $n \in S_i^c$, $x_j/\widehat{x}_j = x_n/\widehat{x}_n$. However, because of Condition (1), we assume that there exists an indirectly connected path from any location to any other location, so that repeating this argument for all j and using the indirect connectivity we can prove that $x_j/\widehat{x}_j = x_n/\widehat{x}_n$ for all $j, n \in S$ i.e. the solutions are the same up-to-scale, a contradiction.

Lemma 4. *If $\phi, \psi \geqslant 0$ or $\phi, \psi \leqslant -1$, the eigenvalue for $|A|$ is*

$$\lambda = \frac{\phi - \psi}{1 + \phi + \psi}, 1,$$

and

$$\left|\frac{\phi-\psi}{1+\phi+\psi}\right| \leqslant 1.$$

Proof. Notice that

$$|A| = \begin{pmatrix} \left|\frac{1+\psi}{1+\psi+\phi}\right| & \left|\frac{1+\phi}{1+\psi+\phi}\right| \\ \left|\frac{\phi}{1+\psi+\phi}\right| & \left|\frac{\psi}{1+\psi+\phi}\right| \end{pmatrix} = \begin{pmatrix} \frac{1+\psi}{1+\psi+\phi} & \frac{1+\phi}{1+\psi+\phi} \\ \frac{\phi}{1+\psi+\phi} & \frac{\psi}{1+\psi+\phi} \end{pmatrix}.$$

Then we can solve the following characteristic functions

$$\lambda^2 - \left(\frac{1+\psi}{1+\psi+\phi} + \frac{\psi}{1+\psi+\phi}\right)\lambda + \frac{1+\psi}{1+\psi+\phi}\frac{\psi}{1+\psi+\phi} - \frac{1+\phi}{1+\psi+\phi}\frac{\phi}{1+\psi+\phi} = 0.$$

Then

$$\lambda = \frac{\phi-\psi}{1+\phi+\psi}, 1.$$

It is easy to show that $\left|\frac{\phi-\psi}{1+\phi+\psi}\right| \leqslant 1$. □

Lemma 5. *Suppose that trade costs, $\tau = (\tau_{ij})$, satisfy quasi-symmetry. Then if $\left|\frac{\phi-\psi}{1+\phi+\psi}\right| \leqslant 1$, there exists a strictly positive solution. If $\left|\frac{\phi-\psi}{1+\phi+\psi}\right| < 1$, the unique solution is computed recursively.*

Proof. With quasi-symmetry assumption, the equilibrium system is reduced to N dimensional (single) integral equation form. Then we can apply the result by Karlin and Nirenberg (1967). Also the unique strictly positive solution is computed recursively. We borrow an argument from Proposition 1 in Online Appendix B.1. Since the destination effects are linear in the origin effects, we have[34]

$$P_i^{\psi-\phi} = C_i \frac{\tau_{2,j}^{-\phi}}{\tau_{1,k}^{-\phi}} p_i^{1+\phi+\psi}.$$

Then we show that 23 is obtained. Define $x_i = p_i^{\frac{(1+\psi+\phi)\phi}{\psi-\phi}}$, then (23) is re-written as follows:

$$x_i = \sum_j K_{ij} x_j^{\frac{\phi-\psi}{1+\psi+\phi}}.$$

From Karlin and Nirenberg (1967), the solution is unique if

$$\left|\frac{\phi-\psi}{1+\psi+\phi}\right| \leqslant 1.$$

[34] We retake the numeraire. Specifically, we choose the numeraire so that the origin effects are exactly the same as the origin effects.

Now starting from any initial point x, compute recursively

$$x_i^{(n)} = \frac{\sum_j K_{ij} \left(x_j^{(n-1)}\right)^{\frac{\phi-\psi}{1+\psi+\phi}}}{\sum_{i,j} K_{ij} \left(x_j^{(n-1)}\right)^{\frac{\phi-\psi}{1+\psi+\phi}}},$$
$$x_i^{(0)} = x.$$

Notice that for all n, $x^{(n)}$ is bounded. □

We obtain the following inequality as in **Part ii)**

$$1 \leqslant \mu^{(n)} \equiv \frac{\max_i \frac{x_i^{(n)}}{x_i^{(n-1)}}}{\min_i \frac{x_i^{(n)}}{x_i^{(n-1)}}} \leqslant \frac{\max \left(\frac{x_i^{(n-1)}}{x_i^{(n-2)}}\right)^{\frac{\phi-\psi}{1+\psi+\phi}}}{\min \left(\frac{x_i^{(n-1)}}{x_i^{(n-2)}}\right)^{\frac{\phi-\psi}{1+\psi+\phi}}}.$$

Then taking the logarithm,

$$\ln \mu^{(n)} \leqslant \left|\frac{\phi-\psi}{1+\psi+\phi}\right| \ln \mu^{(n-1)} \leqslant \left|\frac{\phi-\psi}{1+\psi+\phi}\right|^n \ln \mu^{(0)} \to 0.$$

Therefore $\mu^{(n)} \to 1$. Since $x^{(n)}$ is bounded, $x_i^{(n)} \to x_i^{\infty}$ as $n \to \infty$. Then for any t, we have

$$tx_i^{\infty} = \frac{t^{1-\frac{\phi-\psi}{1+\psi+\phi}}}{\sum_{i,j} K_{ij} \left(x_j^{\infty}\right)^{\frac{\phi-\psi}{1+\psi+\phi}}} \sum_j K_{ij} \left(tx_j^{\infty}\right)^{\frac{\phi-\psi}{1+\psi+\phi}}.$$

By setting t as

$$\frac{t^{1-\frac{\phi-\psi}{1+\psi+\phi}}}{\sum_{i,j} K_{ij} \left(x_j^{\infty}\right)^{\frac{\phi-\psi}{1+\psi+\phi}}} = 1,$$

then $x_i^* = tx_i^{\infty}$ is the unique solution.

B.3 Existence and Uniqueness using Gross Substitutes Methodology (a la Alvarez and Lucas (2007))

In this subsection, we prove the existence and uniqueness of an equilibrium in our universal gravity framework using the gross substitutes methodology employed by Alvarez and Lucas (2007). As we show below, the sufficient conditions here are stronger than we provide in Theorem 1 above.

Proposition 2. *Consider any model within the universal gravity framework. If $\phi > \psi > 0$ and $\tau_{ij} \in (0, \infty)$ for all $i, j \in S$, then the excess demand system of the model satisfies gross substitutes and, as a result, the equilibrium exists and is unique.*

Proof. Recall the equilibrium conditions of the universal gravity framework from equations (1) and

$$p_i C_i \left(\frac{p_i}{P_i}\right)^{\psi} = \sum_{j\in S} \tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} p_j C_j \left(\frac{p_j}{P_j}\right)^{\psi} \forall i \in S \tag{28}$$

$$P_i^{-\phi} = \sum_{j\in S} \tau_{ji}^{-\phi} p_j^{-\phi} \; \forall i \in S \tag{29}$$

Substituting equation (29) into (28) yields a single equilibrium system of equations that depends only on the factor prices in every location:

$$p_i^{1+\phi+\psi} \left(\sum_{j\in S} \tau_{ji}^{-\phi} p_j^{-\phi}\right)^{\frac{\psi}{\phi}} C_i = \sum_{j\in S} \tau_{ij}^{-\phi} C_j p_j^{1+\psi} \left(\sum_{k\in S} \tau_{kj}^{-\phi} p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}} \quad \forall i \in S$$

We define the corresponding excess demand function as:

$$\begin{aligned} Z_i(\mathbf{p}) =& \frac{1}{p_i} \left(\frac{1}{\sum_{k\in S} C_k \left(\sum_{l\in S} \tau_{lk}^{-\phi} (\beta p_l)^{-\phi}\right)^{\frac{\psi}{\phi}} (\beta p_k)^{\psi}} \right) \times \\ & \left[\sum_{j\in S} \tau_{ij}^{-\phi} C_j p_i^{-\phi} p_j^{1+\psi} \left(\sum_{k\in S} \tau_{kj}^{-\phi} p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}} - p_i^{1+\psi} \left(\sum_{j\in S} \tau_{ji}^{-\phi} p_j^{-\phi}\right)^{\frac{\psi}{\phi}} C_i \right], \end{aligned} \tag{30}$$

where P_i is defined by (29).This system written as such needs to satisfy 6 properties to be an excess demand system and the gross substitute property to establish existence and uniqueness. The six conditions are:

1. $Z(\mathbf{p})$ is continuous for $\mathbf{p} \in \Delta\left(R_+^N\right)$
2. $Z(\mathbf{p})$ is homogenous of degree zero.
3. $Z(\mathbf{p}) \cdot \mathbf{p} = 0$ (Walras' Law).
4. There exists a $k > 0$ such that $Z_j(\mathbf{p}) > -k$ for all j.
5. If there exists a sequence $p^m \to p^0$, where $p^0 \neq 0$ and $p_i^0 = 0$ for some i, then it must be that:

$$max_j\{Z_j(p^m)\} \to \infty \tag{31}$$

and the gross-substitute property:

6. Gross substitutes property: $\frac{\partial Z(p_j)}{\partial p_k} > 0$ for all $j \neq k$.

We verify each of these properties in turn. Property 1 is trivial given equation (30) for excess demand. To see property 2, consider multiplying factor prices by a scalar $\beta > 0$, which immediately yields $Z_i(\beta\mathbf{p}) = Z_i(\mathbf{p})$. as required. Property 3 can be seen as follows:

$$\begin{aligned}
Z(\mathbf{p}) \cdot \mathbf{p} = &= \sum_{i\in S} Z_i(\mathbf{p}) p_i \iff \\
&= \left(\frac{1}{\sum_{k\in S} C_k \left(\sum_{l\in S} \tau_{lk}^{-\phi} p_l^{-\phi}\right)^{\frac{\psi}{\phi}} p_k^{\psi}}\right) \times \\
&= \sum_{i\in S}\left(\sum_{j\in S} \tau_{ij}^{-\phi} C_j p_i^{-\phi} p_j^{1+\psi} \left(\sum_{k\in S} \tau_{kj}^{-\phi} p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}} - p_i^{1+\psi}\left(\sum_{j\in S} \tau_{ji}^{-\phi} p_j^{-\phi}\right)^{\frac{\psi}{\phi}} C_i\right) \iff \\
&= 0,
\end{aligned}$$

as required. Property 4 can be seen as follows:

$$\begin{aligned}
Z_i(\mathbf{p}) &= \frac{1}{p_i} \frac{\sum_{j\in S} \tau_{ij}^{-\phi} C_j p_i^{-\phi} p_j^{1+\psi} \left(\sum_{k\in S} \tau_{kj}^{-\phi} p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}}}{\sum_{k\in S} C_k \left(\sum_{l\in S} \tau_{lk}^{-\phi} (\beta p_l)^{-\phi}\right)^{\frac{\psi}{\phi}} (\beta p_k)^{\psi}} - Q_i \implies \\
Z_i(\mathbf{p}) &> -Q_i > \bar{Q}
\end{aligned}$$

since $\frac{1}{p_i}\left(\frac{1}{\sum_{k\in S} C_k \left(\sum_{l\in S} \tau_{lk}^{-\phi} (\beta p_l)^{-\phi}\right)^{\frac{\psi}{\phi}} (\beta p_k)^{\psi}}\right) \sum_{j\in S} \tau_{ij}^{-\phi} C_j p_i^{-\phi} p_j^{1+\psi} \left(\sum_{k\in S} \tau_{kj}^{-\phi} p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}} > 0$ for all $\mathbf{p} \gg 0$ and $Q_i \leq \bar{Q}$ from Condition C.3. Property 5 can be seen as follows: consider any $\mathbf{p} \in \Delta\left(R_+^N\right)$ such that there exists an $l \in S$ where $p_l = 0$ and an $l' \in S$ where $p_{l'} > 0$. Consider any sequence of factor prices such that $\mathbf{p}^n \to \mathbf{p}$ as $n \to \infty$. Then we need to show that:

$$\max_{i\in S} Z_i(\mathbf{p}) \to \infty.$$

To see this note that:

$$\begin{aligned}
\max_{i\in S} Z_i(\mathbf{p}^n) &= \max_{i\in S} \frac{\frac{1}{p_i}\sum_{j\in S} (\tau_{ij} p_i)^{-\phi} C_j p_j^{1+\psi} \left(\sum_{k\in S} (\tau_{kj} p_k)^{-\phi}\right)^{\frac{\psi-\phi}{\phi}}}{\sum_{k\in S} C_k \left(\sum_{l\in S} \tau_{lk}^{-\phi} p_l^{-\phi}\right)^{\frac{\psi}{\phi}} p_k^{\psi}} - Q_i \implies \\
\max_{i\in S} Z_i(\mathbf{p}^n) &> \max_{i,j\in S} \frac{p_j}{p_i} \tau_{ij}^{-\phi} \frac{C_j p_i^{-\phi} p_j^{\psi} \left(\sum_{k\in S} \tau_{kj}^{-\phi} p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}}}{\sum_{k\in S} C_k \left(\sum_{l\in S} \tau_{lk}^{-\phi} p_l^{-\phi}\right)^{\frac{\psi}{\phi}} p_k^{\psi}} - \bar{Q}.
\end{aligned}$$

Hence, if it is the case that $\max_{i,j\in S} \frac{p_j}{p_i} \tau_{ij}^{-\phi} \frac{C_j p_i^{-\phi} p_j^{\psi} \left(\sum_{k\in S} \tau_{kj}^{-\phi} p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}}}{\sum_{k\in S} C_k \left(\sum_{l\in S} \tau_{lk}^{-\phi} p_l^{-\phi}\right)^{\frac{\psi}{\phi}} p_k^{\psi}} \to \infty$, then because $\max_{i\in S} Z_i(\mathbf{p}^n)$ is bounded below it, it must be that $\max_{i\in S} Z_i(\mathbf{p}^n) \to \infty$ as well. Note that:

$$\max_{i,j\in S}\frac{p_j}{p_i}\tau_{ij}^{-\phi}\frac{C_j p_i^{-\phi}p_j^{\psi}\left(\sum_{k\in S}\tau_{kj}^{-\phi}p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}}}{\sum_{k\in S}C_k\left(\sum_{l\in S}\tau_{lk}^{-\phi}p_l^{-\phi}\right)^{\frac{\psi}{\phi}}p_k^{\psi}}>\max_{i,j\in S}\frac{p_j}{p_i}\tau_{ij}^{-\phi}\frac{C_j p_i^{-\phi}p_j^{\psi}\left(\sum_{k\in S}\tau_{kj}^{-\phi}\left(p^{\min}\right)^{-\phi}\right)^{\frac{\psi-\phi}{\phi}}}{\sum_{k\in S}C_k\left(\sum_{l\in S}\tau_{lk}^{-\phi}\left(p^{\min}\right)^{-\phi}\right)^{\frac{\psi}{\phi}}\left(p^{\max}\right)^{\psi}}\implies$$
$$>C_{ij}\min_{l\in S}p_l^{-(\phi-\psi)},$$

where $p^{\min}\equiv\min_{l\in S}p_l$, $p^{\max}\equiv\max_{l\in S}$, and $C_{ij}\equiv\tau_{ij}^{-\phi}\frac{C_j\left(\sum_{k\in S}\tau_{kj}^{-\phi}\left(p^{\min}\right)^{-\phi}\right)^{\frac{\psi-\phi}{\phi}}}{\sum_{k\in S}C_k\left(\sum_{l\in S}\tau_{lk}^{-\phi}\left(p^{\min}\right)^{-\phi}\right)^{\frac{\psi}{\phi}}\left(p^{\max}\right)^{\psi}}$. Since $\phi>\psi>0$ and there exists an $l\in S$ such that $p_l^n\to\infty$ as $n\to\infty$, then we have $\max_{i\in S}Z_i\left(\mathbf{p}^n\right)\to\infty$ as well.

Finally, we verify gross-substitutes. Without loss of generality, we differentiate only the bracketed term (as the term outside the bracket will be multiplied by zero since the bracket term is equal to zero in the equilibrium). We have:

$$\frac{\partial Z_i\left(\mathbf{p}\right)}{\partial p_j}=\frac{\partial}{\partial p_j}\left[\sum_{j\in S}\tau_{ij}^{-\phi}C_j p_i^{-\phi}p_j^{1+\psi}\left(\sum_{k\in S}\tau_{kj}^{-\phi}p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}}-p_i^{1+\psi}\left(\sum_{j\in S}\tau_{ji}^{-\phi}p_j^{-\phi}\right)^{\frac{\psi}{\phi}}C_i\right]\iff$$
$$=\left(1+\psi\right)\tau_{ij}^{-\phi}C_j p_i^{-\phi}p_j^{\psi}\left(\sum_{k\in S}\tau_{kj}^{-\phi}p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}}+$$
$$\left(\phi-\psi\right)p_j^{-\phi-1}\sum_{l\in S}\tau_{il}^{-\phi}C_l p_i^{-\phi}p_l^{\psi}\left(\sum_{k\in S}\tau_{kl}^{-\phi}p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}-1}+\psi p_j^{-\phi-1}p_i^{1+\psi}\left(\sum_{j\in S}\tau_{ji}^{-\phi}p_j^{-\phi}\right)^{\frac{\psi}{\phi}-1}>0$$

because $\phi>\psi>0$ and prices, trade costs, and supply shifters C_l are strictly positive. Because properties 1-6 hold, by Propositions 17.B.2, 17.C.1 and 17.F.3 of Mas-Colell, Whinston, and Green (1995), the equilibrium exists and unique. □

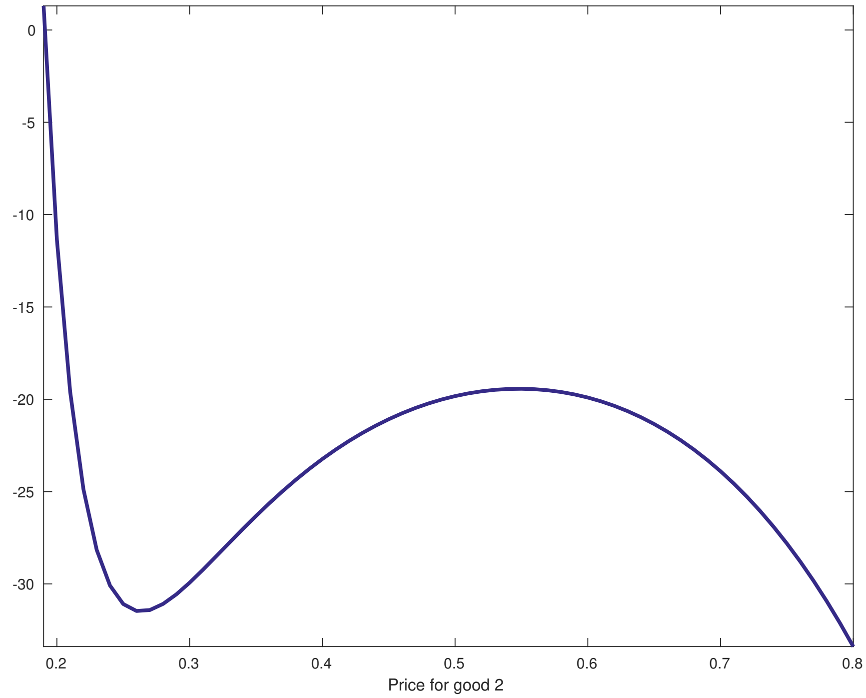
Note that in the case where $\psi>\phi>0$ – which is the ordering we find when we estimate the gravity constants in Section 5 – Theorem 1 still proves existence and uniqueness of the equilibrium. The following example shows that gross substitutes may not be satisfied in this case.

Example 1. (Gross substitution) Consider the three location economy. Take p_3 as the numeraire The gross substitute is violated if there exists $\overline{p}_1$ such that $Z_1\left(\overline{p}_1,p_2,1\right)$ is not monotonic w.r.t. p_2. Consider the following parameter values:

$$\begin{aligned}(\phi,\psi)&=(2,5)\\ \tau_{ij}&=1\quad\text{for } i,j\in\{1,2,3\}\\ C_i&=(.9,.6,.1)^{\mathbf{T}}.\end{aligned}$$

Figure 7 shows that with these parameter values, $Z_1\left(\overline{p}_1,p_2,1\right)$ is not monotonic w.r.t. p_2 when $\overline{p}_1=.5$.

Figure 7: Excess demand function for $1, Z_1(p_2)$



B.4 Tariffs in the universal gravity framework

In this subsection, we show how one can use the tools developed above to analyze the effect of tariffs in a trade model.

Because tariffs introduce an additional source of revenue, they are are not strictly contained within the universal gravity framework. However, it turns out that the equilibrium structure of a tariff model is mathematically equivalent to the equilibrium structure of the universal gravity framework. As a result, we can apply Theorems 1 and 2 almost immediately to the case of tariffs.

To see this, consider a simple Armington trade model with N locations.[35] Each location $i \in \{1, ..., N\}$ is endowed with its own differentiated variety and L_i workers who supply their unit labor inelastically and consume varieties from all locations with CES preferences and an elasticity of substitution σ. Suppose that trade is subject to technological iceberg trade costs $\tau_{ij} \geq 1$ and ad-valorem tariffs $\tilde{t}_{ij} \geq 0$. Define $t_{ij} \equiv 1 + \tilde{t}_{ij}$. Then we can write the value of trade flows from i to j (excluding the tariffs) as:

$$X_{ij} = \tau_{ij}^{1-\sigma} t_{ij}^{-\sigma} A_i^{\sigma-1} w_i^{1-\sigma} P_j^{\sigma-1} E_j, \tag{32}$$

where A_i is the productivity in location $i \in S$, w_i is the wage, P_j is the ideal Dixit-Stiglitz price index, and E_j is expenditure.

[35] We consider an Armington model in order to have an explicit welfare function, the results that follow will hold for any general equilibrium model where the aggregate supply elasticity $\psi = 0$.

Income in location i from trade is equal to its total sales (excluding tariffs):

$$Y_i = \sum_{j \in S} X_{ij}. \tag{33}$$

Total income (and hence expenditure) also includes the revenue earned from tariffs T_i:

$$E_i = Y_i + T_i, \tag{34}$$

where tariff revenue is equal to the bilateral tariff charged on all trade being sent[36]:

$$T_i = \sum_{j \in S} \tilde{t}_{ji} X_{ji}. \tag{35}$$

The total expenditure by consumers in location i is also equal to its total imports plus the tariffs incurred:

$$E_i = \sum_{j \in S} \left(1 + \tilde{t}_{ji}\right) X_{ji}. \tag{36}$$

Combining equations (34), (35), (36), we can demonstrate that trade flows are balanced:

$$\begin{aligned} E_i &= \sum_{j \in S} \left(1 + \tilde{t}_{ji}\right) X_{ji} \iff \\ Y_i + \sum_{j \in S} \tilde{t}_{ji} X_{ji} &= \sum_{j \in S} \left(1 + \tilde{t}_{ji}\right) X_{ji} \iff \\ Y_i &= \sum_{j \in S} X_{ji} \end{aligned} \tag{37}$$

Finally, total expenditure is equal to the payment to workers plus tariff revenue:

$$\begin{aligned} E_i &= w_i L_i + T_i \iff \\ Y_i &= w_i L_i \end{aligned} \tag{38}$$

Define $K_{ij} \equiv \tau_{ij}^{1-\sigma} t_{ij}^{-\sigma}$ as the bilateral "kernel", $B_i \equiv A_i L_i$ as the "income shifter", $\gamma_i \equiv A_i^{\sigma-1} w_i^{1-\sigma}$ as the origin fixed effect, $\delta_j \equiv P_j^{\sigma-1} E_j$ as the destination fixed effect, and $\alpha \equiv \frac{1}{1-\sigma}$. Combining equations (33), (37), and (38) yields the following system of equilibrium equations:

$$\begin{aligned} w_i L_i &= \sum_{j \in S} X_{ij} \iff \\ B_i \gamma_i^{\alpha} &= \sum_{j \in S} K_{ij} \gamma_i \delta_j \end{aligned} \tag{39}$$

[36] If we had instead supposed that tariffs are only levied on goods that actually arrive, we would have $T_i = \sum_j \frac{\tilde{t}_{ji}}{\tau_{ji}} X_{ji}$, which does not change the following analysis in any substantive way.

$$w_i L_i = \sum_{j \in S} X_{ji} \iff$$
$$B_i \gamma_i^{\alpha} = \sum_{j \in S} K_{ji} \gamma_j \delta_i. \tag{40}$$

Equations (39) and (40) can be jointly solved to recover the equilibrium $\{\gamma_i\}_{i \in S}$ and $\{\delta_i\}_{i \in S}$; given $\{\gamma_i\}_{i \in S}$ and $\{\delta_i\}_{i \in S}$, in turn, we can solve for all endogenous variables, as wages can be written as $w_i = \gamma_i^{\frac{1}{1-\sigma}} A_i$, the price index can be written as $P_i = \left(\sum_{j \in S} \tau_{ji}^{1-\sigma} t_{ji}^{1-\sigma} \gamma_j\right)^{\frac{1}{1-\sigma}}$, expenditure can be written as $E_i = \delta_i \left(\sum_{j \in S} \tau_{ji}^{1-\sigma} t_{ji}^{1-\sigma} \gamma_j\right)$, and real expenditure can be written as $W_i \equiv \frac{E_i}{P_i} = \delta_i \left(\sum_{j \in S} \tau_{ji}^{1-\sigma} t_{ji}^{1-\sigma} \gamma_j\right)^{\frac{\sigma}{\sigma-1}}$. As we note at the beginning of Section 3, this equilibrium system is identical in mathematical structure to the universal gravity equilibrium equations 1 and 2. Hence, Theorem 1 applies directly (with existence as long as $\sigma \neq 0$ and uniqueness as long as $\sigma \geq 1$). Moreover, a similar methodology as employed in Theorem 2 can be used to determine how the equilibrium variables γ_i and δ_i respond to shocks that alter the kernel K_{ij} (be they due to changes in iceberg trade frictions or tariffs). In particular:

$$\frac{\partial \ln \gamma_l}{\partial \ln K_{ij}} = X_{ij} \times \left(A_{l,i}^{+} + A_{N+l,j}^{+} - c\right) \tag{41}$$
$$\frac{\partial \ln \delta_l}{\partial \ln K_{ij}} = X_{ij} \times \left(A_{N+l,i}^{+} + A_{l,j}^{+} - c\right), \tag{42}$$

where $\tilde{A}_{i,j}^{-1}$ is the $\langle i, j \rangle$ element of the $2N \times 2N$ matrix the (pseudo) inverse$\tilde{\mathbf{A}}^{-1}$[37]:

$$\tilde{\mathbf{A}}^{-1} = \begin{pmatrix} \frac{\sigma}{1-\sigma}\mathbf{Y} & -\mathbf{X} \\ \frac{1}{1-\sigma}\mathbf{Y} - \mathbf{X}^T & -\mathbf{Y} \end{pmatrix}^{-1}, \tag{43}$$

Because all endogenous variables in the model are simple functions $\{\gamma_i\}_{i \in S}$ and $\{\delta_i\}_{i \in S}$, one can apply equations (41) and (42) to immediately derive any elasticity of interest, e.g. the effect of welfare in location l from changing the tariffs j impose on goods coming from i.

B.5 Global shocks

In this subsection we show that the "exact hat algebra" pioneered by Dekle, Eaton, and Kortum (2008) and extended by Costinot and Rodriguez-Clare (2013) can be applied to any model in the universal gravity framework to calculate the effect of any (possibly large) trade shock. (Note that Section 4 instead showed how to calculate the *elasticity* of endogenous variables to any trade cost shock). We show that the key takeaway from Section 4 holds for all trade shocks: Given observed data, all the gravity models with the same gravity constants imply the same counterfactual predictions for all endogenous variables (i.e. factor prices, price indices, nominal incomes, real incomes, and trade flows).

[37] The psuedo-inverse can be calculated simply by removing the first row and column and taking the inverse; see footnote 16

Consider an arbitrary change in the trade friction matrix $\{\tau_{ij}\}_{S\times S}$. In what follows, we denote with a hat the ratio of the counterfactual to initial value of the variable, i.e. $\hat{x}_i \equiv \frac{x_i^{counterfactual}}{x_i^{initial}}$. The following proposition provides an analytical expression relating the change in the factor price and the associated price index to the change in trade frictions and the initial observed trade flows:

Proposition 3. *Consider any given set of observed trade flows* $\mathbf{X}$*, gravity constants* ϕ *and* ψ*, and change in the trade friction matrix* $\widehat{\tau}$*. Then the percentage change in the exporter and importer shifters,* $\{\hat{p}_i\}$ *and* $\left\{\hat{P}_i\right\}$*, if it exists, will solve the following system of equations:*

$$\widehat{p}_i^{1+\phi+\psi}\widehat{P}_i^{-\psi} = \sum_{j\in S}\frac{X_{ij}}{Y_i}\widehat{\tau}_{ij}^{-\phi}\widehat{P}_j^{\phi}\widehat{p}_j\left(\frac{\widehat{p}_j}{\widehat{P}_j}\right)^{\psi} \quad \text{and} \quad \widehat{P}_i^{-\phi} = \sum_{j\in S}\left(\frac{X_{ji}}{E_j}\right)\hat{\tau}_{ji}^{-\phi}\hat{p}_j^{-\phi},\ \forall i\in S \tag{44}$$

Proof. We first note that equilibrium equations (5) and (2) must hold for both the initial and counterfactual equilibria. Taking ratios of the counterfactual to initial values yields:

$$\widehat{p}_i^{1+\phi+\psi}\widehat{P}_i^{-\psi} = \frac{\sum_{j\in S}\left(\tau'_{ij}\right)^{-\phi}\left(P'_j\right)^{\phi}p'_jC_j\left(\frac{p'_j}{P'_j}\right)^{\psi}}{\sum_{j\in S}\tau_{ij}^{-\phi}P_j^{\phi}p_jC_j\left(\frac{p_j}{P_j}\right)^{\psi}}\forall i\in S$$

$$\widehat{P}_i^{-\phi} = \frac{\sum_{j\in S}\left(\tau'_{ji}\right)^{-\phi}\left(p'_j\right)^{-\phi}}{\sum_{j\in S}\tau_{ji}^{-\phi}p_j^{-\phi}},\ \forall i\in S$$

where we denote the counterfactual equilibrium variables with a prime and the initial equilibrium variables as unadorned. Note that from the gravity equation (5) (and conditions C.3 - C.5) we have $X_{ij} = \tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}p_jC_j\left(\frac{p_j}{P_j}\right)^{\psi}$, where $p_jC_j\left(\frac{p_j}{P_j}\right)^{\psi} = E_j$, so that the above equations become:

$$\widehat{p}_i^{1+\phi+\psi}\widehat{P}_i^{-\psi} = \frac{\sum_{j\in S}\left(\tau'_{ij}\right)^{-\phi}\left(P'_j\right)^{\phi}p'_jC_j\left(\frac{p'_j}{P'_j}\right)^{\psi}}{p_i^{\phi}\sum_{j\in S}X_{ij}}\forall i\in S$$

$$\widehat{P}_i^{-\phi} = \frac{\sum_{j\in S}\left(\tau'_{ji}\right)^{-\phi}\left(p'_j\right)^{-\phi}}{P_i^{-\phi}\frac{1}{E_i}\sum_{j\in S}X_{ji}},\ \forall i\in S$$

Finally, note that from condition C.2 and C.4 we have $E_i = \sum_{j\in S}X_{ij}$ and $Y_i = \sum_{j\in S}X_{ij}$, respectively. Then using our definition $\hat{x}_i \equiv \frac{x_i^{counterfactual}}{x_i^{initial}} \iff x_i^{counterfactual} = \hat{x}_i x_i^{initial}$ we have:

$$\widehat{p}_i^{1+\phi+\psi}\widehat{P}_i^{-\psi} = \sum_{j\in S}\left(\frac{X_{ij}}{Y_i}\right)\hat{\tau}_{ij}^{-\phi}\hat{P}_j^{\phi}\hat{p}_j\left(\frac{\widehat{p}_j}{\widehat{P}_j}\right)^{\psi} \forall i\in S$$

$$\widehat{P}_i^{-\phi} = \sum_{j\in S}\left(\frac{X_{ji}}{E_j}\right)\hat{\tau}_{ji}^{-\phi}\hat{p}_j^{-\phi}\forall i\in S,$$

as required. □

Note that equation (44) inherits the same mathematical structure as equations (1) and (2). As a result, part (i) of Theorem 1 proves that there will exist a solution to equation (44) and part (ii) of Theorem 1 provides conditions for its uniqueness.

B.6 Identification

In this subsection, we show how one can always choose a set of bilateral trade costs to match observed trade flows for any choice of gravity constants, own trade costs, and supply shifters. We first state the result as a proposition before providing a proof.

Proposition 4. *Take as given the set of observed trade flows $\{X_{ij}\}$, an assumed set of supply shifters $\{C_i\}$, aggregate factor supply $\bar{Q}$, and own trade costs $\{\tau_{ii}\}$, and the gravity constants ϕ and ψ, and $\overline{Q}$. Then there exists a unique set of trade frictions $\{\tau_{ij}\}_{i\neq j}$, factor prices $\{p_i\}$, price indices $\{P_i\}$, and factor supplies $\{Q_i\}$ such that the following equilibrium conditions hold:*

1. *For all locations $i\in S$, income is equal to the product of the factor price and the factor quantity:*
$$Y_i = p_i Q_i$$
2. *For all location pairs $\{i,j\}\in S\times S$, the value of trade flows from i to j can be written in the following gravity equation form:*
$$X_{ij} = \tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j$$
3. *For all locations $i\in S$, the factor supply satisfies the following condition:*
$$Q_i = \frac{C_i\left(\frac{p_i}{P_i}\right)^{\psi}}{\sum_{k=1}^{N}C_k\left(\frac{p_k}{P_k}\right)^{\psi}}\bar{Q}$$

Proof. First, note that the income $Y_i = \sum_{j\in S}X_{ij}$, expenditure $E_i = \sum_{j\in S}X_{ji}$, and own expenditure share $\lambda_{jj}\equiv\frac{X_{jj}}{E_j}$, are all immediately derived from the observed trade flow data.

Second, let us define our unknown parameters and endogenous variables as functions of data and known parameters. The trade frictions are defined follows:

$$\tau_{ij} = \tau_{jj} \left(\frac{Y_j}{Y_i}\right) \left(\frac{\lambda_{jj}}{\lambda_{ii}}\right)^{\frac{\psi}{\phi}} \left(\frac{C_i}{C_j}\right) \left(\frac{\tau_{jj}}{\tau_{ii}}\right)^{\psi} \left(\frac{X_{jj}}{X_{ij}}\right)^{\frac{1}{\phi}}$$

for all $i, j \in S$ such that $i \neq j$.

The factor prices are defined as

$$p_i = Y_i \frac{\left(\lambda_{ii}\tau_{ii}^{\phi}\right)^{\frac{\psi}{\phi}} / C_i}{\sum_{j \in S} \left(\lambda_{jj}\tau_{jj}^{\phi}\right)^{\frac{\psi}{\phi}} / C_j}$$

for all $i \in S$.

Given the factor prices and trade costs, the price index is defined as:

$$P_i = \left(\sum_{j \in S} \tau_{ji}^{-\phi} p_j^{-\phi}\right)^{-\frac{1}{\phi}}$$

for all $i \in S$.

Finally, the factor quantities in each location are defined as:

$$Q_i = \frac{C_i \left(\frac{p_i}{P_i}\right)^{\psi}}{\sum_{k=1}^{N} C_k \left(\frac{p_k}{P_k}\right)^{\psi}} \bar{Q}$$

for all $i \in S$.

It is first helpful to note that given the above definitions of the trade costs and factor price indices, we have the following convenient relationship between own expenditure shares and prices:

$$\lambda_{jj} = \left(\tau_{jj} \frac{p_j}{P_j}\right)^{-\phi}$$

To see this, note that we can write:

$$
\begin{aligned}
\lambda_{jj} &= \left(\tau_{jj}\frac{p_j}{P_j}\right)^{-\phi} \iff \\
\frac{X_{jj}}{E_j} &= \frac{\tau_{jj}^{-\phi}p_j^{-\phi}}{\sum_{i\in S}\tau_{ij}^{-\phi}p_i^{-\phi}} \iff \\
\tau_{jj}^{-\phi}p_j^{-\phi} &= \left(\frac{X_{jj}}{E_j}\right)\sum_{i\in S}\tau_{ij}^{-\phi}p_i^{-\phi} \iff \\
\tau_{jj}^{-\phi}p_j^{-\phi} &= \left(\frac{X_{jj}}{E_j}\right)\sum_{i\in S}\left(\tau_{jj}\left(\frac{Y_j}{Y_i}\right)\left(\frac{\lambda_{jj}}{\lambda_{ii}}\right)^{\frac{\psi}{\phi}}\left(\frac{C_i}{C_j}\right)\left(\frac{\tau_{jj}}{\tau_{ii}}\right)^{\psi}\left(\frac{X_{jj}}{X_{ij}}\right)^{\frac{1}{\phi}}\right)^{-\phi} p_i^{-\phi} \iff \\
\tau_{jj}^{-\phi}p_j^{-\phi} &= \sum_{i\in S}\left(\frac{X_{ij}}{E_j}\right)\left(\frac{(Y_i/C_i)^{\phi}\left(\lambda_{ii}\tau_{ii}^{\phi}\right)^{\psi}}{(Y_j/C_j)^{\phi}\left(\lambda_{jj}\tau_{jj}^{\phi}\right)^{\psi}}\right)\tau_{jj}^{-\phi}p_i^{-\phi} \iff \\
(Y_j/C_j)^{\phi}\left(\lambda_{jj}\tau_{jj}^{\phi}\right)^{\psi}p_j^{-\phi} &= \sum_{i\in S}\left(\frac{X_{ij}}{E_j}\right)(Y_i/C_i)^{\phi}\left(\lambda_{ii}\tau_{ii}^{\phi}\right)^{\psi}p_i^{-\phi} \iff \\
p_j^{\phi-\phi} &= \sum_{i\in S}\left(\frac{X_{ij}}{E_j}\right)p_i^{\phi-\phi} \iff \\
E_j &= \sum_{i\in S}X_{ij},
\end{aligned}
$$

which is the definition of E_j.

We now confirm each of the three equilibrium conditions. To see that income is equal to the product of the factor price and the factor quantity, we write:

$$
\begin{aligned}
p_i \times Q_i &= Y_i \times \frac{\left(\left(\lambda_{ii}\tau_{ii}^{\phi}\right)^{\frac{\psi}{\phi}}/C_i\right)}{\sum_{j\in S}\left(\left(\lambda_{jj}\tau_{jj}^{\phi}\right)^{\frac{\psi}{\phi}}/C_j\right)} \times Q_i \iff \\
p_i \times Q_i &= Y_i \times \left(\frac{C_i\left(\frac{p_i}{P_i}\right)^{\psi}}{\sum_{j\in S}C_j\left(\frac{p_j}{P_j}\right)^{\psi}}\right)^{-1} \times Q_i \iff \\
p_i \times Q_i &= Y_i \times \frac{Q_i}{Q_i} \iff \\
p_i \times Q_i &= Y_i,
\end{aligned}
$$

as required.

To see that the value of trade flows can be written in the gravity equation form, we write the gravity equation as follows:

$$\tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j = \left(\tau_{jj}\left(\frac{Y_j}{Y_i}\right)\left(\frac{\lambda_{jj}}{\lambda_{ii}}\right)^{\frac{\psi}{\phi}}\left(\frac{C_i}{C_j}\right)\left(\frac{\tau_{jj}}{\tau_{ii}}\right)^{\psi}\left(\frac{X_{jj}}{X_{ij}}\right)^{\frac{1}{\phi}}\right)^{-\phi} p_i^{-\phi}P_j^{\phi}E_j \iff$$

$$\tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j = X_{ij}\left(\frac{(Y_i/C_i)^{\phi}\,\lambda_{ii}^{\psi}\tau_{ii}^{\phi\psi}}{(Y_j/C_j)^{\phi}\,\lambda_{jj}^{\psi}\tau_{jj}^{\phi\psi}}\right)\left(\frac{p_i}{p_j}\right)^{-\phi}\frac{\tau_{jj}^{-\phi}p_j^{-\phi}P_j^{\phi}E_j}{X_{jj}}$$

Recall from above that we have the following relationship between prices and own expenditure shares:

$$\lambda_{ii} = \left(\tau_{ii}\frac{p_i}{P_i}\right)^{-\phi}$$

so that:

$$\tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j = X_{ij}\left(\frac{(Y_i)^{\phi}\left(\left(\frac{p_i}{P_i}\right)^{\psi}C_i\right)^{-\phi}}{(Y_j)^{\phi}\left(\left(\frac{p_j}{P_j}\right)^{\psi}C_j\right)^{-\phi}}\right)\left(\frac{p_i}{p_j}\right)^{-\phi}\frac{\tau_{jj}^{-\phi}p_j^{-\phi}P_j^{\phi}E_j}{X_{jj}}$$

Furthermore, recall that we have defined our quantities as follows:

$$Q_i = \frac{C_i\left(\frac{p_i}{P_i}\right)^{\psi}}{\sum_{k=1}^{N}C_k\left(\frac{p_k}{P_k}\right)^{\psi}}\bar{Q},$$

which implies that:

$$\tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j = X_{ij}\left(\frac{(Y_i/Q_i)^{\phi}}{(Y_j/Q_j)^{\phi}}\right)\left(\frac{p_i}{p_j}\right)^{-\phi}\frac{\tau_{jj}^{-\phi}p_j^{-\phi}P_j^{\phi}E_j}{X_{jj}}$$

We have shown above that $p_iQ_i = Y_i$, so that we have:

$$\tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j = X_{ij}\frac{\tau_{jj}^{-\phi}p_j^{-\phi}P_j^{\phi}E_j}{X_{jj}}$$

We claim that this implies that observed trade flows are explained by the gravity equation, i.e.:

$$X_{ij} = \tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j$$

To see this, suppose not. Then we have

$$\tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j = X_{ij}\frac{\tau_{jj}^{-\phi}p_j^{-\phi}P_j^{\phi}E_j}{X_{jj}}$$

but $X_{ij} \neq \tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j$. Then without loss of generality we can write $X_{ij} = \tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j\varepsilon_{ij}$, where $\varepsilon_{ij} \neq 1$.

$$\begin{aligned}
\tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} E_j &= \left(\tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} E_j \varepsilon_{ij}\right) \frac{\tau_{jj}^{-\phi} p_j^{-\phi} P_j^{\phi} E_j}{\left(\tau_{jj}^{-\phi} p_j^{-\phi} P_j^{\phi} E_j \varepsilon_{jj}\right)} \iff \\
1 &= \frac{\varepsilon_{ij}}{\varepsilon_{jj}} \iff \\
\varepsilon_{ij} &= \varepsilon_{jj} \equiv \varepsilon_j \ \forall i \in S
\end{aligned}$$

which then implies that we have:

$$X_{ij} = \tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} E_j \varepsilon_j$$

however, we know that:

$$\begin{aligned}
\sum_{i \in S} X_{ij} &= E_j \iff \\
\sum_{i \in S} \tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} E_j \varepsilon_j &= E_j \iff \\
\frac{\sum_{i \in S} \tau_{ij}^{-\phi} p_i^{-\phi}}{\sum_{i \in S} \tau_{ij}^{-\phi} p_i^{-\phi}} &= \frac{1}{\varepsilon_j} \iff \\
\varepsilon_j &= 1,
\end{aligned}$$

which is a contradiction. Hence, the observed trade flows are explained by the gravity equation.

Finally, we note that the third equilibrium condition trivially holds by the definition of Q_i:

$$Q_i = \frac{C_i \left(\frac{p_i}{P_i}\right)^{\psi}}{\sum_{k=1}^{N} C_k \left(\frac{p_k}{P_k}\right)^{\psi}} \bar{Q}.$$

Hence, given our definitions, we have found a unique set of trade frictions $\{\tau_{ij}\}_{i \neq j}$, factor prices $\{p_i\}$, price indices $\{P_i\}$, and factor supplies $\{Q_i\}$ such that the equilibrium conditions hold for any set of observed trade flows $\{X_{ij}\}$, an assumed set of supply shifters $\{C_i\}$ and own trade costs $\{\tau_{ii}\}$, and the gravity constants ϕ and ψ. □

B.7 Real expenditure and the openness to trade

In this subsection, we show we can express real expenditure in any model within the universal gravity framework as a function of openness to trade and the gravity constants, as in Arkolakis, Costinot, and Rodríguez-Clare (2012).

We begin by defining $\lambda_{ii} \equiv \frac{X_{ii}}{E_i}$ as location i's own expenditure share. From equation (5), we can express the real factor price in a location as a function of its own expenditure share:

$$
\begin{aligned}
X_{ij} &= \frac{p_{ij}^{-\phi}}{\sum_{k\in S} p_{kj}^{-\phi}} E_j \implies \\
\lambda_{ii}^{-\frac{1}{\phi}} &= \left(\frac{p_i}{P_i}\right).
\end{aligned} \tag{45}
$$

Moreover, given conditions C.3, C.4 and C.5, we can write total real expenditure as a function of the real factor price as follows:

$$
\begin{aligned}
W_i &= \frac{E_i}{P_i} \iff \\
W_i &= \left(\frac{p_i}{P_i}\right) Q_i \iff \\
W_i &= \left(\frac{p_i}{P_i}\right) \left(\frac{C_i \left(\frac{p_i}{P_i}\right)^{\psi}}{\sum_{k\in S} C_k \left(\frac{p_k}{P_k}\right)^{\psi}} \bar{Q}\right) \iff \\
W_i &= \left(\frac{\bar{Q}}{\sum_{k\in S} C_k \left(\frac{p_k}{P_k}\right)^{\psi}}\right) C_i \left(\frac{p_i}{P_i}\right)^{1+\psi}.
\end{aligned} \tag{46}
$$

Combining equations (45) and (46) yields:

$$
W_i = \lambda C_i \left(\lambda_{ii}\right)^{-\frac{1+\psi}{\phi}},
$$

where $\lambda \equiv \left(\frac{\bar{Q}}{\sum_{k\in S} C_k \lambda_{kk}^{-\frac{\psi}{\phi}}}\right)$ is an (endogenous) scalar common to all locations. Note that a positive aggregate factor supply elasticity ($\psi > 0$) increases the elasticity of total real expenditure to own expenditure share, thereby amplifying the gains from trade. Note too that the derivations above imply that:

$$
\frac{\partial \ln \left(\frac{E_i}{P_i}\right)}{\partial \ln \tau_{ij}} = (\psi + 1) \frac{\partial \ln \left(\frac{p_i}{P_i}\right)}{\partial \ln \tau_{ij}} + \frac{\partial \ln \lambda}{\partial \ln \tau_{ij}},
$$

i.e. we can recover the elasticity of the total real expenditure (to-scale) to the trade cost shock from the elasticity of the real factor price to the trade cost shock by simply multiplying by $\psi + 1$.