# Appendix to the paper "A Unified Theory of Firm Selection and Growth"*

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**Abstract**

This appendix provides the detailed proof for Lemma 7 of the original paper

**Lemma 1** *Let* $y > x > 0$. *Then*

$$y\frac{m'(y)}{m(y)} > x\frac{m'(x)}{m(x)} \ ,$$

**Proof.** Notice that $m' = m(m-x)$. Using that,

$$\begin{aligned} \left(x\frac{m'}{m}\right)' &= \frac{m'}{m} + x\left(\frac{m'}{m}\right)' \\ &= \frac{m'}{m} + x(m-x)' \\ &= \frac{m'}{m} + x(m(m-x)-1) \\ &= \frac{m'}{m} + mx(m-x) - x \\ &= mx(m-x) - 2x + m \ . \end{aligned}$$

Thus, I need to show that

$$mx(m-x) - 2x + m > 0 \implies$$
$$mx(m-x) - 3m > 2x - 4m \ . \tag{1}$$

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It is also true that $m - x > 0$ from property F**??** and that (see a lenghty proof by [**?**] p. 406),

$$\begin{aligned} mx\,(m-x)^2 - 3m\,(m-x) + 2 &> 0 \implies \\ (m-x)\,[mx\,(m-x) - 3m] + 2 &> 0\ , \end{aligned}$$

and therefore

$$mx\,(m-x) - 3m > -\frac{2}{m-x}\ .$$

In combination with 1 the last equation implies that it suffices to show

$$-\frac{2}{m-x} > 2x - 4m$$

Rearranging,

$$-1 > (x-2m)\,(m-x) \implies$$

$$1 < -\,(x-2m)\,(m-x)$$

$$0 < (2m-x)\,(m-x) - 1$$

which is an inequality that has been proven by [**?**], which completes the proof of the lemma. ■