Universal Gravity*

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Abstract

This paper studies the theoretical properties and counterfactual predictions of a large class of general equilibrium trade and economic geography models. We begin by presenting a framework that combines aggregate supply and demand equations with market clearing conditions. We prove that existence, uniqueness and – given observed trade flows – the counterfactual predictions for trade flows, incomes, and real output prices of any model within this framework depend only on the demand and supply elasticities (the "gravity constants"). We propose a new strategy to estimate these gravity constants using an instrumental variables approach that relies on the general equilibrium structure of the model. Finally, we use these estimates to compute the impact of a trade war between US and China.

1 Introduction

Over the past fifteen years, there has been a quantitative revolution in spatial economics. The proliferation of general equilibrium gravity models incorporating flexible linkages across many locations now gives researchers the ability to conduct a rich set of real world analyses. However, the complex general equilibrium interactions and the variegated assumptions underpinning different models has resulted in our understanding of the models' properties to lag behind. As a result, many important questions remain either partially or fully unresolved,

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including: When does an equilibrium exists and when is it unique? Do different models have different counterfactual implications?

In this paper, we characterize the theoretical and empirical properties common to a large class of gravity models spanning the fields of international trade and economic geography. We first provide a "universal gravity" framework combining aggregate demand and supply equations with standard market clearing conditions that incorporates many workhorse trade and economic geography models.[1] We show that existence and uniqueness of the equilibria of all models under the auspices of our framework can be characterized solely based on their aggregate demand and supply elasticities (the "gravity constants"). Moreover, the counterfactual predictions for trade flows, incomes, and real output prices of these models can be expressed solely as a function of the gravity constants and observed data. Hence, the key theoretical properties and positive counterfactual predictions of all gravity models depend ultimately on the value of two parameters – the elasticities of supply and demand. We show how these gravity constants can be estimated using an instrumental variables approach that relies on the general equilibrium structure of the model. Finally, we use these estimates to compute the impact of a trade war between US and China.

To construct our framework, we consider a representative economy in which an aggregate good is traded across locations subject to the following six economic conditions: 1) "iceberg" type bilateral trade frictions; 2) a constant elasticity of substitution (CES) aggregate demand function; 3) a CES aggregate supply function; 4) market clearing; 5) balanced trade; and 6) a choice of the numeraire. Any model in which the equilibrium can be represented in a way that satisfies these conditions is said to be contained within the universal gravity framework. Moreover, these conditions impose sufficient structure to completely characterize all general equilibrium interactions of trade flows, incomes, and real output prices. It turns out that the aggregate demand elasticity from condition 2 and the aggregate supply elasticity from condition 3 play a particularly important role in this characterization.

We first provide sufficient conditions for the existence, uniqueness, and interiority of the equilibrium of the model that depend solely on the gravity constants. Existence occurs everywhere except for a knife-edge constellation of parameters (corresponding e.g. to Leontief preferences in an Armington trade model or when agglomeration forces are just strong enough to create a "black hole" equilibrium in an economic geography model). An equilibrium is

[1]Examples of gravity trade models included in our framework are perfect competition models such as Anderson (1979), Anderson and Van Wincoop (2003), Eaton and Kortum (2002),Dekle et al. (2008), Caliendo and Parro (2010) monopolistic competition models such as Krugman (1980), Melitz (2003) as specified by Chaney (2008), Arkolakis et al. (2008), Di Giovanni and Levchenko (2008), , and the Bertrand competition model of Bernard et al. (2003). Economic geography models incorporated in our framework include Allen and Arkolakis (2014) and Redding (2016). See Table 1 for the mapping from work-horse trade and economic geography models into the universal gravity framework.

unique as long as the demand elasticity is (weakly) negative and the supply elasticity is (weakly) positive (or vice versa and both elasticities are greater than one in magnitude); moreover, if the inequalities are strict, an iterative algorithm is guaranteed to converge to the the unique equilibrium from any interior starting point. Multiplicity may occur if demand and supply elasticities are both negative (for example, in an economic geography model if agglomeration forces are sufficiently strong) or if demand and supply elasticities are both positive (for example, in a trade model if goods are complementary). We also show that these sufficient conditions can be extended further if trade frictions are "quasi" symmetric – a common assumption in the literature and provide conditions under which an equilibrium exists and an iterative algorithm is guaranteed to converge to the equilibrium.

We then examine how a shock to bilateral trade frictions affects equilibrium trade flows, incomes, and real output prices. To do so, we derive an analytical expression for the counterfactual elasticities of these endogenous variables to changes in all bilateral trade frictions that elucidates the networks effects of trade. In particular, we show how can this expression be written as series of terms expressing how a shock propagates through the trading network, e.g. the direct effect of a shock, the effect of the shock on all locations' trading partners, the effect on all locations' trading partners' trading partners, etc. Importantly, we show that this expression depends only on observed trade flows and the gravity constants, demonstrating that conditional on these two model parameters, the positive macro-economic implications for all gravity models are the same.[2] Moreover, we analytically prove that when trade frictions are "quasi" symmetric, the impact of a trade friction shock on the real output prices and real expenditure in directly affected locations will always exceed the impact on other indirectly-affected locations.

We proceed by estimating the gravity constants using a novel procedure that can be applied to any model contained within the universal gravity framework. We show that the supply and demand elasticities can be estimated by regressing a location's fixed effect (recovered from a gravity equation) on its own expenditure share (the coefficient of which is the supply elasticity) and its income (the coefficient of which is the demand elasticity). Identifying the elasticities requires a set of instruments that are correlated with own expenditure share and income, but uncorrelated with unobserved supply shifters (such as productivity) in the residual. We construct such instruments using the general equilibrium structure of the model by calculating the equilibrium own expenditure shares and incomes of a hypothetical world where no such unobserved supply shifters exist and bilateral trade frictions are only

[2]While the implications for real output prices are the same for all gravity models, the mapping from real output prices to welfare will in general depend on the particular model. As a result, the normative (welfare) implications will vary across different models, as we discuss in detail below.

a function of distance. Using this procedure, we estimate a demand elasticity in line with previous estimates from the trade literature (e.g. Simonovska and Waugh (2014)) and a supply elasticity that is larger than is typically (implicitly) calibrated to in trade models but appears reasonable given estimates from the economic geography literature.

Finally, we use the estimated gravity constants along with the expression for comparative statics to evaluate the effect of a trade war between the U.S. and China on the real expenditure of all countries in the world. Given our large estimated supply elasticity, we find modest declines in real output prices but large declines in real expenditure. Third country effects are also substantial, with important trading partners of China (e.g. Vietnam and Japan) and the U.S. (e.g. Canada and Mexico) being especially adversely affected.

This paper is related to a number of strands of literature in the fields of international trade, economic geography, and general equilibrium theory. There is a small but growing literature examining the structure of general equilibrium models of trade and economic geography. In particular, Arkolakis et al. (2012a) provide conditions under which a model yields a closed form expression for changes in welfare as a function of changes in openness, while in a recent paper Adao et al. (2017) show how to conduct counterfactual predictions in neoclassical trade models without imposing gravity. In contrast, our paper incorporates models with elastic aggregate supply curves, thereby allowing analysis of both economic geography models and trade models with intermediate "round-about" production. A key characteristic of the class of models we study is that the "gravity constants" are the same across all locations; while strong, this assumption imposes sufficient structure to completely characterize all general equilibrium interactions while retaining tractability even in the presence of a large number of locations.[3]

In terms of the theoretical properties of the equilibrium, Alvarez and Lucas (2007) use the gross substitutes property to establish sufficient conditions for uniqueness for gravity trade models. We instead generalize results from the study of nonlinear integral equations (see e.g. Karlin and Nirenberg (1967); Zabreyko et al. (1975); Polyanin and Manzhirov (2008)) to systems of nonlinear integral equations. As a result, the sufficient conditions we provide are strictly weaker than those derived by Alvarez and Lucas (2007). In particular, our conditions allows the supply elasticity to be larger in magnitude than the demand elasticity (in which case gross substitutes may not hold), which is what we find when we estimate the elasticities. In previous work, Allen and Arkolakis (2014) provide sufficient conditions for existence and

[3]In contrast, the literature on Computable General Equilibrium models typically focuses on models with a large number of elasticities (e.g. location or region specific) but only a small number of regions; for a review of these models see Menezes et al. (2006). Although outside the purview of this paper, it would be perhaps be interesting future work to determine whether some of the tools developed below could be applied to those models.

uniqueness for economic geography models. Unlike those results, our conditions do not require symmetric trade frictions nor do we require finite trade frictions between all locations. Unlike both Alvarez and Lucas (2007) and Allen and Arkolakis (2014), our theoretical results cover both trade and economic geography models simultaneously.

Our analytical characterization of the counterfactual predictions is related to the "exact hat algebra" methodology pioneered by Dekle et al. (2008) and extended in Costinot and Rodriguez-Clare (2013) (and many others). Unlike that approach, we characterize the elasticity of endogenous variables to trade shocks (i.e. we examine local shocks instead of global shocks). There are several advantages of our local approach: first, all possible counterfactuals can be calculated simultaneously through a single matrix inversion. Second, our analytical characterization holds for local shocks around the observed equilibria even if there are other possible equilibria (in which case we are unaware of a procedure that ensures the solution to the "exact hat" approach that corresponds to the observed equilibria). Third, the local analytical expression admits a simple economic interpretation as a shock propagating through the trading network. In this regard, our paper is related to the recent working paper by Bosker and Westbrock (2016) which examines how shocks propagate through global production networks. Fourth, our analytical derivation allows us to characterize the relative size of the elasticity of real output prices and real output in different locations from a trade friction shock, providing (to our knowledge) one of the first analytical results about the relative size of the direct and indirect impacts of a trade friction shock in a model with many locations and arbitrary bilateral frictions.[4]

Our estimation strategy uses equilibrium income and own expenditure shares from a hypothetical economy as instruments to identify the demand and supply elasticities. Following Eaton and Kortum (2002), we use the fixed effects of a gravity equation as the dependent variable in an instrumental variables regression (although we use the regression to estimate the supply elasticity along with the demand elasticity). One advantage of our approach is the simplicity of calculating our instruments using bilateral distances and observed geographic variables; in this regard, we owe credit to Frankel and Romer (1999) who instrument for trade with geography (albeit not in a general equilibrium context).

The idea of using the general equilibrium structure of the gravity model to recover key parameters is originally due to Anderson and Van Wincoop (2003). Following this, several papers have sought to improve the typical gravity equation estimation by accounting for equilibrium conditions. For example, Anderson and Yotov (2010) pursues an estimation strategy imposing that the equilibrium "adding up constraints" of the multilateral resistance terms are satisfied, whereas Fally (2015) proposes the use of a Poisson Pseudo-Maximum-

[4]Mossay and Tabuchi (2015) prove a similar result in a three country world.

Likelihood estimator whose fixed effects ensure that such constraints are satisfied, and Egger and Nigai (2015) develops a two-step model consistent approach that overcomes bias arising from general equilibrium forces and unobserved trade frictions. Unlike these papers, here our focus is on recovering the demand and supply elasticities rather than estimating trade friction coefficients in a model consistent manner.

Recent work by Anderson et al. (2016) explores the relationship between trade and growth examined by Frankel and Romer (1999) in a structural context. They recover the demand (trade) elasticity from a regression of income on a multilateral resistance term, where endogeneity concerns are addressed by calculating multilateral resistance based on international linkages only. Our estimation strategy, in contrast, recovers both the demand and supply elasticities from a gravity regression and overcomes endogeneity concerns using an instrumental variables approach based on the general equilibrium structure of the model.

Finally, we should note that the brief literature review above is by no means complete and refer the interested reader to the excellent review articles by Baldwin and Taglioni (2006), Head and Mayer (2013), Costinot and Rodriguez-Clare (2013) and Redding and Rossi-Hansberg (2017), where the latter two focus especially on quantitative spatial models.

The remainder of the paper is organized as follows. In the next section, we present the universal framework and discuss how it nests existing general equilibrium gravity models. In Section 3, we present the theoretical results for existence and uniqueness. In Section 4, we present the results concerning the counterfactual predictions of the model. In Section 5, we estimate the gravity constants. In Section 6 we calculate the effects of a U.S. - China trade war. Section 7 concludes.

2 A universal gravity framework

Before turning to the universal gravity framework, we present two variants of the simple Armington gravity model to provide a concrete example of the type of models that fall within our framework. Suppose there are N locations each producing a a differentiated good and in what follows we define the set $S \equiv \{1, ..., N\}$. The only factor of production is labor, where we denote the allocation of labor in location $i \in S$ as L_i and assume the total world labor endowment is $\sum_{i \in S} L_i = \bar{L}$. Shipping the good from $i \in S$ to final destination j incurs an iceberg *trade friction*, where $\tau_{ij} \geq 1$ units must be shipped in order for one unit to arrive. Consumers have CES preferences with elasticity of substitution $\sigma \geq 0$.

In the first variant, which we call the "trade" model, suppose that the labor endowed to a location is exogenous and perfectly inelastic, as in Anderson (1979) and Anderson and Van Wincoop (2003). Suppose too that there is roundabout production, as in Eaton and

Kortum (2002), that combines labor and an intermediate input in a Cobb-Douglas fashion. Thus, the quantity of output produced in location i is $Q_i = (A_i L_i)^{\zeta} I_i^{1-\zeta}$, with $\zeta \in (0,1]$ the labor share, A_i is the labor productivity in location $i \in S$ and I_i is an intermediate input equal to a CES aggregate of the differentiated varieties in all locations with the same elasticity of substitution σ as final demand. In this case, the output price in location i is $p_i = (w_i/A_i)^{\zeta} P_i^{1-\zeta}$, where w_i is the wage and $P_j \equiv \left(\sum_{k\in S} (p_j \tau_{kj})^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$ is both the CES price index for the consumer and the price per unit of intermediate input.

In the second variant, the "economic geography" model, we suppose instead that the labor supplied to a location is perfectly elastic so that welfare is equalized across locations, as in Allen and Arkolakis (2014).[5] Welfare in this model is the product of the real expenditure of labor and the amenity value of living in a location, denoted by , and . welfare equalization implies $\frac{w_i}{P_i} u_i = \frac{w_j}{P_j} u_j$ for all $i, j \in S$. We further assume that productivities and amenities are subject to spillovers: $A_i = \bar{A}_i L_i^a$ and $u_i = \bar{u}_i L_i^b$. In this variant of the model, the quantity of output produced in location i is $Q_i = \bar{A}_i L_i^{1+a}$ and the output price is $p_i = w_i / \left(\bar{A}_i L_i^a\right)$.[6]

In both variants of the model, CES consumer preferences for the goods from each location yields a gravity equation that characterizes the aggregate demand in location j for the differentiated variety from location i:

$$X_{ij} = \frac{(p_i \tau_{ij})^{1-\sigma}}{\sum_{k\in S} (p_j \tau_{kj})^{1-\sigma}} E_j, \quad \text{for all } j, \tag{1}$$

where $E_j = \sum_{j\in S} X_{ji}$ is the expenditure in location j.

More subtly, both variants of the model also feature an aggregate supply for the quantity of output produced in each location. In the trade variant of the model – despite the labor supply being perfectly inelastic – we can use the fact that a constant share of revenue is paid to both workers and intermediates to write the output of location i as:

$$Q_i = A_i L_i \left(\frac{p_i}{P_i}\right)^{\frac{1-\zeta}{\zeta}}. \tag{2}$$

Similarly, in the economic geography variant of the model we can use the welfare equalization condition to write:

$$Q_i = \kappa \bar{A}_i^{\frac{b-1}{a+b}} \bar{u}_i^{-\frac{1+a}{a+b}} \left(\frac{p_i}{P_i}\right)^{-\frac{1+a}{a+b}}, \tag{3}$$

[5]In addition, this formulation incorporates many prominent economic geography models, e.g. Helpman (1998); Donaldson and Hornbeck (2012); Bartelme (2014); Redding (2016).

[6]It is straightforward to add round-about production into the economic geography variant of the model (see Table 1); we omit to do so here to keep our illustrative examples as simple as possible.

where $\kappa \equiv \left(\bar{L}/\left(\sum_{i\in S}\left(\bar{A}_i\bar{u}_i\right)^{-\frac{1}{a+b}}\left(\frac{p_i}{P_i}\right)^{-\frac{1}{a+b}}\right)\right)^{1+a}$ is an (endogenous) scalar that depends on the aggregate labor endowment $\bar{L}$ and we refer to $\frac{p_i}{P_i}$ as the *real output price* in location $i \in S$.[7] Finally, in both variants, we close the model by requiring that the value of total output equals total sales (market clearing), i.e.

$$Y_i \equiv p_i Q_i = \sum_{j\in S} X_{ij}, \tag{4}$$

and that total expenditure equals total output (balanced trade), i.e.:

$$E_i = p_i Q_i. \tag{5}$$

Substituting the CES demand (equation 1) and supply equations (equations 2 or 3) into the market clearing and balanced trade conditions yields the following identical system of equilibrium equations for both variants of the model. In particular,

$$p_i^{1+\phi} C_i \left(\frac{p_i}{P_i}\right)^{\psi} = \sum_{j\in S} \tau_{ij}^{-\phi} P_j^{\phi} p_j C_j \left(\frac{p_j}{P_j}\right)^{\psi} \forall i \in S \tag{6}$$

$$P_i^{-\phi} = \sum_{j\in S} \tau_{ji}^{-\phi} p_j^{-\phi} \ \forall i \in S, \tag{7}$$

where in the trade variant of the model $\psi \equiv \frac{1-\zeta}{\zeta}$ and $C_i \equiv A_i L_i$, in the economic geography variant of the model $\psi \equiv -\frac{1+a}{a+b}$ and $C_i \equiv \bar{A}_i^{\frac{b-1}{a+b}} \bar{u}_i^{-\frac{1+a}{a+b}}$, and in both models $\phi \equiv \sigma - 1$. Note in both models the constants $\{C_i\}_{i\in S}$ are exogenous model location-specific fundamentals, which we refer to as *supply shifters* in what follows, and ϕ, ψ are global parameters. Given supply shifters, trade frictions, and the two parameters, one can use equations (6) and (7) to solve for output prices p_i and prices indices P_i (up-to-scale). One can then use a normalization that total world income is equal to one, i.e. $\sum_{i\in S} Y_i = 1$ and the gravity equation (equation 1) to calculate trade flows X_{ij}. Given trade flows, income Y_i can then be recovered from market clearing (equation 4). Note that although the endogenous scalar κ from the economic geography model does not enter the equilibrium system of equations (and hence does not affect trade flows or incomes), it does affect the level of output, a point we return to below.

[7] In these two examples – as in most of the analysis that follows – we focus on interior equilibria where production is positive in all locations. In the Online Appendix B.2 we generalize our setup to allow for the possibility of non-interior solutions where production is zero in some locations, which allows e.g. for the case that welfare in unpopulated locations may be lower than populated locations. In Theorem 1 below, we provide sufficient conditions under which all equilibria are guaranteed to be interior.

This example highlights the close relationship between trade and geography models and suggests the possibility for a unified analysis of the properties of such spatial gravity models. In what follows, we present a framework comprising six simple economic conditions about aggregate trade flows of a representative good between many locations. We show that the equilibrium of any model that satisfies these conditions can be represented by the solution to equations (6) and (7).

To proceed with out universal gravity framework, it is helpful to first introduce some terminology. Define the *output* $Q_i \geq 0$ to be the quantity of the representative good produced in location $i \in S$; the *quantity traded* $Q_{ij} \geq 0$ be the quantity of the representative good in location $i \in S$ that is consumed in location $j \in S$; the *output price* $p_i \geq 0$ to be the (factory gate) price per unit of the representative good in location $i \in S$; the *bilateral price* $p_{ij} \geq 0$ to be the cost of the representative good from location $i \in S$ in location $j \in S$; the *income* $Y_i \equiv p_i Q_i$ to be the total value of the representative good in location $i \in S$; the *trade flows* $X_{ij} \equiv p_{ij} Q_{ij}$ to be the value of the good in $i \in S$ sold to $j \in S$; the *expenditure* $E_i \equiv \sum_{j \in S} X_{ji}$ to be the total value of imports in $i \in S$; the *real expenditure* $W_i \equiv E_i / P_i$ is a measure of expenditure in location $i \in S$, where P_i is a *price index* defined below; and the *real output price* to be p_i / P_i.

We say that an equilibrium is *interior* if output and output prices are strictly positive in all locations, i.e. $Q_i > 0$ and $p_i > 0$ for all $i \in S$. In what follows, we focus our attention to interior equilibria and disregard the trivial equilibrium where $Q_i = 0$ for all $i \in S$. We provide sufficient conditions to ensure all equilibria are interior below and examine non-interior solutions in depth in Online Appendix B.2. Clearly, because of the presence of complementarities there is a possibility of multiple interior equilibria. This is true in the economic geography model because of labor mobility and agglomeration externalities or even in the trade model when complementarities in consumption are large (low σ).

We first start with a condition that describes the relationship between the output price in location i and the bilateral price:

Condition 1. The bilateral price is equal to the product of the output price and a bilateral scalar:

$$p_{ij} = p_i \tau_{ij}, \tag{8}$$

where, as above, $\{\tau_{ij}\}_{i,j \in S} \in \overline{\mathbb{R}}_{++}$ are referred to as *trade frictions*.[8]

Given prices, the next condition can be used to derive aggregate demand.

[8] $\overline{\mathbb{R}}_{++}$ is defined as $\mathbb{R}_{++} \cup \{\infty\}$. If $\tau_{ij} = \infty$, then there is no trade between i and j.

Condition 2. (CES Aggregate Demand). There exists an exogenous (negative of the) *demand elasticity* $\phi \in \mathbb{R}$ such that the expenditure in location $j \in S$ can be written as:

$$E_j = \left(\sum_{i \in S} p_{ij}^{-\phi} \right)^{-\frac{1}{\phi}} W_j, \tag{9}$$

where W_j is the real expenditure and the associated price index is $P_j \equiv \left(\sum_{i \in S} p_{ij}^{-\phi} \right)^{-\frac{1}{\phi}}$. By Shephard's lemma, condition 2 (or, for short, C.2 thereafter) implies that the trade flows from $i \in S$ to $j \in S$ can be written as::

$$X_{ij} = \frac{p_{ij}^{-\phi}}{\sum_{k \in S} p_{kj}^{-\phi}} E_j. \tag{10}$$

We refer to equation (10) as the *aggregate demand* of the universal gravity model. The aggregate demand equation (10) combined with C.1 yields a *gravity equation* equivalent to equation (2) in Anderson and Van Wincoop (2004), Condition R3' in Arkolakis et al. (2012a) and the CES factor demand specification considered in Adao et al. (2017). Accordingly, we note that the demand elasticity ϕ is often referred to as the "trade elasticity" in the literature.

It is important to emphasize that real expenditure $W_i = \frac{E_i}{P_i}$ and real output prices $\frac{p_i}{P_i}$ are distinct concepts from welfare, as neither necessarily correspond to the welfare of the underlying factor of production (such as labor) of a particular model. In the models above, for example, the welfare of a worker corresponds to her real wage, which is equal to the marginal product of a worker divided by the price index. Because of the presence of roundabout production (in the trade model) or externalities (in the economic geography model), a workers marginal product is not equal to the price per unit (gross) output. [9]

We furthermore assume that output in a location is potentially endogenous and specify the following supply-side equation:

Condition 3. (CES Aggregate Supply) There exists exogenous *supply shifters* $\{C_i\} \in \mathbb{R}_{++}^N$, an exogenous *aggregate supply elasticity* $\psi \in \mathbb{R}$, and an endogenous scalar $\kappa > 0$ such that output in each location $i \in S$ can be written as: (11)

$$Q_i = \kappa C_i \left(\frac{p_i}{P_i} \right)^{\psi}. \tag{11}$$

In what follows, we refer to equation (11) as the *aggregate supply* of the universal gravity

[9]The relationship between real output prices and welfare for a number of seminal models are summarized in the last column of Table 1 and discussed in detail in Online Appendix B.11.

model and the pair of demand and supply elasticities $(-\phi, \psi)$ as the *gravity constants*.

In general, the value of the endogenous scalar κ will depend on the particular model; for example, as we saw above, in the trade model $\kappa = 1$, whereas in the economic geography model κ is endogenously determined. Without taking a particular stance on the underlying model (and the implied value of κ), the scale of output is unspecified. However, we show below that we can still identify the equilibrium trade flows, incomes, and real output prices – including their level – without knowledge of κ.

Finally, to close the model, we impose two standard conditions and choose our numeraire:

Condition 4. (Output market clearing). For all $i \in S$, $Q_i = \sum_{j \in S} \tau_{ij} Q_{ij}$.

Note that by multiplying both sides of C.4 by the output price we have that income is equal to total sales as in equation (4) in our example economy.[10]

Condition 5. (Balanced trade). For all $i \in S$, $E_i = p_i Q_i$.

Balanced trade is a standard assumption in (static) gravity models, despite trade imbalances being a common occurrence empirically. When we combine the general equilibrium structure of the model with data to characterize the counterfactual implications of gravity models, we relax C.5 to allow for exogenous trade deficits.

Our final condition is a normalization:

Condition 6. World income equals to one:

$$\sum_i Y_i = 1. \tag{12}$$

In the absence of a normalization, the level of prices are undetermined because equations (6) and (7) are homogeneous of degree 0 in $\{p_i, P_i\}_{i \in S}$. Moreover, without specifying κ in equation (11), the level of output is also unknown. The choice of normalizing world income to one in C.6 addresses both these issues simultaneously by pining down the product of the level of these two unknown scalars. As a result, we can determine the equilibrium level (i.e. including scale) of nominal incomes and trade flows. However, the cost of doing is that both the level of output (in quantities) and prices remain unknown. As a result, the primary focus in the following analysis is on three endogenous model outcomes for which we can pin down the levels: incomes, trade flows, and real output prices $\{p_i/P_i\}_{i \in S}$ (which are invariant to the both κ and the scale of prices and hence determined including scale).

[10] As Anderson and Van Wincoop (2004) show, one can combine C.1, C.2, and C.4 to derive a gravity equation of the form $X_{ij} = \left(\frac{\tau_{ij}}{\Pi_i P_j}\right)^{-\phi} Y_i E_j$, where $\Pi_i^{-\phi} \equiv \sum_{j \in S} \left(\frac{\tau_{ij}}{P_j}\right)^{-\phi} E_j$ and $P_j^{-\phi} \equiv \sum_{i \in S} \left(\frac{\tau_{ij}}{\Pi_i}\right)^{-\phi} Y_i$ are outward and inward multilateral resistance terms, respectively.

Given any gravity constants $\{\phi, \psi\}$, supply shifters, $\{C_i\}_{i \in S}$, and bilateral trade frictions $\{\tau_{ij}\}_{i,j \in S}$, we define an *equilibrium of the universal gravity framework* to be a set of endogenous outcomes determined up-to-scale, namely: outputs $\{Q_i\}_{i \in S}$, quantities traded $\{Q_{ij}\}_{i,j \in S}$, output prices $\{p_i\}_{i \in S}$, bilateral prices $\{p_{ij}\}_{i,j \in S}$, price indices $\{P_i\}_{i \in S}$, and real expenditures, as well as a set of endogenous outcomes for which the scale is known, namely: incomes $\{Y_i\}_{i \in S}$, expenditures $\{E_i\}_{i \in S}$, trade flows $\{X_{ij}\}_{i,j \in S}$ and real output prices $\{p_i/P_i\}_{i \in S}$ that together satisfy C.2-C.6.

As Table 1 summarizes, many well-known trade and economic geography models are contained within the universal gravity framework. On the demand side, it is well known (see e.g. Arkolakis et al. (2012b) and Adao et al. (2017)) that many trade models imply an aggregate CES demand system as specified in C.2.[11] For example, in the Armington perfect competition model, a CES demand combined with linear production functions implies $\phi = \sigma - 1$, in the Eaton and Kortum (2002) model, a Ricardian model with endogenous comparative advantage across goods and Frechet distributed productivities across sectors with elasticity θ implies that $\phi = \theta$. Similarly, a class of monopolistic models with CES or non-CES demand, linear production function, and Pareto distributed productivities with elasticity θ, summarized in Arkolakis et al. (2012b), also implies $\phi = \theta$. Economic geography models delivering gravity equations for trade flows such as Allen and Arkolakis (2014) and Redding (2016) also satisfy C.2.

As discussed in the example above, labor mobility across locations generates a CES aggregate supply satisfying C.3, with a supply elasticity of $\psi = -\frac{1+a}{a+b}$. In this case, the supply elasticity depends on the strength of the agglomeration / dispersion forces summarized by $a + b$. Assuming $a > -1$, if dispersion forces dominate $(a + b < 0)$, the supply elasticity is positive, whereas when agglomeration forces dominate $(a + b > 0)$, the supply elasticity is negative.

Perhaps more surprising, trade models incorporating "round-about" trade with intermediates goods also exhibit an aggregate CES supply, even though workers are immobile across locations. As discussed in the example above, the supply elasticity is $\psi = \frac{1-\zeta}{\zeta}$ and hence positive and increasing in the share of intermediates in the production. In the next two sections, we show that any trade and economic geography models sharing the same gravity constants will also share the same theoretical properties and counterfactual implications.

What types of models are not contained within the universal gravity framework? C.2 and C.3 are violated by models that do not exhibit constant demand and supply elasticities,

[11] The class of trade models considered by Arkolakis et al. (2012a) (under their CES demand assumption R3') are a strict subset of the models which fall within the universal gravity framework, corresponding to the case of $\psi = 0$.

which include Novy (2010), Head et al. (2014), Melitz and Redding (2014), Fajgelbaum and Khandelwal (2013) and Adao et al. (2017). Models with multiple factors of production with non-constant factor intensities will generally not admit a single aggregate good representation and hence are also not contained within the universal gravity framework (although the tools developed below can often be extended to analyze such models depending on the particular functional forms). C.5 is violated both by dynamic models in which the trade deficits are endogenously determined and by models incorporating additional sources of revenue (like tariffs); hence these models are not contained within the universal gravity framework. However, we show in Online Appendix B.8 how the results below can be applied to a simple Armington trade model with tariffs.[12]

3 Existence, uniqueness, and interiority of equilibria

We proceed by deriving a number of theoretical properties of the equilibria of all models contained within the universal gravity framework.

To begin, we note that we can combine C.1 through C.5 to write the equilibrium output prices and price indices (to-scale) as the solution to equations (6) and (7). These equations are sufficient to recover the equilibrium level of real output prices and – given the normalization in C.6 – the equilibrium level of incomes, expenditures, and trade flows as well as all other endogenous variables up-to-scale.[13] As a result, equations (6) and (7) (together with the normalization in C.6) are sufficient to characterize the equilibrium of the universal gravity framework.

Before proceeding, we impose two mild conditions on bilateral trade frictions $\{\tau_{ij}\}_{i,j\in S}$:

Assumption 1. *The following parameter restrictions hold:*

i) $\tau_{ii} < \infty$ *for all* $i \in S$.

ii) The graph of the matrix of trade frictions $\{\tau_{ij}\}_{i,j\in S}$ *is strongly connected.*

The first part of the assumption imposes strictly positive diagonal elements of the matrix of bilateral trade frictions. The second part of the assumption – strong connectivity –

[12] It is important to note that while the universal gravity framework can admit tariffs, how tariffs affect the model implications will in general depend on the micro-economic foundations of a model. In particular, the Armington model presented in Online Appendix B.8 abstracts from two additional complications that may arise with the introduction of tariffs. First, the elasticity of trade to tariffs may be different than the elasticity of trade to trade frictions depending on the model; second, if one does not impose that tariffs are uniform for all trade flows between country pairs, the construction of (good-varying) optimal tariffs will depend on the particular micro-economic structure of the model; see Costinot et al. (2016) for a detailed discussion of these issues.

[13] See Online Appendix B.1 for these derivations.

requires that there is a sequential path of finite bilateral trade frictions that can link any two locations i and j for any $i \neq j$. This condition has been applied previously in general equilibrium analysis as a condition for existence in McKenzie (1959, 1961), Arrow et al. (1971), invertibility by Cheng (1985); Berry et al. (2013), and uniqueness by Arrow et al. (1971), Allen (2012). In our case these two assumptions are the weakest assumptions on the matrix of trade frictions we can accommodate in order to analyze existence and uniqueness of interior equilibrium.

We mention briefly (but do not need to assume) a third condition. We say that trade frictions are *quasi-symmetric* if there exist a pair of strictly positive vectors $\left(\tau_i^A, \tau_i^B\right) \in \mathbb{R}_{++}^{2N}$ such that for any $i, j \in S$, we can write $\tau_{ij} = \tilde{\tau}_{ij}\tau_i^A\tau_j^B$, where $\tilde{\tau}_{ij} = \tilde{\tau}_{ji}$. Quasi-symmetry is a common assumption in the literature (see for example Anderson and Van Wincoop (2003), Eaton and Kortum (2002), Waugh (2010), Allen and Arkolakis (2014)), and we prove in Online Appendix B.3 that C.1, C.2, C.4, and C.5 taken together imply that the origin and destination-specific terms in the bilateral trade flow expression are equal up to scale, i.e. $p_i^{-\phi} \propto p_i^{1+\psi} P_i^{\phi-\psi} C_i$, which in turn implies that equilibrium trade flows will be symmetric, i.e. $X_{ij} = X_{ji}$ for all $i, j \in S$. The only way the trade can be balanced when trade frictions are quasi-symmetric is to make trade flows bilaterally balanced. As a result, equations (6) and (7) simplify to a single set of equilibrium equations, which allows allows us to relax the conditions on the following theorem regarding existence and uniqueness:

Theorem 1. *Consider any model contained within the universal gravity framework satisfying Assumption 1. Then:*

(i) If $1 + \psi + \phi \neq 0$, then there exists an interior equilibrium.

(ii) If $\phi \geq -1$, and $\psi \geq 0$ then all equilibria are interior.

(iii) If $\{\phi \geq 0, \psi \geq 0\}$ or $\{\phi \leq -1, \psi \leq -1\}$ (or, if trade frictions are quasi-symmetric and either $\left\{\phi \geq -\frac{1}{2}, \psi \geq -\frac{1}{2}\right\}$ or $\left\{\phi \leq -\frac{1}{2}, \psi \leq -\frac{1}{2}\right\}$) then there is a unique interior equilibrium.

(iv) If $\{\phi > 0, \psi > 0\}$ or $\{\phi < -1, \psi < -1\}$ (or, if trade frictions are quasi-symmetric and either $\left\{\phi > -\frac{1}{2}, \psi > -\frac{1}{2}\right\}$ or $\left\{\phi < -\frac{1}{2}, \psi < -\frac{1}{2}\right\}$).

Proof. See Appendix A.1 for parts (i) and (iii) and Online Appendix B.2 for part (ii). □

A key advantage of Theorem 1 is that despite the large dimensionality of the parameter space (N supply shifters $\{C_i\}_{i\in S}$ and N^2 trade frictions $\{\tau_{ij}\}_{i,j\in S}$), the conditions are only stated in terms of the two gravity constants. Of course, since we provide sufficient conditions, there may be certain parameter constellations such as particular geographies of trade frictions where uniqueness may still occur even if the conditions of Theorem 1 are not satisfied.[14,15]

The sufficient conditions for existence, interiority, and uniqueness from Theorem 1 are illustrated in Figure 1. In the case of existence, standard existence theorems (see e.g. Mas-Colell et al. (1995)) guarantee existence for endowment economies when preferences are strictly convex. This is also true in the universal gravity framework: existence of an interior equilibrium may fail only when $1 + \psi + \phi = 0$, which corresponds to the Armington trade model (without intermediate goods) where $\sigma = 0$, i.e. with Leontief preferences that are not strictly convex. Moreover, in the economic geography example above, an interior equilibrium does not exist in the knife-edge case where $\sigma = \frac{1+a}{a+b}$, as agglomeration forces lead to the concentration of all economic activity in one location (see Allen and Arkolakis (2014)).

As long as the partial elasticity of aggregate demand with respect to own output price is greater than negative 1 and the partial elasticity of supply with respect to the real output price is positive, all equilibria are interior. For example, in the economic geography model above, if these conditions are satisfied, one can show that the welfare of an uninhabited location approaches infinity as its population approaches zero, ensuring that all locations will be populated in equilibrium.

An equilibrium is unique as long as the partial elasticity of aggregate demand to output prices is negative (i.e. $\phi \geq 0$) and the partial elasticity of aggregate supply is positive (i.e. $\psi \geq 0$). There is also a unique interior equilibrium the demand elasticity is positive and the supply elasticity is negative and both elasticities have magnitudes greater than one, although such parameter constellations are less economically meaningful (and there may also exist non-interior equilibria). Multiplicity of interior equilibria may arise in cases when supply and demand elasticities are both positive (which occurs e.g. in trade models when goods are complements) or when supply and demand elasticities are both negative (which occurs e.g. in economic geography models when agglomeration forces are stronger than dispersion forces). Such examples of multiplicity are easy to construct - Appendix B.7 provides examples of multiplicity in a two location world where either the demand elasticity is negative (in which

[14] Alvarez and Lucas (2007) provide an alternative approach based on the gross substitute property to provide conditions for uniqueness of the Eaton and Kortum (2002) model. In Online Appendix B.6, we show that the gross substitutes property directly applied to our system may fail if the supply elasticity ψ is larger in magnitude than the demand elasticity ϕ, i.e. in ranges $\psi > \phi \geq 0$ or $\psi < \phi \leq -1$. Theorem 1 provides strictly weaker sufficient conditions in that regard. Such parameter constellations are consistent with economic geography models with weak dispersion forces or trade models with large intermediate goods shares. Importantly, in Section 5, we estimate that $\psi > \phi > 0$ empirically.

[15] Theorem 1 generalizes Theorem 2 of Allen and Arkolakis (2014) in three ways: 1) it allows for asymmetric trade frictions; 2) it allows for infinite trade frictions between certain locations; and 3) it applies to a larger class of general equilibrium spatial model, including notably trade models with inelastic labor supplies (i.e. models in which $\psi = 0$). Theorem 1 also provides a theoretical innovation, as it shows how to extend the mathematical argument of Karlin and Nirenberg (1967) to multi-equation systems of non-linear integral equations.

case the relative demand and supply curves are both upward sloping) or the supply elasticity is negative (in which case the relative demand and supply curves are both downward sloping). Finally, quasi-symmetric trade frictions allow us to extend the range of gravity constants for which uniqueness is guaranteed, but do not qualitatively change the intuition for the results.

4 The network effects of a trade shock

We now turn to how the universal gravity framework can be used to make predictions of how a change in trade frictions alter equilibrium trade flows, incomes, and real output prices in each location.[16]

To begin, we define two $N \times 1$ vectors (which, with some abuse of language, we will call "curves"): define the supply curve $\mathbf{Q}^s$ to be the set of supply equations (11) from C.3 (multiplied by output prices and divided by κ); and define the demand curve $\mathbf{Q}^d$ to be the set of market clearing (demand) equations combining C.1, C.2, C.4, and C.5, i.e.:

$$\mathbf{Q}^s(\mathbf{p},\mathbf{P}) \equiv \left(p_i \times C_i \left(\frac{p_i}{P_i}\right)^{\psi}\right)_{i\in S} \tag{13}$$

$$\mathbf{Q}^d(\mathbf{p},\mathbf{P};\boldsymbol{\tau}) \equiv \left(\sum_{j\in S} \tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} p_j C_j \left(\frac{p_j}{P_j}\right)^{\psi}\right)_{i\in S}, \tag{14}$$

where $\mathbf{p} \equiv (p_i)_{i\in S}$ and $\mathbf{P} \equiv \left(\left[\sum_j \tau_{ji}^{-\phi} p_j^{-\phi}\right]^{-\frac{1}{\phi}}\right)_{i\in S}$ are $N \times 1$ vectors and $\tau \equiv (\tau_{ij})_{i,j\in S}$ is an $N^2 \times 1$ vector. Note that we express both the supply and demand curves in value terms, which will prove helpful in deriving the comparative statics in terms of observed trade flows.

In equilibrium, supply is equal to demand, i.e. $\mathbf{Q}^s(\mathbf{p},\mathbf{P}) = \mathbf{Q}^d(\mathbf{p},\mathbf{P};\boldsymbol{\tau})$. We fully differentiate this equation, along with the definition of the price index, to yield the following system of $2N$ linear equations relating a small change in trade costs, $D\ln\boldsymbol{\tau}$, to a small change in output prices and price indices, $D\ln\mathbf{p}$ and $D\ln\mathbf{P}$, respectively:

$$\left(\underbrace{\begin{pmatrix} D_{\ln\mathbf{p}}\mathbf{Q}^s & \mathbf{0} \\ \mathbf{0} & \mathbf{I}\end{pmatrix}}_{\equiv\mathbf{S}} - \underbrace{\begin{pmatrix} D_{\ln\mathbf{p}}\mathbf{Q}^d & D_{\ln\mathbf{P}}\mathbf{Q}^d - D_{\ln\mathbf{P}}\mathbf{Q}^s \\ D_{\ln\mathbf{p}}\ln\mathbf{P} & \mathbf{0}\end{pmatrix}}_{\equiv\mathbf{D}}\right)\begin{pmatrix} D\ln\mathbf{p} \\ D\ln\mathbf{P}\end{pmatrix} = \underbrace{\begin{pmatrix} D_{\ln\boldsymbol{\tau}}\mathbf{Q}^d \\ D_{\ln\boldsymbol{\tau}}\ln\mathbf{P}\end{pmatrix}}_{\equiv\mathbf{T}} D\ln\boldsymbol{\tau},$$

where $\mathbf{S}$ (*the supply matrix*) and $\mathbf{D}$ (*the demand matrix*) are $2N \times 2N$ matrices capturing

[16] In what follows, we focus on the policy shocks that alter bilateral trade frictions $\{\tau_{ij}\}_{i,j\in S}$. In Online Appendix B.8, we show how one can apply similar tools to characterize the theoretical properties and conduct counterfactuals in an Armington trade model with tariffs.

the marginal effects of a change in the output price on the supply and demand curves (where the demand matrix also captures the net effect of a change in the price index), respectively, and $\mathbf{T}$ is a $2N \times N^2$ matrix capturing the marginal effects of a change in trade costs on the demand curve and price index.

Given expressions (13) and (14), we can write all three matrices solely as a function of the gravity constants and observables as follows:

$$\mathbf{S} = \begin{pmatrix} (1+\psi)\,\mathbf{Y} & ,\mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix},\ \mathbf{D}= \begin{pmatrix} -\phi\mathbf{Y} + (1+\psi)\,\mathbf{X} & (\phi-\psi)\,\mathbf{X} + \psi\mathbf{Y} \\ \mathbf{E}^{-1}\mathbf{X}^T & \mathbf{0} \end{pmatrix},\ \mathbf{T} = \begin{pmatrix} -\phi\,(\mathbf{X}\otimes\mathbf{1})\circ(\mathbf{I}\otimes\mathbf{1}) \\ \left(\mathbf{E}^{-1}\mathbf{X}^T\otimes\mathbf{1}\right)\circ(\mathbf{1}\otimes\mathbf{I}) \end{pmatrix}, \tag{15}$$

where $\mathbf{X}$ is the (observed) $N \times N$ trade flow matrix whose $\langle i,j\rangle^{th}$ element is X_{ij}, $\mathbf{Y}$ is the $N \times N$ diagonal income matrix whose i^{th} diagonal element is Y_i, $\mathbf{E}$ is the $N \times N$ diagonal income matrix whose i^{th} diagonal element is E_i, $\mathbf{I}$ is the $N \times N$ identity matrix and $\mathbf{1}$ is an $1 \times N$ matrix of ones,$\mathbf{I}_i$ is the standard i-th basis for $\mathbb{R}^N$, and where $\otimes$ represents the Kronecker product and $\circ$ represents the element-wise multiplication (i.e. Hadamard product). [17]

A simple application of the implicit function theorem allows us to characterize the elasticity of prices and price indices to any trade cost shock. Define the $2N \times 2N$ matrix $\mathbf{A} \equiv \mathbf{S} - \mathbf{D}$ and, with a slight abuse of notation, let $A^{-1}_{k,l}$ denote the $\langle k,l\rangle^{th}$ element of the (pseudo) inverse of $\mathbf{A}$. Then:

Theorem 2. *Consider any model contained in the universal gravity framework. Suppose that $\mathbf{X}$ satisfies strong connectivity. If $\mathbf{A}$ has rank $2N-1$, then:*

(i) The elasticities of output prices and output price indices are given by:

$$\frac{\partial \ln p_l}{\partial \ln \tau_{ij}} = -\phi X_{ij} A^{-1}_{l,i} + \frac{X_{ij}}{E_j} A^{-1}_{l,N+j} \quad \text{and} \quad \frac{\partial \ln P_l}{\partial \ln \tau_{ij}} = -\phi X_{ij} A^{-1}_{N+l,i} + \frac{X_{ij}}{E_j} A^{-1}_{N+l,N+j}. \tag{16}$$

(ii) If the largest absolute value of eigenvalues of $\mathbf{S}^{-1}\mathbf{D}$ is less than one, then $\mathbf{A}^{-1}$ has the following series expansion:

$$\mathbf{A}^{-1} = \sum_{k=0}^{\infty} \left(\mathbf{S}^{-1}\mathbf{D}\right)^k \mathbf{S}^{-1},$$

[17] In what follows (apart from part (iii) of Theorem 2), we do not assume that C.5 holds in the data, i.e. that income is necessarily equal to expenditure; rather, we allow for income and expenditure to differ by a location-specific scalar, i.e. we allow for (exogenous) deficits.

(iii) If trade frictions are quasi-symmetric and $\phi, \psi \geq 0$ then for all $i, l \in S$ and $j \neq i, l$,

$$\frac{\partial \ln (p_i/P_i)}{\partial \ln \tau_{il}}, \frac{\partial \ln (p_l/P_l)}{\partial \ln \tau_{li}} < \frac{\partial \ln (p_j/P_j)}{\partial \ln \tau_{il}}$$
$$\frac{\partial \ln (p_i Q_i/P_i)}{\partial \ln \tau_{il}}, \frac{\partial \ln (p_l Q_l/P_l)}{\partial \ln \tau_{li}} < \frac{\partial \ln (p_j Q_j/P_j)}{\partial \ln \tau_{il}}.$$

and the inequalities have the opposite sign ($>$) if ($\phi, \psi \leq -1$).

Proof. See Appendix A.2. □

Recall from Section 3 that knowledge of the output prices and price indices up-to-scale is sufficient to recover real output prices and – along with the normalization C.6 – is sufficient to recover equilibrium trade flows, expenditures, and incomes.[18] As a result, part (i) of Theorem 2 states that given gravity constants and observed data, the (local) counterfactuals of these variables for all models contained in the university gravity framework are the same.[19]

The second part of Theorem 2 provides a simple interpretation of the counterfactuals as a shock propagating through the trade network. Consider a shock that decreases the trade cost between i and j by a small amount $\partial \ln \tau_{ij}$ and define $\left(\mathbf{S}^{-1}\mathbf{D}\right)^k \mathbf{S}^{-1}$ as the k^{th} *degree effect* of the shock. It turns out the k^{th} degree effect is simply the effect of the $k-1^{th}$ degree shock on the output prices and price indices of all locations' trading partners, holding constant their trading partners' prices and price indices. To see this, consider first the 0^{th} degree effect. Holding constant the prices and price indices in all other locations, the direct effect of a decrease in $\partial \ln \tau_{ij}$ is a shift of the demand curve upward in i by $\phi X_{ij} \times \partial \ln \tau_{ij}$ and a decrease in the price index in j by $\frac{X_{ij}}{E_j} \times \partial \ln \tau_{ij}$. To re-equilibriate supply and demand (holding constant prices and price indices in all other locations), we then trace along the

[18] Because of homogeneity of degree 0, we can without loss of generality normalize one price; moreover, from Walras' law, if $2N-1$ equilibrium conditions hold, then the last equation holds as well. As a result, $\mathbf{A}$ will have at most $2N-1$ rank and $\mathbf{A}^{-1}$ can be calculated by simply eliminating one row and column of $\mathbf{A}$ and then calculating its inverse. The values of the eliminated row can then be determined using the normalization C.6. For example, if one removes the first row and column, $\frac{\partial \ln p_1}{\partial \ln \tau_{ij}}$ can be chosen to ensure that $\sum_{i \in S} \frac{\partial \ln Y_i}{\partial \ln \tau_{ij}} = 0$ so that C.6 is satisfied.

[19] In Online Appendix B.9, we show how the "exact hat algebra" (Dekle et al. (2008), Costinot and Rodriguez-Clare (2013)) can be applied to any model in the universal gravity framework to calculate the effect of any (possibly large) trade shock. The key takeaway – that counterfactual predictions depend only on observed data and the value of the gravity constants – remains true globally. However, if the uniqueness conditions of Theorem 1 do not hold, we are unaware of any procedure that guarantees that the solution found using the "exact hat algebra" approach corresponds to the counterfactual of the observed equilibrium. Indeed, it is straightforward to construct a simple example where in the presence of multiple equilibria, iterative algorithms used to solve the "exact hat algebra" system of equations will converge to qualitatively different equilibria than what is observed in the data even for arbitrarily small shocks, implying arbitrarily large counterfactual elasticities. In contrast, the elasticities in Theorem 2 will provide the correct local counterfactual elasticities around the observed equilibrium even in the presence of multiple equilibria.

supply curve to where supply equals demand by scaling the effect by $\mathbf{S}^{-1}$, for a total effect of $\mathbf{S}^{-1}\partial \ln \tau$. Consider now the 1^{st} degree effect. We first take the resulting changes in the price and price index from the 0^{th} degree effect and calculate how they shift the demand curve (and alter the price index) in all i and j trading partners by multiplying the 0^{th} degree effect by the demand matrix, i.e. $\mathbf{D}\left(\mathbf{S}^{-1}\partial \ln \tau\right)$. To find how this changes the price and price index in each trading partner, (holding constant the prices and price indices in the trading partners' trading partners), we then trace along the supply curve by again scaling the shock by $\mathbf{S}^{-1}$, for a combined effect of $\mathbf{S}^{-1}\mathbf{D}\mathbf{S}^{-1}\partial \ln \tau$. The process continues iteratively, with the k^{th} degree effect shifting the demand curve and price index according to the $k-1$ shock and then re-equilibriated supply and demand by tracing along the supply curve (holding constant the prices and price indices in all trading partners), for an effect of $\left(\mathbf{S}^{-1}\mathbf{D}\right)^k \mathbf{S}^{-1}\partial \ln \tau$, as claimed.[20] The total change in prices and price indices is the infinite sum of all k^{th} degree shocks.

The third part of Theorem 2 says that the direct impact of a symmetric decline in trade frictions $\partial \ln \tau_{il}$ and $\partial \ln \tau_{li}$ on real output prices (and real expenditure) in the directly affected locations i and l will be larger than the impact of that shock in any other indirectly affected location $j \neq i, l$. If the demand and supply elasticities are positive, then a decline in trade frictions will cause the real output prices in the directly affected locations to rise more than any indirectly affected location (the ordering is reversed if the demand and supply elasticities are negative). This analytical result characterizes the relative impact of a trade friction shock on different locations in a model with many locations and arbitrary bilateral frictions.[21]

5 Estimating the gravity constants

In the previous section, we saw that the impact of a trade friction shock on trade flows, incomes, expenditures, and real output prices in any gravity model can be determined solely from observed trade flow data and the value the demand and supply elasticities. In this section, we show how these gravity constants can be estimated. We use data on international trade flows, so for the remainder of the paper we refer to a location as a country.

[20] One can also derive the alternative representation $\mathbf{A}^{-1} = -\sum_{k=0}^{\infty} \mathbf{D}^{-1}\left(\mathbf{S}\mathbf{D}^{-1}\right)^k$, in which the ordering is reversed: the k^{th} degree effect is calculated by first shifting the supply curve by the $k-1$ degree shock and then tracing along the demand curve to re-equilibriate supply and demand.

[21] Mossay and Tabuchi (2015) prove a similar result in a three country world.

5.1 Methodology

We first derive an equation that shows that the relationship between three observables – relative trade shares, relative incomes, and relative own expenditure shares – are governed by the two gravity constants. We then show how this relationship under minor assumptions can be used as an estimating equation to recover the gravity constants. We begin by combining C.1 and C.2 to express the expenditure share of country j on trade from i relative to its expenditure on its own goods as a function of the trade frictions, the output prices in i and j, and the aggregate demand elasticity:

$$\frac{X_{ij}}{X_{jj}} = \left(\frac{\tau_{jj} p_j}{\tau_{ij} p_i}\right)^{\phi}.$$

We then use the relationship $p_i = Y_i/Q_i$ to re-write this expression in terms of incomes and aggregate quantities and rely on C.3 to write the equilibrium output as a function of output prices and the output price index:

$$\frac{X_{ij}}{X_{jj}} = \left(\frac{\tau_{jj}\left(\frac{Y_j}{C_j}\right)\left(\frac{p_i}{P_i}\right)^{\psi}}{\tau_{ij}\left(\frac{Y_i}{C_i}\right)\left(\frac{p_j}{P_j}\right)^{\psi}}\right)^{\phi}. \tag{17}$$

We now define $\lambda_{jj} \equiv X_{jj}/E_j$ to be the fraction of income country j spends on its own goods (j's "own expenditure share"). By combining C.1 and C.2, we note j's own expenditure share can be written as $\lambda_{jj} = \left(\tau_{jj}\frac{p_j}{P_j}\right)^{-\phi}$, which allows us to write equation (17) (in log form) as:

$$\ln \frac{X_{ij}}{X_{jj}} = -\phi \ln \frac{\tau_{ij}}{\tau_{jj}} + \phi \ln \frac{Y_j}{Y_i} + \psi \ln \frac{\lambda_{jj}}{\lambda_{ii}} - \phi \ln \frac{C_j}{C_i} + \phi\psi \ln \frac{\tau_{jj}}{\tau_{ii}}. \tag{18}$$

Equation (18) shows that the demand elasticity ϕ is equal to the partial elasticity of trade flows to relative incomes, whereas the supply elasticity ψ is equal to the partial elasticity of trade flows to the relative own expenditure shares. Intuitively, the greater j's income relative to i (holding all else equal, especially the relative supply shifters $\ln \frac{C_j}{C_i}$), the greater the price in j relative to i and hence the more it would demand from i relative to j; the greater the demand elasticity ϕ, the greater the effect of the price difference on expenditure. Conversely, because the real output price is inversely related to a country's own expenditure share, the greater j's own expenditure share relative to i, the lower the relative aggregate supply to j and hence the more j will consume from i relative to j; the larger the supply elasticity ψ, the more responsive supply will be to differences in own expenditure share.

Equation (18) forms the basis of our strategy for estimating the gravity elasticities ϕ

and ψ. However, it also highlights two important challenges in estimation. First, equation (18) suggests that for any observed set of trade flows $\{X_{ij}\}$ and any assumed set of gravity elasticities $\{\phi, \psi\}$, own trade frictions $\{\tau_{ii}\}$, and supply shifters $\{C_i\}$, there will exist a unique set of trade frictions $\{\tau_{ij}\}_{i \neq j}$ for which the observed trade flows are the equilibrium trade flows of the model.[22] As a result, trade flow data alone will not provide sufficient information to estimate the gravity elasticities. Second, equation (18) highlights that the gravity elasticities are partial elasticities holding the (unobserved) relative supply shifters $\{C_i\}$ fixed. Because both income and own expenditure shares are correlated with supply shifters through the equilibrium structure of the model, any estimation procedure must contend with this correlation between observables and unobservables.

In order to address both concerns, we combine plausibly exogenous observed geographic variation with the general equilibrium structure of the model to estimate the gravity elasticities. We proceed in a two-stage procedure.[23] First, we re-write equation (18) as:

$$\ln \frac{X_{ij}}{X_{jj}} = -\phi \ln \frac{\tau_{ij}}{\tau_{jj}} - \ln \pi_i + \ln \pi_j,$$

where $\ln \pi_i \equiv \phi \ln Y_i + \psi \ln \lambda_{ii} - \phi \ln C_i + \phi\psi \ln \tau_{ii}$ is a country-specific fixed effect. We assume relative trade frictions scaled by the trade elasticity can be written as a function of their continent of origin c, continent of destination d, and the decile of distance between the origin and destination countries, l:

$$-\phi \ln \frac{\tau_{ij}}{\tau_{jj}} = \beta_{cd}^l + \varepsilon_{ij},$$

where ε_{ij} is a residual assumed to be independent across origin-destination pairs. The country-specific fixed effect can then be recovered from the following the following equation:

$$\ln \frac{X_{ij}}{X_{jj}} = \beta_{cd}^l - \ln \pi_i + \ln \pi_j + \varepsilon_{ij}, \tag{19}$$

where we estimate β_{cd}^l non-parametrically using a set of 360 dummy variables (10 distances deciles $\times$ 6 origin continents $\times$ 6 destination continents). Let $\ln \hat{\pi}_i$ denote the estimated fixed effect and define $\hat{\nu}_i \equiv \ln \hat{\pi}_i - \ln \pi_i$ to be its estimation error.

[22] See Online Appendix B.10 for a formal proof of this result.

[23] While the two step procedure we follow resembles the procedure used in Eaton and Kortum (2002) to recover the trade elasticity from observed wages, there are two important differences. First, our procedure applies to a large class of trade and economic geography models and allows us to simultaneously estimate both the demand (trade) elasticity and the supply elasticity (rather than assuming e.g. that the population of a country is exogenous and calibrating the model to a particular intermediate good share). Second, our procedure relies on the general equilibrium structure of the model to generate the identifying variation (rather than e.g. instrumenting for wages with the local labor supply, which would be inappropriate for economic geography models).

In the second stage, we write the estimated fixed effect as a function of income and own expenditure share:

$$\ln \hat{\pi}_i = \phi \ln Y_i + \psi \ln \lambda_{ii} + \nu_i, \tag{20}$$

where $\nu_i \equiv -\phi \ln C_i + \phi\psi \ln \tau_{ii} + \hat{\nu}_i$ is a residual that combines the unobserved supply shifter, the unobserved own trade friction, and the estimation error from the first stage. As mentioned above, it is not appropriate to estimate equation (20) via ordinary least squares, as variation in the supply shifter will affect income and the own expenditure share through the equilibrium structure of the model, creating a correlation between the residual and the observed covariates. Intuitively, the larger the supply shifter of a country, the greater its output and hence the greater the trade flows for a given observed income; since the country-specific fixed effect $\ln \pi_i$ is decreasing in relative trade flows, the OLS estimate of ϕ will be biased downwards.

To overcome this bias, we pursue an instrumental variables (IV) strategy, where we use the general equilibrium structure of the model to construct a valid instrument. To do so, we calculate the equilibrium trade flows of a hypothetical world where the bilateral trade frictions and supply shifters depend only on observables. We then use the incomes and relative own expenditure shares of this hypothetical world as instruments for the observed incomes and own expenditure shares. These counterfactual variables are valid instruments as long as (1) they are correlated with their observed counterparts (which we can verify); and (2) the observable components of the bilateral trade frictions and supply shifters are uncorrelated with unobserved supply shifters.

Because the first-stage estimation of (19) provides an unbiased estimate of $-\phi \ln \frac{\tau_{ij}}{\tau_{jj}}$, we use the estimated origin-continent-destination-continent-decile coefficients $\hat{\beta}^l_{cd}$ to create our counterfactual measure of bilateral trade frictions (normalizing own trade frictions $\tau_{jj} = 1$). In the simplest version of our procedure, we then calculate the equilibrium income and own expenditure share given these bilateral trade frictions, assuming that the supply shifter C_i is equal in all countries. In this version of the procedure, the instrument is valid as long as the the general equilibrium effects of distance on the origin fixed effects of a gravity equation are uncorrelated with unobserved heterogeneity in supply shifters (or own trade frictions). Because we calculate the equilibrium of the model in a counterfactual world where there is no heterogeneity in supply shifters, it seems reasonable to assume that the resulting equilibrium income and own expenditure shares that we use as instruments are uncorrelated with any real world heterogeneity. However, our instrument would be invalid if there were a correlation between unobserved supply shifters and the observed geography of a country (e.g. if countries more remotely located were also less productive or less attractive places to reside).

To mitigate such a concern (and to allow for more realistic variation across countries in supply), we extend the approach to allow the supply shifter to vary across countries depending on a vector of (exogenous) observables X_i^c, e.g. land controls like the amount of fertile land, geographic controls like the distance to nearest coast, institutional controls like the rule of law, historical controls like the population in 1400, and schooling and R&D controls like average years of schooling. Given a set of supply shifters $\{C_i\}$ that depend only these observables and the set of trade frictions that depend only on our non-parametric estimates from above, we re-calculate the equilibrium income and own expenditure share in each country. We then use the equilibrium values from this hypothetical world as our instruments, while and control directly for the observables X_i^c in equation (20). As a result, the identifying variation from the instruments only arises through the general equilibrium structure of the model.[24] Intuitively, differences in observables like land area in neighboring countries generates variation in the demand that a country faces for its production, as well as variation in the price it faces for its consumption, even conditional on its own observables.

There are two things to note about the above procedure. First, to construct the hypothetical equilibrium incomes and own expenditure shares requires assuming values of the gravity constants ϕ and ψ for the hypothetical world. In what follows, we choose a demand elasticity $\phi = 8.28$ and a supply elasticity $\psi = 3.76$, which correspond to the (estimated) demand elasticity estimated and (implicitly calibrated) supply elasticity in Eaton and Kortum (2002). We should note that while the particular choice of the these parameters will affect the strength of the constructed instruments, they will not affect the consistency of our estimates of the gravity constants under the maintained assumption that bilateral distances are uncorrelated with the unobserved supply shifters conditional on observables.[25]

The second thing to note about the estimation procedure is more subtle. As mentioned in Section 3 and discussed in detail in Online Appendix B.3, when bilateral trade frictions are "quasi-symmetric" the equilibrium origin and destination shifters will be equal up to scale.

[24] Calculating the counterfactual equilibrium income and own expenditure share in each country when the supply shifters depend on observables requires assuming a particular mapping between the observables X_i^c and the supply shifter C_i. We assume that $C_i = X_i^c \beta^c$ and note that the theory implies the following equilibrium condition:

$$\ln Y_i = \frac{\phi}{\phi - \psi} \ln C_i + \frac{1+\psi}{\psi - \phi} \ln \gamma_i + \frac{\psi}{\psi - \phi} \ln \delta_i .$$

As a result, we choose the β^c that arise from the OLS regression $\ln Y_i = \frac{\phi}{\phi - \psi} X_i^c \beta^c + \epsilon_i$. Although our estimates of β^c may be biased due to the correlation between X_i^c and ϵ_i, this bias only affects the strength of the instrument, because if each X_i^c is uncorrelated with the residual ν_i in equation (20) (i.e. X_i^c is exogenous), then any linear combination of X_i^c will also be uncorrelated with the residual.

[25] In principle, we could search over different values of the gravity constants to find the constellation that maximizes the power of our instruments. In practice, however, our estimates vary only a small amount across different values of the gravity constants.

In this case, there will be a perfect log linear relationship between the income of a country, its own expenditure share and its supply shifter.[26] As a result, if we were to impose quasi-symmetric bilateral trade frictions in the hypothetical world, the equilibrium income and expenditure shares generated would be perfectly collinear, preventing us from simultaneously identifying the demand and supply elasticities in the second stage. Intuitively, identification of the demand elasticity requires variation in a country's supply curve (its destination fixed effect), whereas identification of the supply elasticity requires variation in a country's demand curve (its origin fixed effect); when trade frictions are quasi-symmetric, however, the two co-vary perfectly. Our choice to allow distance to affect trade frictions differently depending on the continent of origin and continent of destination introduces the necessary asymmetries in the trade frictions to allow the model constructed instruments to vary separately, allowing for identification of both the supply and demand elasticities simultaneously. To address concerns about the extent to which these asymmetries are sufficient to separately identify the two, we report the Sanderson-Windmeijer F-test (see Sanderson and Windmeijer (2016)) in the results that follow.

5.2 Data

We now briefly describe the data we use to estimate the gravity constants.

Our trade data comes from the Global Trade Analysis Project (GTAP) Version 7 (Narayanan, 2008). This data provides bilateral trade flows between 94 countries for the year 2004. To construct own trade flows, we subtract total exports from the total sales of domestic product, i.e. $X_{ii} = X_i - \sum_{j \neq i} X_{ij}$. We use the bilateral distances between countries from the CEPII gravity data set of Head et al. (2010) to construct deciles of distance between two countries. We rely on the data set of Nunn and Puga (2012) to provide a number of country level characteristics that plausibly affect supply shifters, including "land controls" (land area interacted with the fraction of fertile soil, desert, and tropical areas), "geographic controls" (distance to the nearest coast and the fraction of country within 100 kilometers of an ice free coast), "historical controls" (log population in 1400 and the percentage of the population of European descent), "institutional controls" (the quality of the rule of law). Finally, following Eaton and Kortum (2002), we also consider "schooling and R&D controls" including the average years of schooling from UNESCO (2015) and the R&D stocks from Coe et al. (2009), where a dummy variable is included if the country is not in each respective data set.

[26] In particular, $(1 + 2\phi) \ln E_i = (2\phi) \ln C_i + (1 - 2\psi) \ln \lambda_{ii} + C$.

5.3 Estimation results

Table 2 presents the results of our estimation of equation (18). The first column presents the ordinary least squares regression; we estimate a positive supply elasticity and negative demand elasticity, consistent with the discussion above that the OLS estimate of the demand elasticity is biased downward. Column 2 presents the instrumental variable estimation where the counterfactual income and own expenditure shares comprising our instrument are constructed assuming equal supply shifters. After correcting for the bias arising from the correlation between the unobserved supply shifters and observed incomes and own expenditure shares, we find positive supply and demand elasticities, although the demand elasticity is not statistically significant. Columns 3 through 7 sequentially allows the supply shifter in the construction of the instrument to vary across countries depending on an increasing number of observables (while including these same observables as controls in both the first and second stages of the IV estimation of equation (18)). Including these observables both increases the strength of the instruments and reduces the concern that the instruments are correlated with unobserved supply shifters. Reassuringly, our estimated demand and supply elasticities vary only slightly with the inclusion of additional controls.[27]

In our preferred specification (column 7), we estimate a demand elasticity of $\phi = 3.72$ (95% confidence interval [1.14,6.29] and a supply elasticity $\psi = 68.49$ (95% confidence interval [5.38,131.60]).[28] Hence, our demand elasticity estimate is somewhat lower than the preferred estimate of Eaton and Kortum (2002) of 8.28 (although similar to their estimate using variation in wages of 3.6), as well as similar to estimates of trade elasticity around 4 in Anderson and Van Wincoop (2004), Simonovska and Waugh (2014), and Donaldson (forthcoming). Unlike these papers, however, we also estimate the supply elasticity. Our point estimate, while noisily estimated, is substantially larger than and statistically different (at the 5% level) from the supply elasticity to which Eaton and Kortum (2002) implicitly calibrate. Moreover, our estimated value is consistent with recent estimates of labor mobility from the migration literature. To see this, consider an economic geography framework with intermediate goods, agglomeration forces, and Frechet distributed preferences over location (see the last row of Table 1). If we match the labor share in production of 0.21 in Eaton and Kortum (2002) and the agglomeration force of $\alpha = 0.10$ in Rosenthal and Strange (2004), then our point estimate of ψ is consistent with a migration elasticity (Fréchet shape param-

[27]Figure 4 in the online appendix shows that our instrumental variables of counterfactual income and own expenditure shares are positively correlated with their observed counterparts, even after differencing out the observables in the supply shifters.

[28]While the p-value of the Sanderson-Windmeijer F-test is statistically significant in the first stage for income, it is only marginally statistically significant for expenditure shares, suggesting that the wide confidence interval for the supply elasticity may be due in part to a weak instrument.

eter) of 1.4. This is similar to estimates from the migration literature using observed labor flows and about one-third to one-half the size of within-country estimates.[29]

6 The impact of a U.S.-China trade war

We now apply the estimates from Section 5 to evaluate the impact of a trade war between the U.S. and China. We model the trade war as an increase in the trade frictions between the U.S. and China (holding constant all other trade frictions). We then characterize how such a trade war propagates through the trade network using the methodology developed in Section 4.[30]

There are two 0^{th} degree effects of the trade war: first, the U.S. and China export less to each other, causing the output prices in both countries to fall; second, the the cost of importing increases, causing the price index in both countries to rise. Both effects cause the real output price to decline, with a greater decline in China because both its export and import shares with the U.S. are relatively larger.

The top panel of Figure 2 depicts the 1^{st} degree effect on the real output price in all countries. The effect in the U.S. and China is positive, as the degree 0 decline in output price reduces the cost of own expenditure (causing the price index to fall in both countries). In other countries, however, the degree 1 effect is negative, as the U.S. and China demand less of their goods, causing their trading partner's output prices to fall. The most negatively affected countries are those who export the most to the U.S. and China.

Summing across all degree shocks yields the total elasticity of real output prices in each country to the trade war shock, which the bottom panel of Figure 2 depicts.[31] Not surprisingly, the two countries hurt most by a trade war are the U.S. and China. Moreover, while all countries are made worse off, the countries who are closely linked through the trading network with the U.S. and China (e.g. Canada, Mexico, Vietnam, and Japan) are hurt more than those countries that are less connected (e.g. India). All told, we estimate that a 10%

[29] Ortega and Peri (2013) estimates an migration elasticity to destination country income of 0.6 using international migration flows and an estimate of 1.8 for the sub-sample of migration flows within the European Union, albeit not using a log-linear gravity specification. Within countries (aud with log-linear gravity specifications), Monte et al. (2015) estimate a migration elasticity of 4.4 in the U.S.; Tombe et al. (2015) estimate a migration elasticity of 2.54 in China, and Morten and Oliveira (2014) estimate a migration elasticity of 3.4 in Brazil.

[30] In the counterfactuals that follow, we accommodate the deficits observed in the data by assuming that the observed ratio of expenditure to income for each country remains remains constant and impose an aggregate market clearing condition that total income is equal to total expenditure. The results are qualitatively similar if we instead solve for the (unique) set of balanced trade flows that match the observed import shares and treat these balanced trade flows as the data.

[31] Figures 5 through 9 in the Online Appendix depict the impact of the degrees 0, 1, 2, and higher on the relative prices, relative output, income, the relative price index, and real output prices in each country.

increase in bilateral trade frictions is associated with a decline in real output price of 0.04% in the U.S. and 0.14% in China. These modest changes in the real output price are due to the large supply elasticity, causing the aggregate output to reallocate away from the U.S. and China in response to the trade war. The converse of this result, however, is that the reallocation of the aggregate output results in large changes to *total* real expenditure: for example, in the Armington trade model interpretation, a 10% increase in bilateral trade frictions causes the total real expenditure to fall by 2.7% in the U.S. and by 9.8% in China.[32]

There are two potential concerns about these estimated effects. First, because the elasticities correspond to an infinitesimal shock, one may worry that the effects of a large trade war may differ. To address this concern, we calculate the effect of a 50% increase in bilateral trade frictions using the methodology discussed in Online Appendix B.9. The correlation between the local elasticities and global changes exceeds 0.99, indicating that the local *relative* effect of the trade war is virtually the same as the global effect.[33] However, the local effect does overstate the global effect of such a shock, as we find that log first differences implied by the global shock are roughly 80% the size of those implied by the local elasticities. Second, the effects of the trade war above were calculated given the gravity constants estimated in Section 5; one may be concerned that the effects of the trade wars may differ substantially across alternative values of these elasticities. To address this concern, we calculate the effects of a trade war for a large number of different combinations of supply and demand elasticities.[34] Across all constellations in the 95% confidence interval of the two estimated gravity constants, the calculated elasticities are quite similar, with a 10% increase in bilateral trade frictions associated with a decline in real output price between 0.03% and 0.05% in the U.S. and 0.07% and 0.26% in China. Of course, as Section 4 emphasizes, the particular value of the gravity constants may substantially affect the impact of counterfactuals more generally.

7 Conclusion

In this paper, we provide a framework that unifies a large set of trade and geography models. We show that the properties of models within this framework depend crucially on the value of two gravity constants: the aggregate supply and demand elasticities. Sufficient conditions for the existence and uniqueness of the equilibria depend solely on the gravity constants. Moreover, given observed trade flows, these gravity constants are sufficient to determine the

[32]Recall from Section 2 that while the changes in real output prices are identified from the value of trade flows alone, without specifying κ in equation (11), the change in total real expenditure is only identified up to scale. In Armington trade models with intermediates, however, this is not a problem, as $\kappa = 1$.

[33]See Figure 10 in the Online Appendix.

[34]See Figure 11 in the Online Appendix.

effect of a trade friction shock on trade flows, incomes, and real output price without needing to specify a particular underlying model.

We then develop a novel instrumental variables approach for estimating the gravity constants using the general equilibrium structure of the framework. Using our estimates, we find potentially large losses may arise due to a trade war between U.S. and China occur.

By providing a universal framework for understanding the general equilibrium forces in trade and geography models, we hope that this paper provides a step toward unifying the quantitative general equilibrium approach with the gravity regression analysis common in the empirical trade and geography literature. Toward this end, we have developed a toolkit that operationalizes all the theoretical results presented in this paper.[35] We also hope the tools developed here can be extended to understand other general equilibrium spatial systems, such as those incorporating additional types of spatial linkages beyond trade frictions.

[35] The toolkit is available for download on Allen's website.

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Table 1: Examples of models in the universal gravity framework

Model	Demand elasticity (ϕ)	Supply elasticity (ψ)	Model parameters	Welfare of labor
Armington (1969), Anderson (1979), Anderson and Van Wincoop (2003) (with intermediates)	$\sigma - 1$	$\frac{1-\zeta}{\zeta}$	σ subs. param. ζ labor share	$B_i \times \left(\frac{p_i}{P_i}\right)^{1+\psi}$
Krugman (1980) (with intermediates)	$\sigma - 1$	$\frac{1-\zeta}{\zeta}$	σ subs. param. ζ labor share	$B_i \times \left(\frac{p_i}{P_i}\right)^{1+\psi}$
Eaton and Kortum (2002) (with intermediates)	θ	$\frac{1-\zeta}{\zeta}$	θ heterogeneity param. ζ labor share	$B_i \times \left(\frac{p_i}{P_i}\right)^{1+\psi}$
Melitz (2003), Di Giovanni and Levchenko (2013)	$\theta\left(1+\frac{\theta-(\sigma-1)}{\theta(\sigma-1)}\right)$	$\frac{1-\zeta}{\zeta}$	σ subs. param. θ heterogeneity param.	$B_i \times \left(\frac{p_i}{P_i}\right)^{1+\psi}$
Allen and Arkolakis (2014)	$\sigma - 1$	$-\frac{1+a}{a+b}$	σ subs. param. a productivity spillover b amenity spillover	$\left(\sum_i B_i \left(\frac{p_i}{P_i}\right)^{-\frac{1}{a+b}}\right)^{-(a+b)}$
Redding (2016)	θ	$\frac{\alpha\varepsilon}{1+\varepsilon(1-\alpha)}$	θ heterogeneity (goods) param. ε heterogeneity (labor) param. α goods expenditure share	$\left[\sum_i B_i \left(\frac{p_i}{P_i}\right)^{\alpha\varepsilon}\right]^{\frac{1}{\varepsilon}}$
Redding and Sturm (2008) (A variant of Helpman (1998))	$\sigma - 1$	$\frac{\alpha}{(1-\alpha)(\sigma-1)-\alpha}$	σ subs. param. α share spent on goods	$\left(\sum_i B_i \left(\frac{p_i}{P_i}\right)^{\psi\phi}\right)^{\frac{1+\psi}{1+\psi\phi+\psi}}$
Economic geography with intermediate goods and spillovers	θ	$\frac{1-\zeta}{\zeta} - \frac{1}{\zeta b+a}\left(\frac{a+\zeta}{\zeta}\right)$	θ heterogeneity (goods) param. ζ intermediate good share a productivity spillover b amenity spillover	$\left(\sum_i B_i \left(\frac{p_i}{P_i}\right)^{\frac{1}{\zeta}}\right)$
Economic geography with intermediate goods, idiosyncratic preferences, and spillovers	θ	$\frac{1-\zeta}{\zeta} + \frac{\varepsilon}{\zeta-\varepsilon(a+\zeta\beta)}\left(\frac{a+\zeta}{\zeta}\right)$	θ heterogeneity (goods) param. ε heterogeneity (labor) param. ζ intermediate good share a productivity spillover b amenity spillover	$\left[\sum_i B_i \left[\left(\frac{p_i}{P_i}\right)^{\frac{1}{\zeta}}\right]^{\frac{\varepsilon}{1-\varepsilon\left(\frac{a}{\zeta}+b\right)}}\right]^{\frac{1-\varepsilon\left(\frac{a}{\zeta}+b\right)}{\varepsilon}}$

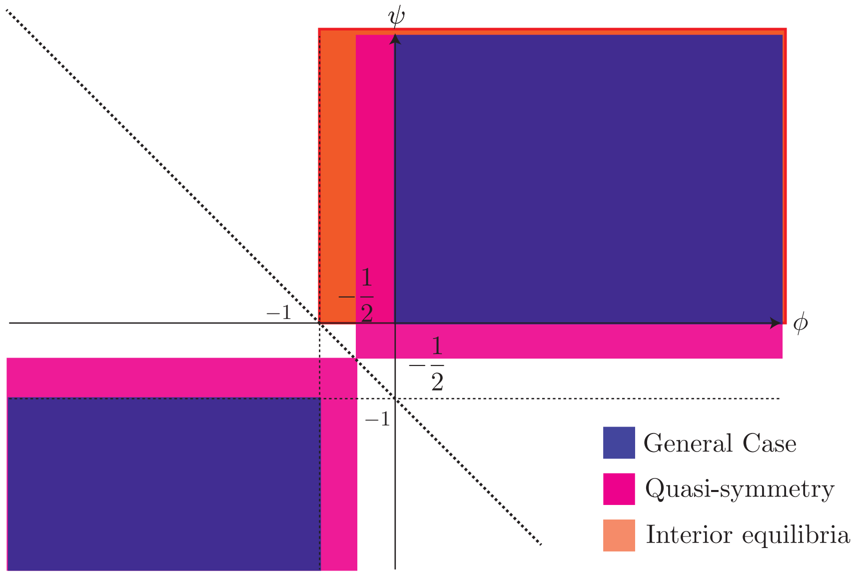
Notes: This table includes a (non-exhaustive) list of trade and economic geography models that can be examined within the universal gravity framework, the mapping of their structural parameters to the gravity constants, and the relationship between the welfare of workers and the real output prices. B_i is an exogenous location specific parameter whose interpretation depends on the particular model. λ is an endogenous variable which affects every country simultaneously.

Table 2: ESTIMATING THE GRAVITY CONSTANTS

	(1) OLS	(2) IV	(3) IV	(4) IV	(5) IV	(6) IV	(7) IV
Log Income (Demand	-0.403**	1.484	3.278	4.364*	3.882**	3.539***	3.715***
elasticity)	(0.171)	(1.157)	(2.674)	(2.371)	(1.838)	(1.356)	(1.312)
Log Own Expenditure	3.381**	92.889***	108.592**	116.649**	71.859**	64.968**	68.488**
Share (Supply Elasticity)	(1.600)	(13.417)	(48.104)	(47.944)	(35.883)	(33.014)	(32.198)
Land controls	No	No	Yes	Yes	Yes	Yes	Yes
Geographic controls	No	No	No	Yes	Yes	Yes	Yes
Historical controls	No	No	No	No	Yes	Yes	Yes
Institutional controls	No	No	No	No	No	Yes	Yes
Schooling and R&D controls	No	No	No	No	No	No	Yes
First stage Sanderson-Windmeijer F-test:							
Income		25.909	3.994	6.349	20.095	34.198	25.763
(p-value)		0.004	0.102	0.053	0.007	0.002	0.004
Own expenditure share		72.702	4.388	4.923	3.561	4.577	5.205
(p-value)		0.000	0.090	0.077	0.118	0.085	0.071
Observations	94	94	94	94	94	94	94

Notes: The dependent variable is the estimated country fixed effect of a gravity regression of the log ratio of bilateral trade flows to destination own trade flows on categorical deciles of distance variables, where the coefficient is allowed to vary by continent of origin and destination. Hence, each observation in the regressions above is a country. Instruments for income and own expenditure share are the equilibrium values from a trade model where the bilateral trade frictions are those predicted from the same gravity equation and countries are either identical in their supply shifters (column 2) or their supply shifters are estimated from a regression of observed income on observables (columns 3 through 7). In the latter case, the observables determining the supply shifters are controlled for directly in the first and second stage regressions, so identification of the demand and supply elasticities arise only from the general equilibrium effect on income and own expenditure shares. Land controls include land area interacted with fraction fertile soil, desert, and tropical areas. Geographic controls include the distance to nearest coast and the fraction of country within 100 km of an ice free coast. Historical controls include the log population in 1400 and the percentage of the population of European descent. Institutional controls include the quality of the rule of law. Schooling and R&D controls are average years of schooling (from UNESCO) and the R&D stocks (from Coe et al. (2009)), where a dummy variable is included if the country is not in each respective data set. Land, geographic, and historical control are from Nunn and Puga (2012). Standard errors clustered at the continent level are reported in parentheses. Stars indicate statistical significance: * $p<.10$ ** $p<.05$ *** $p<.01$.

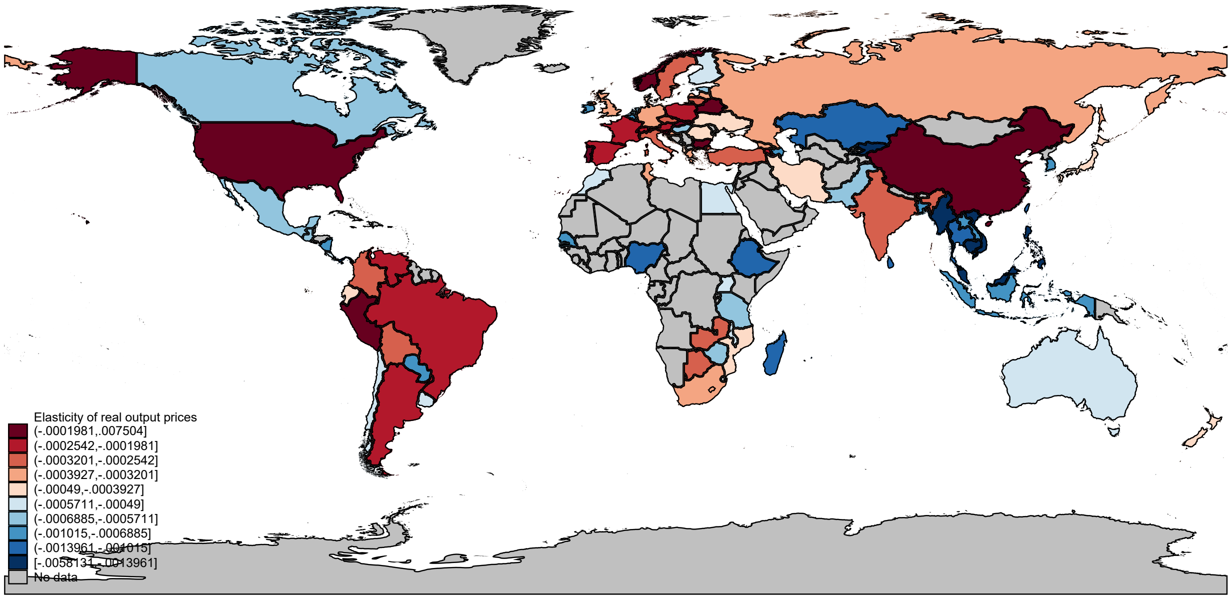
Figure 1: Existence and uniqueness



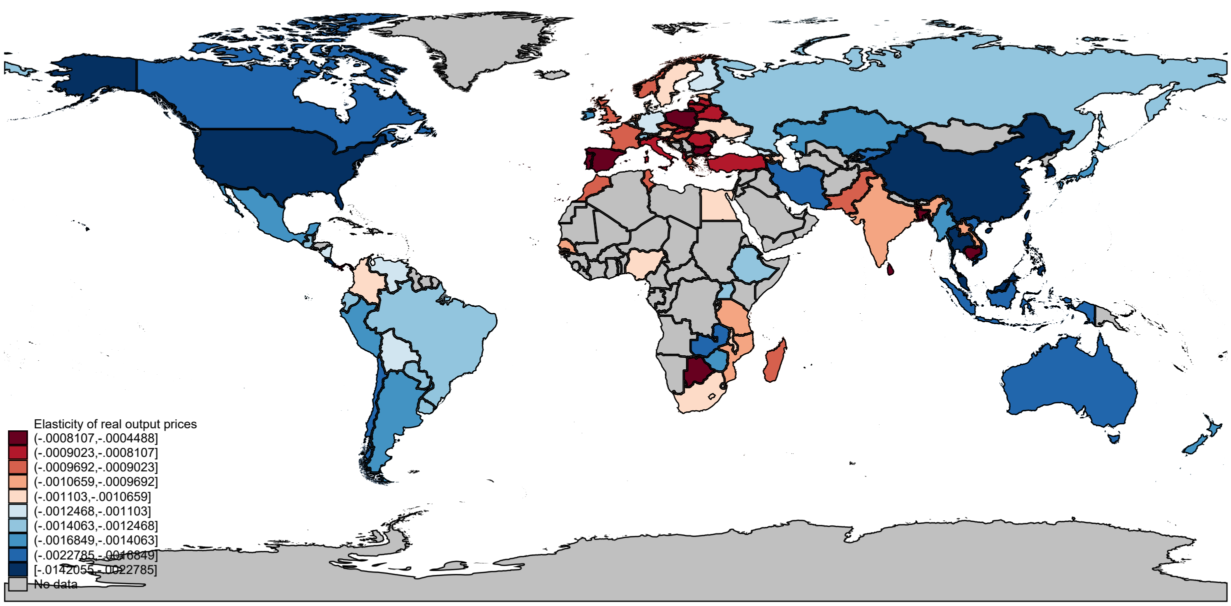
Notes: This figure shows the regions in (ϕ, ψ) space for which the gravity equilibrium is unique and interior. Existence can be guaranteed throughout the entire region except for the case $1 + \phi + \psi = 0$.

Figure 2: The network effect of a U.S.-China trade war

(a) Degree 1 Effect



(b) Total Effect



Notes: This figure depicts the elasticity of real output prices to an increase in the bilateral trade frictions between the U.S. and China (a "trade war") in all countries. The top panel depicts the "Degree 1" effect, which is the effect of the direct shock on the U.S. and China on all countries through the trade network, holding constant the output prices and quantities of their trading partners fixed. The bottom panel shows the total effect of the trade war on the real output price in each country.

A Proofs

A.1 Proof of Theorem 1

Proof. **Part i)** The proof proceeds as follows. First we transform the equilibrium conditions to the associated non-linear integral equations form. However, we cannot directly apply the fixed point theorem for the non-linear integral equations since the system does not map to a compact space. Therefore we need to "scale" the system so that we can apply the fixed point, which implies that there exists a fixed point for the scaled system. Finally we construct a fixed point for the original non-linear integral equations. In this subsection, we show how to set up in the associated integral equation form, and apply the fixed point theorem. The other technical parts are proven in Online Appendix B.4. Note that our result proposition is a natural generalization of Karlin and Nirenberg (1967) to a system of non-linear integral equations.

Define z as follows:

$$z \equiv \begin{pmatrix} (x_i)_i \\ (y_i)_i \end{pmatrix} \equiv \begin{pmatrix} \left(p_i^{1+\psi+\phi} P_i^{-\psi}\right)_i \\ \left(P_i^{-\phi}\right)_i \end{pmatrix}.$$

Then the system of equations (6) and (7) of the general equilibrium gravity model is re-written in vector form:

$$\begin{pmatrix} (x_i)_i \\ (y_i)_i \end{pmatrix} = \begin{pmatrix} \sum_j K_{ij} C_i^{-1} C_j x_j^{a_{11}} y_j^{a_{12}} \\ \sum_j K_{ji} x_j^{a_{21}} y_j^{a_{22}} \end{pmatrix}, \tag{21}$$

where $A = (a_{ij})_{i,j}$ is given by

$$A = \begin{pmatrix} \frac{1+\psi}{1+\psi+\phi} & -\frac{1+\phi}{1+\psi+\phi} \\ -\frac{\phi}{1+\psi+\phi} & \frac{\psi}{1+\psi+\phi} \end{pmatrix}.$$

Also the kernel, K_{ij}, is given by $K_{ij} = \tau_{ij}^{-\phi}$. Notice that we cannot directly apply Browser's fixed point theorem for equation (21) since there are no trivial compact domain for equation (21). Therefore consider the following "scaled" version of equation (21).

$$z = \begin{pmatrix} (x_i)_i \\ (y_i)_i \end{pmatrix} = \begin{pmatrix} \frac{\sum_j K_{ij} C_i^{-1} C_j x_j^{a_{11}} y_j^{a_{12}}}{\sum_{i,j} K_{ij} C_i^{-1} C_j x_j^{a_{11}} y_j^{a_{12}}} \\ \frac{\sum_j K_{ji} x_j^{a_{21}} y_j^{a_{22}}}{\sum_{i,j} K_{ji} x_j^{a_{21}} y_j^{a_{22}}} \end{pmatrix} \equiv F(z), \tag{22}$$

and F is defined over the following compact set$C$:

$$C = \left\{ x \in \Delta\left(R_+^N\right); x_i \in [\underline{x}, \overline{x}]\, \forall i \right\} \times \left\{ y \in \Delta\left(R_+^N\right); y_i \in [\underline{y}, \overline{y}]\, \forall i \right\}, \tag{23}$$

where the bounds for x and y are respectively given as follow:

$$\bar{x} \equiv \max_{i,j} \frac{K_{ij} C_i^{-1} C_j}{\sum_{i,j} K_{ij} C_i^{-1} C_j} \quad \underline{x} \equiv \min_{i,j} \frac{K_{ij} C_i^{-1} C_j}{\sum_{i,j} K_{ij} C_i^{-1} C_j}$$
$$\overline{y} \equiv \max_{i,j} \frac{K_{ji}}{\sum_{i,j} K_{ji}} \quad \underline{y} = \min_{i,j} \frac{K_{ji}}{\sum_{i,j} K_{ji}}.$$

It is trivial to show that F maps from C to C and continuous over the following compact set C, so that we can apply Brouwer's fixed point and there exists an fixed point $z^* \in C$.

There are two technical points needed to be proven; first, there exists a fixed point for the original (un-scaled) system (21); second, the equilibrium z^* is strictly positive. These two claims are proven in Lemma 1 and Lemma 2 in Online Appendix B.4, respectively.

Part (iii) It suffices to show that there exists a unique interior solution for equation (21). Suppose that there are two strictly positive solutions (x_i, y_i) and $(\widehat{x}_i, \widehat{y}_i)$ such that there does not exist $t, s > 0$ satisfying

$$(x_i, y_i) = (t\widehat{x}_i, s\widehat{y}_i) .$$

Namely, two solutions are "linearly independent." First note that for any $i \in S$, we can evaluate one of equation (21).

$$\frac{x_i}{\widehat{x}_i} = \frac{1}{\widehat{x}_i} \sum_{j \in S} K_{ij} C_i^{-1} C_j \left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{11}} \left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{12}} (\widehat{x}_j)^{\alpha_{11}} (\widehat{y}_j)^{\alpha_{12}} \tag{24}$$

$$\leqslant \max_{j \in S} \left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{11}} \max_{j \in S} \left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{12}} . \tag{25}$$

Taking the maximum of the left hand side,

$$\max_{i \in S} \frac{x_i}{\widehat{x}_i} \leqslant \max_{j \in S} \left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{11}} \max_{j \in S} \left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{12}} . \tag{26}$$

Lemma 3 in Online Appendix B.4 shows that the inequality is actually strict. Analogously, we obtain

$$\min_{i \in S} \frac{x_i}{\widehat{x}_i} \geqslant \min_{j \in S} \left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{11}} \min_{j \in S} \left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{12}} . \tag{27}$$

Dividing equation (26) by equation (27), it is shown that

$$1 \leqslant \mu_x \equiv \frac{\max_{i \in S} \frac{x_i}{\widehat{x}_i}}{\min_{i \in S} \frac{x_i}{\widehat{x}_i}} < \frac{\max_{j \in S} \left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{11}}}{\min_{j \in S} \left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{11}}} \times \frac{\max_{j \in S} \left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{12}}}{\min_{j \in S} \left(\frac{y_j}{\widehat{y}_j}\right)^{a_{12}}} = \mu_x^{|\alpha_{11}|} \times \mu_y^{|\alpha_{12}|},$$

where

$$\mu_y \equiv \frac{\max_{i \in S} \frac{y_i}{\widehat{y}_i}}{\min_{i \in S} \frac{y_i}{\widehat{y}_i}}.$$

The same argument is applied to obtain the following inequality

$$1 \leqslant \mu_y \equiv \frac{\max_{i \in S} \frac{y_i}{\widehat{y}_i}}{\min_{i \in S} \frac{y_i}{\widehat{y}_i}} < \frac{\max_{j \in S} \left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{21}}}{\min_{j \in S} \left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{21}}} \times \frac{\max_{j \in S} \left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{22}}}{\min_{j \in S} \left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{22}}} = \mu_x^{|\alpha_{21}|} \times \mu_y^{|\alpha_{22}|}.$$

Taking logs in the two inequalities and exploiting the restriction we can write

$$\begin{pmatrix} \ln \mu_x \\ \ln \mu_y \end{pmatrix} < \underbrace{\begin{pmatrix} |\alpha_{11}| & |\alpha_{12}| \\ |\alpha_{21}| & |\alpha_{22}| \end{pmatrix}}_{=|A|} \begin{pmatrix} \ln \mu_x \\ \ln \mu_y \end{pmatrix} , \tag{28}$$

which from the Collatz–Wielandt formula, equation (28) implies that the largest eigenvalue of $|A|$ is greater than one:

$$\rho\left(|A|\right) > 1.$$

However, we prove in Lemma 4 in Online Appendix B.4 that the sufficient condition in part (ii) of Theorem 1 guarantees that the largest absolute eigenvalue is 1. As a result, this is a contradiction.

Quasi-symmetry) When the bilateral trade frictions satisfy quasi-symmetry, then we can reduce the system to$N$ dimensional integral system (see Online Appendix B.3). Then the same logic used above can be applied to show there exists a unique strictly positive solution. As mentioned above, this result follows directly from Karlin and Nirenberg (1967) and is summarized in Theorem 2.19 of Zabreyko et al. (1975). The same argument for (iv) is used for convergence. □

A.2 Proof of Theorem 2

Proof. **Part (i)** Equation (16) is a direct application of the implicit function theorem. Define a function $F : R^{2N} \to R^{2N}$ as follows.

$$F_i\left(\left(\ln p_i\right)_{i=1}^N, \left(\ln P_i\right)_{i=1}^N\right) = \kappa C_i p_i^{1+\psi} P_i^{-\psi} - \kappa \sum_k \tau_{ik}^{-\phi} p_i^{-\phi} C_k P_k^{\phi-\psi} p_k^{1+\psi}$$

$$F_{N-1+i}\left(\left(\ln p_i\right)_{i=1}^N, \left(\ln P_i\right)_{i=1}^N\right) = P_i^{-\phi} - \sum_k \tau_{ki}^{-\phi} p_k^{-\phi}$$

Applying the implicit function theorem for F, we obtain the comparative static (16). As in Dekle et al. (2008), the matrix $\mathbf{A}$ and $\mathbf{T}$ can be expressed in terms of observables.

Part (ii) Notice that $\mathbf{A}$ is written as follows:

$$\mathbf{A} = \mathbf{S}\left(\mathbf{I} - \mathbf{S}^{-1}\mathbf{D}\right),$$

where $\mathbf{S}$ and $\mathbf{D}$ are defined by equation (15). If the largest absolute eigenvalue for $\mathbf{S}^{-1}\mathbf{D}$ is less than 1, then $\mathbf{A}^{-1}$ is expressed as $\sum_{k=0}^{\infty}\left(\mathbf{S}^{-1}\mathbf{D}\right)^k \mathbf{S}^{-1}$. Note that we could have similarly written $\mathbf{A} = -\left(\mathbf{I} - \mathbf{S}\mathbf{D}^{-1}\right)\mathbf{D}$, so that if the largest eigenvalue for $\mathbf{S}\mathbf{D}^{-1}$ is less than 1, $\mathbf{A}^{-1}$ can be expressed as $-\sum_{k=0}^{\infty}\mathbf{D}^{-1}\left(\mathbf{S}\mathbf{D}^{-1}\right)^k$, as noted in footnote 20.

Part (iii) When quasi-symmetric assumption is imposed, destination effects are proportional to the associated origin effects. Therefore as shown in Online Appendix B.3, the equilibrium is characterized by the following single non-linear system of equations:

$$\underbrace{p_i^{1+\psi-\psi\frac{1+\psi+\phi}{\psi-\phi}}\left(\frac{\tau_i^A}{\tau_i^B}\right)^{-\psi\frac{\phi}{\psi-\phi}}\left(C_i\right)^{\frac{\phi}{\psi-\phi}}}_{=Y_i/\kappa} = \sum_{j\in S}\underbrace{\tilde{\tau}_{ij}^{-\phi} p_i^{-\phi}\left(\tau_i^A\right)^{-\phi}\left(\tau_j^A\right)^{-\phi} p_j^{-\phi}}_{=X_{ij}/\kappa} \tag{29}$$

As before, define z_i for all $i \in S$ as follows:

$$z_i\left(p;\tau\right) = \kappa p_i^{1+\psi-\psi\frac{1+\psi+\phi}{\psi-\phi}}\left(\frac{\tau_i^A}{\tau_i^B}\right)^{-\psi\frac{\phi}{\psi-\phi}}\left(C_i\right)^{\frac{\phi}{\psi-\phi}} - \kappa\sum_{j\in S}\tilde{\tau}_{ij}^{-\phi} p_i^{-\phi}\left(\tau_i^A\right)^{-\phi}\left(\tau_j^A\right)^{-\phi} p_j^{-\phi}.$$

Then apply the implicit function theorem to (29),

$$\frac{\partial \ln p}{\partial \ln \tau_{il}} = -2\left(\underbrace{\frac{\partial z}{\partial \ln p}}_{N\times N}\right)^{-1}\underbrace{\frac{\partial z}{\partial \ln \tau_{il}}}_{N\times 1}. \tag{30}$$

Note that numerical number 2 shows up to preserve quasi-symmetry of trade frictions. As in the general trade friction case, $\frac{\partial z}{\partial \ln p}$ is expressed as observables:

$$\frac{\partial z}{\partial \ln p} = \left[\phi\frac{1+\psi+\phi}{\phi-\psi}\right]\left[\mathbf{Y}+\frac{\phi-\psi}{1+\psi+\phi}\mathbf{X}\right],$$

where $\mathbf{Y} = diag\left(Y_i\right)$ and $\mathbf{X} = \left(X_{ij}\right)_{i,j\in S}$. Define $\mathbf{A}$ as follows:

$$\mathbf{A} = \mathbf{Y}+\frac{\phi-\psi}{1+\psi+\phi}\mathbf{X}.$$

From Lemma 5, $\mathbf{A}$ has positive diagonal elements and is dominant of its rows. Equation (30) is

$$\frac{\partial \ln p_i}{\partial \ln \tau_{il}} = -2\frac{\phi-\psi}{1+\psi+\phi}A_{ii}^{-1}X_{il}, \quad \frac{\partial \ln p_j}{\partial \ln \tau_{il}} = -2\frac{\phi-\psi}{1+\psi+\phi}A_{ji}^{-1}X_{il}.$$

Since the price index is log-linear w.r.t. the associated output price, we have

$$\frac{\partial \ln P_i}{\partial \ln \tau_{il}} = \frac{1+\psi+\phi}{\psi-\phi}\frac{\partial \ln p_i}{\partial \ln \tau_{il}}.$$

Therefore, the real output price is

$$\frac{\partial \ln\left(p_i/P_i\right)}{\partial \ln \tau_{il}} = \left(\frac{2\phi+1}{\phi-\psi}\right)\frac{\partial \ln p_i}{\partial \ln \tau_{il}} = -2\frac{2\phi+1}{1+\psi+\phi}A_{ii}^{-1}X_{il}.$$

Then the ordering of the real output price follows from part (iii) of Theorem 2 , $A_{ii}^{-1} > A_{ji}^{-1}$ for $j \in S-i$. The result for real expenditure then follows immediately from C.5 and equation (11), as $E_i/P_i \propto C_i\left(p_i/P_i\right)^{1+\psi}$:

$$\frac{\partial \ln\left(p_iQ_i/P_i\right)}{\partial \ln \tau_{il}} = -2\frac{2\phi+1}{1+\psi+\phi}\left(1+\psi\right)A_{ii}^{-1}X_{il} + \underbrace{\frac{\partial \ln \kappa}{\partial \ln \tau_{il}}}_{\text{common}}.$$

By the same argument, the ordering of $\left(\frac{\partial \ln(p_iQ_i/P_i)}{\partial \ln \tau_{il}}\right)$ follows. $\square$