Sovereign Debt and Default

Costas Arkolakis
Teaching fellow: Federico Esposito

Economics 407, Yale

February 2014
Outline

- Sovereign debt and default
- A brief history of default episodes
- A Simple Model of Default
- Managing Sovereign Debt
Sovereign Debt and Default
Sovereign Debt

Not only investors but also governments can borrow or lend.

- In fact, governments typically accumulate debt (called government or public debt).

- **Sovereign Debt**: Is a contingent claim on a nation’s assets. Governments will repay depending on whether it is more beneficial to repay than to default.

- **Sovereign Default**: Occurs when a sovereign government (i.e., one that is autonomous or independent) fails to meet its legal obligations to payments on debt.
Debt to GDP ratio across countries

**Figure:** Debt to GDP ratio, 1998-2015 (projections for 2013-15). Source: AMECO database
Sometimes the Debt Grows Large...
...Typically Followed by the Interest Rate

Figure: Greek Spread over German Bonds, (10 Yr maturity bonds). Source: Bloomberg
A History of Default Episodes
Default Episodes

First Recorded Default: 4 century BC. Hellenic City-States defaulted on loans from Delian league (Winkler 1933).

Other episodes:

- 1343, Edward III of England
- Spain 7 times in the 19th century
- 46 European defaults between 1501-1900
- US states defaulted in the 1800s
Default Episodes

First Recorded Default: 4 century BC. Hellenic City-States defaulted on loans from Delian league (Winkler 1933).

Other episodes:

- 1343, Edward III of England
- Spain 7 times in the 19th century
- 46 European defaults between 1501-1900
- US states defaulted in the 1800s

In modern times, Greece has defaulted five times - in 1826, 1843, 1860, 1893, and 1932

- We are no match for the Spanish the last 300 years (but we are getting better at it!)
Default Episodes

In the past, defaults would sometime lead to conflicts.

- Luckily, not in fashion any more.

Today, no particular way to enforce repayment.

- But, there are costs to defaulting.
- If there were not, none would lend in the first place!
Default Episodes

In the past, defaults would sometime lead to conflicts.

- Luckily, not in fashion any more.

Today, no particular way to enforce repayment.

- But, there are costs to defaulting.
- If there were not, none would lend in the first place!

Costs of Default

- Financial market penalties: markets will not lend to you anymore. Lose consumption smoothing opportunities.
- Macroeconomic implications: disruption in financial markets may bring economic downturn, export/import declines etc
The Latin-American Debt crisis

Evolution of Debt to GDP in some emerging economies

**Figure:** The evolution of the debt/GNP ratio in selected countries

<table>
<thead>
<tr>
<th></th>
<th>1980</th>
<th>1982</th>
<th>1985</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>.48</td>
<td>.84</td>
<td>.84</td>
</tr>
<tr>
<td>Brazil</td>
<td>.31</td>
<td>.36</td>
<td>.49</td>
</tr>
<tr>
<td>Mexico</td>
<td>.30</td>
<td>.53</td>
<td>.55</td>
</tr>
</tbody>
</table>

Interest Payments in Latin American Countries

**Figure:** Interest payments in selected Latin American countries. Average 1980-81.

<table>
<thead>
<tr>
<th>Country</th>
<th>Percent of Debt at floating rate</th>
<th>Interest Payment to Exports ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>58</td>
<td>15</td>
</tr>
<tr>
<td>Brazil</td>
<td>64</td>
<td>28</td>
</tr>
<tr>
<td>Colombia</td>
<td>39</td>
<td>16</td>
</tr>
<tr>
<td>Chile</td>
<td>58</td>
<td>28</td>
</tr>
<tr>
<td>Mexico</td>
<td>73</td>
<td>19</td>
</tr>
<tr>
<td>All Latin America</td>
<td>65</td>
<td>28</td>
</tr>
</tbody>
</table>


Some Latin American countries faced Debt crisis

- They initially faced low interest rates: borrowed a lot
Trade Balance in Latin America

To repay debts requires running trade surpluses.

- Also implementation of austerity measures (lower wages, decrease fiscal deficit)

**Figure:** Trade Balance in the Latin America
A Simple Model of Default
A Simple Model of Default: Goal

We saw a series of interesting facts about debt and defaults.

We want a simple model that will explain these facts.
A Simple Model of Default: Goal

We saw a series of interesting facts about debt and defaults.

We want a simple model that will explain these facts.

- 1. High debt arises due to adverse shocks.
A Simple Model of Default: Goal

We saw a series of interesting facts about debt and defaults.

We want a simple model that will explain these facts.

- 1. High debt arises due to adverse shocks.
- 2. High debt leads to higher interest rates.
We saw a series of interesting facts about debt and defaults.

We want a simple model that will explain these facts.

- 1. High debt arises due to adverse shocks.
- 2. High debt leads to higher interest rates.
- 3. A combination of the above leads some times to default.
A Simple Model of Default

Two period model:

- Period 1: a country gets a loan
- Period 2: the country decides whether to repay the loan or not

→ Given the decisions in period 1, the country takes action only in period 2
A Simple Model of Default

**Two period model:**

- Period 1: a country gets a loan
- Period 2: the country decides whether to repay the loan or not

→ Given the decisions in period 1, the country takes action only in period 2

**A country sells bonds** $d'$ in a price $q = 1/(1 + r)$ to receive $d = qd'$ in period 1. Needs to pay 1 dollar to lender in period 2. World interest rate prevails $r = r^*$. **If the country defaults, it loses fraction** $c$ of its output.
A Simple Model of Default

Two period model:

- Period 1: a country gets a loan
- Period 2: the country decides whether to repay the loan or not

→ Given the decisions in period 1, the country takes action only in period 2

A country sells bonds \(d'\) in a price \(q = 1/(1 + r)\) to receive \(d = qd'\) in period 1. Needs to pay 1 dollar to lender in period 2. World interest rate prevails \(r = r^*\). If the country defaults, it loses fraction \(c\) of its output.

- The effective interest rate that the government pays is \(1/q = (1 + r)\)
A Simple Model of Default

Two period model:

- Period 1: a country gets a loan
- Period 2: the country decides whether to repay the loan or not

Given the decisions in period 1, the country takes action only in period 2

A country sells bonds $d'$ in a price $q = 1 / (1 + r)$ to receive $d = qd'$ in period 1. Needs to pay 1 dollar to lender in period 2. World interest rate prevails $r = r^*$. If the country defaults, it loses fraction $c$ of its output.

- Output, $y'(s)$, is stochastic for different states of the world $s$. 
A Simple Model of Default

Two period model:

- Period 1: a country gets a loan
- Period 2: the country decides whether to repay the loan or not

Given the decisions in period 1, the country takes action only in period 2

A country sells bonds \( d' \) in a price \( q = 1/(1 + r) \) to receive \( d = qd' \) in period 1. Needs to pay 1 dollar to lender in period 2. World interest rate prevails \( r = r^* \). **If the country defaults, it loses fraction** \( c \) **of its output.**

- Output, \( y'(s) \), is stochastic for different states of the world \( s \).
- If the country decides to repay the loan in the next period, consumes \( y'(s) - d' \)
A Simple Model of Default

**Two period model:**

- Period 1: a country gets a loan
- Period 2: the country decides whether to repay the loan or not

→ Given the decisions in period 1, the country takes action only in period 2

A country sells bonds $d'$ in a price $q = 1 / (1 + r)$ to receive $d = qd'$ in period 1. Needs to pay 1 dollar to lender in period 2. World interest rate prevails $r = r^*$. **If the country defaults, it loses fraction $c$ of its output.**

- **Output**, $y'(s)$, is stochastic for different states of the world $s$.
- If the country decides to repay the loan in the next period, consumes $y'(s) - d'$
- But, if the country defaults, consumes $y'(s) (1 - c)$ where $c \in (0, 1)$. 
A Simple Model of Default

- When does country default?
  
  \[ y'(s) - d' < y'(s)(1 - c) \]
A Simple Model of Default

- When does country default?
  - $\Rightarrow$ In the states of the world that $y'(s) - d' < y'(s)(1 - c)$
  - Solve for $\tilde{y}'$ such that $\tilde{y}' - d' = \tilde{y}'(1 - c)$. 
A Simple Model of Default

- When does country default?
  
  - $\Rightarrow$ In the states of the world that $y'(s) - d' < y'(s)(1 - c)$

  - Solve for $\tilde{y}'$ such that $\tilde{y}' - d' = \tilde{y}'(1 - c)$.

  - If $y'(s) < \tilde{y}'$, the country defaults (adverse shock may trigger default).
A Simple Model of Default

- When does country default?
  
  • ⇒ In the states of the world that \( y' (s) - d' < y' (s) (1 - c) \)

  • Solve for \( \tilde{y}' \) such that \( \tilde{y}' - d' = \tilde{y}' (1 - c) \).

  • If \( y' (s) < \tilde{y}' \), the country defaults (adverse shock may trigger default).

  • If \( d' \) is high, \( \tilde{y}' \) is also high (high \( d' \) may trigger default).
A Simple Model of Default

- When does country default?

  - ⇒ In the states of the world that \( y' (s) - d' < y' (s) (1 - c) \)

  - Solve for \( \tilde{y}' \) such that \( \tilde{y}' - d' = \tilde{y}' (1 - c) \).

  - If \( y' (s) < \tilde{y}' \), the country defaults (adverse shock may trigger default).

  - If \( d' \) is high, \( \tilde{y}' \) is also high (high \( d' \) may trigger default).

  - If \( r^* \) is high, \( \tilde{y}' \) increases (Increases \( d' \) in order to achieve a certain level \( d' q = d' / (1 + r^*) \))
A Simple Model of Default

- When does country default?
  - In the states of the world that $y'(s) - d' < y'(s)(1 - c)$
  
  - Solve for $\tilde{y}'$ such that $\tilde{y}' - d' = \tilde{y}'(1 - c)$.
  
  - If $y'(s) < \tilde{y}'$, the country defaults (adverse shock may trigger default).
  
  - If $d'$ is high, $\tilde{y}'$ is also high (high $d'$ may trigger default).
  
  - If $r^*$ is high, $\tilde{y}'$ increases (Increases $d'$ in order to achieve a certain level $d'q = d'/(1 + r^*)$)

Limitation of the model: This model ignores completely lenders’ expectations. In reality, $r \neq r^*$ and in fact $r = r(d')$. 
The Eaton-Gersovitz Model of Default

Now, we will make the simple model a tad more exciting.

⇒ Accomodate the possibility that bonds’ prices depend on the expectation that the country defaults on its debt.

- Two periods. Period 1: country sells a bond at price $q$; Period 2: it decides whether to pay the lenders 1 dollar or default.
- Output is stochastic in period 2, $y'(s)$. 
The Eaton-Gersovitz Model of Default

Now, we will make the simple model a tad more exciting.

⇒ Accomodate the possibility that bonds’ prices depend on the expectation that the country defaults on its debt.

• Based on Eaton-Gersovitz model, 1981, Review of Economic Studies

• Two periods. Period 1: country sells a bond at price $q$; Period 2: it decides whether to pay the lenders 1 dollar or default.

• Output is stochastic in period 2, $y'(s)$.

• No consumption in period 1, but there is some debt, $d$, that needs to be rolled-over using new debt, $d'$.

• In period 2, the government has to decide whether (i) to repay the debt $d'$ (consume $y'(s) - b'$) or, (ii) to default (consume $y'(s) (1 - c)$).
  - $c$ is the fraction of output reduction caused by the default (e.g., due to political unrest etc)
Government problem

The government chooses the debt level for the next period:

$$\max_{d'} E \{ u (y' - d') , u (y' (1 - c)) \} \quad \text{s.t.} \quad d = q (d') d'$$

where the price of the bond, $q (d')$, is determined in equilibrium by

$$q (d') = \Pr \left\{ u (y' - d') \geq u (y' (1 - c)) \right\} \frac{1}{1 + r^*}$$

Notice that we can directly substitute out $d' = d / q (d')$. 

Government problem

The government chooses the debt level for the next period:

$$\max_{d'} E \left\{ u \left( y' - d' \right), u \left( y' \left( 1 - c \right) \right) \right\} \quad \text{s.t. } d = q \left( d' \right) \cdot d'$$

where $q \left( d' \right)$ is determined in equilibrium by

$$q \left( d' \right) = \frac{Pr \left\{ u \left( y' - d' \right) \geq u \left( y' \left( 1 - c \right) \right) \right\}}{1 + r^*} = \frac{Pr \left\{ y' - d' \geq y' \left( 1 - c \right) \right\}}{1 + r^*}$$

Notice that we can directly substitute out $d' = d / q \left( d' \right)$.

- For example, if there are 3 states with equal probabilities and the country defaults only in the worst state: $q \left( d' \right) = \frac{2}{3} \frac{1}{1+r^*}$.
- Effective interest rate: $\frac{1}{q(d')} = \left( 1 + r^* \right) \cdot 3/2 > 1 + r^*$

  - Probability of default affects the interest rate!
Government problem

The government chooses the debt level for the next period:

$$ \max_{d'} E \left\{ u \left( y' - d' \right), u \left( y' \left(1 - c\right) \right) \right\} \quad \text{s.t.} \quad d = q \left( d' \right) d' $$

where $q \left( d' \right)$ is determined in equilibrium by

$$ q \left( d' \right) = \frac{\Pr \left\{ u \left( y' - d' \right) \geq u \left( y' \left(1 - c\right) \right) \right\}}{1 + r^*} = \frac{\Pr \left\{ y' - d' \geq y' \left(1 - c\right) \right\}}{1 + r^*} $$

Notice that we can directly substitute out $d' = d / q \left( d' \right)$.

- But if the initial debt, $d$, is high, default may happen in 2/3 states:
  $$ q \left( d' \right) = \frac{1}{3} \frac{1}{1 + r^*}. $$
- Effective interest rate (spread) is higher.
Debt and Reputation

The government chooses the debt level for the next period:

$$\max_{d'} E \left\{ u \left( y' - d' \right), u \left( y' \left(1 - c\right)\right) \right\} \quad \text{s.t. } d = q \left( d' \right) d'$$

where $q \left( d' \right)$ is determined in equilibrium by

$$q \left( d' \right) = \frac{\Pr \left\{ u \left( y' - d' \right) \geq u \left( y' \left(1 - c\right)\right) \right\}}{1 + r^*} = \frac{\Pr \left\{ y' - d' \geq y' \left(1 - c\right) \right\}}{1 + r^*}$$

Notice that we can directly substitute out $d' = d / q \left( d' \right)$.

- Notice that if $c = 0$ the country would always default! $q \left( d' \right) \to 0$ so that the effective interest rate is $\infty$!

- Famous result by Bulow and Roggoff 1989, American Economic Review: You cannot sustain debt simply by reputation, and with no additional penalty for default!
How the Model Fits the Facts

1. High debt arises due to adverse shocks.
   - Interest payments increase and in a multiperiod setup adverse shocks imply need for borrowing.
2. High debt leads to higher interest rates.
   - Investors discount defaulty probability
3. A combination of the above leads some times to default.
   - High debt and high interest rate makes it more difficult to repay
Managing Sovereign Debt
Debt Reduction Schemes

Solutions for excessive sovereign debt:

1. Unilateral Debt Forgiveness.
Debt Reduction Schemes

Solutions for excessive sovereign debt:

1. Unilateral Debt Forgiveness.
2. Third party buy-backs (other entities - e.g., governments, institutions etc. - might be willing to buy out all the debt in current low prices and ask only for partial repayments of the bonds)

3. Debt Restructuring (renegotiate part of your debt with the lenders, also called a 'haircut')

4. Debt swaps (issuance of new debt that has seniority – i.e. is served before– the old debt)

5. (Partial) Unilateral Default! (the so-called nuclear option)
Debt Reduction Schemes

Solutions for excessive sovereign debt:

1. Unilateral Debt Forgiveness.

2. Third party buy-backs (other entities - e.g., governments, institutions etc. - might be willing to buy out all the debt in current low prices and ask only for partial repayments of the bonds)

3. Debt Restructuring (renegotiate part of your debt with the lenders, also called a ‘haircut’)
Debt Reduction Schemes

Solutions for excessive sovereign debt:

1. Unilateral Debt Forgiveness.
2. Third party buy-backs (other entities - e.g., governments, institutions etc. - might be willing to buy out all the debt in current low prices and ask only for partial repayments of the bonds)
3. Debt Restructuring (renegotiate part of your debt with the lenders, also called a ‘haircut’)
4. Debt swaps (issuance of new debt that has seniority – i.e. is served before– the old debt)
Debt Reduction Schemes

Solutions for excessive sovereign debt:

1. Unilateral Debt Forgiveness.
2. Third party buy-backs (other entities - e.g., governments, institutions etc. - might be willing to buy out all the debt in current low prices and ask only for partial repayments of the bonds)
3. Debt Restructuring (renegotiate part of your debt with the lenders, also called a ‘haircut’)
4. Debt swaps (issuance of new debt that has seniority – i.e. is served before – the old debt)
5. ...(Partial) Unilateral Default! (the so-called nuclear option)
The Greek “Debt Reduction” Scheme

Jan 21, 2010: Greek-German spread for 10-year debt reaches 300 basis points.

- At that point, without international help, only option Unilateral Default.
The Greek “Debt Reduction” Scheme

Jan 21, 2010: Greek-German spread for 10-year debt reaches 300 basis points.

- At that point, without international help, only option Unilateral Default.

May 2, 2010: Troika (EC: European Commission, IMF, ECB) agree with Greek gov to a $143 bil bailout package (amount increased soon!).
The Greek “Debt Reduction” Scheme

Jan 21, 2010: Greek-German spread for 10-year debt reaches 300 basis points.

- At that point, without international help, only option Unilateral Default.

May 2, 2010: Troika (EC: European Commission, IMF, ECB) agree with Greek gov to a $143 bil bailout package (amount increased soon!).

- Guarantee greek public debt (lenders & new issuance). At that point, a debt swap; troika pays expiring bonds in exchange of seniority.
The Greek “Debt Reduction” Scheme

October 27, 2011: Major private bond holders agreed on a 50% ‘haircut’. Ultimately 83.5% of Greek bond holders will participate. By then, only small part of the debt is private.

- The debt to the participating bond holders was backed by Troika.
The Greek “Debt Reduction” Scheme

October 27, 2011: Major private bond holders agreed on a 50% ‘haircut’. Ultimately 83.5% of Greek bond holders will participate. By then, only small part of the debt is private.

- The debt to the participating bond holders was backed by Troika.

2012-2014: Slowly, arrangement becomes a third-party partial buy-back. ECB buys out large fraction of greek bonds; EC lowers interest rates.
The Greek “Debt Reduction” Scheme

**October 27, 2011:** Major private bond holders agreed on a 50% ‘haircut’. Ultimately 83.5% of Greek bond holders will participate. By then, only small part of the debt is private.

- The debt to the participating bond holders was backed by Troika.

**2012-2014:** Slowly, arrangement becomes a third-party partial buy-back. ECB buys out large fraction of greek bonds; EC lowers interest rates.

**February 2014:** Greek debt/GDP $>170\%$. Clearly, unsustainable... Greece hopes for partial Debt Forgiveness from Troika.
Debt Reduction Schemes

If probability of repayment is low, it could be realistic for lenders to adjust the value of the debt.

- The free rider problem arises: how can you ensure that all the lenders reduce the debt at the same time?
Debt Reduction Schemes

If probability of repayment is low, it could be realistic for lenders to adjust the value of the debt.

- The free rider problem arises: how can you ensure that all the lenders reduce the debt at the same time?
- From an individual lender’s point of view, it might be better if he does not forgive.
Debt Reduction Schemes

If probability of repayment is low, it could be realistic for lenders to adjust the value of the debt.

⇒ Debt Overhang.

- Let $D$ be the debt. Consider the possibility that part of the debt is forgiven.
- Let $\pi$ be the probability that the good state occurs and in the bad state the government defaults and only repays $aD$, where $a < 1$ is the fraction of the money that the country will pay if there is a default; this probability is a function of the state, $\pi = \pi(D)$, and $\frac{d\pi(D)}{dD} < 0$. Total expected revenues of the lender are

$$\pi(D) D + (1 - \pi(D)) aD$$

- It might make sense for the lenders to lower the debt $D$ in order to maximize the above expression by lowering the default probability.